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Short Communication

Separating contribution from two coherent and interfering inputs

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1. Multiple input–multiple output problem

Conventional methods for the separation of inputs contribution to a signal of interest involve the extraction of statistically independent virtual inputs (principal component analysis and other techniques [1–5]). However, in practical applications, measured inputs are often coherent at many frequencies. Some researchers suggest further separation of incoherent inputs into non-interfering and interfering ones [6]. If a system is characterized by multiple partly coherent and interfering inputs, their separation into coherent non-interfering inputs without the use of advanced diagnostic techniques is almost impossible. Frequently, this type of task requires acquisition of additional data, which is difficult to perform in a non-laboratory environment [7,8]. The complexity of the problem substantially gets higher as the number of input increases. Nonetheless it is possible to solve the problem in some particular cases just utilizing the originally measured data.

For the purpose of simplicity, let us consider a system with two inputs X_1 and X_2, multiple outputs $Y_1, Y_2, \ldots, Y_i, \ldots, Y_n$, the action of noise components $N_1, N_2, \ldots, N_i, \ldots, N_n$, and where $H_{11}, H_{1i}, \ldots, H_{2i}$ are transfer functions with the first and second indices representing the input

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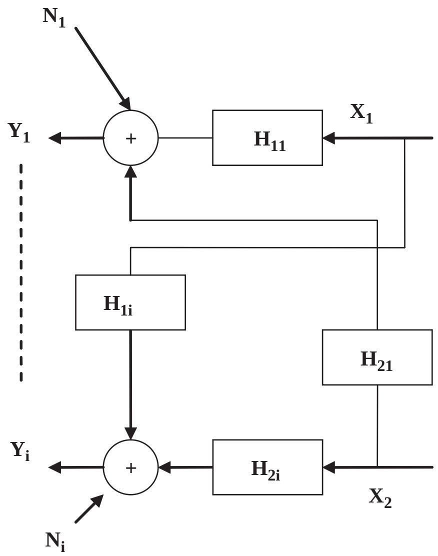

Fig. 1. Block diagram for two input/multiple output problem.

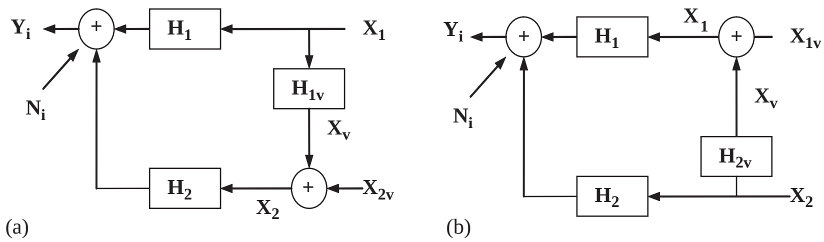

Fig. 2. Probable variants for interfering inputs. (a) The first input interferes the second; (b) the second input interferes the first.

and output, respectively (Fig. 1). Here, the principle component analysis or other engineering rationales indicate that most likely one of the inputs influences the other.

The situation has an ambiguity since the independent input is not known. A couple of probable cases are shown in Fig. 2, where H_1 and H_2 are transfer functions between measured inputs and outputs, H_{1v} and H_{2v} are transfer functions of the probable interfering paths. The task involves extraction of the virtual input matrix $\mathbf{S}_V$ from the measured input matrix $\mathbf{S}_M$. Note that $\mathbf{S}_V$ corresponds to partly coherent non-interfering (independent) inputs. We have:

$$\mathbf{S}_V(f) = \begin{bmatrix} S_{11}(f) & S_{12v}(f) \\ S^*_{12v}(f) & S_{2v2v}(f) \end{bmatrix}$$

or

$$\mathbf{S}_V(f) = \begin{bmatrix} S_{1v1v}(f) & S_{1v2}(f) \\ S^*_{1v2}(f) & S_{22}(f) \end{bmatrix}, \tag{1}$$

where S_{11} and S_{22} are the measured input autospectrums, S_{1v1v} and S_{2v2v} are the virtual input autospectrums of non-interfering inputs, S_{12v} and S_{1v2} are the virtual input cross-spectrums, f is frequency and superscript $*$ denotes complex conjugate. Also other unknowns in the block diagrams in Fig. 2 should be calculated.

