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Journal of Sound and Vibration 288 (2005) 653–667
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

A consistent concept for high- and low-frequency dynamics based on stochastic modal analysis

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Accepted 5 July 2005
Available online 1 September 2005

Abstract

Accurate expressions for the kinetic energy in substructure excited by white noise and broad-band spectra, based on classical random vibration theory and modal analysis, are presented. The approach is accurate, general and valid for all frequency ranges, since no simplifying are needed to arrive at the presented power flow relations. Strong coupling, local energies and energies in substructures can be analyzed for uncorrelated as well as for correlated excitation. The results are compared with statistical energy analysis (SEA) which is applicable for the high-frequency range. It is shown, that the SEA representations is only suitable for very weak coupling between substructures, while an inverse representation does not show the observed limitations. Energies in substructures are not sensitive to variations of the eigenfrequencies or mode shapes due to the summation over frequency ranges and over the domain of the substructure. Hence, modal analysis will lead to accurate estimates even in case FE analysis fails to provide accurate eigenfrequencies and mode shapes, since the coupling between substructures is still represented with acceptable accuracy. Uncertain structural properties will affect the coupling between substructures and therefore the power flow. It is suggested to assess this influence by using Monte Carlo simulation.

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doi:10.1016/j.jsv.2005.07.011

1. Introduction

Unavoidable uncertainties of structural properties and the finite resolution of mathematical (FE) models do not allow to predict with confidence the modal properties in the higher-frequency range. To estimate the vibrations in structures due to random noise inputs, procedures have been developed which do not require mode shapes and eigenfrequencies, but only estimates for the number of modes within frequency bands (modal densities) and averaged modal damping factors (dissipation factors). Using simplifying assumptions like very high-frequency ranges and white-noise excitation, statistical energy analysis (SEA) has the ability to focus on essentials like vibrational energy, dissipation and power input. These relations between input power and kinetic energies in substructures are well documented, and Refs. [1,2] summarize essential developments in this field. Applying standard SEA procedures, the a priori introduced simplifying assumptions as weak conservative coupling between substructures or ''rain of the roof'' white-noise excitation, do not allow to study cases where the underlying assumptions are not valid, or to assess the quality of SEA predictions where the prepositions are only met approximately.

The question arises as to whether a similar power flow relation can be obtained for all frequencies ranges, a formulation which is not based on the standard simplifying assumptions which are justifiable only in the high-frequency domain and do not apply in the low and intermediate range. This paper shows that such a simple representation is indeed possible; the approach is based on a global mode representation employing closed form expressions for white-noise excitation. The expressions are exact only for white-noise conditions. They are then generalized for band-limited noise excitation. Apart from this generalization, no further restrictions need to be introduced. Instead of a delta correlated 'rain-on-the-roof' force excitation, correlated excitation is treated. The suggested approach uses a basis similar to that described in Ref. [3], who introduced the so-called energy influence coefficient (EIC) method. The authors of Ref. [3] use global modes and arrives at linear matrix relations between spectral densities of the excitation and the kinetic energies in subsystems. More recently, the so-called energy finite element method (EFEM) has been developed [4], which uses also a modal approach in context with component mode synthesis.

We establish linear relations between the energies in subsystems and the powers introduced by the external excitation, and give relations between the spectral densities and the power input. The derived expressions provide a rigorous basis for treating such uncertainties by analytical or numerical means, e.g. Monte Carlo simulation. The derived relations show that the power flow relations are not sensitive to accurate mode shapes and eigenfrequencies. Hence, estimates for the modal properties obtained by standard FE analysis suffice in the low and intermediate frequency range to obtain robust results. Uncertainties regarding the energy dissipation and coupling of substructures, however, will introduce a considerable uncertainty in the energy distributions of the substructures. We believe that in a first step the physics of a phenomenon should be captured as accurately as possible. Only then, should the uncertainties of all parameters involved be considered in a next step. To make this two-step procedure feasible, the deterministic relations should be simple and straightforward and operate with the required independent variables, i.e. the energies in subsystems, the damping characteristics and the input powers, respectively.

The use of global modes provides the means to overcome some of the limitations of the standard SEA approach, particularly when we deal with strong and non-conservative coupling,

coupling by concentrated elements and indirect coupling. However, standard SEA procedures are not based on global modes, but on modal properties of the substructures. If we use the well developed component mode synthesis (e.g. Ref. [5]), we could in fact obtain the global modes on basis of such modal properties. This topic, however, is beyond the scope of the present paper.

2. Method of analysis

2.1. Stationary stochastic modal response

For structures excited by random noise, the response can usually be approximated as linear, ignoring small nonlinearities as they might be introduced by material and geometrical nonlinearities, contacts, etc. The following considerations are confined to a linear structural system discretized by n degrees of freedom (dof), of which the equation of motion reads,

$$\mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{C} \cdot \dot{\mathbf{x}} + \mathbf{K} \cdot \mathbf{x} = \mathbf{f}(t), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ denotes the mass, viscous damping and stiffness matrices, and $\mathbf{x}$ and $\mathbf{f}(t)$ represent the generalized displacement and force vector, respectively.

For the derivation of the stochastic response, it is advantageous to rewrite the equation of motion (1) in a first-order form,

$$\dot{\mathbf{y}} = \mathbf{A} \cdot \mathbf{y} + \mathbf{g} \tag{2}$$

introducing the state vector $\mathbf{y}$, the vector $\mathbf{g}$ and the matrix $\mathbf{A}$ as specified in the following where $\mathbf{U}$ denotes the identity matrix

$$\mathbf{y} = \begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix}; \quad \mathbf{g} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{f} \end{Bmatrix}; \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{U} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}. \tag{3}$$

There are well-established procedures for determining the stochastic response of linear systems (see e.g. Ref. [6]). The main features of the stochastic response can be characterized by its first two moments, i.e. the mean vector $E\{\mathbf{y}\}$ and the covariance matrix $\mathbf{D} = E\{\mathbf{y}\mathbf{y}^{\mathrm{T}}\} - E\{\mathbf{y}\} \cdot E\{\mathbf{y}\}^{\mathrm{T}}$, where $E\{\cdot\}$ denotes the mathematical expectation or average.

