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Short Communication

Free vibration studies of simply supported non-homogeneous functionally graded magneto-electro-elastic finite cylindrical shells

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Abstract

In this article, free vibration studies on Functionally Graded Materials (FGM) magneto-electro-elastic cylindrical shells have been carried out. A series solution is assumed in circumferential and axial direction such that the three-dimensional character of the solution is preserved. The finite element model is derived based on constitutive equation of magneto-electro-elastic material accounting for coupling between elasticity, electric and magnetic effect. Frequency behavior with the variation of power law index is analyzed and explained on the basis of constitutive material. One of the aims of the study is to evaluate the influence of magnetic and piezoelectric effect on the structural frequency. In addition influence of length to radius ratio and radius to thickness ratio on the frequency behaviour of such shells have been attempted.

1. Introduction

Literature dealing with research on the behaviour of magneto-electro-elastic structures has gained more importance recently as these smart or intelligent materials have ability of converting energy from one form to the other (among magnetic, electric and mechanical energy). Studies on

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static and dynamic behaviour on plates as well as infinite cylinder has been dealt in literature. Recently, Pan [1] derived an exact closed-form solution for the simply supported and multilayered plate made of anisotropic piezoelectric and piezomagnatic materials under a static mechanical load, Pan and Heyliger [2] solved the corresponding vibration problem. Piezoelectric and piezomagnetic composites exhibit coupling effect of electric and magnetic fields. In most of the studies these composite materials has been used as layers or as multiphase. The behaviour of finitely long cylindrical shells under uniform internal pressure has been studied by Wang and Zhong [3]. According to Wang and Zhong [3] piezoelectric and piezomagnetic composites in general have the coupling effect which is two orders higher than that of single-phase magnetoelectric constituent materials. Micro-mechanical analysis of fully coupled electro-magneto-thermo-elastic composites has been carried out by Aboudi [4] for the prediction of the effective moduli of magneto-electro-elastic composites. Li [5] studied the multi-inclusion and inhomogenity problems in a magneto-electro-elastic solid. From the literature survey, it is found that only few studies have been reported on magneto-electro-elastic structures by finite element analysis. Buchanan [6] has studied the behaviour of layered versus multiphase magneto-electro-elastic composites. Lage et al. [7] developed a layerwise partial mixed finite element model for magneto-electro-elastic plate. The studies on non-homogeneous magneto-electro-elastic structure are less in literature. Chen and Lee [8] adopted state space formulation to derive equations for non-homogeneous transversely isotropic magneto-electro-elastic plates. According to Chen et al. [9], like other advanced material the functionally graded magneto-electro-elastic structures will appear soon and he has carried out free vibration analysis of non-homogeneous transversely isotropic magneto-electro-elastic plates.

Recently, Buchanan [10] studied the free vibration behaviour of infinitely long magneto-electro-elastic solid cylinder by using semi-analytical finite element method. From literature it is found that no work is available on finite magneto-electro-elastic cylindrical shells. Hence in the present study, free vibration analysis of functionally graded non-homogeneous magneto-electro-elastic cylindrical shell are carried out by using series solution in conjunction with finite element approach.

2. Constitutive equations and analytical model of FGM materials properties

The coupled constitutive equations for anisotropic and linearly magneto-electro-elastic solids can be written as [6]

$$\sigma_k = C_{jk} S_k - e_{jk} E_k - q_{kj} H_k, \tag{1}$$

$$D_k = e_{jk} S_k + \varepsilon_{jk} E_k + m_{jk} H_k, \tag{2}$$

$$B_j = q_{jk} S_k + m_{jk} E_k + \mu_{jk} H_k, \tag{3}$$

where σ_k denotes stress, D_j is electric displacement and B_j is magnetic induction. C_{jk}, ε_{jk} and μ_{jk} are the elastic, dielectric and magnetic permeability coefficients. e_{kj}, q_{kj} and m_{jk} are piezoelectric,