The virtual input is understood as an input extracted from the original measurement data. It is refined from the contribution of the interfering signal, i.e. the virtual input spectra do not depend on another input (however there can be partial coherence irrespective of interference between virtual inputs). One of the measured inputs is independent since it is not influenced by the contribution of another input via the interfering path.

2. Problem criterion and mathematical background

The measured input matrix $\mathbf{S}_M$ can be represented as an eigenvalue decomposition [4,9]:

$$\mathbf{S}_M = \begin{bmatrix} S_{11} & S_{12} \\ S^*_{12} & S_{22} \end{bmatrix} = \mathbf{U}^{\mathrm{H}} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \mathbf{U}, \tag{2}$$

where S_{12} is the measured input cross-spectrum, $\mathbf{U}$ is the normalized eigenvector matrix, $\lambda_{1,2}$ are eigenvalues and superscript H denotes Hermitian transpose.

From Eq. (2), the eigenvalues for the two input case can be obtained in the form:

$$\lambda_{1,2}(f) = \frac{S_{11} + S_{22}}{2} \pm \sqrt{\frac{(S_{11} - S_{22})^2}{4} + |S_{12}|^2}. \tag{3}$$

If both inputs are contributing, the eigenvalues must have approximately equal values [4,9]. The relative difference between the eigenvalues is calculated as

$$\delta(f) = \frac{|\lambda_1 - \lambda_2|}{\lambda_m},$$
$$\lambda_m(f) = \frac{\lambda_1 + \lambda_2}{2}, \tag{4}$$

where λ_m is the mean of the eigenvalues. We can calculate δ_1 for the case in Fig. 2a and δ_2 for the case in Fig. 2b. A correct separation of the interfering inputs gives the minimal value for δ. Thus it is proposed to employ $\chi(f)$, the ratio of relative differences between the eigenvalues, as a criterion for the detection of the independent input:

$$\chi(f) = \frac{\delta_1(f)}{\delta_2(f)}. \tag{5}$$

Note that if $\chi < 1$, input 1 contributes to input 2. Otherwise, input 2 is independent while input 1 is influenced by input 2.

The criterion χ can be expressed with the elements of virtual input spectra matrix shown in Fig. 2 in the form:

$$\chi = \frac{S_{1v1v} + S_{22}}{S_{11} + S_{2v2v}} \sqrt{\frac{0.25(S_{11} - S_{2v2v})^2 + |S_{12v}|^2}{0.25(S_{1v1v} - S_{22})^2 + |S_{1v2}|^2}}. \tag{6}$$

The criterion is always real and positive, and the value does not depend on the cross-spectral phase. Note that χ is a function of the absolute magnitudes of virtual and measured input spectrums. It is needed to identify virtual autospectrums and cross-spectrums absolute magnitude that are used to detect the non-interfering input. The virtual spectrums can be calculated from the considerations of diagrams in Fig. 2. It is possible to utilize equations for transfer function detection by minimizing noise on output signals technique [1,9]:

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ H_{1v} & 1 \end{bmatrix} &= \begin{bmatrix} S_{11} & S_{12v} \\ S_{21} & S_{22v} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12v} \\ S_{12v}^* & S_{2v2v} \end{bmatrix}^{-1}, \\ \begin{bmatrix} 1 & H_{2v} \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} S_{11v} & S_{12} \\ S_{21v} & S_{22} \end{bmatrix} \begin{bmatrix} S_{1v1v} & S_{1v2} \\ S_{1v2}^* & S_{22} \end{bmatrix}^{-1}, \end{aligned} \tag{7}$$

where virtual cross-spectrums S_{1v2} and S_{12v} are found for the diagrams in Fig. 2a and b, respectively. Equations associated with the second matrix row for the first case and with the first row for the second case are a matter of interest. Thus there are two independent equations with four unknown variables. The matrix component S_{22v} or S_{11v} can be found through other variables [1,2]:

$$\begin{aligned} S_{22v} &= S_{12v} H_{1v} + S_{2v2v}, \\ S_{11v} &= S_{21v} H_{2v} + S_{1v1v}. \end{aligned} \tag{8}$$

In spite of the fact that the number of unknowns in Eqs. (7) and (8) is greater than the number of equations, they can be resolved with respect to the cross-spectral component of the virtual input matrix (S_{12v} or S_{1v2}) to obtain solution expressed by known variables only:

$$\begin{aligned} S_{12v} &= -\det \begin{bmatrix} S_{11} & S_{12v} \\ S_{12v}^* & S_{2v2v} \end{bmatrix} \Big/ S_{12}^*, \\ S_{1v2} &= -\det \begin{bmatrix} S_{1v1v} & S_{1v2} \\ S_{1v2}^* & S_{22} \end{bmatrix} \Big/ S_{12}. \end{aligned} \tag{9}$$

