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Short Communication

Acoustic wave propagation predictions based on two different particle/fluid two-phase flow models

Janaki Gadiyaram, John Peddieson*, Sastry Munukutla

Department of Mechanical Engineering and Center for Electric Power, Tennessee Technological University, Cookeville, TN 38505, USA

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Abstract

Acoustic wave propagation predictions are reported based on two frequently used models of particle/fluid two phase flow. It is shown that the predictions of one model are qualitatively correct while those of the other are qualitatively incorrect. These results provide a concrete example of the extreme sensitivity of two phase flow predictions to modeling assumptions.

1. Introduction

The purpose of this note is to point out a situation in which two equally plausible continuum models of fluid/particle two-phase flow produce qualitatively (not just quantitatively) different predictions concerning acoustic wave propagation. As discussed by Jean and Peddieson [1] and illustrated by the numerous references cited therein, considerable uncertainty remains in the continuum approach to mathematical modeling of particle/fluid two-phase flows. One unresolved issue involves the role of the fluid stress tensor gradient in the momentum balance equations for the two phases. Thus, the recent papers of Neri and Gidaspow [2] and Matonis et al. [3] have the fluid phase stress tensor gradient acting only on the fluid phase (no sharing) while those by Jin and Campbell [4] and Johri and Glasser [5] have the fluid phase stress tensor gradient shared by the

*Corresponding author. Tel.: +1 931 372 3274; fax: +1 615 372 6340.
E-mail address: jpeddieson@tntech.edu (J. Peddieson).

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two phases in proportion to their volume fractions (stress sharing). Other formulations also appear in the literature such as that of Celik and Gel [6]. The no sharing and stress sharing models are compared herein in the context of one-dimensional linearized acoustic wave propagation. Both approximate and exact closed form solutions and exact numerical solutions are given for wave speeds and both temporal and spatial damping factors.

The literature on wave propagation in particle/fluid two-phase systems is extensive and will not be reviewed in this note. Two recent representative contributions are the papers by Chung et al. [7] and Song [8]. Both of these contributions also illustrate the sensitivity of predictions to modeling assumptions.

2. Governing equations

Consider a particle/inviscid fluid dispersion performing a one-dimensional motion. The flow direction is denoted by z and the time t. For an inviscid fluid, the stress tensor gradient reduces to a pressure gradient and stress sharing becomes pressure sharing. The appropriate one-dimensional mass balance and linear momentum balance equations are applicable to either the continuous (fluid) phase ($i = 1$) or the dispersed (particulate) phase ($i = 2$) and have the respective forms:

$$\partial_t \rho_i + \partial_z(\rho_i w_i) = 0, \quad i = 1, 2 \tag{1}$$

and

$$\rho_i(\partial_t w_i + w_i \partial_z w_i) = -\partial_z p_i + F_i, \quad i = 1, 2, \tag{2}$$

where each phase has mass density ρ_i, velocity w_i, pressure p_i, and body force per unit volume F_i.

Let

$$\rho_1 = \rho_c(1 - \varphi), \quad \rho_2 = \rho_d \varphi,$$

$$p_1 = (1 - \delta\varphi)p, \quad p_2 = \delta\varphi p,$$

$$F_1 = -f + b_c - \delta p \partial_z \varphi, \quad F_2 = f + b_d + \delta p \partial_z \varphi, \tag{3}$$

where ρ_c is the true fluid density, ρ_d is the true particulate density, φ is the volume fraction of particle material, p is the fluid pressure, δ is the pressure sharing factor (to be discussed subsequently), f is the interphase force per unit volume applied to the dispersed phase by the continuous phase, and b_c and b_d are the respective fluid phase and particle phase external body forces per unit volume. Substituting Eqs. (3) into Eqs. (1) and (2) yields

$$\begin{aligned}
&\partial_t(\rho_c(1-\varphi)) + \partial_z(\rho_c(1-\varphi)w_c) = 0,\\
&\partial_t(\rho_d\varphi) + \partial_z(\rho_d\varphi w_d) = 0,\\
&\rho_c(1-\varphi)(\partial_t w_c + w_c \partial w_c) = (1-\delta\varphi)\partial_z p - f + b_c,\\
&\rho_d\varphi(\partial_t w_d + w_d \partial_t w_d) = -\delta\varphi\partial_z p + f + b_d.
\end{aligned} \tag{4a--d}$$

