Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Journal of Sound and Vibration 289 (2006) 54–65
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Stability in parametric resonance of axially accelerating beams constituted by Boltzmann's superposition principle

Xiao-Dong Yang[a], Li-Qun Chen[b,*]

[a]Department of Engineering Mechanics, Shenyang Institute of Aeronautical Engineering, Shenyang 110034, China
[b]Department of Mechanics, Shanghai University, Shanghai 200436, China

Received 27 August 2004; received in revised form 17 December 2004; accepted 28 January 2005
Available online 9 April 2005

Abstract

Stability in transverse parametric vibration of axially accelerating viscoelastic beams is investigated. The governing equation is derived from Newton's second law, Boltzmann's superposition principle, and the geometrical relation. When the axial speed is a constant mean speed with small harmonic variations, the governing equation can be treated as a continuous gyroscopic system with small periodically parametric excitations and a damping term. The method of multiple scales is applied directly to the governing equation without discretization. The stability conditions are obtained for combination and principal parametric resonance. Numerical examples demonstrate that the increase of the viscosity coefficient causes the lager instability threshold of speed fluctuation amplitude for given detuning parameter and smaller instability range of the detuning parameter for given speed fluctuation amplitude. The instability region is much bigger in lower order principal resonance than that in the higher order.

1. Introduction

Many engineering devices can be modeled as axially moving beams [1,2]. One major problem is the occurrence of large transverse vibrations, termed as parametric vibration, due to tension or axial speed variation.

*Corresponding author.
E-mail address: lqchen@online.sh.cn (L.-Q. Chen).

doi:10.1016/j.jsv.2005.01.035

Transverse parametric vibration of axially accelerating beams has been extensively analyzed. Although Pasin [3] first studied the problem as early as in 1972, much progress was achieved recently. Öz et al. [4] applied the method of multiple scales to study dynamic stability of an axially accelerating beam with small bending stiffness. Özkaya and Pakdemirli [5] applied the method of multiple scales and the method of matched asymptotic expansions to construct non-resonant boundary layer solutions for an axially accelerating beam with small bending stiffness. Öz and Pakdemirli [6] and Öz [7] used the method of multiple scales to calculate analytically the stability boundaries of an axially accelerating tensioned beam under simply supported conditions and fixed–fixed conditions, respectively. Parker and Lin [8] adopted a 1-term Galerkin discretization and the perturbation method to study dynamic stability of an axially accelerating beam subjected to a tension fluctuation. Özkaya and Öz [9] applied artificial neural network algorithm to determine stability of an axially accelerating beam. Yang and Chen [10] and Chen et al. [11] applied the averaging method to analyze the stability of axially accelerating linear beams on simple or fixed supports. Yang and Chen [12] studied numerically bifurcation and chaos of an axially accelerating nonlinear beam. Their investigations were based on two-term Galerkin truncation. Chen and Yang [13] applied directly the method of multiple scales to study the axially moving viscoelastic beam with variable speed on simple or fixed supports.

All above-mentioned researchers considered elastic beams [3–9] or viscoelastic beams constituted by the Kelvin model, a differential constitutive law [11–13]. There is no investigation on transverse vibrations of axially accelerating beams constituted by the viscoelastic constitutive law of an integral type. To address the lack of research in this aspect, the authors investigate parametric resonance of an axially accelerating viscoelastic beam constituted by Boltzmann's superposition principle. The stability conditions are obtained for combination and principal parametric resonance by using the multiple scales method. The numerical examples for stability of beams with simple supports and fixed supports are presented and the effect of viscoelasticity is discussed.

2. The governing equation

A uniform axially moving viscoelastic beam, with density ρ, cross-sectional area A, moment of inertial I and initial tension P_0, travels at the time-dependent axial transport speed $v(T)$ between two prismatic ends separated by distance L. Consider only the bending vibration described by the transverse displacement $V(X,T)$, where T is the time and X is the axial coordinate. Newton's second law of motion yields

$$\rho A\left(\frac{\partial^2 U}{\partial T^2}+2v\frac{\partial^2 U}{\partial X\,\partial T}+\frac{\mathrm{d}v}{\mathrm{d}T}\frac{\partial U}{\partial X}+v^2\frac{\partial^2 U}{\partial X^2}\right)=P_0\frac{\partial^2 U(X,T)}{\partial X^2}-\frac{\partial^2 M(X,T)}{\partial X^2}, \tag{1}$$