It can be proved that if the measured and virtual signals have a negligible noise contribution, the determinants of the measured and virtual matrices are equal (see Appendix A):

$$\det \begin{bmatrix} S_{11} & S_{12v} \\ S_{12v}^* & S_{2v2v} \end{bmatrix} = \det \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^* & S_{22} \end{bmatrix}$$

or

$$\det\begin{bmatrix} S_{1v1v} & S_{1v2} \\ S^*_{1v2} & S_{22} \end{bmatrix} = \det\begin{bmatrix} S_{11} & S_{12} \\ S^*_{12} & S_{22} \end{bmatrix}. \tag{10}$$

Thus unknown cross-spectral components can be found from the measured input matrix.

One can see from the expressions in Eq. (9) that the virtual cross-spectrums for probable interference are distinct by phase only since the same real number is divided by complex conjugates. It means that modulus of the virtual cross-spectrum is invariant and sign of the phase is determined by the direction of the interfering signal propagation (whether from the first input to the second or vice versa). It is also obvious that the probable virtual cross-spectrums are complex conjugates:

$$S_{12v} = S^*_{1v2}. \tag{11}$$

By utilizing the relations in Eq. (10), the unknown autospectra can be expressed in the form:

$$S_{2v2v} = \frac{|S_{12v}|^2 - |S_{12}|^2}{S_{11}} + S_{22}$$

or

$$S_{1v1v} = \frac{|S_{1v2}|^2 - |S_{12}|^2}{S_{22}} + S_{11}. \tag{12}$$

Now, the independent input is obtained by substitution of the virtual spectrums into formula (6). Further operations are to be undertaken to express the matrix $\mathbf{S}_V$ in terms of the separated inputs. As it was shown above, the modulus of the virtual cross-spectrum does not depend on the presumed independent input. Thus Eq. (6) can be expressed via the measured items only.

The interfering power can be identified from the following conventional equation:

$$S_{vv} = S_{11}|H_{1v}|^2$$

or

$$S_{vv} = S_{22}|H_{2v}|^2. \tag{13}$$

The virtual transfer function can be easily found from Eq. (17) after the calculation of all virtual spectral components. It is possible to find the modulus square of the virtual transfer function using different expressions. One of them can be represented as follows:

$$|H_{1v}|^2 = S_{11}^{-1}(S_{22} - S_{2v2v} - 2S_{11}^{-1}(\mathrm{Re}(S_{12}S^*_{12v}) - |S_{12v}|^2)$$

or

$$|H_{2v}|^2 = S_{22}^{-1}(S_{11} - S_{1v1v} - 2S_{22}^{-1}(\mathrm{Re}(S_{12}S^*_{1v2}) - |S_{1v2}|^2).$$

By taking into account expressions in Eqs. (9) and (10), the modulus of virtual transfer function can be written in a different way:

$$\begin{aligned} |H_{1v}|^2 &= S_{11}^{-2}(|S_{12}|^2 + |S_{12v}|^2 + 2\det[S_M]\cos 2\varphi_{12}), \\ |H_{2v}|^2 &= S_{22}^{-2}(|S_{12}|^2 + |S_{1v2}|^2 + 2\det[S_M]\cos 2\varphi_{12}), \end{aligned} \tag{14}$$

where φ_{12} is the phase of cross-spectrum S_{12}. It can be seen from formula described above and statement in Eq. (11) that the right-hand sides of the expressions between parentheses are equal. It follows from Eqs. (13) and (14) that the ratio of the possible virtual power contributions is the inverse of the measured autospectra ratio. The same conclusion can be obtained with respect to the ratio of probable virtual autospectra (S_{1v1v}/S_{2v2v}) from the consideration of relations in Eqs. (10) and (11).