Inspection of Eqs. (4c,d) shows that the fluid phase pressure gradient acts only on the fluid phase for $\delta = 0$, but is shared by the phases in proportion to their volume fractions for $\delta = 1$. Thus, in the present work $\delta = 0$ corresponds to the no sharing model while $\delta = 1$ corresponds to the pressure sharing model. While Eqs. (4) are formally valid for any value of δ, only $\delta = 0$ and $\delta = 1$ will be considered subsequently.

3. Acoustic wave propagation

It is desired to apply Eqs. (4) to small amplitude acoustic wave propagation with body forces neglected ($b_c = 0,\ b_d = 0$). Toward this end consider small disturbances imposed on a uniform quiescent initial state. Thus

$$\rho_c = \rho_{c,0} + \rho_{c,p}, \quad p = p_0 + p_p, \quad w_c = 0 + w_{c,p},$$
$$w_d = 0 + w_{d,p}, \quad f = 0 + f_p, \quad \varphi = \varphi_0 + \varphi_p. \tag{5}$$

The first terms on the right-hand side of Eqs. (5) represent the initial state and the second terms on the right-hand side are the disturbances (perturbations). Substituting Eqs. (5) into Eqs. (4), neglecting all terms containing products of perturbations (to achieve linearization about the initial state), and rewriting the results in terms of the original variables leads to

$$\begin{aligned} &(1-\varphi_0)\partial_t\rho_c - \rho_{c,0}\partial_t\varphi + \rho_{c,0}(1-\varphi_0)\partial_z w_c = 0, \\ &\partial_t\varphi + \varphi_0\partial_z w_d = 0, \\ &\rho_{c,0}(1-\varphi_0)\partial_t w_c - (1-\delta\varphi_0)\partial_z p + f = 0, \\ &\rho_d\varphi_0\partial_t w_d + \delta\varphi_0\partial_z p - f = 0. \end{aligned} \tag{6a–d}$$

Implicit in the work leading to Eqs. (6) is the restriction that the constitutive equations for p and f must have linearized forms. Here it is assumed that

$$\begin{aligned} &p = p_0 + a^2(\rho_c - \rho_{c,0}), \\ &f = \rho_d\varphi_0 N(w_c - w_d) + \rho_{c,0}\varphi_0 M(\partial_t w_c - \partial_t w_d), \end{aligned} \tag{7a, b}$$

where a is the speed of sound associated with the quiescent initial state of the continuous phase, N is a drag modulus, and M is an added mass modulus. In Eq. (7a), it is assumed that the fluid pressure is a function of fluid density only (to avoid the necessity of involving the phasic energy equations). This is a better assumption for liquids than for gases but, in any case, does not fundamentally alter the nature of the results. Eq. (7b) represents the interphase force as a linear combination of a term proportional to relative velocity (steady drag contribution) and a term proportional to relative acceleration (added mass contribution). Using Eq. (6b) to eliminate φ from Eqs. (6a,c,d) and substituting Eqs. (7) into Eqs. (6c,d) leads to the three equations

$$\begin{aligned} &(1-\varphi_0)(\partial_t\rho_c + \partial_z w_c) + \rho_{c,0}\varphi_0\partial_z w_d = 0, \\ &\rho_{c,0}((1-\varphi_0+\varphi_0 M)\partial_t w_c - \varphi_0 M\partial_t w_d) + (1-\delta\varphi_0)a^2\partial_z\rho_c + \rho_d\varphi_0 N(w_c - w_d) = 0, \\ &(\rho_d + \rho_{c,0}M)\partial_t w_d - \rho_{c,0}M\partial_t w_c + \delta a^2\partial_z\rho_c + \rho_d N(w_d - w_c) = 0 \end{aligned} \tag{8}$$

containing only the dependent variables w_c, w_d, and ρ_c.