where $M(X,T)$ is the bending moment given by

$$M(X,T)=-\int_A Z\sigma(X,Z,T)\,\mathrm{d}A, \tag{2}$$

where Z, X plane is the principal plane of bending, and $\sigma(X, Z, T)$ is the disturbed normal stress. The viscoelastic material of the beam obeys the Kelvin model, with the constitution relation

$$\sigma(X, Z, T) = e(X, Z, T)E(0) + \int_0^t \dot{E}(T - T')e(X, Z, T')\,\mathrm{d}T', \tag{3}$$

where $e(X, Z, T)$ is the axial strain, $E(T)$ is the relaxation modulus, and η is the viscosity coefficient. For small deflections, the strain–displacement relation is

$$e(X, Z, T) = -Z\frac{\partial^2 U(X, T)}{\partial X^2}. \tag{4}$$

The relaxation modulus E is assumed as

$$E(T) = (E_0 - a) + a\mathrm{e}^{-\varepsilon\eta T}, \tag{5}$$

where bookkeeping device ε is a small dimensionless parameter accounting for the fact the viscosity coefficient is very small.

Substitution of Eqs. (4) and (5) into Eq. (3) and substitution of the resulting equation into Eq. (2) lead to

$$M(X, T) = E_0 I\frac{\partial^2 U(X, T)}{\partial X^2} - \varepsilon\eta a I\int_0^t \mathrm{e}^{-\varepsilon\eta(T-T')}\frac{\partial^2 U(X, T)}{\partial X^2}\,\mathrm{d}T'. \tag{6}$$

Substitution of Eq. (6) into Eq. (1) leads to

$$\rho A\left(\frac{\partial^2 U}{\partial T^2} + 2v\frac{\partial^2 U}{\partial X\,\partial T} + \frac{\mathrm{d}v}{\mathrm{d}T}\frac{\partial U}{\partial X} + v^2\frac{\partial^2 U}{\partial X^2}\right) - P_0\frac{\partial^2 U}{\partial X^2} + E_0 I\frac{\partial^4 U(X, T)}{\partial X^4} - \varepsilon\eta I a\int_0^t \mathrm{e}^{-\varepsilon\eta T}\frac{\partial^4 U(X, T)}{\partial X^4}\,\mathrm{d}T' = 0. \tag{7}$$

Introduce the dimensionless variables and parameters

$$u = \frac{U}{L},\quad x = \frac{X}{L},\quad t = T\sqrt{\frac{P_0}{\rho A L^2}},\quad \gamma = v\sqrt{\frac{\rho A}{P_0}},\quad D_0 = \frac{E_0 I}{P_0 L^2},\quad D_1 = \frac{aI}{P_0 L^2},\quad \alpha = \eta\sqrt{\frac{\rho A L^2}{P_0}}. \tag{8}$$

Eq. (7) can be cast into the dimensionless form

$$\frac{\partial^2 u}{\partial t^2} + 2\gamma\frac{\partial^2 u}{\partial x\,\partial t} + \frac{\mathrm{d}\gamma}{\mathrm{d}t}\frac{\partial u}{\partial x} + (\gamma^2 - 1)\frac{\partial^2 u}{\partial x^2} + D_0\frac{\partial^4 u}{\partial x^4} = \varepsilon\alpha D_1 s, \tag{9}$$

where

$$s = \int_0^t \mathrm{e}^{-\varepsilon\alpha(t-t')}\frac{\partial^4 u(X, T)}{\partial x^4}\,\mathrm{d}t'. \tag{10}$$

