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Active robust vibration control of flexible structures

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Abstract

Flexible structures are extensively used in many space applications, for example, space-based radar antennae, space robotic systems, and space station, etc. The flexibility of these space structures results in problems of structural vibration and shape deformation, etc. In recent years, active control methods have been developed to suppress structural vibration and improve the performance of these flexible space structures. In this paper, we developed an approach for active vibration control of flexible structures with integrated piezoelectric actuators using control theory. First, dynamic models for a flexible circular plate with integrated piezoelectric actuators and sensors are derived using the Rayleigh–Ritz method. An active robust controller is designed to suppress vibration of the circular plate. Robustness of the control system of the circular plate is discussed for the model parameter uncertainty. This active robust vibration control method is tested via experimental implementation. The experimental results show that the proposed robust active control method is efficient for active vibration suppression.

1. Introduction

Flexible structures are extensively used in many space applications, for example, space-based radar antennae, space robotic systems, and space station, etc. The flexibility of these space structures results in problems of structural vibration and shape deformation, etc. In recent years, active control methods have been developed to suppress structural vibration and improve the performance of flexible structures [1–11]. Crawley and de Luis [2] investigated piezoelectric

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actuators as elements of smart structures and derived the static and dynamic models for segmented piezoelectric actuators bonded or embedded in the flexible structures. Bailey and Hubbard [1] designed an active distributed parameter damper for a cantilever beam using piezoelectric actuators, and developed an algorithm to actively control the damper using the Lyapunov second method. Hagood and Chung [9] established an analytical models for general structures with piezoelectric materials coupling the mechanical and electrical coordinates using Hamilton principle and demonstrated the applications of the general models using a cantilevered beam. Dosch et al. [5] presented a method for synthesizing an antenna model using identified single input and single output transfer functions. The identified model was used to design a positive position feedback and H_∞ controller and the damping in all the modes in the targeted frequency range were increased. Garcia et al. [7] developed an active control method to suppress vibration of flexible ribbed antenna structures using piezoceramic components as both sensor and actuator simultaneously. Most published papers in the area of active vibration control design vibration controllers based on dynamic models of flexible structures without consideration of uncertainties. However, the system model uncertainties from parameter variation, mode truncation, etc., have a significant influence on the system performance.

In recent years, the research topic of robust active control for flexible structures has received considerable attention [4,11,12], etc. Damaren and Le-Ngoc [4] developed an active vibration control method for a bandsaw blade using H_∞ control theory, and analyzed the robust stability of the closed-loop system. Kar et al. [11] presented a H_∞ robust method for controlling the bending and torsional vibration of a plate structure using a reduced order model which is derived from the first three vibration modes. Sadri et al. [12] developed a robust control approach for active vibration control of plate-like structures based on H_∞ control theory, experimentally implemented the proposed robust controller for cantilever plate with two piezoelectric actuators and two non-collocated sensors, and compared the performance of the H_∞ controller and linear quadratic Gaussian controller. It is found that H_∞ control theory is an attractive method to be used to design robust vibration controllers for the suppression of flexible structures. However, the H_∞ controller was found to be very sensitive to controller parameter variations if the order of the controller is reduced to a certain range, and the controller implementation requires powerful computer processing [12]. In some space applications, computer processing power is limited, for example, microsatellite on-board computer. Thus, it is necessary to develop robust control methods which can deal with system uncertainties and be easily implemented using limited computer processing power. In this paper, a robust active vibration control method is developed based on robust control theory. Robustness of the active control system is analyzed and the stability of the closed-loop system is proved based on Lyapunov stability theory.

2. System modeling

In this section, a circular plate structure with bonded piezoelectric actuators and sensors is considered, and the circular plate is clamped in the center circular area. The circular plate structure is described as in Fig. 1. In order to establish the mathematical model of the circular plate structure, it is assumed that the piezoelectric elements are perfectly bonded to the structure with zero glue thickness.

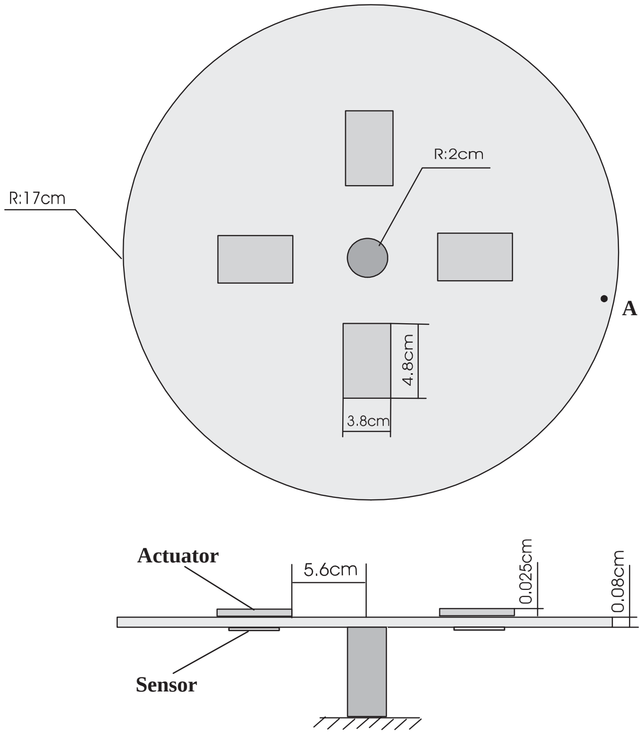

Fig. 1. Configuration of circular plate with bonded actuators and sensors.