To investigate harmonic wave propagation let

$$\rho_c = \mathrm{Re}(A\exp(\mathrm{i}(\ell z - \omega t))), \quad w_c = \mathrm{Re}(B\exp(\mathrm{i}(\ell z - \omega t))), \quad w_d = \mathrm{Re}(C\exp(\mathrm{i}(\ell z - \omega t))), \tag{9}$$

where A, B and C are constants, Re means "real part of", and $\mathrm{i} = \sqrt{-1}$. The quantities ℓ and ω appearing in Eqs. (9) will be referred to herein as the respective complex circular wave number and complex circular frequency. Substituting Eqs. (9) into Eqs. (8) produces three homogeneous algebraic equations in A, B, and C. Equating the corresponding determinate of the coefficients to zero yields

$$\rho_{c,0}(1-\varphi_0)(\rho_d(1-\varphi_0)+(\rho_{c,0}(1-\varphi_0)+\rho_d\varphi_0)M)\omega^3+\mathrm{i}\rho_d(1-\varphi_0)(\rho_{c,0}(1-\varphi_0)+\rho_d\varphi_0)N\omega^2 \\ -\rho_{c,0}((1-\varphi_0)(\rho_{c,0}\delta\varphi_0+\rho_d(1-\delta\varphi_0))+\rho_{c,0}M)(a\ell)^2\omega-\mathrm{i}\rho_{c,0}\rho_d N(a\ell)^2=0. \tag{10}$$

These equations hold for both ℓ regarded as a known real quantity with ω to be determined and ω regarded as a known real quantity with ℓ to be determined. For the former

$$\omega=\omega_R+\mathrm{i}\omega_I \tag{11}$$

and

$$\zeta=-\omega_I, \quad \lambda=2\pi/\ell, \quad c=\omega_R/\ell \tag{12}$$

are the respective temporal attenuation coefficient, wavelength, and wave speed of a temporally damped right propagating harmonic wave. For the latter

$$\ell=\ell_R+\mathrm{i}\ell_I \tag{13}$$

and

$$\eta=\ell_I, \quad \tau=2\pi/\omega, \quad c=\omega/\ell_R \tag{14}$$

are the respective spatial attenuation coefficient, period, and wave speed of a spatially damped right propagating harmonic wave. Eq. (10) can be written in the dimensionless form

$$(1-\varphi_0)(1-\varphi_0+((1-\varphi_0)/r+\varphi_0)M)\Omega^3+\mathrm{i}(1-\varphi_0)(1-\varphi_0+r\varphi_0)\Omega^2 \\ -((1-\varphi_0)(\delta\varphi_0/r+1-\delta\varphi_0)+M/r)L^2\Omega-\mathrm{i}L^2=0, \tag{15}$$

where

$$r=\rho_d/\rho_{c,0}, \quad \Omega=\omega/N, \quad L=a\ell/N. \tag{16}$$

With L a known real quantity, Eq. (15) is a cubic equation to solve for Ω. Since the cubic formula is unwieldy, numerical solutions of Eq. (15) will be reported subsequently. Before doing this, it is useful to seek an approximate closed form solution to Eq. (15) for small volume fractions ($\varphi_0 \ll 1$). This is accomplished by rewriting Eq. (15) in the approximate form

$$(\Omega-L)(\Omega+L)((1+M/r)\Omega+\mathrm{i})=\varphi_0\Omega((2(1+M/r)-M)\Omega^2+\mathrm{i}(2-r)\Omega \\ -(1+\delta(1-1/r))L^2) \tag{17}$$