Using Eq. (8) and its derivative, one can get

$$\dot{s} = -\varepsilon\alpha s + \frac{\partial^4 u}{\partial x^4}. \tag{11}$$

3. Stability condition via the method of multiple scales

In the present investigation, the axial speed is assumed to be a small simple harmonic variation, with the amplitude $\varepsilon\gamma_1$ and the frequency ω, about the constant mean speed γ_0,

$$\gamma(t) = \gamma_0 + \varepsilon\gamma_1 \sin \omega t. \tag{12}$$

Here the bookkeeping device ε is used to indicate the fact that the fluctuation amplitude is small, with the same order as the dimensionless viscosity coefficient. Substitution of Eq. (12) into Eq. (9) and neglect higher order ε terms in the resulting equation yield

$$M\frac{\partial^2 u}{\partial t^2} + G\frac{\partial u}{\partial t} + Ku = -2\varepsilon\gamma_1 \sin \omega t\frac{\partial^2 u}{\partial x\,\partial t} - 2\varepsilon\gamma_0\gamma_1 \sin \omega t\frac{\partial^2 u}{\partial x^2} - \varepsilon\omega\gamma_1 \cos \omega t\frac{\partial u}{\partial x} + \varepsilon\alpha D_1 s, \tag{13}$$

where the mass, gyroscopic, and linear stiffness operators are, respectively, defined as

$$M = I, \quad G = 2\gamma_0\frac{\partial}{\partial x}, \quad K = (\gamma_0^2 - 1)\frac{\partial^2}{\partial x^2} + D_0\frac{\partial^4}{\partial x^4}. \tag{14}$$

The method of multiple scales will be employed to solve Eqs. (9) and (11). A first-order uniform approximation is sought in the form

$$u(x, t; \varepsilon) = u_0(x, T_0, T_1) + \varepsilon u_1(x, T_0, T_1) + \cdots \tag{15}$$

and a zero order to s is

$$s(x, t; \varepsilon) = s_1(x, T_0, T_1; \varepsilon), \tag{16}$$

where $T_0 = \tau$ and $T_1 = \varepsilon\tau$ are, respectively, the fast and slow time scales. Substitution of Eqs. (15), (16) and the following relationship

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial T_0} + \varepsilon\frac{\partial}{\partial T_1} + \cdots, \quad \frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial T_0^2} + 2\varepsilon\frac{\partial^2}{\partial T_0\,\partial T_1} + \cdots \tag{17}$$

into Eqs. (9) and (11) and then equalization of coefficients of ε^0 and ε in the resulting equation lead to

$$M\frac{\partial^2 u_0}{\partial T_0^2} + G\frac{\partial u_0}{\partial T_0} + Ku_0 = 0, \tag{18}$$

$$\dot{s} = -\varepsilon\alpha s + \frac{\partial^4 u}{\partial x^4}, \tag{19}$$

$$M\frac{\partial^2 u_1}{\partial T_0^2} + G\frac{\partial u_1}{\partial T_0} + Ku_1 = -2\frac{\partial^2 u_0}{\partial T_0\,\partial T_1} - 2\gamma_0\frac{\partial^2 u_0}{\partial x\,\partial T_1} - 2\gamma_1 \sin \omega t\left(\frac{\partial^2 u_0}{\partial x\,\partial T_0} + \gamma_0\frac{\partial^2 u_0}{\partial x^2}\right) - \gamma_1\omega \cos \omega t\frac{\partial u_0}{\partial x} - \alpha D_1 s_1. \tag{20}$$

Wickert and Mote [14] have obtained the solution to Eq. (18)

$$u_0(x,T_0,T_1)=\sum_{k=0,1,\ldots}\left[\phi_k(x)A_k(T_1)e^{i\omega_k T_0}+\bar{\phi}_k(x)\bar{A}_k(T_1)e^{-i\omega_k T_0}\right], \tag{21}$$

where the over bar denotes complex conjugation, and the kth natural frequency and the kth complex eigenfunction can be determined by the boundary conditions.

To investigate the summation parametric response, Eq. (21) can be expressed as

$$u_0(x,T_0,T_1)=\phi_n(x)A_n(T_1)e^{i\omega_n T_0}+\phi_m(x)A_m(T_1)e^{i\omega_m T_0}+\text{cc}, \tag{22}$$

where cc stands for the complex conjugate of all preceding terms on the right hand of an equation. Substituting Eq. (22) into Eq. (19) and neglecting higher order ε terms in the resulting equation yield

$$s_1=A_n\phi_n''''\frac{1}{i\omega_n}e^{i\omega_n T}+A_m\phi_m''''\frac{1}{i\omega_m}e^{i\omega_m T}+\text{cc}. \tag{23}$$

If the variation frequency ω approaches the sum of any two natural frequencies of system (13), summation parametric resonance may occur. A detuning parameter σ is introduced to quantify the deviation of ω from $\omega_n+\omega_m$, and ω is described by

$$\omega=\omega_n+\omega_m+\varepsilon\sigma, \tag{24}$$

where ω_n and ω_m are, respectively, the nth and mth natural frequencies of system (13).

