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Journal of Sound and Vibration 287 (2005) 611–613
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

Non-harmonic Fourier analysis available for detecting very low-frequency components

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Received 7 October 2003; received in revised form 28 July 2004; accepted 31 January 2005
Available online 31 May 2005

The limitation of the spectrum estimation based on the procedure using the fast Fourier transform (FFT) is that of frequency resolution [1]. This limitation is particularly prominent when analyzing short data records. In practice, short data records occur because many measured processes have time-varying spectra and are considered constant only for short intervals. The following non-harmonic Fourier analysis is available for analyzing short data records.

Let $a(f)$ and $b(f)$ be Fourier coefficients which are given by

$$a(f) = \sum_{m=0}^{M} W(m)S(m,f) \Big/ \sum_{m=0}^{M} S(m,f)^2, \quad (1)$$

$$b(f) = \sum_{m=0}^{M} W(m)C(m,f) \Big/ \sum_{m=0}^{M} C(m,f)^2, \quad (2)$$

where $W(m)$ is the data sequence for $m = 0,1,\ldots,M$ sampled from a waveform by the frequency F, f is the arbitrary frequency and

$$S(m,f) = \sin[2\pi(m - M/2)f/F], \quad (3)$$

$$C(m,f) = \cos[2\pi(m - M/2)f/F]. \quad (4)$$

doi:10.1016/j.jsv.2005.01.025

Obtain a temporary sinusoidal component $H(m,f)$ such that

$$H(m,f) = a(f)S(m,f) + b(f)C(m,f). \tag{5}$$

Then, subtracting the temporary sinusoidal component from the data sequence, we have a residual data sequence:

$$R(m,f) = W(m) - H(m,f). \tag{6}$$

Since

$$\sum_{m=0}^{M} S(m,f)C(m,f) = 0 \tag{7}$$

for all f, we have

$$\sum_{m=0}^{M} R(m,f)^2 = \sum_{m=0}^{M} W(m)^2 - \sum_{m=0}^{M} H(m,f)^2. \tag{8}$$

Thus, if the power of $H(m,f)$ is maximum at $f = f_1$, that of $R(m,f)$ becomes minimum at f_1, so that we attain a least-squares fit of $W(m)$ to a sinusoid by the first sinusoidal component $H(m,f_1)$.

The same procedure is carried out substituting the weakest residual data sequence $R(m,f_1)$ for $W(m)$ to find the second sinusoidal component $H(m,f_2)$, and so on. Thus, detecting sinusoidal components one by one, after N process iterations, we have

$$W(m) = \sum_{n=1}^{N} H(m,f_n) + R(m,f_N) \tag{9}$$

and

$$\sum_{m=0}^{M} W(m)^2 = \sum_{n=1}^{N} \sum_{m=0}^{M} H(m,f_n)^2 + \sum_{m=0}^{M} R(m,f_N)^2. \tag{10}$$

The power of residual data sequence $R(m,f_N)$ decreases as N increases. Unlike conventional Fourier analysis, a data sequence reconstructed using sinusoidal components obtained by the new method is generally nonperiodic, enabling prediction by extrapolation of a reconstructed data sequence ($m<0$, $m>M$).

The non-harmonic Fourier analysis yields the accurate estimate of the spectrum of sinusoids in noise. An example of the analysis applied to the detection of a sinusoid in white noise is illustrated in Fig. 1, where (a) shows a 512-point data sequence for analysis and (b) the detected sinusoid which is compared with an original one. The period of the sinusoid normalized by $1/F$ is 1700, so that the duration of the available data record is 0.3 period which limits the frequency resolution of the harmonic analysis, i.e., normalized periods estimated by the FFT are 512, 256, and so on. The non-harmonic analysis described above reduces to the conventional Fourier analysis when the arbitrary frequencies are harmonic. Thus, we can reduce the computational burden of the non-harmonic Fourier analysis by employing the FFT for rough analysis to limit arbitrary frequency assignments within a certain band of frequencies indicated by the maximum FFT power spectrum

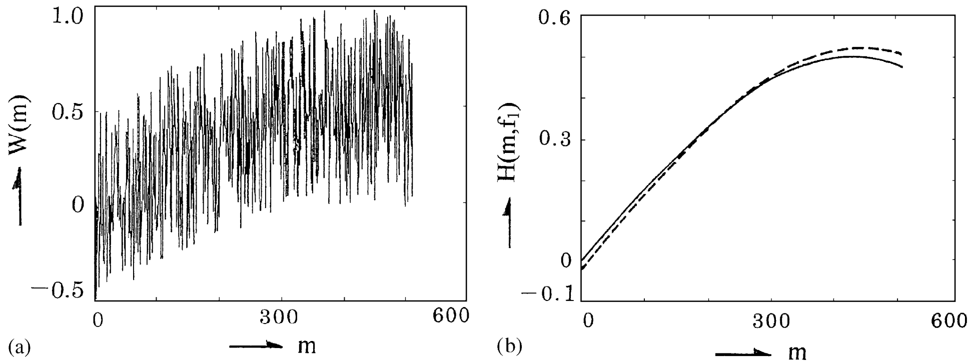

Fig. 1. (a) A 512-point data sequence $W(m)$ for analysis and (b) the detected sinusoid $H(m,f_1)$ (broken line) comparing with an original one (solid line; the duration of 0.3 period).

frequency of the data sequence [2]. It should be noted that one can use the same procedure to obtain sinusoidal components until the error (the power of a residual data sequence) is reduced to a desired value.

References

[1] S.M. Kay, S.L. Marple, Spectrum analysis—a modern perspective, *Proceedings of the IEEE* 69 (11) (1981) 1380–1419.

[2] Y. Hirata, T. Koike, Speech band compression using a generalized harmonic analysis, *Technical Report of IEICE* ES 98-61 (1998) 17–24.