The mean vector $E\{\mathbf{y}\}$ is obtained by taking the expectation of Eq. (2). For a stationary response the mean $E\{\mathbf{y}\}$ must not change w.r.t. time; it follows that the mean vector is determined as in the static case, with the loading replaced by its time-invariant expectation. As is common practice in random vibration analysis, it will be assumed, that the response $\mathbf{y}$ and the excitation $\mathbf{f}$ or $\mathbf{g}$ have zero mean, i.e. $E\{\mathbf{y}\} = E\{\mathbf{g}\} = \mathbf{0}$; the static part must be added if the total response is required.

The covariance matrix $\mathbf{D}$ can be determined in a straightforward manner. The differentiation of the symmetric matrix $\mathbf{D}$ w.r.t. time, and the introduction of Eq. (2),

$$\dot{\mathbf{D}} = \frac{\mathrm{d}}{\mathrm{d}t} E\{\mathbf{y}\mathbf{y}^{\mathrm{T}}\} = E\{\dot{\mathbf{y}}\mathbf{y}^{\mathrm{T}} + \mathbf{y}\dot{\mathbf{y}}^{\mathrm{T}}\} = \mathbf{A} \cdot \mathbf{D} + \mathbf{D} \cdot \mathbf{A}^{\mathrm{T}} + \mathbf{B} = \mathbf{0} \tag{4}$$

lead to the so-called Lyapunov equation for the stationary case ($\dot{\mathbf{D}} = \mathbf{0}$), where the matrix $\mathbf{B}$ denotes the symmetric matrix

$$\mathbf{B} = E\{\mathbf{g}\mathbf{y}^{\mathrm{T}}\} + E\{\mathbf{y}\mathbf{g}^{\mathrm{T}}\} = \begin{bmatrix} \mathbf{0} & E\{\mathbf{x}\mathbf{f}^{\mathrm{T}}\} \cdot \mathbf{M}^{-1} \\ \mathbf{M}^{-1}E\{\mathbf{f}\mathbf{x}^{\mathrm{T}}\} & \mathbf{M}^{-1}E\{\mathbf{f}\dot{\mathbf{x}}^{\mathrm{T}}\} + E\{\dot{\mathbf{x}}\mathbf{f}^{\mathrm{T}}\}\mathbf{M}^{-1} \end{bmatrix}. \tag{5}$$

Eq. (4) is a linear equation system which uniquely determines all $2n^2 + n$ components of the symmetric covariance matrix $\mathbf{D}$, provided the matrix $\mathbf{B}$ and the matrices defining the structure are known.

Note that no restricting assumptions have been made in Eq. (4) with Eq. (5) regarding the stochastic characteristics of the excitation. Considerable effort is needed to determine the submatrices $E\{\mathbf{f}\mathbf{x}^{\mathrm{T}}\}$ and $E\{\mathbf{f}\dot{\mathbf{x}}^{\mathrm{T}}\}$ for the general case, in which the random loading $\mathbf{f}(t)$ and the stochastic response $\mathbf{x}(t)$ are correlated. Only for the special case of a purely random process, i.e. so-called white noise, can these two matrices be formulated independently from the specific response $\mathbf{x}(t)$ and $\dot{\mathbf{x}}(t)$. Let the random excitation be represented as

$$\mathbf{f}(t) = \mathbf{G} \cdot \boldsymbol{\xi}(t), \tag{6}$$

where $\boldsymbol{\xi}(t)$ represents a vector of M uncorrelated white-noise components $\xi_k(t)$ and $\mathbf{G}$ a matrix of dimension $n \times M$ with constant coefficients. With such a representation, spatially correlated white-noise excitation can be described. The components $\xi_k(t)$ are assumed to be uncorrelated since it is always possible to introduce a linear transformation leading to the above representation. In most cases, the distributions of the excitation are close to a normal one for which the components $\xi_k(t)$ can be regarded as independent. Each white-noise component $\xi_k(t)$ is characterized by the Dirac delta correlated auto-covariance function $R_{kk}(\tau)$

$$R_{kk}(\tau) = E\{\xi_k(t)\xi_k(t+\tau)\} = I_{kk} \cdot \delta(\tau) \tag{7}$$

or its Fourier transform with constant spectral density $S_{kk} = I_{kk}/(2\pi)$.

$$S_{kk}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{kk}(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau = \text{const.} = \frac{1}{2\pi} I_{kk}. \tag{8}$$

White noise is a fairly accurate substitute for the wide band excitation. However, colored noise can also be treated by using filtered white-noise. Since a generalization to colored noise is possible, the excitation will be regarded in the following as white noise only. For white-noise excitation, the force vector $\mathbf{f}(t)$ is statistically independent of the displacement vector $\mathbf{x}(t)$, and also independent of the velocity $\dot{\mathbf{x}}(\tau < t)$, and $E\{\mathbf{f}\dot{\mathbf{x}}^{\mathrm{T}}\}$ is a function only of the white-noise intensity matrix $\mathbf{I}$ and the mass matrix $\mathbf{M}$,

$$E\{\mathbf{f}\mathbf{x}^{\mathrm{T}}\} = E\{\mathbf{f}\} \cdot E\{\mathbf{x}\}^{\mathrm{T}} = \mathbf{0}; \quad E\{\mathbf{f}\dot{\mathbf{x}}^{\mathrm{T}}\} = \tfrac{1}{2}\mathbf{G} \cdot \mathbf{I} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{M}^{-1}, \tag{9}$$

where the white-noise intensity matrix $\mathbf{I}$ is a diagonal matrix and the matrix $\mathbf{B}$ defined in Eq. (5) simplifies to