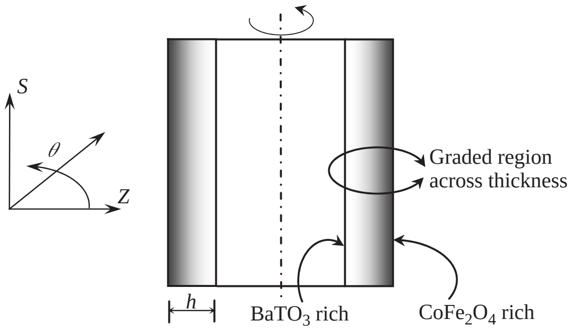

Fig. 1. Schematic configuration and coordinate of FGM magneto-electro-elastic shell analysed in present study.

piezomagnetic and magnetoelectric material coefficients. The details of the constitutive equation, strain–displacement, electric field–electric potential and magnetic field–magnetic potential are given by Buchanan [10] and hence not repeated here. All the constants for FGM material are evaluated by using effective property model.

The present study considers functionally graded material composed of piezoelectric and magnetostrictive material. The grading is accounted across the thickness of the shell.

Consider an FGM cylindrical shell radius r, and thickness h as shown in Fig. 1. In present analysis it is assumed that the composition is varied from the inner to the outer surfaces, i.e., the inner surface of the shell is metal piezoelectric-rich, whereas the outer surface is magnetostrictive-rich. In addition, material properties are graded throughout the thickness direction according to the volume fraction power law distribution. Present study considers smooth and continuous variation of the volume fraction of either piezo or magno based on the power law index. A simple power law type definition for the volume fraction of the metal across the radial direction of the shell is assumed as given in Eq. (4)

$$V_f = \left(\frac{r_z - r_i}{r_o - r_i}\right)^n, \tag{4}$$

where r_z represents radius at any point along the radial direction of the shell, r_i is the inner radius, r_o is the outer radius of the shell. Based on the above definition it follows that the inner surface of the cylindrical shell will be piezo-rich. The simplest is the law of mixtures originally proposed by Voigt is used in the present study. The sum total volume of the constituent materials, $BaTiO_3$ (B) and $CoFe_2O_4$ (C) should be

$$V_B + V_C = 1. \tag{5}$$

Based on the volume fraction definition and law of mixtures, the effective material property definition follows,

$$(\mathrm{MP})_{\mathrm{eff}} = (\mathrm{MP})_{\mathrm{ot}} V_B + (\mathrm{MP})_{\mathrm{in}} V_C, \tag{6}$$

where 'MP' is general notation for material property. Making use of equation (4)–(6) the following effective mechanical and thermal properties definitions can be written as:

$$C_{\text{eff}} = (C_{\text{ot}} - C_{\text{in}})\left(\frac{r_z - r_i}{r_o - r_i}\right)^n + C_{\text{in}}, \tag{7a}$$

$$e_{\text{eff}} = (e_{\text{ot}} - e_{\text{in}})\left(\frac{r_z - r_i}{r_o - r_i}\right)^n + e_{\text{in}}, \tag{7b}$$

$$q_{\text{eff}} = (q_{\text{ot}} - q_{\text{in}})\left(\frac{r_z - r_i}{r_o - r_i}\right)^n + q_{\text{in}} \tag{7c}$$

$$\varepsilon_{\text{eff}} = (\varepsilon_{\text{ot}} - \varepsilon_{\text{in}})\left(\frac{r_z - r_i}{r_o - r_i}\right)^n + \varepsilon_{\text{in}} \tag{7d}$$

$$\mu_{\text{eff}} = (\mu_{\text{ot}} - \mu_{\text{in}})\left(\frac{r_z - r_i}{r_o - r_i}\right)^n + \mu_{\text{in}} \tag{7e}$$

In the above equations (7a)–(7e), the subscripts 'ot' stands for outer surface and 'in' stands for inner surface of the cylindrical shell. 'eff' stands for effective material properties obtained by above equations for particular power law index n.