As modulus is a positive number, the number inside the brackets in the right-hand side in formula (14) must be positive too. It was proved above that the modulus of the virtual cross-spectrum is independent of the way of interference, i.e. there is no difference if the first input influences the other or vice versa. It follows from Eqs. (11) and (14) that positive transfer function modulus should be invariant as well. The condition can be treated as necessary for the detection if the measured inputs interfere with each other:

$$|S_{12}|^2 + |S_{12v}|^2 > -2\det[S_M]\cos 2\varphi_{12}$$

or

$$S_{11}(S_{22} + S_{2v2v}) > 4\det[S_M]\sin^2\varphi_{12}. \tag{15}$$

Here the formula is written for the first independent input (Fig. 2a). If the second input influences the first one, indices 1 and 2 should be exchanged. The first formula in Eq. (15) is satisfied at any spectral component if (in a phase determined within $\pm\pi$ interval):

$$0 \leqslant |\varphi_{12}| \leqslant \frac{\pi}{4}$$

or

$$\frac{3\pi}{4} \leqslant |\varphi_{12}| \leqslant \pi, \tag{16}$$

i.e., the interference is possible at arbitrary values of the absolute magnitudes with the phase given in Eq. (16). When the phase takes other values, the spectral component modulus becomes of greater importance.

3. Illustration of the proposed method implementation

To illustrate the proposed technique, let us consider a hydraulic system where the noise radiation is substantially coherent with the pressure fluctuation inside two communicating pipelines. The pipelines interact through the bypass line with a control valve. Static pressure in the second line is higher than in the first one so the flow direction is clear. As it is known, pressure waves can propagate both upstream and downstream. Thus it is not obvious which line interferes the other. Both pipelines are originally pressurized from the same source, so their pressure fluctuations are significantly coherent for the frequencies of interest.

The system is equipped by embedded dynamic pressure sensors and the most intensive pressure fluctuation harmonics correlating with the outer noise are estimated. The measured input matrix components are shown in Table 1. It should be noted that not all cross-spectra in Table 1 satisfy condition given in Eq. (16).

Table 1
Measured input matrix components

Frequency (Hz)	S_{11} (Pa^2)	S_{22} (Pa^2)	S_{12} (Pa^2)
235	153742	43139	$70225e^{0.78j}$
649	2903616	1645	$68906.3e^{-1.315j}$
708	173806	25027	$65792.3e^{3.045j}$

Table 2
Virtual input matrix components

Frequency (Hz)	1st line		2nd line		Independent contributor	Contributed power, S_{vv} (Pa^2)
	Virtual S_{1v1v} (Pa^2)	Virtual S_{1v2} (Pa^2)	Virtual S_{2v2v} (Pa^2)	Virtual S_{12v} (Pa^2)		
236	53020	$-24218e^{-0.78j}$	14877	$-24218e^{0.78j}$	Line 2, $\chi = 1.697$	128765
649	17354	$-411.8e^{1.315j}$	9.8	$-411.8e^{-1.315j}$	Line 2, $\chi = 1.208$	2896384
708	852.1	$-322.6e^{-3.045j}$	122.7	$-322.6e^{3.045j}$	Line 2, $\chi = 1.069$	175479

Virtual spectra and transfer function are calculated in accordance with formulas in Eqs. (9), (12)–(14), and the results are summarized in Table 2. In accordance with Eq. (6), the pressure disturbances from pipeline 2 interfere with the pressure fluctuations from pipeline 1. Moreover contribution from the interfering input S_{vv} surpasses the independent power input S_{1v1v}. Coherence between the virtual non-interfering inputs for frequencies 649 and 708 Hz is insignificant (less than 0.6%) since the magnitude of virtual cross-spectrum is negligible. Hence approximately the same data (except data for the identification of the independent contributor) could be obtained through the use of traditional procedures for statistically independent inputs. However the calculated coherence becomes 25.64% at 236 Hz and exploration of the contribution from the inputs with zero coherence would bring a noticeable error to the calculation.

Choosing an incorrect variant with the first independent input brings only minor changes to the virtual input matrix in comparison with the measured one, i.e. the huge difference between λ_1 and λ_2 is almost kept the same. Thus it can be considered as an additional evidence of the appropriate independent contributor detection.