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{-1} \cdot \mathbf{G} \cdot \mathbf{I} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{M}^{-1} \end{bmatrix}. \tag{10}$$

The second part of Eq. (9) can be derived by representing white noise as a sequence of independent random impulses acting at instances $t_i = i\Delta t$. The magnitudes J_i of these impulses are normal distributed, have zero mean and the variance $I \cdot \Delta t$. Consider for simplicity a single mass m and $G = 1$. If an impulse arrives at time t_i, this impulse is solely responsible for a jump $\dot{x}(t_i + \varepsilon) - \dot{x}(t_i - \varepsilon)$ equal to the strength of the impulse J_i divided by the mass m, where ε is an arbitrary small positive number. Independent from the shape of the force, the average power transfered to the mass within the duration Δt is $E\{\xi \dot{x}\} = E\{J_i^2\}/(2m\Delta t) = I/(2m)$. This basic result can be generalized for white noise of the form in Eq. (6) and general mass matrix $\mathbf{M}$ as shown in Eqs. (9) and (10).

The Lyapunov equation (4) can be solved in closed form for the white-noise case as shown in the following. Assuming modal damping, we may decouple the equation of motion (1) by using n modal coordinates $\mathbf{z}(t)$,

$$\mathbf{x}(t) = \mathbf{\Phi} \cdot \mathbf{z}(t), \tag{11}$$

$$\ddot{z}_j(t) + 2\zeta_j\omega_j\dot{z}_j(t) + \omega_j^2 z_j(t) = q_j(t) = \boldsymbol{\phi}_j^{\mathrm{T}} \cdot \mathbf{f}, \quad 1 \leqslant j \leqslant n, \tag{12}$$

where the matrix $\mathbf{\Phi}$ contains all n eigenvectors of the characteristic equation

$$\mathbf{K} \cdot \mathbf{\Phi} = \mathbf{M} \cdot \mathbf{\Phi} \cdot \mathbf{\Lambda} \tag{13}$$

and $\mathbf{\Lambda} = [\mathrm{diag}(\omega_j^2)]$ is a diagonal matrix comprising all eigenvalues. The eigenvectors are orthogonal with respect to the stiffness matrix $\mathbf{K}$ and mass matrix $\mathbf{M}$ and are normalized to satisfy the relations

$$\mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{K} \cdot \mathbf{\Phi} = \mathbf{\Lambda}, \quad \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{\Phi} = \mathbf{U}. \tag{14}$$

The damping matrix $\mathbf{C}$ is assumed to decouple by using the transformation

$$\mathbf{\Gamma} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{C} \cdot \mathbf{\Phi} = [\mathrm{diag}(\gamma_j)]; \quad \gamma_j = 2\zeta_j\omega_j. \tag{15}$$

To derive the Lyapunov equation (4) in modal coordinates, Eqs. (2) and (3) are now written in a slightly different form

$$\mathbf{M\Phi}\begin{Bmatrix} \dot{\mathbf{z}} \\ \ddot{\mathbf{z}} \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ -\mathbf{K} & -\mathbf{C} \end{bmatrix} \mathbf{\Phi} \begin{Bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{f} \end{Bmatrix}. \tag{16}$$

Pre-multiplying the above equation by $\mathbf{\Phi}^{\mathrm{T}}$ and introducing the modal properties (14) and (15), leads to the following equation of motion in modal coordinates:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{Bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{U} \\ -\mathbf{\Lambda} & -\mathbf{\Gamma} \end{bmatrix} \begin{Bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}} f \end{Bmatrix}, \tag{17}$$

where all submatrices are diagonal matrices. Applying further the approach as shown for deriving (4), the Lyapunov equation must satisfy the following three matrix equations:

$$E\{\mathbf{z}\dot{\mathbf{z}}^{\mathrm{T}}\} + E\{\dot{\mathbf{z}}\mathbf{z}^{\mathrm{T}}\} = \mathbf{0}, \tag{18}$$

$$\mathbf{\Lambda} \cdot E\{\mathbf{z}\mathbf{z}^{\mathrm{T}}\} - \mathbf{\Gamma} \cdot E\{\mathbf{z}\dot{\mathbf{z}}^{\mathrm{T}}\} - E\{\dot{\mathbf{z}}\dot{\mathbf{z}}^{\mathrm{T}}\} = \mathbf{0}, \tag{19}$$

$$\mathbf{\Lambda} \cdot E\{\mathbf{z}\dot{\mathbf{z}}^{\mathrm{T}}\} - E\{\mathbf{z}\dot{\mathbf{z}}^{\mathrm{T}}\} \cdot \mathbf{\Lambda} + \mathbf{\Gamma} \cdot E\{\dot{\mathbf{z}}\dot{\mathbf{z}}^{\mathrm{T}}\} + E\{\dot{\mathbf{z}}\dot{\mathbf{z}}^{\mathrm{T}}\} \cdot \mathbf{\Gamma} = \hat{\mathbf{I}} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{G} \cdot \mathbf{I} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{\Phi}. \quad (20)$$

From the above relations, we can obtain an explicit solution in closed form for all the components of the covariance matrix **D** as shown in the following. Considering the jth row and kth column of Eq. (19) and then the kth row and jth column we obtain the following two equations:

$$\omega_j^2 E\{z_j z_k\} - \gamma_j E\{z_j \dot{z}_k\} - E\{\dot{z}_j \dot{z}_k\} = 0, \quad (21)$$

$$\omega_k^2 E\{z_j z_k\} + \gamma_k E\{z_j \dot{z}_k\} - E\{\dot{z}_j \dot{z}_k\} = 0, \quad (22)$$

where $E\{z_j \dot{z}_k\} = -E\{z_k \dot{z}_j\}$ (see Eq. (18)) has been used in the last Eq. (22). The terms $E\{z_j z_k\}$ and $E\{z_j \dot{z}_k\}$ can be expressed as function of $E\{\dot{z}_j \dot{z}_k\}$:

$$\left\{ \begin{array}{c} E\{z_j z_k\} \\ E\{z_j \dot{z}_k\} \end{array} \right\} = \frac{E\{\dot{z}_j \dot{z}_k\}}{\gamma_j \omega_k^2 + \gamma_k \omega_j^2} \left\{ \begin{array}{c} \gamma_j + \gamma_k \\ \omega_j^2 - \omega_k^2 \end{array} \right\}. \quad (23)$$