3. Finite element formulation

Recently Buchanan [10] has analysed free vibration behaviour for infinitely long solid cylinder. In the present work similar to circumferential direction shape functions satisfying axial boundary conditions for simply supported shells has been adopted. These series has been assumed in circumferential direction and axial direction such a way that modes becomes decoupled. The finite element model has been used in the radial direction. Hence the shape functions are as follows;

$$\begin{aligned}
u(r,\theta,z) &= U(r)\cos m\theta \cos\left(\frac{n\pi z}{l}\right),\\
v(r,\theta,z) &= V(r)\sin m\theta \cos\left(\frac{n\pi z}{l}\right),\\
w(r,\theta,z) &= W(r)\cos m\theta \sin\left(\frac{n\pi z}{l}\right),\\
\varphi(r,\theta,z) &= \Phi(r)\cos m\theta \sin\left(\frac{n\pi z}{l}\right),\\
\psi(r,\theta,z) &= \Psi(r)\cos m\theta \sin\left(\frac{n\pi z}{l}\right),
\end{aligned} \tag{8}$$

where m is an integer and is the circumferential harmonic mode number, n is the axial harmonic mode number. In the end the analysis has been reduced for finite element in radial direction still retaining the three-dimensional dependence the solution based on the choice of m and n. The analysis is carried out by two noded finite element and the assumed shape functions are

$$U_i = [N_u]\{U\}, \quad \Phi = [N_\varphi]\{\Phi\}, \quad \Psi = [N_\varphi]\{\Psi\}, \tag{9}$$

where

$$N_1 = \left(\frac{r_{i+1} - r}{r_{i+1} - r_i}\right) \quad , \quad N_2 = \left(\frac{r - r_i}{r_{i+1} - r_i}\right)$$

For a coupled filed problem finite element equations are as follows:

$$\begin{aligned} &\left[[K_{uu}] - \omega^2[M]\right]\{U\} + \left[K_{u\phi}\right]\{\phi\} + \left[K_{u\psi}\right]\{\psi\} = 0 \\ &\left[K_{u\phi}\right]^{\mathrm{T}}\{U\} - \left[K_{\phi\phi}\right]\{\phi\} - \left[K_{\phi\psi}\right]\{\psi\} = 0 \\ &\left[K_{u\psi}\right]^{\mathrm{T}}\{U\} - \left[K_{\phi\psi}\right]^{\mathrm{T}}\{\phi\} - \left[K_{\psi\psi}\right]\{\psi\} = 0 \end{aligned} \tag{10}$$

Various stiffness matrices are defined as shown below.

$$\begin{aligned} &[K_{uu}] = c\int [B_u]^{\mathrm{T}}[C][B_u] r\,\mathrm{d}r, \\ &\left[K_{u\phi}\right] = c\int [B_u]^{\mathrm{T}}[e]\left[B_\phi\right] r\,\mathrm{d}r, \\ &\left[K_{u\psi}\right] = c\int [B_u]^{\mathrm{T}}[q]\left[B_\psi\right] r\,\mathrm{d}r, \\ &\left[K_{\phi\phi}\right] = c\int \left[B_\phi\right]^{\mathrm{T}}[\varepsilon]\left[B_\phi\right] r\,\mathrm{d}r, \\ &\left[K_{\psi\psi}\right] = c\int \left[B_\psi\right]^{\mathrm{T}}[\mu]\left[B_\psi\right] r\,\mathrm{d}r, \\ &\left[K_{\phi\psi}\right] = c\int \left[B_\phi\right]^{\mathrm{T}}[m]\left[B_\psi\right] r\,\mathrm{d}r, \\ &[M] = c\int [N]^{\mathrm{T}}[\rho][N] r\,\mathrm{d}r, \end{aligned} \tag{11}$$

where $c = 0.5 * \pi L$.

$[B_u]$, $[B_\phi]$, $[B_\psi]$ represent the strain–displacement, electric field–electric potential and magnetic field–magnetic potential relations, respectively. The details are given by Buchanan [10]. In the present study the Gaussian integration scheme has been implemented to evaluate integrals involved in different matrices. The functionally graded material properties are accounted by evaluating the material properties at Gaussian points. In Eq. (10), eliminating electric and magnetic potential terms by condensation techniques to get K_{eq}.

$$\left[K_{\mathrm{eq}}\right]\{U\} + [M]\{\ddot{U}\} = 0, \tag{12}$$

where $\left[K_{\mathrm{eq}}\right] = [K_{uu}] + \left[K_{u\phi}\right][K_{II}]^{-1}[K_I] + \left[K_{u\psi}\right][K_V]^{-1}[K_{IV}]$.

The details of the various component matrices are described in Ref. [10] and hence not repeated here.