First, the constitutive relations for piezoelectric ceramic are introduced. The constitutive equations of piezoelectric ceramic are expressed as

$$\sigma = \mathbf{C}_p\varepsilon - \mathbf{D}_p^{\mathrm{T}}\mathbf{E}, \tag{1}$$

$$\mathbf{D} = \mathbf{D}_p\varepsilon + \mathbf{\Lambda}\mathbf{E}, \tag{2}$$

where σ and ε are stress and strain vectors of the piezoelectric ceramic, $\mathbf{D}$ and $\mathbf{E}$ are vectors of electrical displacement and field, $\mathbf{C}_p$ is the elastic stiffness matrix of piezoelectric ceramic, $\mathbf{D}_p$ is the piezoelectric stress coefficient matrix, and $\mathbf{\Lambda}$ is the permittivity matrix. Eqs. (1) and (2) describe the converse and direct effect of piezoelectric ceramic.

In order to derive the dynamic equation of the circular plate structure, the transverse displacement is approximately expressed as

$$w(r,\theta,t) = \sum_{i=1}^{n} \phi_i(r,\theta)q_i(t) = \phi(r,\theta)\mathbf{q}(t), \tag{3}$$

where r and θ are polar coordinates with origin at the center, $\mathbf{q}(t) = [q_1^{\mathrm{T}}(t)q_2^{\mathrm{T}}(t)\ldots q_n^{\mathrm{T}}(t)]^{\mathrm{T}}$ are the generalized coordinates of the structure, and $\phi(r,\theta) = [\phi_1(r,\theta)\phi_2(r,\theta)\ldots\phi_n(r,\theta)]$ are displacement shape functions. Using the Rayleigh–Ritz method and consideration of damping, the motion equation of the circular plate structure with piezoelectric actuators is obtained as [9]

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{B}_v\mathbf{v}, \tag{4}$$

where $\mathbf{q}$ is the generalized coordinate vector of the structure, $\mathbf{v}$ is the vector of applied voltage on piezoelectric actuators, $\mathbf{M}$ is the structure inertia matrix, $\mathbf{C}$ is the structures damping matrix, $\mathbf{K}$ is the structure stiffness matrix, $\mathbf{B}_v$ is the input matrix which is used to apply forces to the structure by piezoelectric actuators.

3. Controller design

In the previous section, the mathematical model of the circular plate with bonded piezoelectric actuators is established. In this section, we develop an active robust vibration control method for the circular plate structures with model uncertainty. In order to design a controller to suppress structural vibration, the dynamic model of the circular plate is expressed as the state space form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{v}, \tag{5}$$

where

$$\mathbf{x} = [\mathbf{q}^{\mathrm{T}}\dot{\mathbf{q}}^{\mathrm{T}}]^{\mathrm{T}}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_v \end{bmatrix},$$

and where we assume the pair $(\mathbf{A}, \mathbf{B})$ is controllable. The above state-space form of the flexible structure is the model representation of the flexible structure without consideration of parameter perturbation. A number of control methods have been developed to control structural vibration without consideration of uncertainty, for example, linear quadratic Gaussian [8], positive position feedback [6], etc. For the dynamic model with uncertainties, the desired performance of the system cannot be reached using the control methods without consideration of uncertainties. Thus, it is necessary to develop robust control approaches to suppress vibration of flexible structures with model uncertainties.

Model uncertainties of flexible structures include two categories, one is parameter perturbation which is known as structured uncertainty, the other is mode truncation which is known as unstructured uncertainty. In this paper, the parameter perturbation uncertainty is considered, and the model of flexible structures with parameter uncertainty is expressed in the state-space form

$$\dot{\mathbf{x}} = (\hat{\mathbf{A}} + \Delta\mathbf{A}(t))\mathbf{x} + (\hat{\mathbf{B}} + \Delta\mathbf{B}(t))\mathbf{v}. \tag{6}$$