Eq. (20) can also be represented by its components which read

$$(\omega_j^2 - \omega_k^2) E\{z_j \dot{z}_k\} + (\gamma_j + \gamma_k) E\{\dot{z}_j \dot{z}_k\} = \hat{I}_{jk}. \quad (24)$$

Eqs. (23) and (24) yield

$$E\{\dot{z}_j \dot{z}_k\} = \frac{\hat{I}_{jk}}{\gamma_j + \gamma_k + ((\omega_j^2 - \omega_k^2)^2)/(\gamma_j \omega_k^2 + \gamma_k \omega_j^2)}. \quad (25)$$

This relation completes the solution for the second moments in modal coordinates and will be the basis for deriving the following power flow relations.

2.2. Spatial distribution of kinetic energy for white noise

An interesting approach relates the power **p** introduced into the system by the external excitation and dissipated by damping, to the vibrational energy **E** of the structural components. To simplify the derivation, we will assume that the mass matrix is diagonal; this corresponds to lumped mass instead of a continuous mass distribution. This simplification has a negligible effect on the deterministic and stochastic responses of a structure. To derive such a relation, the white-noise intensity matrix **I** in physical as well as in model coordinates must be related to the power of the excitation introduced into the system. If the excitation $\xi_k(t)$ acts only on the kth (physical) dof, the following relation can be derived by modal analysis:

$$p_k = E\{f_k \dot{x}_k\} = \frac{1}{2} I_{kk} \sum_{s=1}^{n} \phi_{ks}^2 = \pi S_{kk} \sum_{s=1}^{n} \phi_{ks}^2, \quad (26)$$

where the first index k in ϕ_{ks} refers to the dof and the second index s to the mode. For the general case where the possible correlated excitation is defined by Eq. (6), the input power reads

$$p_k = \sum_{i=1}^{n} E\{f_i \dot{x}_i[\xi_k]\} = \frac{1}{2} I_{kk} \sum_{i=1}^{n} \left(\sum_{s=1}^{n} g_{ik} \phi_{is} \right)^2. \quad (27)$$

Next, the modal density $\hat{I}_{jk}$ is related to the power p_k of the external white-noise excitation.

$$\hat{I}_{jk} = \boldsymbol{\phi}_j^{\mathrm{T}} \cdot \mathbf{G} \cdot \mathbf{I} \cdot \mathbf{G}^{\mathrm{T}} \cdot \boldsymbol{\phi}_k = \sum_{i=1}^{M} I_{ii} \left(\sum_{l=1}^{n} g_{li}\phi_{lj} \right) \left(\sum_{l=1}^{n} g_{li}\phi_{lk} \right). \tag{28}$$

The white-noise intensity I_{ii} is related to the power p_i by Eq. (27). Therefore, the above relation can be expressed alternatively by

$$\hat{I}_{jk} = 2 \sum_{i=1}^{M} c_{i(j,k)} p_i, \tag{29}$$

where the dimensionless coefficients $c_{i(j,k)}$ are determined from Eq. (27).

$$c_{i(j,k)} := \frac{\left(\sum_{l=1}^{n} g_{li}\phi_{lj} \right) \left(\sum_{l=1}^{n} g_{li}\phi_{lk} \right)}{\sum_{l=1}^{n} \left(\sum_{s=1}^{n} g_{li}\phi_{ls} \right)^2}. \tag{30}$$

By introducing Eq. (29) into Eq. (25), the relation between the input power p_i and the covariance of the modal velocities is established

$$E\{\dot{z}_j \dot{z}_k\} = \frac{2}{\gamma_{(j,k)}} \sum_{i=1}^{M} c_{i(j,k)} p_i, \tag{31}$$

where the abbreviation

$$\gamma_{(j,k)} = \gamma_j + \gamma_k + \frac{(\omega_j^2 - \omega_k^2)^2}{\gamma_j \omega_k^2 + \gamma_j \omega_k^2} \tag{32}$$

is introduced.

Denoting the averaged kinetic energy of each dof l as e_l and recalling the modal representation in Eq. (11) we find

$$e_l = \frac{m_l}{2} E\{\dot{x}_l^2\} = \frac{m_l}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \phi_{lj}\phi_{lk} E\{\dot{z}_j \dot{z}_k\}. \tag{33}$$

For small damping ratios ξ_j, a few percent and less, the main contribution to the kinetic energy of the structure stems from the diagonal terms $E\{\dot{z}_j^2\}$ and closely spaced modes with $\omega_j^2 \approx \omega_k^2$ as it can be recognized by Eqs. (31) and (32). We order the frequencies so that

$$\omega_1^2 \leqslant \omega_2^2 \leqslant \cdots \leqslant \omega_n^2. \tag{34}$$

Then Eq. (33) can be replaced by the following expression:

$$e_l = \sum_{j=1}^{n} \frac{m_l}{2} \phi_{lj}^2 E\{\dot{z}_j^2\} + 2 \sum_{l=1}^{n-1} \frac{m_l}{2} \phi_{lj}\phi_{l,j+1} E\{\dot{z}_j \dot{z}_{j+1}\} + \cdots . \tag{35}$$