To study the piezoelectric effect on frequency due to $BaTiO_3$ material, the stiffness matrix $\left[K_{\mathrm{eq\_}\phi\phi}\right]$ is derived and is given by

$$\left[K_{\mathrm{eq\_}\phi\phi}\right] = [K_{uu}] + \left[K_{u\phi}\right]\left[K_{\phi\phi}\right]^{-1}\left[K_{u\phi}\right]^{\mathrm{T}}. \tag{13}$$

To study the magnetic effect on frequency due to magnetic $CoFe_2O_4$ material $\left[K_{\mathrm{eq}\_\psi\psi}\right]$ is used as stiffness matrix and is shown below.

$$\left[K_{\mathrm{eq}\_\psi\psi}\right] = [K_{uu}] + \left[K_{u\psi}\right]\left[K_{\psi\psi}\right]^{-1}\left[K_{u\psi}\right]^{\mathrm{T}}. \tag{14}$$

4. Results and discussion

4.1. Validation of the code and present formulation

As there is no open literature available on free vibration studies on magneto-electro-elastic FGM finite cylindrical shell it is felt that result can be compared for infinite cylindrical shell for $BaTiO_3$ and $CoFe_2O_4$ given by Buchanan [10] by incorporating power law index as $n = 0.0$ and 1000.0 in the computer code developed for FGM magneto-electro-elastic shell. From Table 1 it is seen that there is an excellent correlation between the present results and those reported by Buchanan [10] for the both the materials.

Table 2 gives values of frequency obtained by including, piezoelectric effect, piezomagnetic effect and combination effect. Here power law index $n = 0.0$ corresponds to an isotropic shell with properties corresponding to that of homogeneous $BaTiO_3$ (piezoelectric) cylindrical shell and $n = 1000.0$ corresponds to cylindrical shell Magnetostrictive material ($CoFe_2O_4$). The power law index value n other than two extreme values governs the distribution of properties of piezo-magnetic mixture in FGM shell. Variation of the composition of piezoelectric and magnetos-trictive is linear for power law index $n = 1.0$. From Table 2 it is found that as the value of power law index increases the natural frequency increases as it approaches towards the homogeneous

Table 1
Comparison of frequencies Ω for an infinite solid cylinder of piezoelectric ($BaTiO_3$) and magnetostrictive cobalt iron oxide ($CoFe_2O_4$) $\Omega = \omega a\sqrt{(\rho/C_{44})}$

Mode	$k = 4.0$, $m = 1$			
	$BaTiO_3$		$CoFe_2O_4$	
	Present	Buchanan [10]	Present	Buchanan [10]
1	4.820	4.708	4.88	4.869
2	5.743	5.763	5.85	5.984
3	7.784	7.619	7.98	7.905
4	8.540	7.937	8.65	8.535
5	9.287	9.752	10.28	10.008
6	11.193	10.876	11.92	11.807
7	11.394	11.521	12.88	12.505
8	12.896	13.608	13.43	13.566
9	14.369	13.978	15.37	15.247
10	14.506	15.914	15.65	15.748

Table 2
Variation of natural frequency (Hz) with power law exponent 'n' for a simply supported FGM ($BaTiO_3$–$CoFe_2O_4$) cylindrical shell ($r = 1.0$; $H = 0.6$, $l/r = 4.0\,m$)