$\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are the estimated or available values of $\mathbf{A}$ and $\mathbf{B}$, and the pair $(\hat{\mathbf{A}}, \hat{\mathbf{B}})$ is controllable, $\Delta\mathbf{A}(t)$ and $\Delta\mathbf{B}(t)$ are the model uncertainties of system and input matrices $\mathbf{A}$ and $\mathbf{B}$. $\Delta\mathbf{A}(t)$ and $\Delta\mathbf{B}(t)$ are assumed to be expressed as

$$\Delta\mathbf{A}(t) = \hat{\mathbf{B}}\Delta\mathbf{H}(t), \tag{7}$$

$$\Delta\mathbf{B}(t) = \hat{\mathbf{B}}\Delta\mathbf{G}(t), \tag{8}$$

where $\Delta\mathbf{H}(t)$ and $\Delta\mathbf{G}(t)$ are two matrices which represent the uncertainties of matrices $\mathbf{A}$ and $\mathbf{B}$.

The state-space form of the flexible structure with parameter perturbations is obtained in Eq. (6). It is noted that the linear quadratic regulator method is successfully used to control the vibration of flexible structures if the model of flexible structures is exactly known. For the system with parameter perturbations, it is difficult to get the desired performance. Thus, a robust vibration control scheme is developed based on the flexible structure model (6). The robust vibration controller for flexible structures is defined as

$$\mathbf{v} = \mathbf{v}_0 + \Delta\mathbf{v}, \tag{9}$$

where $\mathbf{v}_0$ is the controller part for the nominal system, and $\Delta\mathbf{v}$ is the part to overcome the effects of model uncertainty. The controller part for a nominal system without model uncertainty is described as a standard linear quadratic regulator [13],

$$\mathbf{v}_0 = \mathbf{K}\mathbf{x}, \tag{10}$$

where the feedback gain $\mathbf{K}$ is calculated based on the standard algebraic Riccati equation, and $\hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{K}$ is a Hurwitz matrix, i.e. all eigenvalues of $\hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{K}$ are located in the open left half plane. By Lyapunov theory, there exist symmetric positive definite matrices $\mathbf{P}$ and $\mathbf{Q}$ which satisfy the Lyapunov equation

$$(\hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{K})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{K}) = -\mathbf{Q}. \tag{11}$$

To analyze the stability of the closed-loop system, the state-space form (6) can be described as using Eqs. (7)–(10)

$$\dot{\mathbf{x}} = (\hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{K})\mathbf{x} + \hat{\mathbf{B}}\Delta\mathbf{v} + \hat{\mathbf{B}}\Delta\varphi(\mathbf{x}, t), \tag{12}$$

where

$$\Delta\varphi(\mathbf{x}, t) = (\Delta\mathbf{H}(t) + \Delta\mathbf{G}(t)\mathbf{K})\mathbf{x} + \Delta\mathbf{G}(t)\Delta\mathbf{v}. \tag{13}$$

It is noted that the system perturbation is described by $\Delta\varphi(\mathbf{x}, t)$. In order to design the control part $\Delta\mathbf{v}$ to compensate for the effect of the term $\Delta\varphi(\mathbf{x}, t)$ on the system performance, the control signal $\Delta\mathbf{v}$ is designed based on nonlinear robust theory [14]:

$$\Delta\mathbf{v} = -\frac{\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}\rho(\mathbf{x}, t)}{\|\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}\|\rho(\mathbf{x}, t) + \varepsilon}\rho(\mathbf{x}, t), \tag{14}$$

where ε is a positive scalar constant control parameter, $\mathbf{P}$ is defined in Eq. (11), and $\rho(\mathbf{x}, t)$ is a positive scalar function of the system state which is defined in the following. From the expression of $\Delta\varphi(\mathbf{x}, t)$, the norm of $\Delta\varphi(\mathbf{x}, t)$ can be written as

$$\|\Delta\varphi(\mathbf{x}, t)\| \leqslant \|\Delta\mathbf{H}(t)\| \cdot \|\mathbf{x}\| + \|\Delta\mathbf{G}(t)\mathbf{K}\| \cdot \|\mathbf{x}\| + \|\Delta\mathbf{G}(t)\| \cdot \|\Delta\mathbf{v}\|. \tag{15}$$

It is noted that the input matrix $\mathbf{B}$ consists of two parts, the estimated part $\hat{\mathbf{B}}$ and the model error part $\Delta\mathbf{B}(t)$, and $\mathbf{B}$ can be expressed as $\mathbf{B} = \hat{\mathbf{B}}(\mathbf{I} + \Delta\mathbf{G}(t))$ using Eq. (8). It is reasonable to assume that all elements of the matrices $\mathbf{A}$ and $\mathbf{B}$ are bounded and have less than 100% parameter errors. Thus, it is assumed that $\|\Delta\mathbf{G}(t)\| < \|\mathbf{I}\| = 1$. Defining $\|\Delta\mathbf{H}(t)\| = \delta_1(t)$, $\|\Delta\mathbf{G}(t)\mathbf{K}\| = \delta_2(t)$, $\|\Delta\mathbf{G}(t)\| = \delta_3(t)$, noting $1 - \delta_3(t) > 0$, and using Eq. (14), the norm of $\Delta\varphi(\mathbf{x}, t)$ is obtained as

$$\|\Delta\varphi(\mathbf{x}, t)\| \leqslant (\delta_1(t) + \delta_2(t))\|\mathbf{x}\| + \delta_3(t)\rho(\mathbf{x}, t) \triangleq \rho(\mathbf{x}, t). \tag{16}$$