Introducing the notation,

$$h_{l(j,k)} = m_l \phi_{lj} \phi_{lk} \tag{36}$$

and making use of Eq. (31), we find

$$e_l = \sum_{j=1}^{n} \frac{h_{l(j,j)}}{\gamma_{(j,j)}} \sum_{i=1}^{M} c_{i(j,j)} p_i + 2 \sum_{j=1}^{n-1} \frac{h_{l(j,j+1)}}{\gamma_{(j,j+1)}} \sum_{i=1}^{M} c_{i(j,j+1)} p_i + \cdots \ . \tag{37}$$

Introducing further the vectors,

$$\mathbf{e}^{\mathrm{T}} = \{e_1, e_2, \ldots, e_n\}; \quad \mathbf{h}^{\mathrm{T}}_{(j,k)} = \{h_{1(j,k)}, h_{2(j,k)}, \ldots, h_{n(j,k)}\},$$
$$\mathbf{p}^{\mathrm{T}} = \{p_1, p_2, \ldots, p_M\}; \quad \mathbf{c}^{\mathrm{T}}_{(j,k)} = \{c_{1(j,k)}, c_{2(j,k)}, \ldots, c_{M(j,k)}\}, \tag{38}$$

we can generalize the above equation to establish a relation for the spatial distribution of the energy $\mathbf{e}$ for all dof of the structure subjected to white-noise excitation $\mathbf{p}$

$$\mathbf{e} = \left[\sum_{j=1}^{n} \frac{\mathbf{h}_{(j,j)} \cdot \mathbf{c}^{\mathrm{T}}_{(j,j)}}{\gamma_{(j,j)}} + 2 \sum_{j=1}^{n-1} \frac{\mathbf{h}_{(j,j+1)} \cdot \mathbf{c}^{\mathrm{T}}_{(j,j+1)}}{\gamma_{(j,j+1)}} + \cdots \right] \cdot \mathbf{p}. \tag{39}$$

This relation specifies in a unique manner the kinetic energy in the system as a linear function of the input power.

First consider the vector $\mathbf{h}_{(j,k)}$ defined in Eq. (36). Since the eigenvectors $\boldsymbol{\phi}_j$ or modes are normalized w.r.t. the mass matrix, the vector $\mathbf{h}_{(j,k)}$ satisfies the property:

$$\sum_{l=1}^{n} h_{l(j,j)} = 1; \quad 0 \leqslant h_{l(j,j)} \leqslant 1, \tag{40}$$

$$\sum_{l=1}^{n} h_{l(j,k)} = 0; \quad -1 \leqslant h_{l(j,k)} \leqslant 1 \quad \text{for } j \neq k. \tag{41}$$

Looking at the total kinetic energy e_{tot} of the system,

$$e_{\mathrm{tot}} = \sum_{l=1}^{n} e_l = \left[\sum_{j=1}^{n} \frac{\mathbf{c}^{\mathrm{T}}_{(j,j)}}{\gamma_{(j,j)}} \right] \cdot \mathbf{p}, \tag{42}$$

we see that only the first term in Eq. (39) contributes to the total energy of the system, while the second and higher terms vanish due to Eq. (41). Thus, the second term in Eq. (39) accounts for a redistribution of the kinetic energy within the system, keeping the total energy constant. The effect of the redistribution is in most cases negligible if the eigenfrequencies are well separated, and becomes important only for pairs of closely spaced eigenfrequencies.

For the special case in which independent excitations act on single masses m_i and there is external power $E\{f_i \dot{x}_i\}$ coming into the structural system, we have $I_{ii} = 2m_i p_i$ and $\hat{I}_{j,k} = \phi_{ij} \phi_{ik} I_{ii}$, and

$$c_{i(j,k)} = m_i \phi_{ij} \phi_{ik} = h_{i(j,k)} \tag{43}$$

so that Eq. (39) yields

$$\mathbf{e} = \left[\sum_{j=1}^{n} \frac{\mathbf{h}_{(j,j)} \cdot \mathbf{h}^{\mathrm{T}}_{(j,j)}}{\gamma_{(j,j)}} + 2 \sum_{n=1}^{n-1} \frac{\mathbf{h}_{(j,j+1)} \cdot \mathbf{h}^{\mathrm{T}}_{(j,j+1)}}{\gamma_{(j,j+1)}} + \cdots \right] \cdot \hat{\mathbf{p}}, \tag{44}$$

where the kth component of the vector $\hat{\mathbf{p}}$ denotes the input power $\hat{p}_k = E\{f_k \dot{x}_k\}$.

2.3. Spatial distribution of kinetic energy for band-limited noise excitation

White-noise excitation is a mathematical abstraction which assumes constant spectral density S_{ii} in an infinite frequency range. White noise is physically not realizable, but a suitable approximation for wide band spectra $S_{ii}(\omega)$, where the frequency content varies slowly over the frequency range. By using the assumption that each mode ϕ_j is excited with constant spectral density $S_{ii}(\omega_j)$, Eq. (26) can be generalized for wide band excitation:

$$p_i = \pi \sum_{j=1}^{m} S_{ii}(\omega_j)\phi_{ij}^2. \tag{45}$$

For the general case where the possible correlated excitation is defined by Eq. (6), the input power reads

$$p_i = \pi \sum_{l=1}^{n} E\{f_l \dot{x}_l[\xi_i]\} = \pi \sum_{j=1}^{m} S_{ii}(\omega_j)\left(\sum_{l=1}^{n} g_{li}\phi_{lj}\right)^2. \tag{46}$$

Next, expanding Eq. (28), the white-noise intensity I_{ii} at frequencies ω_j and ω_k is replaced by $2\pi\sqrt{S_{ii}(\omega_j)S_{ii}(\omega_k)}$, resulting in

$$\hat{I}_{jk} = \sum_{i=1}^{M} \sqrt{S_{ii}(\omega_j)S_{ii}(\omega_k)}\left(\sum_{l=1}^{n} g_{li}\phi_{lj}\right)\left(\sum_{l=1}^{n} g_{li}\phi_{lk}\right), \tag{47}$$

where the dimensionless coefficients $c_{i(j,k)}$ in Eq. (29) are determined from Eqs. (46) and 47.