(where all terms that are nonlinear in φ_0 have been neglected), finding a first approximation by neglecting the right-hand side, and then finding a second approximation by iteration. The results are

$$\Omega_{1,2}=(\pm1+\varphi_0(\pm(2-r+(1+\delta(1/r-1)+M(2/r-1))(1+M/r)L^2) \\ +\mathrm{i}(1-r+\delta(1-1/r))L)/(2(1+(1+M/r)^2L^2)))L \tag{18}$$

and

$$\Omega_3 = -\mathrm{i}(1+\varphi_0(r+(1+\delta(1-1/r))(1+M/r)^2L^2)/((1+M/r) \times(1+(1+M/r)^2L^2)))/(1+M/r). \tag{19}$$

The corresponding dimensionless temporal attenuation factors are

$$Z_{1,2} = \varphi_0(r-1)(1-\delta/r)L^2/(2(1+M/r)^2L^2) \tag{20}$$

and

$$Z_3 = (1+\varphi_0(r+(1+\delta(1-1/r))(1+M/r)^2L^2)/((1+M/r)(1+(1+M/r)^2L^2)))/(1+M/r) \tag{21}$$

while the corresponding wave speeds are

$$C_{1,2} = \pm(1+\varphi_0((2-r+(1+\delta(1/r-1)+M(2/r-1)(1+M/r)L^2)/(2(1+(1+M/r)^2L^2))) \tag{22}$$

and

$$C_3 = 0, \tag{23}$$

where

$$Z = -\Omega_\mathrm{I} = \zeta/N, \quad C = \Omega_R/L = c/a. \tag{24}$$

Since the wave speeds and the attenuation factors all depend on the wave number, harmonic wave propagation is found to be dispersive.

Eqs. (20) and (22) describe waves propagating in the positive ($C_1>0$) and negative ($C_2<0$) directions with equal wave speeds in the infinite region $-\infty<z<+\infty$ while Eqs. (21) and (23) describe a non-propagating wave. Physically all the three temporal attenuation factors should be positive (since the quiescent state cannot be unstable). To the order of approximation of the solution process, this condition is fulfilled for Z_3 because the term multiplied by φ_0 must always be regarded as small compared to unity. On the other hand, Eq. (20) indicates that $Z_{1,2}\geqslant 0$ for all r with $\delta=1$ but only for $r\geqslant 1$ with $\delta=0$. This is true regardless of the value of M.

Some representative sets of results obtained solving Eq. (15) numerically are given in Table 1. A number of combinations of δ, φ, r and M are represented. All the results shown in Table 1 correspond to $L=3$, but several other values of L were also employed and all produced qualitatively similar results. In all cases it can be seen that $Z_{1,2}>0$ for all r with $\delta=1$ while $Z_{1,2}\geqslant 0$ only for $r\geqslant 1$ with $\delta=0$. Numerous other calculations (not reported for the sake of brevity) exhibited the same behavior.

With Ω a known real quantity Eq. (15) is a quadratic equation to solve for L. It can be rewritten as

$$L^2 = \Omega^2(1-\varphi_0)(R+\mathrm{i}I), \tag{25}$$

where

$$R = (1-\varphi_0+r\varphi_0+(1-\varphi_0)(1-\varphi_0+((1-\varphi_0)/r+\varphi_0)M)(1-\delta\varphi_0+\delta\varphi_0/r + M/(r(1-\varphi_0)))\Omega^2)/(1+((1-\delta\varphi_0+\delta\varphi_0/r)(1-\varphi_0)+M/r)^2\Omega^2), \tag{26}$$