Thus, $\rho(\mathbf{x}, t)$ is well defined as

$$\rho(\mathbf{x}, t) = (1 - \delta_3(t))^{-1}(\delta_1(t) + \delta_2(t))\|\mathbf{x}\|. \tag{17}$$

In the follow section, robustness analysis of the vibration control system is given. The convergence of the system state is analyzed with respect to the uniform ultimately boundedness [14] in the following theorem.

Theorem. *Given the flexible structure system described in Eq.* (12). *For a given* $\varepsilon > 0$ *and* $\rho(\mathbf{x}, t)$ *in Eq.* (17), *if the control signal* $\Delta\mathbf{v}$ *is chosen as Eq.* (14), *then the system state variable* $\mathbf{x}$ *is uniformly ultimately bounded with respect to the set S defined as*

$$S = \{\mathbf{x} \in R^{2n} \mid \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{x} \leqslant \lambda_{\max}(\mathbf{P})\sigma\}, \tag{18}$$

where $\lambda_{\max}(\mathbf{P})$ *is the maximum eigenvalue of* $\mathbf{P}$ *and* σ *is defined as*

$$\sigma = \left(\frac{2\varepsilon}{\lambda_{\min}(\mathbf{Q})}\right)^{1/2}. \tag{19}$$

Proof. The Lyapunov function candidate is chosen as

$$V(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{x}. \tag{20}$$

Differentiating V with respect to time, using Eq. (12), and noting $\mathbf{P}$ defined in Eq. (11) is a constant symmetric positive definite matrix, the derivative of the Lyapunov function candidate is obtained as

$$\dot{V}(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}((\hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{K})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{K}))\mathbf{x} + 2(\Delta\mathbf{v} + \Delta\varphi(\mathbf{x}, t))^{\mathrm{T}}\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}. \tag{21}$$

Using Eqs. (11) and (16), we obtain

$$\begin{aligned}\dot{V}(\mathbf{x}) &= -\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + 2(\Delta\mathbf{v} + \Delta\varphi(\mathbf{x}, t))^{\mathrm{T}}\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x} \\ &\leqslant -\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + 2\Delta\mathbf{v}^{\mathrm{T}}\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x} + 2\|\Delta\varphi(\mathbf{x}, t)\| \cdot \|\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}\| \\ &\leqslant -\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + 2(\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x})^{\mathrm{T}}\left(\Delta\mathbf{v} + \frac{\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}}{\|\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}\|}\rho(\mathbf{x}, t)\right).\end{aligned} \tag{22}$$

Substituting Eq. (14) into Eq. (22), the derivative of $V(\mathbf{x})$ can be expressed as

$$\begin{aligned}\dot{V}(\mathbf{x}) &\leqslant -\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + 2\varepsilon\frac{\|\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}\|\rho(\mathbf{x}, t)}{\|\hat{\mathbf{B}}^{\mathrm{T}}\mathbf{P}\mathbf{x}\|\rho(\mathbf{x}, t) + \varepsilon} \\ &\leqslant -\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + 2\varepsilon.\end{aligned} \tag{23}$$

Note that

$$\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 \leqslant \mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} \leqslant \lambda_{\max}(\mathbf{Q})\|\mathbf{x}\|^2, \tag{24}$$

where $\lambda_{\min}(\mathbf{Q})$ and $\lambda_{\max}(\mathbf{Q})$ are the minimum and maximum eigenvalues of $\mathbf{Q}$, and $\lambda_{\min}(\mathbf{Q}) > 0$ since $\mathbf{Q}$ is a positive definite matrix. Using Eq. (23), we obtain

$$\dot{V}(\mathbf{x}) \leqslant -\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 + 2\varepsilon. \tag{25}$$

Thus, $\dot{V}(\mathbf{x}) < 0$ for all t only if

$$\|\mathbf{x}\| > \left(\frac{2\varepsilon}{\lambda_{\min}(\mathbf{Q})}\right)^{1/2} = \sigma. \tag{26}$$

Define a closed ball, $B(\sigma)$, centered at $\mathbf{x} = 0$ and with radius σ defined in the above equation. Thus, $\dot{V}(\mathbf{x}) < 0$ for all $t \in R^1$ and all $\mathbf{x} \notin B(\sigma)$. This implies that the system state $\mathbf{x}$ is bounded. Furthermore, following Ref. [14], the uniform ultimately boundedness of the system state $\mathbf{x}$ can be proved easily.