$$c_{i(j,k)} = \frac{\sqrt{S_{ii}(\omega_j)S_{ii}(\omega_k)}\left(\sum_{l=1}^{n} g_{li}\phi_{lj}\right)\left(\sum_{l=1}^{n} g_{li}\right)\phi_{lk}}{\sum_{s=1}^{m} S_{ii}(\omega_s)\left(\sum_{l=1}^{n} g_{li}\phi_{ls}\right)^2}. \tag{48}$$

For the special case in which independent excitations act on single masses m_k and there is external power $E\{f_k \dot{x}_k\}$ coming into the structural system, the previous relations in Eqs. (43) and (44) do not apply, since band-limited noise does not excite all modes with the same intensity. The above equation lead in this case to

$$c_{i(j,k)} = \frac{\sqrt{S_{ii}(\omega_j)S_{ii}(\omega_k)}\phi_{ij}\phi_{ik}}{\sum_{s=1}^{m} S_{ii}(\omega_s)\phi_{is}^2} \neq h_{i(j,k)}, \tag{49}$$

$$c_{i(j,j)} = \frac{S_{ii}(\omega_j)\phi_{ij}^2}{\sum_{s=1}^{m} S_{ii}(\omega_s)\phi_{is}^2} < h_{i(j,j)} = m_i\phi_{ij}^2. \tag{50}$$

All other relations in Section 2.2, especially the result in Eq. (39) are still applicable for band-limited spectra.

2.4. Kinetic energy in coupled structural components

SEA can be regarded as a procedure for determining the vibration energy balance in complex coupled systems. The subsystems in the SEA model are either weakly coupled structural components or "blocks of similar modes". The external excitation introduces an input power Π_{in}

into each subsystem which is dissipated by internal damping, but also transmitted to neighboring subsystems by the coupling. The dissipation of the power $\Pi_{i,\mathrm{in}}$ of the ith subsystem is expressed by the so-called loss factor η_i multiplied by the kinetic energy E_i of the subsystem. The power flow Π_{ij} from the ith to the jth neighboring subsystems is measured by the coupling loss factor η_{ij} multiplied by the difference $(E_i - E_j)$ of the energies. Hence, SEA states that power is transmitted only from subsystems of higher kinetic energy to neighboring systems with lower energy. It will be demonstrated that this law is not valid for low and intermediate frequency ranges. If the system consists of L subsystems then the typical energy flow equation is

$$\omega_c \begin{bmatrix} \eta_1 + \sum_{j\neq 1}\eta_{1j} & -\eta_{12} & \cdots & -\eta_{1L} \\ -\eta_{21} & \eta_2 + \sum_{j\neq 2}\eta_{2j} & \cdots & -\eta_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ -\eta_{L1} & -\eta_{L2} & \cdots & \eta_L + \sum_{j\neq L}\eta_{Lj} \end{bmatrix} \cdot \begin{Bmatrix} E_1 \\ E_2 \\ \vdots \\ E_L \end{Bmatrix} = \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \vdots \\ \Pi_L \end{Bmatrix}. \tag{51}$$

Here the frequency band is $[\omega_l, \omega_u]$ with $\omega_c = (\omega_l + \omega_u)/2$, and the coupling loss factors are symmetric, i.e. $\eta_{ij} = \eta_{ji}$. For substructures S_i and S_j which are not directly coupled, the coupling loss factors are assumed to vanish, i.e. $\eta_{ij} = 0$.

This SEA power flow relation can be expressed in matrix notation,

$$\mathbf{Q} \cdot \{\mathbf{E}\} = \{\mathbf{\Pi}_{\mathrm{in}}\} \tag{52}$$

with

$$q_{ij} \in \mathbf{Q}; \quad q_{ii} = \omega_c\left(\eta_i + \sum_{j\neq i}\eta_{ij}\right); \quad q_{ij} = -\omega_c\eta_{ij} \text{ for } j\neq i. \tag{53}$$

The SEA equation (52) is now compared with the accurate solution stated in Eq. (39). The solution in Eq. (39) is given in the form

$$\mathbf{e} = \mathbf{Q}^{-1} \cdot \mathbf{p} \tag{54}$$

corresponding to an inverse formulation of Eq. (52). Eq. (39) can be employed to specify for energies E_i in substructures $S_i, i = 1, 2, \ldots, L$. Let $\{J_i\}$ be the set of dof belonging to the substructure S_i. Since the kinetic energy E_i of substructure S_i is the sum of kinetic energies e_j of all the dof j belonging to S_i, Eq. (39) reads for substructures

$$\mathbf{E} = \left[\sum_{j=1}^{m} \frac{\bar{\mathbf{h}}_{(j,j)} \cdot \mathbf{c}^{\mathrm{T}}_{(j,j)}}{\gamma_{(j,j)}} + 2\sum_{j=1}^{m-1} \frac{\bar{\mathbf{h}}_{(j,j+1)} \cdot \mathbf{c}^{\mathrm{T}}_{(j,j+1)}}{\gamma_{(j,j+1)}} + \cdots\right] \cdot \mathbf{p}, \tag{55}$$

where m modes are excited within the frequency range $[\omega_l, \omega_u]$ by using the notation

$$E_i = \sum_{j\in\{J_i\}} e_j, \tag{56}$$

$$\bar{h}_{i(j,k)} = \sum_{l\in\{J_i\}} h_{l(j,k)}; \quad \bar{\mathbf{h}}^{\mathrm{T}}_{(j,k)} = \{\bar{h}_{1(j,k)}, \bar{h}_{2(j,k)}, \ldots, \bar{h}_{L(j,k)}\}. \tag{57}$$