Mode no.	f_{uu}	$f_{\text{equivalent}}$	$f_{\text{eq\_}\psi\psi}$	$f_{\text{eq\_}\phi\phi}$
Pure $BaTiO_3$ ($n = 0.0$)				
(1,1)	201.88	202.36	201.01	202.49
(2,1)	349.35	349.65	349.57	349.87
(3,1)	759.38	760.21	759.86	760.69
(4,1)	1209.78	1210.84	1210.55	1211.61
(5,1)	1661.83	1662.98	1662.89	1664.04
$n = 0.2$				
(1,1)	203.56	203.70	203.57	203.93
(2,1)	362.60	363.26	362.79	363.55
(3,1)	789.53	790.91	790.02	791.43
(4,1)	1253.98	1255.57	1254.78	1256.38
(5,1)	1716.46	1718.08	1717.55	1719.18
$n = 0.5$				
(1,1)	205.38	205.26	205.30	205.60
(2,1)	372.62	373.78	372.80	374.10
(3,1)	812.79	814.74	813.29	815.28
(4,1)	1289.61	1291.71	1290.43	1292.54
(5,1)	1762.71	1764.82	1763.83	1765.95
$n = 1.0$				
(1,1)	207.44	207.14	207.25	207.59
(2,1)	380.52	382.19	380.69	382.51
(3,1)	831.57	833.98	832.09	834.54
(4,1)	1319.92	1322.43	1320.76	1323.28
(5,1)	1804.22	1806.77	1805.36	1807.92
$n = 5.0$				
(1,1)	212.94	212.53	212.56	213.19
(2,1)	393.23	394.88	393.43	395.19
(3,1)	859.98	862.03	860.51	862.59
(4,1)	1367.34	1369.51	1368.17	1370.41
(5,1)	1872.59	1875.05	1873.69	1876.29
$n = 10.0$				
(1,1)	214.70	214.24	214.28	214.94
(2,1)	396.60	397.72	396.82	398.02
(3,1)	865.85	867.22	866.37	867.80
(4,1)	1376.01	1377.51	1376.83	1378.44
(5,1)	1884.35	1886.12	1885.41	1887.41
Pure $CoFe_2O_4$ ($n = 1000.0$)				
(1,1)	217.30	216.68	216.82	217.44
(2,1)	402.25	402.24	402.49	402.51
(3,1)	874.97	874.93	875.49	875.53
(4,1)	1388.59	1388.48	1389.36	1389.48
(5,1)	1900.32	1900.10	1901.30	1901.54

magnetostrictive material which corresponds to $n = 1000.0$. The following notations are used in Table 2 and in subsequent discussion.

f_{uu} = structural frequency including elastic constants only;
$f_{\text{equivalent}}$ = structural frequency including magnetic and piezoelectric effect;
$f_{\text{eq}\_\psi\psi}$ = structural frequency including magnetic effect;
$f_{\text{eq}\_\phi\phi}$ = structural frequency including piezoelectric effect.

From Table 2 it is seen that for the case of $BaTiO_3$ (Piezoelectric) the frequency evaluator using the influence of the both piezo and magnetic effect is higher than that of the conventional structural frequency. It is also seen from Table 2 that the magnetic effect reduces the frequency by very little amount. In contrast the piezoelectric effect has higher effect and increases the frequency marginally compared to structural frequency. As the power law index n increases the influence of the magnetic effect is felt more compared to the piezoelectric effect as the material approaches homogeneous magnetostrictive. In general the frequency evaluated by using influence of piezo-magnetic effect is lower than that of structural frequency. Influence is more pronounce at the fundamental mode. It is also seen that as n increases the system frequency increases this is to be expected as the elastic constants are higher in the case of magnetic material. In general the conventional shells have their lowest frequency corresponding to the higher circumferential modes. In contrast from Table 2 it is seen that the shell frequencies increases with circumferential mode. This is due to the fact that the shell dimensions of study correspond to very thick solid cylindrical shell.

4.2. Free vibration studies on thin FGM magneto-electro-elastic cylindrical shells

After validating the present formulation studies have been carried out for FGM ($BaTiO_3$–$CoFe_2O_4$) cylindrical shells. The first axial mode frequencies associated with first 20 circumferential modes for a simply supported boundary condition are presented in Fig. 2 for

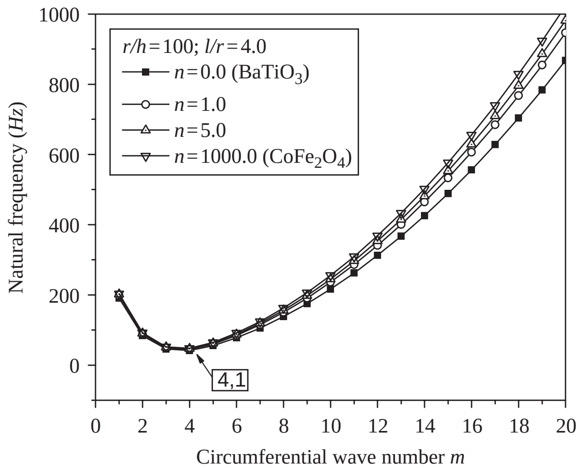

Fig. 2. Free vibration frequency of $BaTiO_3$–$CoFe_2O_4$ FGM cylindrical shell for different power law index, $l/r = 4.0$, $r/h = 100$. Lowest natural frequency mode (4,1).

different power law index. The frequency characteristic is typical of homogeneous isotropic or orthotropic shells, depicting a bathtub curve. The frequency characteristics do not change with the power law index and follows the same behaviour.