In this section, a robust vibration control method for flexible structures has been developed. This control scheme can be used to suppress the vibration of flexible structures with model parameter perturbations. The robustness of the control system is analyzed based on nonlinear robust control theory. The proposed controller is described by Eqs. (9), (10), and (14).

4. Experimentation

Theoretical analysis and computer simulation are important but not sufficient for investigating the usefulness of new results, as practical factors such as measurement noise, unmodeled truncated modes, actuators saturation, etc., are neglected in the theoretical and simulation analysis. The ultimate justification for the value and applicability of the proposed controller lies in the actual hardware implementation. To examine the performance of robust vibration control method for flexible structures, an experimental evaluation of the proposed control scheme was conducted on a setup consisting of a thin circular plate integrated with piezoceramic actuators and piezofilm sensors.

4.1. Description of experimental system

The experimental system consists of a thin circular aluminium plate with bonded eight piezoceramic actuators and two piezo film sensors, five power amplifiers, two charge amplifiers, one non-contact laser displacement sensor, an interface unit, and a control system implemented on a Pentium III PC host computer. The circular aluminium plate with radius 0.17 m and thickness 0.0008 m is clamped at the center area with radius 0.02 m. The eight piezoceramic actuators, PZT BM532 manufactured by the Sensor Technology Limited, and two piezo film sensors, SDT1-028K manufactured by the Measurement Specialties Incorporated, are bonded on both surfaces of the plate symmetrically as shown in Fig. 2. In this experiment, only four piezoceramic patches as shown in Fig. 1 are used as control actuators to suppress the plate vibration. One of the other four piezoceramic patches is used for vibration excitation and three of the other four piezoceramic patches are not used in this experiment. The material properties of the thin circular plate and piezoceramics are presented in Table 1. The first five natural frequencies of the circular plate integrated with actuators and sensors determined by the experiment are 26.4, 27.9, 34.5, 81.6, and 141.5 Hz. The power amplifiers, BOP 500M manufactured by KEPCO Inc., are bipolar amplifiers with ±500 V output voltage used to supply driving signals for the piezoceramic actuators. The charge amplifiers, 5010B manufactured by the Kistler Instrument

Fig. 2. Overview of experimental setup.

Table 1
Material properties of plate and piezoceramics

Properties	Aluminium plate	PZT BM532
Young's modulus, E ($\mathrm{N\,m^{-2}}$)	69×10^9	71.4×10^9
Density, ρ ($\mathrm{kg\,m^{-3}}$)	2730	7350
Poisson ratio, ν	0.33	0.3
Piezoelectric constant, d_{31} ($\mathrm{mV^{-1}}$)		2.00×10^{-10}
Electric permittivity, ε ($\mathrm{Fm^{-1}}$)		1.504×10^{-8}

Corporation, are used to amplify and filter the output signals from the piezo film sensors. The noncontact laser displacement sensors, LB-72 manufactured by Keyence Corporation, are used to measure the displacement of the surface of the thin circular plate. The view of the overall experimental setup for the active robust vibration control of flexible structures at the Canadian Space Agency (CSA) is shown in Fig. 2.

4.2. Experimental results

In this section, experimental evaluation of the performance of the proposed control scheme is presented. To show the performance of the active vibration control method, two experiments were conducted in the following two cases: (1) the circular plate structures is excited by an impulse signal, (2) the circular plate structure is excited by a continuous sinusoidal signal with a constant amplitude. In the first case, the impulse signal load is applied to the plate by a hammer. In the second case, the plate vibration is excited by applying a constant amplitude, sinusoidal voltage signal to a piezoceramic patch bonded on the surface of the plate. To control the plate vibration in the above two cases, four piezoceramic actuators as shown in Fig. 1 are used to apply control signals to suppress the plate vibration. To compare the performance of the vibration control in