In the next step, the power input Π_i in a substructure S_i is defined as the sum of single sources $p_j, j \in K_i$, where $\{K_i\}$ denotes the set of independent excitations action on substructure S_i

$$\Pi_i = \sum_{l \in \{K_i\}} p_l. \tag{58}$$

Introducing the quantities

$$\bar{c}_{i(j,k)} = \frac{\sum_{l \in \{K_i\}} c_{l(j,k)} p_l}{\sum_{l \in \{K_i\}} p_l}; \quad \bar{\mathbf{c}}^{\mathrm{T}}_{(j,k)} = \{\bar{c}_{1(j,k)}, \bar{c}_{2(j,k)}, \ldots, \bar{c}_{L(j,k)}\}, \tag{59}$$

we find that the kinetic energy in the substructures is determined by the linear relation,

$$\mathbf{E} = \mathbf{B} \cdot \mathbf{\Pi} \tag{60}$$

or equivalently in the standard SEA form

$$\bar{\mathbf{Q}} \cdot \mathbf{E} = \mathbf{\Pi} \quad \text{with } \bar{\mathbf{Q}} = \mathbf{B}^{-1}, \tag{61}$$

where the matrix $\mathbf{B}$ reads

$$\mathbf{B} = \left[\sum_{j=1}^{m} \frac{\bar{\mathbf{h}}_{(j,j)} \cdot \bar{\mathbf{c}}^{\mathrm{T}}_{(j,j)}}{\gamma_{(j,j)}} + 2 \sum_{j=1}^{m-1} \frac{\bar{\mathbf{h}}_{(j,j+1)} \cdot \bar{\mathbf{c}}^{\mathrm{T}}_{(j,j+1)}}{\gamma_{(j,j+1)}} + \cdots \right]. \tag{62}$$

Please note, that the matrix $\mathbf{B}$ and consequently $\bar{\mathbf{Q}}$ is non-symmetric, unlike the matrix SEA matrix $\mathbf{Q}$ in Eq. (52). Only for white noise "rain on the roof" excitation, these matrices are symmetric as derived in Eq. (44).

3. Numerical example

3.1. Structural system

We investigate the power flow relation in an elastic beam shown in Fig. 1. Three simply supported beams are coupled by a rotational spring k_1 and k_2. This spring will be varied to

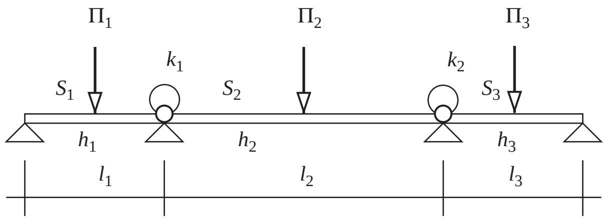

Fig. 1. Linear elastic beam structure.

consider weak and strong coupling between the substructure S_1, S_2 and S_3. It is assumed that torsional deflections are not excited. The system is assumed to have random properties in order to account for unavoidable uncertainties of the mechanical properties. The nominal values for the length are $l_1 = 3.0\,\mathrm{m}$, $l_2 = 6.0\,\mathrm{m}$, $l_3 = 3.0\,\mathrm{m}$, for the beam height $h_1 = 0.01\,\mathrm{m}$, $h_2 = 0.02\,\mathrm{m}$, $h_3 = 0.01\,\mathrm{m}$, constant width $w = 0.30\,\mathrm{m}$, constant density $\rho = 7800\,\mathrm{kg/m^3}$ and constant Young's modulo $E = 2.1 \cdot 10^{11}\,\mathrm{N/m^2}$. A rectangular cross-section is assumed. The following coefficient of variation have been assumed $\mathrm{CoV}(l_1) = \mathrm{CoV}(l_2) = \mathrm{CoV}(l_3) = 0.02$, $\mathrm{CoV}(h_1) = \mathrm{CoV}(h_2) = \mathrm{CoV}(h_3) = 0.05$, and for the rotational springs $\mathrm{CoV}(k_1) = \mathrm{CoV}(k_2) = 0.10$. All variables are considered to be statistically independent and normally distributed. The structure is modeled by FE using 160 dof.

3.2. Energy distributions

In the following, it is assumed that the power Π_i will be transmitted in the middle of the beams. Two frequency ranges, namely case (a) [10–100 Hz] and (b) [100–1000 Hz] will be investigated. Constant spectral densities are assumed within these frequency ranges. Case (a) includes 16 modes and case (b) 45 modes. According to Eq. (62), the results depend mainly on the specific modal damping $\gamma_{(j,j)} = 4\zeta_j\omega_j$. These quantities are generally highly uncertain. In the following results, a constant coefficient of variation $\mathrm{CoV}(\gamma_{(j,j)}) = 0.15$ has been assumed, where the variation is independent for each mode. All results are obtained by Monte Carlo simulation using a sample size of 2500.

Weak coupling: Weak coupling is obtained for the nominal value $k_1 = k_2 = 1.0 \cdot 10^4\,\mathrm{N\,m}$. The mean values of the matrix B (see Eq. (62)) and subsequently the inverse matrix $\bar{\mathbf{Q}}$ (see Eq. (61)) is shown below. Subsequently, the standard deviation for each term of the matrix $\mathbf{B}$ and its inverse $\bar{\mathbf{Q}}$ are listed. It can be observed that the standard deviations of $\bar{\mathbf{Q}}$ is considerably higher, and hence less robust when compared with the standard deviations of matrix $\mathbf{B}$.