Substitution of Eqs. (22)–(24) into Eq. (20) yields

$$\begin{aligned}&M\frac{\partial^2 u_1}{\partial T_0^2}+G\frac{\partial u_1}{\partial T_0}+Ku_1\\&=\left\{-2\dot{A}_n(i\omega_n\phi_n+\gamma_0\phi_n')+\gamma_1\left[\frac{1}{2}(\omega_m-\omega_n)\bar{\phi}_m'+i\gamma_0\bar{\phi}_m''\right]\bar{\phi}_m e^{i\sigma T_1}+\frac{\alpha D_1}{i\omega_n}A_n\phi_n''''\right\}e^{i\omega_n T_0}\\&\quad+\left\{-2\dot{A}_m(i\omega_m\phi_m+\gamma_0\phi_m')+\gamma_1\left[\frac{1}{2}(\omega_n-\omega_m)\bar{\phi}_n'+i\gamma_0\bar{\phi}_n''\right]\bar{\phi}_n e^{i\sigma T_1}+\frac{\alpha D_1}{i\omega_m}A_m\phi_m''''\right\}e^{i\omega_m T_0}\\&\quad+\text{cc}+\text{NST},\end{aligned} \tag{25}$$

where the dot and the prime denote derivation with respect to the slow time variable T_1 and the dimensionless spatial variable x, respectively, and NST stands for the terms that will not bring secular terms into the solution. Eq. (25) has a bounded solution only if a solvability condition holds. The solvability condition demands the orthogonal relationships

$$\begin{aligned}&\left\langle -2\dot{A}_n(i\omega_n\phi_n+\gamma_0\phi_n')+\gamma_1\left[\frac{1}{2}(\omega_m-\omega_n)\bar{\phi}_m'+i\gamma_0\bar{\phi}_m''\right]\bar{\phi}_m e^{i\sigma T_1}+\frac{\alpha D_1}{i\omega_n}A_n\phi_n'''',\phi_n\right\rangle=0,\\&\left\langle -2\dot{A}_m(i\omega_m\phi_m+\gamma_0\phi_m')+\gamma_1\left[\frac{1}{2}(\omega_n-\omega_m)\bar{\phi}_n'+i\gamma_0\bar{\phi}_n''\right]\bar{\phi}_n e^{i\sigma T_1}+\frac{\alpha D_1}{i\omega_m}A_m\phi_m'''',\phi_m\right\rangle=0,\end{aligned} \tag{26}$$

where the inner product is defined for complex functions on $[0,1]$ as

$$\langle f,g\rangle=\int_0^1 f\bar{g}\,dx. \tag{27}$$

Application of the distributive law of the inner product to Eq. (26) leads to

$$\dot{A}_n + \alpha c_{nn} A_n + \gamma_1 d_{nm} \bar{A}_m e^{i\sigma T_1} = 0,$$

$$\dot{A}_m + \alpha c_{mm} A_m + \gamma_1 d_{mn} \bar{A}_n e^{i\sigma T_1} = 0, \tag{28}$$

where

$$c_{kk} = -\frac{(D_1/i\omega_k)\int_0^1 \phi_k'''' \bar{\phi}_k \, dx}{2\left(i\omega_k \int_0^1 \phi_k \bar{\phi}_k \, dx + \gamma_0 \int_0^1 \phi_k' \bar{\phi}_k \, dx\right)} \quad (k = n, m),$$

$$d_{kj} = -\frac{(\omega_j - \omega_k)\int_0^1 \bar{\phi}_j' \bar{\phi}_k \, dx + 2i\gamma_0 \int_0^1 \bar{\phi}_j'' \bar{\phi}_k \, dx}{4\left(i\omega_k \int_0^1 \phi_k \bar{\phi}_k \, dx + \gamma_0 \int_0^1 \phi_k' \bar{\phi}_k \, dx\right)} \quad (k = n, m, \ j = m, n). \tag{29}$$

These coefficients are determined by the eigenfunctions, the natural frequencies and the constant part of the axial speed, and are independent of parametric excitation due to the variation of axial speed.