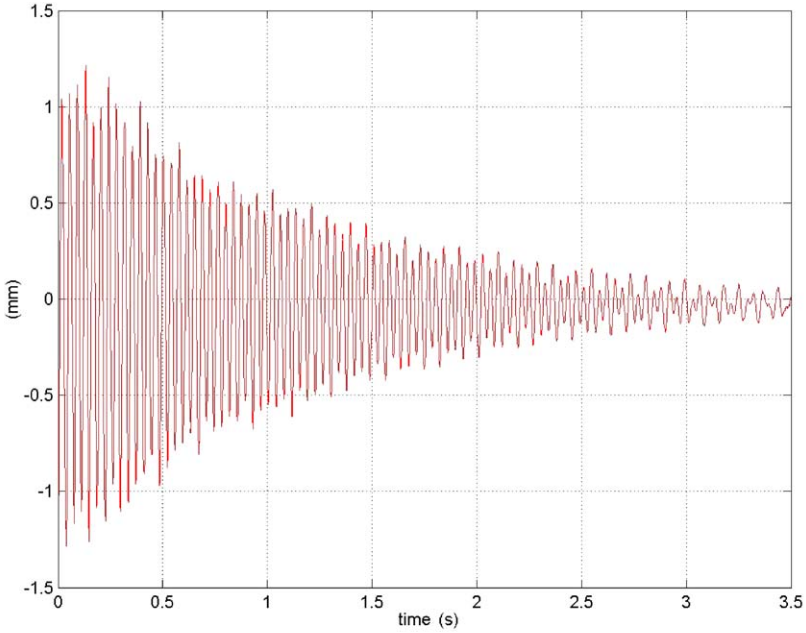

Fig. 3. Displacement response of impulse signal without control.

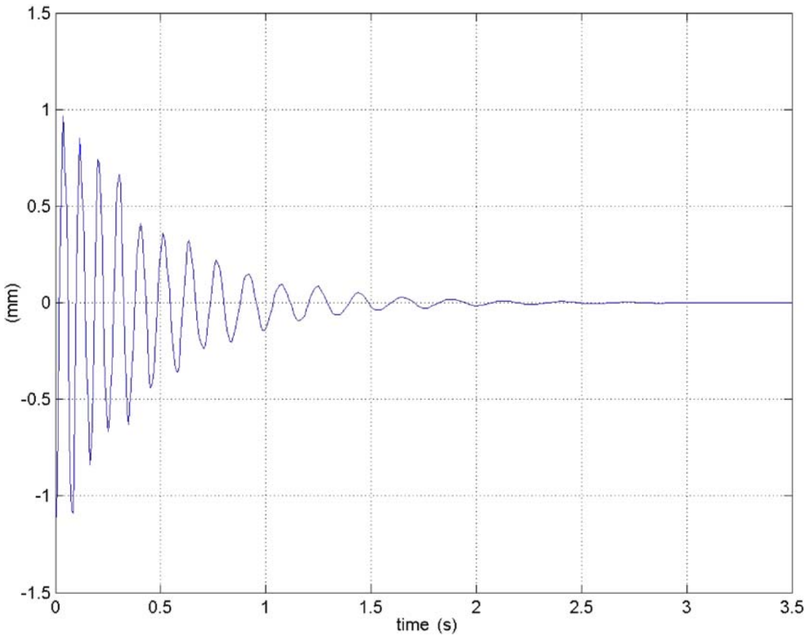

Fig. 4. Displacement response of impulse signal with control.

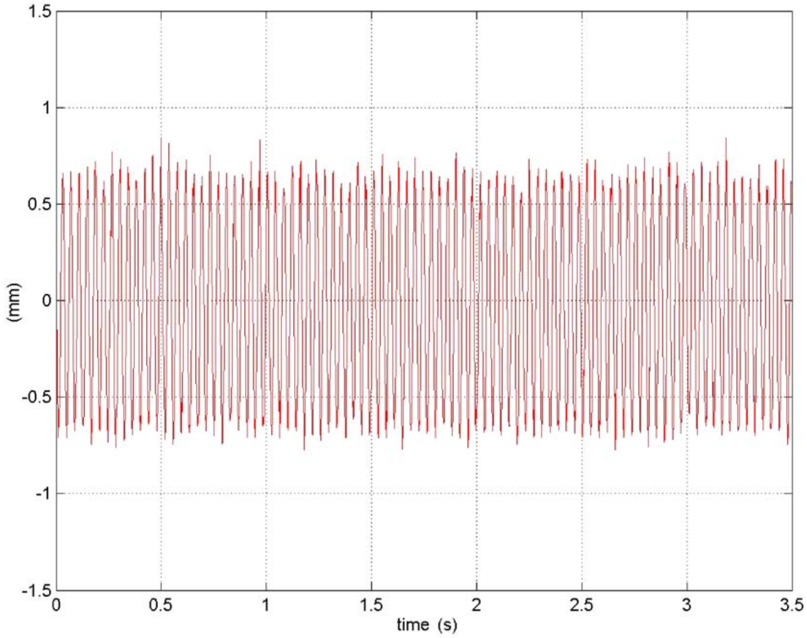

Fig. 5. Displacement response of sinusoidal signal without control.