Table 1
Attenuation factors and wave speeds

φ_0	r	M	δ	$Z_{1,2}$	Z_3	$C_{1,2}$
0.05	0.5	0.5	0	−0.00308	0.51265	±1.03278
0.05	0.5	0.5	1	+0.00286	0.50077	±1.04496
0.1	0.5	0.5	0	−0.00623	0.52598	±1.06858
0.1	0.5	0.5	1	+0.00537	0.50276	±1.09297
0.5	0.5	0.5	0	−0.03266	0.66533	±1.55071
0.5	0.5	0.5	1	+0.01410	0.57179	±1.67277
0.1	0.1	0.5	0	−0.00126	0.16948	±1.09677
0.1	0.1	0.5	1	+0.00895	0.14906	±1.16948
0.1	1	0.5	0	0	0.71429	±1.05409
0.1	1	0.5	1	0	0.71429	±1.05409
0.1	10	0.5	0	+0.37568	1.15819	±0.973904
0.1	10	0.5	1	+0.32035	1.26885	±0.932029
0.1	0.5	0	0	−0.02475	1.10506	±1.05694
0.1	0.5	0	1	+0.02078	1.01399	±1.10340
0.1	0.5	0.5	0	−0.00623	0.52598	±1.06858
0.1	0.5	0.5	1	+0.00537	0.50276	±1.09297
0.1	0.5	1	0	−0.00274	0.34476	±1.07285
0.1	0.5	1	1	+0.00239	0.33449	±1.08919

$$I = \Omega\varphi_0(1-\varphi_0)(r-1)(1-\delta\varphi_0-\delta(1-\varphi_0)/r)/(1+((1-\delta\varphi_0+\delta\varphi_0/r)(1-\varphi_0)+M/r)^2\Omega^2). \tag{27}$$

The exact solution of Eq. (25) can be written as

$$L = \pm(L_R + \mathrm{i}L_I), \tag{28}$$

where

$$L_R = \Omega(((1-\varphi_0)(R^2+I^2)^{1/2}+R)/2)^{1/2}, \quad L_I = \Omega(((1-\varphi_0)(R^2+I^2)^{1/2}-R)/2)^{1/2}\mathrm{sgn}(I). \tag{29}$$

The respective corresponding spatial attenuation factors and wave speeds are

$$Y_{1,2} = \pm L_I, \quad C_{1,2} = \pm\Omega/L_R, \tag{30}$$

where

$$Y = L_I = a\eta/N, \quad C = \Omega/L_R = c/a. \tag{31}$$

When the positive signs are selected, Eqs. (30) describe a harmonic wave propagating in the positive direction in the semi-infinite region $z \geqslant 0$. When the negative signs are selected, Eqs. (30) describe a harmonic wave propagating in the negative direction in the semi-infinite region $z \leqslant 0$. Because of the dependence of both Y and C on frequency, the wave propagation is again dispersive. Physically, the spatial attenuation in either case must be in the direction of propagation, requiring $I > 0$. Eq. (27) indicates that this condition will be violated for $\delta = 0$ and $r < 1$ (regardless of the value of M), but for $\delta = 1$ will be satisfied for all r.

It is important to state that the characteristics of Eqs. (6) can be shown to be real. Thus the non-physical behavior discussed above is not the result of an ill-posed system of equations.

4. Conclusion

In the foregoing section it was shown that the no sharing continuum fluid/particle two-phase flow model employed herein predicted qualitatively incorrect results for acoustic wave propagation while the corresponding pressure sharing model predicted qualitatively correct results. It would be wrong, however, to claim superiority for the pressure sharing model on the basis of this single example for at least two reasons. First, several aspects of the present models are highly simplified. (There is no intrinsic particulate phase pressure, for example). It is interesting, nonetheless, that the inclusion of the added mass effect does not change the qualitative results, even though added mass effects are thought to be most important for $r<1$ where the difficulty occurs. Second, there may well be other situations in which the predictions of qualitatively correct and incorrect results would be reversed. The present analysis illustrates the sensitivity of continuum fluid/particle two-phase flow equations to modeling details. This sensitivity may not manifest itself when numerical solutions to complicated flow problems are being computed. It is possible, for example, that the artificial dissipation present in all two phase flow codes could damp out the unphysical behavior described heretofore. The results presented herein are an indication of the fact that the task of developing reliable general continuum models of fluid/particle two-phase flow is far from completed.

Many of the earlier studies of the acoustics of particle/fluid dispersions were based on the so-called "dusty gas" model (see, for instance, Marble [9]). In the present notation this corresponds to $r \gg 1$, $\varphi_0 \ll 1$ simultaneously. It can be seen from Eqs. (20) and (26) that the non-physical predictions pointed out herein do not occur in this limit.

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