Eq. (28) takes the same form as Eq. (21) in Ref. [13], although the coefficients are different. Eq. (31) in Ref. [13] gives the analytical expression of the stability boundary in summation parametric resonance

$$\left[\frac{\sigma}{2}\left(c_{nn}^R - c_{mm}^R\right)\right]^2 + \left(c_{nn}^R + c_{mm}^R\right)^2\left[\frac{\sigma^2}{4} + \alpha^2 c_{nn}^R c_{mm}^R + \gamma_1^2 d_{nm} \bar{d}_{mn}\right] = 0, \tag{30}$$

where superscript R denotes the real of part of the coefficient. Therefore, the instability region is given as

$$-2\sqrt{\frac{\gamma_1^2 \operatorname{Re}(d_{nm}\bar{d}_{mn}) - \alpha^2 c_{nn}^R c_{mm}^R}{1+\kappa^2}} < \sigma < 2\sqrt{\frac{\gamma_1^2 \operatorname{Re}(d_{nm}\bar{d}_{mn}) - \alpha^2 c_{nn}^R c_{mm}^R}{1+\kappa^2}}, \tag{31}$$

where

$$\kappa = \frac{c_{nn}^R - c_{mm}^R}{c_{nn}^R + c_{mm}^R}. \tag{32}$$

Now we consider the principal parametric resonance where the variation frequency ω approaches two times of a natural frequency of system (13). Denote

$$\omega = 2\omega_n + \varepsilon\sigma. \tag{33}$$

Let $m = n$ in Eq. (30). Then the resulting equation gives the stability boundary in the nth principal parametric resonance.

$$-2\sqrt{\gamma_1^2 |d_{nn}|^2 - \alpha^2 c_{nn}^{R^2}} < \sigma < 2\sqrt{\gamma_1^2 |d_{nn}|^2 - \alpha^2 c_{nn}^{R^2}}. \tag{34}$$

The stability in difference parametric resonance can be treated similarly. Denote

$$\omega = \omega_n - \omega_m + \varepsilon\sigma. \tag{35}$$

The stability boundaries are expressed by Eq. (30), while the coefficients in them are given by

$$c_{nn} = -\frac{(D_1/\mathrm{i}\omega_n)\int_0^1 \phi_n''''\bar{\phi}_n\,\mathrm{d}x}{2(\mathrm{i}\omega_n\int_0^1 \phi_n\bar{\phi}_n\,\mathrm{d}x + \gamma_0\int_0^1 \phi_n'\bar{\phi}_n\,\mathrm{d}x)}, \quad c_{mm} = -\frac{(D_1/\mathrm{i}\omega_m)\int_0^1 \phi_m''''\bar{\phi}_m\,\mathrm{d}x}{2(\mathrm{i}\omega_m\int_0^1 \phi_m\bar{\phi}_m\,\mathrm{d}x - \gamma_0\int_0^1 \phi_m'\bar{\phi}_m\,\mathrm{d}x)},$$

$$d_{nm} = \frac{(\omega_m+\omega_n)\int_0^1 \bar{\phi}_m'\bar{\phi}_n\,\mathrm{d}x - 2\mathrm{i}\gamma_0\int_0^1 \bar{\phi}_m''\bar{\phi}_n\,\mathrm{d}x}{4(\mathrm{i}\omega_n\int_0^1 \phi_n\bar{\phi}_n\,\mathrm{d}x + \gamma_0\int_0^1 \phi_n'\bar{\phi}_n\,\mathrm{d}x)}, \quad d_{mn} = \frac{(\omega_n+\omega_m)\int_0^1 \bar{\phi}_n'\bar{\phi}_m\,\mathrm{d}x + 2\mathrm{i}\gamma_0\int_0^1 \bar{\phi}_n''\bar{\phi}_m\,\mathrm{d}x}{4(\mathrm{i}\omega_m\int_0^1 \phi_m\bar{\phi}_m\,\mathrm{d}x - \gamma_0\int_0^1 \phi_m'\bar{\phi}_m\,\mathrm{d}x)}. \tag{36}$$