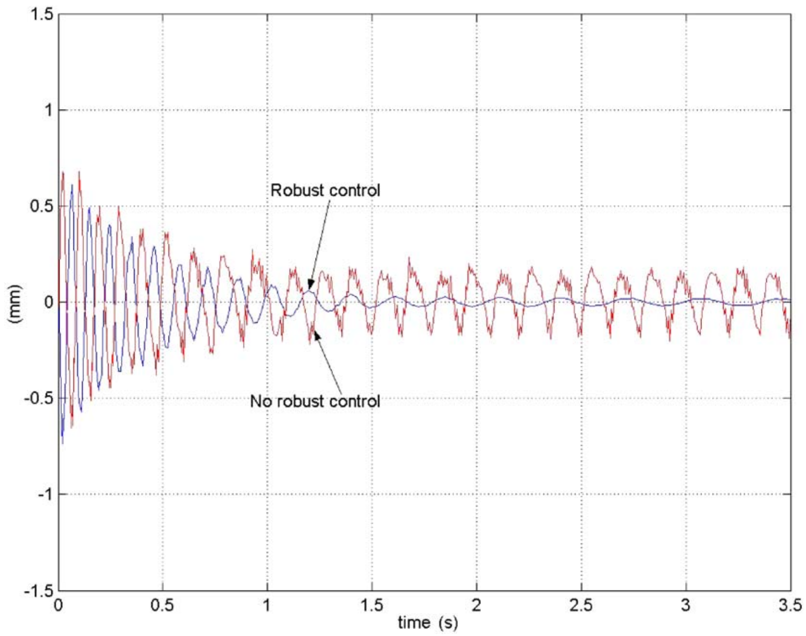

Fig. 6. Displacement response of sinusoidal signal with control.

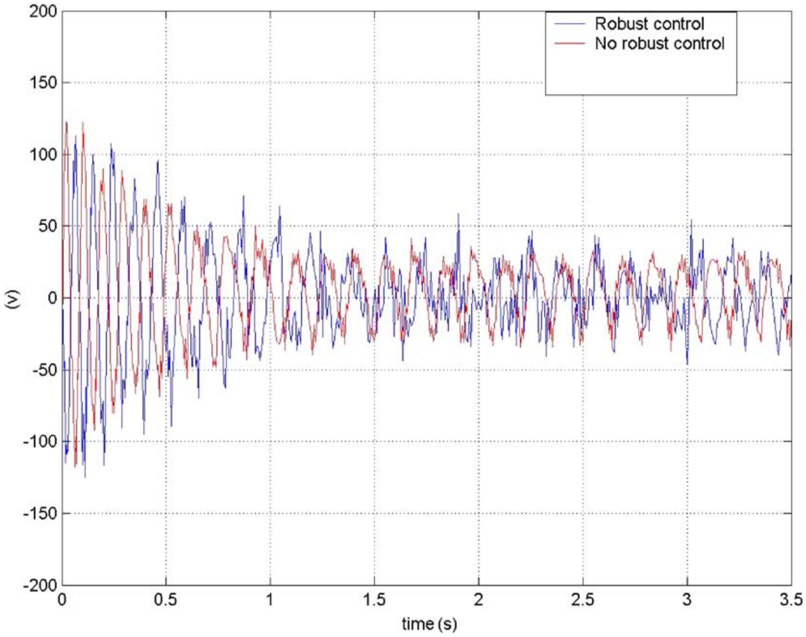

Fig. 7. Input voltage of actuator 1.

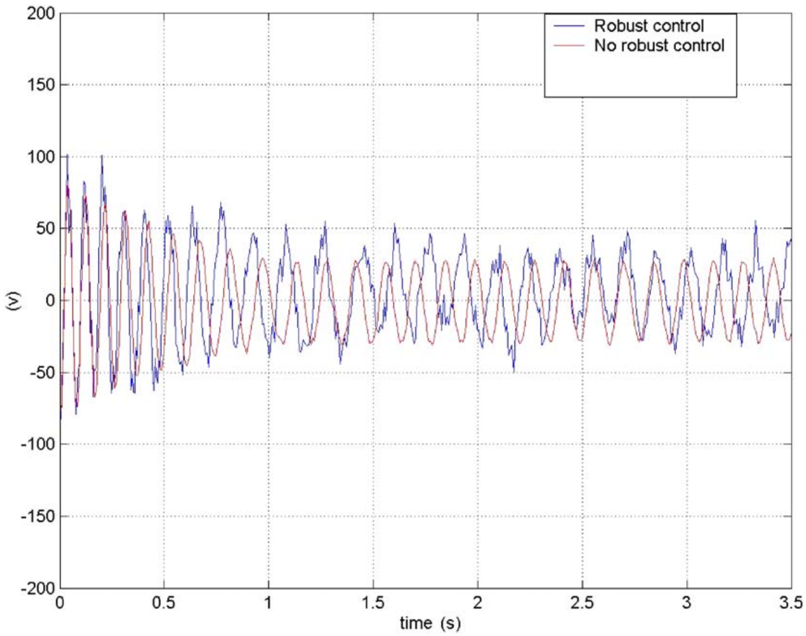

Fig. 8. Input voltage of actuator 2.