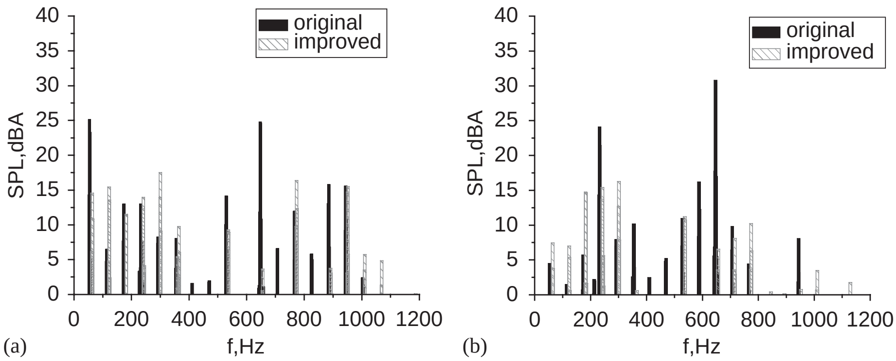

Fig. 3. A weighted sound pressure level associated with the pressure pulsations in the original and improved hydraulic systems. (a) Point 1, (b) point 2.

This information enables us to predict the effect of muffler installation in different locations of the hydraulic system (in the first or the second pipe line, before the bypass junction, after the bypass junction, in the bypass line, etc.) using the block diagram in Fig. 2b. There are two outputs Y_i, which correspond to measurement of noise at two check-up points. Exploration of probable ways to implement a muffler indicates that it is sufficient to install a single muffler in the first line downstream after the bypass line junction since the modulus of transfer function H_1 is significantly higher than the modulus of H_2 and the power contribution to the output from the corresponding path substantially prevails over the contribution from line 2. The expected noise reduction is 5 dBA that has been experimentally verified afterwards. Reduction of some particular noise components reaches up to 24 dBA (Fig. 3). Thus the utilization of the proposed technique eliminates the need to execute extra diagnostic measurements and the consecutive path analysis allows optimizing the number of mufflers and locations for the installation.

4. Conclusions

This paper describes a new technique to extract the non-interfering (virtual) input spectral matrix from the measured data without any additional data acquisition. The proposed technique is based on the analytical solution of spectrum relations for a multiple output system with two interfering inputs. To avoid ambiguity that could influence the independent contributor detection, the ratio of the relative difference between eigenvalues of the virtual input spectra matrix is proposed as a criterion that can be utilized for the identification of the independent input. Additional data such as virtual transfer function and interfering power can be calculated by conventional procedures from the consideration of corresponding block diagram. The method is easy to implement and does not require any complex computation procedure.

Appendix A

If noise in a linear system is negligible in comparison with signals of interest, input and output matrices are related as follows [1,2,5,6]:

$$\begin{bmatrix} 1 & 0 \\ H_{1v} & 1 \end{bmatrix}\begin{bmatrix} S_{11} & S_{12v} \\ S^{*}_{12v} & S_{2v2v} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ H_{1v} & 1 \end{bmatrix}^{\mathrm{H}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$
$$\begin{bmatrix} 1 & H_{2v} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} S_{1v1v} & S_{12v} \\ S^{*}_{12v} & S_{22} \end{bmatrix}\begin{bmatrix} 1 & H_{2v} \\ 0 & 1 \end{bmatrix}^{\mathrm{H}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}, \tag{17}$$

hence it can be written with respect to determinants:

$$\det\begin{bmatrix} 1 & 0 \\ H_{1v} & 1 \end{bmatrix}\det\begin{bmatrix} S_{11} & S_{12v} \\ S^{*}_{12v} & S_{2v2v} \end{bmatrix}\det\begin{bmatrix} 1 & 0 \\ H_{1v} & 1 \end{bmatrix}^{\mathrm{H}} = \det\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$
$$\det\begin{bmatrix} 1 & H_{2v} \\ 0 & 1 \end{bmatrix}\det\begin{bmatrix} S_{1v1v} & S_{12v} \\ S^{*}_{12v} & S_{22} \end{bmatrix}\det\begin{bmatrix} 1 & H_{2v} \\ 0 & 1 \end{bmatrix}^{\mathrm{H}} = \det\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}.$$

As the determinant of the transfer function matrix equals unity, one can write:

$$\det\begin{bmatrix} S_{11} & S_{12v} \\ S^{*}_{12v} & S_{2v2v} \end{bmatrix} = \det\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$
$$\det\begin{bmatrix} S_{1v1v} & S_{12v} \\ S^{*}_{12v} & S_{22} \end{bmatrix} = \det\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$

The same relation can be obtained through the consideration of algebraic relations between measured and virtual spectral components [2,9] with the assumption that multiple coherence between measured inputs and virtual inputs is unity.

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