The influence of the power law index is mainly to change the magnitude of the first axial mode frequency. As the power law index increases the frequencies increase. This fact can be understood easily, as the elastic properties of magnetostrictive are higher compared to piezoelectric counterpart. The power law index does not have a great influence in shifting the associated circumferential mode number to the lowest of the first axial mode frequency. But, however, the change in the thickness of the FGM shell does alter the mode number of the lowest frequency.

In order to understand the behaviour of magneto-electro-elastic thin shells analysis is carried out for ratio of $l/r = 4.0$ for different radius to thickness ratio. The study has been carried out for different power law index. Figs. 3 to 4 contain the frequency results obtained for different power law index for different l/r ratios. It is seen from the Fig. 3 as r/h ratios increases the lowest frequency of the shells occurs at higher circumferential mode. This behaviour is similar to conventional shells. It is also seen that the frequency does not vary much with r/h ratios for lower circumferential modes. This is also to be expected, as the membrane effect is more predominant at lowest circumferential mode. As expected at higher circumferential mode with the increase of r/h

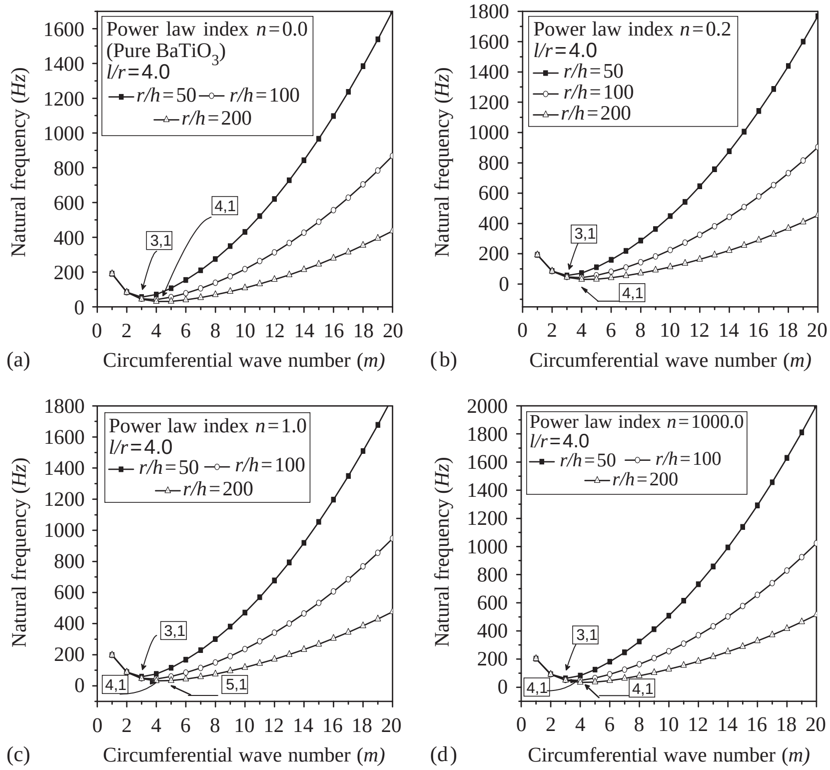

Fig. 3. Variations of the first axial mode natural frequencies associated with 20 circumferential harmonics for FGM $BaTiO_3$–$CoFe_2O_4$ cylindrical shells having $l/r = 4.0$ for different r/h ratios for (a) $n = 0.0$, (b) $n = 0.2$, (c) $n = 1.0$ and (d) $n = 1000.0$.