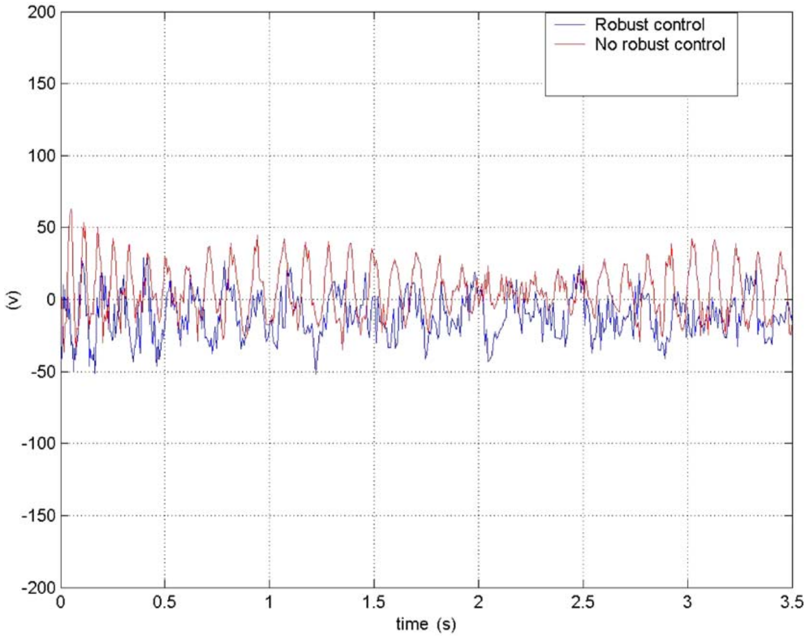

Fig. 9. Input voltage of actuator 3.

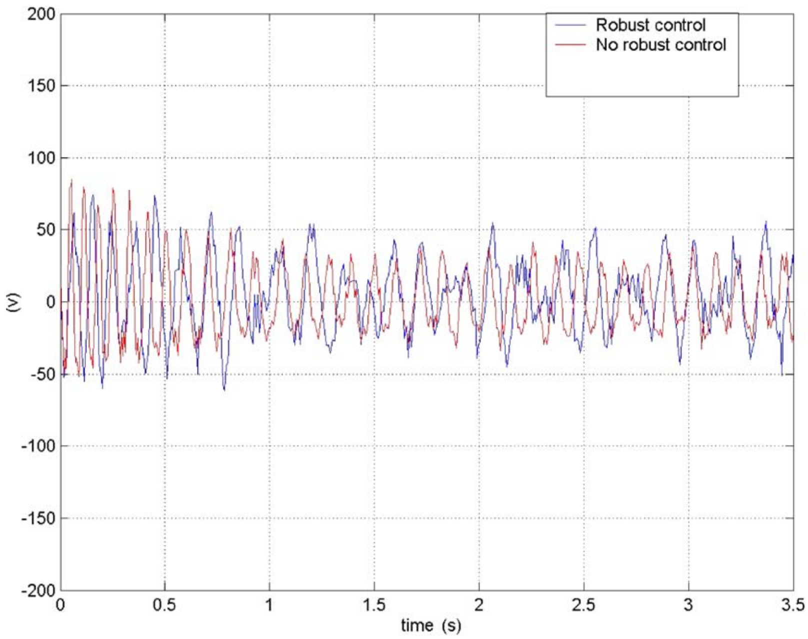

Fig. 10. Input voltage of actuator 4.

two cases, the reference point on the plate is chosen as point A (see Fig. 1). For the impulse excitation, the displacement responses of point A with and without the proposed active vibration controller are shown in Figs. 3 and 4. It is shown that the vibration caused by the impulse load is damped quickly with the active controller, and without control it takes much longer to damp the vibration of the circular plate structure. For the second case, to verify the robustness of the control algorithm against parameter uncertainty, an additional mass (20% of the plate mass) is attached to the circular plate. The displacement response of the point A without control is shown in Fig. 5 and the displacement responses of the point A under controllers with and without robustness are shown in Fig. 6. The control inputs of actuators 1–4 with and without robustness are shown in Figs. 7–10. It is clear that the vibration caused by the continuous sinusoidal excitation is effectively suppressed by applying the proposed active robust vibration control scheme. Comparing the control inputs of actuators 1–4 with and without robust control, the energy requirement of actuators 1–4 for robust and non-robust control is almost the same.

5. Conclusions

In this paper, a robust active vibration control scheme has been developed to suppress the vibration of the circular plate structure with integrated piezoelectric actuators. The controller consists of two parts, the standard linear quadratic regulator and nonlinear control signal to overcome uncertainty of the system model. Robustness of the control system has been analyzed for the model parameter uncertainty based on Lyapunov stability theory. It is proved that the system state is uniformly ultimately bounded. Although the thin circular plate is used as an example for design of the controller, the proposed control scheme is applicable for all structures.

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