Case (a):

$$\begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.923 & 0.095 & 0.025 \\ 0.077 & 0.835 & 0.078 \\ 0.025 & 0.096 & 0.923 \end{bmatrix} \cdot \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$4\langle\zeta\omega\rangle \begin{bmatrix} +1.139 & -0.134 & -0.038 \\ -0.113 & +1.251 & -0.113 \\ -0.038 & -0.134 & +1.139 \end{bmatrix} \cdot \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$\text{st.dev.}\{\mathbf{B}\} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.138 & 0.051 & 0.055 \\ 0.067 & 0.105 & 0.070 \\ 0.055 & 0.051 & 0.138 \end{bmatrix},$$

$$\text{st.dev.}\{\bar{\mathbf{Q}}\} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.251 & 0.095 & 0.182 \\ 0.125 & 0.183 & 0.131 \\ 0.182 & 0.095 & 0.251 \end{bmatrix}.$$

Case (b):

$$\begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.988 & 0.023 & 0.002 \\ 0.033 & 0.977 & 0.033 \\ 0.002 & 0.023 & 0.988 \end{bmatrix} \cdot \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$4\langle\zeta\omega\rangle \begin{bmatrix} +1.019 & -0.024 & -0.002 \\ -0.036 & +1.029 & -0.036 \\ -0.002 & -0.024 & +1.019 \end{bmatrix} \cdot \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$\text{st.dev.}\{\mathbf{B}\} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.070 & 0.020 & 0.009 \\ 0.032 & 0.105 & 0.032 \\ 0.009 & 0.020 & 0.070 \end{bmatrix},$$

$$\text{st.dev.}\{\bar{\mathbf{Q}}\} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.075 & 0.023 & 0.013 \\ 0.039 & 0.062 & 0.039 \\ 0.013 & 0.023 & 0.075 \end{bmatrix}.$$

Strong coupling: Strong coupling is obtained for the nominal value $k_1 = k_2 = 1.0 \cdot 10^6$ N m. Below the mean values of the matrix **B** (see Eq. (62)) and the inverse matrix $\bar{\mathbf{Q}}$ (see Eq. (61)) followed by the standard deviations of the two matrices. Note that the standard deviations for the inverse matrix $\bar{\mathbf{Q}}$ is unacceptable large, while the standard deviations in matrix **B** are still within a reasonable range.

Case (a):

$$\begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.655 & 0.196 & 0.149 \\ 0.221 & 0.632 & 0.221 \\ 0.149 & 0.196 & 0.655 \end{bmatrix} \cdot \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$4\langle\zeta\omega\rangle \begin{bmatrix} +4.320 & -0.508 & -2.754 \\ -0.583 & +2.001 & -0.583 \\ -2.754 & -0.508 & +4.320 \end{bmatrix} \cdot \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$\text{st.dev.}\{\mathbf{B}\} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 0.129 & 0.037 & 0.115 \\ 0.052 & 0.076 & 0.052 \\ 0.115 & 0.037 & 0.129 \end{bmatrix},$$

$$\text{st.dev.}\{\bar{\mathbf{Q}}\} = \frac{1}{4\langle\zeta\omega\rangle} \begin{bmatrix} 11.41 & 0.312 & 11.44 \\ 0.361 & 0.411 & 0.361 \\ 11.44 & 0.312 & 11.41 \end{bmatrix}.$$

Case (b):

$$\begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \frac{1}{4\langle\zeta\omega\rangle}\begin{bmatrix} 0.698 & 0.165 & 0.082 \\ 0.244 & 0.693 & 0.244 \\ 0.082 & 0.165 & 0.698 \end{bmatrix} \cdot \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$4\langle\zeta\omega\rangle\begin{bmatrix} +3.848 & -0.375 & -2.308 \\ -0.561 & +1.729 & -0.561 \\ -2.308 & -0.375 & +3.848 \end{bmatrix} \cdot \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{Bmatrix},$$

$$\text{st.dev.}\{\mathbf{B}\} = \frac{1}{4\langle\zeta\omega\rangle}\begin{bmatrix} 0.093 & 0.023 & 0.079 \\ 0.049 & 0.062 & 0.049 \\ 0.079 & 0.023 & 0.093 \end{bmatrix},$$

$$\text{st.dev.}\{\bar{\mathbf{Q}}\} = \frac{1}{4\langle\zeta\omega\rangle}\begin{bmatrix} 40.14 & 0.248 & 40.16 \\ 0.412 & 0.206 & 0.412 \\ 40.16 & 0.248 & 40.14 \end{bmatrix}.$$

Please note, that contrary to the SEA matrix **Q**, the matrix $\bar{\mathbf{Q}}$ is not symmetric. A symmetric matrix can only be obtained for the white-noise case (see Eq. (44)) if the excitation excites equally all modes. It can also be observed that the standard deviations of the matrix $\bar{\mathbf{Q}}$ are quite high and the term $\bar{Q}_{13}$ differs from zero. However, for weak coupling SEA is capable to approximate the kinetic energies with acceptable accuracy.

4. Conclusions and outlook

The following conclusions can be drawn from the results:

(1) Accurate expressions for the power flow in structures and substructures have been obtained. These expressions are exceptionally simple and moreover generally valid since the only assumption introduced is a linear structural system excited by a stationary random process with a smooth power spectrum.
(2) The derived expressions show that energies in substructures are not sensitive to variations of the eigenfrequencies or of mode shapes due to the summation over frequency ranges and over the domain of the substructure. Modal analysis will lead to accurate estimates for the coupling properties between substructures even in case FE analysis fails to provide accurate eigenfrequencies and mode shapes.
(3) Uncertain structural properties influence the coupling properties and consequently the energy flow. Its variability can be assessed conveniently by Monte Carlo simulation.
(4) Strong or dissipative coupling between substructures can be considered in a straight forward manner.

(5) The matrix relating linearly the power input with the energy in substructures is in general not symmetric. White noise "rain of the roof" excitation is required to arrive at a symmetric matrix.
(6) The standard SEA relations do not lead to a physical meaningful representation for strong coupling, while the inverse form does remain meaningful also for strong coupling.
(7) The presented approach is especially suitable for the frequency domain for which modes are obtainable, i.e. below the high-frequency domain.

Acknowledgements

This research was partially supported by the Austrian Science Fund (WFF) under Contract No. 16769-N12 which is gratefully acknowledged by the authors.

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