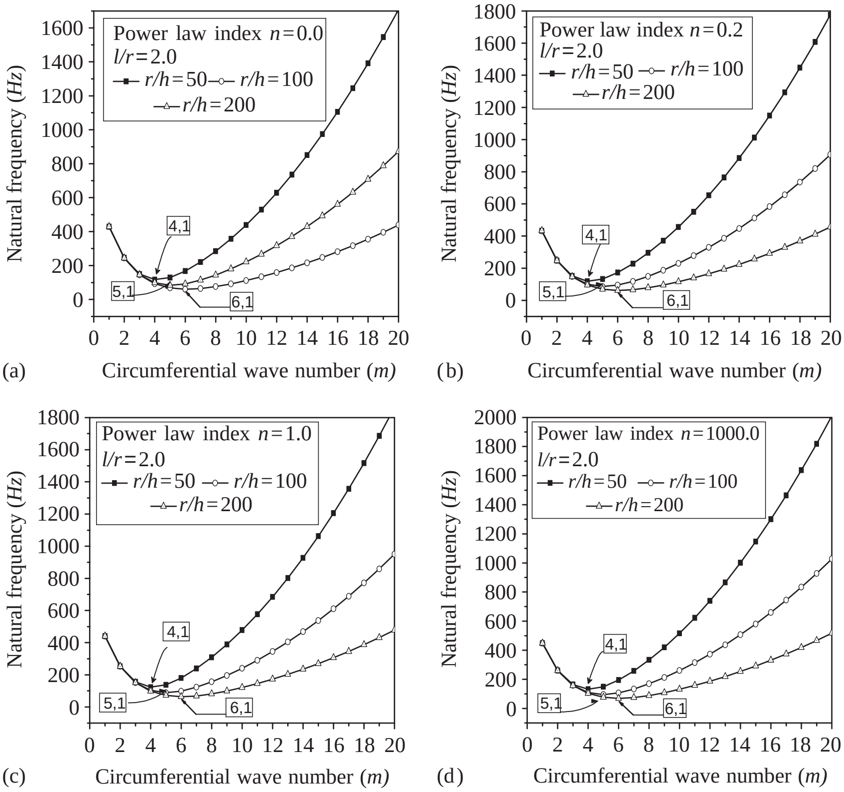

Fig. 4. Variations of the first axial mode natural frequencies associated with twenty circumferential harmonics for FGM $BaTiO_3$–$CoFe_2O_4$ cylindrical shells having $l/r = 2.0$ for different r/h ratios for (a) $n = 0.0$, (b) $n = 0.2$, (c) $n = 1.0$ and (d) $n = 1000.0$.

ratios the shell frequencies comes down and this is due to the fact that the bending stiffness reduces as r/h ratios increases. In addition it is also seen that as the r/h ratio increases the lowest circumferential mode gets shifted. From Fig. 4 it is noticed that the decrease in l/r cause the shift in lowest circumferential mode to the right-hand side of the bathtub curve. This fact can be understood as lower the l/r ratios the membrane effect is more predominant as compared to bending.

5. Conclusions

A study on the free vibration of functionally graded (FG) magneto-electro-elastic cylindrical shell composed of piezoelectric material barium titanate ($BaTiO_3$) and magnetostrictive cobalt iron oxide ($CoFe_2O_4$) have been done. Material properties are graded in the thickness direction of the shell accounting to a volume fraction power law distribution. The analysis was carried out by using series solution in circumferential, axial direction and in conjunction finite element approach in radial direction. The study shows that frequency characteristics of the functionally graded magneto-electro-elastic shells are similar to homogeneous isotropic cylindrical shells. Following the conclusions based on the present study.

1. Influence of piezoelectric effect is to increase the structural frequency marginally and the magnetic effect is to reduce the same marginally.
2. Influence of piezoelectric effect and magnetic effect on structural frequencies is higher at fundamental mode and its influence decreases at higher modes.
3. The variation of free vibration natural frequencies of FGM magneto-electro-elastic shells with respect to circumferential mode are typically characterized by the bathtub curve, similar to conventional shells.
4. Variation of frequencies with respect to radius to thickness ratio and length to radius ratio is similar to that of conventional shells.
5. As power law index increases the frequency of magneto-electro-elastic of the system increases. This is due to the fact that magnetostrictive material has got higher elastic constants compared to piezoelectric counterpart.

It is felt that the present study is highly useful for subsequent study of response of functionally graded magneto-electro-elastic system subjected to mechanical, electrical, and magnetic loading.

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