4. Stability boundaries of beams with simple supports

The boundary conditions of an axially moving beam with simple supports in dimensionless form are

$$u(0,t) = u(1,t) = 0, \quad \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=0} = \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=1} = 0. \tag{37}$$

The eigenfunction corresponding to the kth natural frequency ω_k of that boundary conditions is given by Eq. (19) in Ref. [6]

$$\begin{aligned}\phi_k(x) = {} & \mathrm{e}^{\mathrm{i}\beta_{1k}x} - \frac{(\beta_{4k}^2-\beta_{1k}^2)(\mathrm{e}^{\mathrm{i}\beta_{3k}}-\mathrm{e}^{\mathrm{i}\beta_{1k}})}{(\beta_{4k}^2-\beta_{2k}^2)(\mathrm{e}^{\mathrm{i}\beta_{3k}}-\mathrm{e}^{\mathrm{i}\beta_{2k}})}\mathrm{e}^{\mathrm{i}\beta_{2k}x} - \frac{(\beta_{4k}^2-\beta_{1k}^2)(\mathrm{e}^{\mathrm{i}\beta_{3k}}-\mathrm{e}^{\mathrm{i}\beta_{1k}})}{(\beta_{4n}^2-\beta_{3n}^2)(\mathrm{e}^{\mathrm{i}\beta_{2k}}-\mathrm{e}^{\mathrm{i}\beta_{3k}})}\mathrm{e}^{\mathrm{i}\beta_{3k}x} \\ & - \left[1 - \frac{(\beta_{4k}^2-\beta_{1k}^2)(\mathrm{e}^{\mathrm{i}\beta_{3n}}-\mathrm{e}^{\mathrm{i}\beta_{1k}})}{(\beta_{4k}^2-\beta_{2k}^2)(\mathrm{e}^{\mathrm{i}\beta_{3k}}-\mathrm{e}^{\mathrm{i}\beta_{2k}})} - \frac{(\beta_{4k}^2-\beta_{1k}^2)(\mathrm{e}^{\mathrm{i}\beta_{2k}}-\mathrm{e}^{\mathrm{i}\beta_{1k}})}{(\beta_{4k}^2-\beta_{3k}^2)(\mathrm{e}^{\mathrm{i}\beta_{2k}}-\mathrm{e}^{\mathrm{i}\beta_{3k}})}\right]\mathrm{e}^{\mathrm{i}\beta_{4k}x},\end{aligned} \tag{38}$$

where β_{jk} $(j = 1,2,3,4)$ are eigenvalues of simple supported case.

Consider an axially moving beam with $D_0 = 2.0$, $D_1 = 0.1$ and $\gamma = 3.5$. In summation parametric resonance, Eq. (29) gives $c_{11} = 0.0615, c_{22} = 0.0282, d_{12} = 1.3960 + 0.9334\mathrm{i}$, and $d_{21} = 0.3051 + 0.2040\mathrm{i}$. The stability boundaries for the summation resonance of first two modes in plane $\sigma - \gamma_1$ are shown in Fig. 1 for $\alpha = 0, 0.5, 0.8$. The increasing viscosity coefficient makes the stability boundaries move towards the increasing direction of γ_1 in plane (ω, γ_1) and the instability regions become narrow. The larger viscosity coefficient leads to the larger instability threshold of γ_1 for given σ, and the smaller instability range of σ for given γ_1.

In principal parametric resonance, Eq. (29) gives $d_{11} = -1.2001 + 1.2562\mathrm{i}$, $d_{22} = -0.4330 + 0.9798\mathrm{i}$. Eq. (34) yields the stability boundaries for the first and second principal resonance in plane $\sigma - \gamma_1$ shown, respectively, in Figs. 2 and 3 for $\alpha = 0, 0.5, 0.8$. In both cases, the increasing viscosity coefficient makes the stability boundaries move towards the increasing direction of γ_1 in plane (ω, γ_1) and the instability regions become narrow.

In difference parametric resonance, there is no instability region found.

To compare the difference of the three kinds of resonance instability, we use the same scale of coefficients and also the viscosity is chosen as the same in Figs. 1–3. These figures indicate that the area of the stability boundary in the first principal resonance is the biggest and that of the summation resonance is the smallest. But the stability boundary for the summation resonance is the most sensitive to the change of the viscosity coefficient and that for the first principal the least.

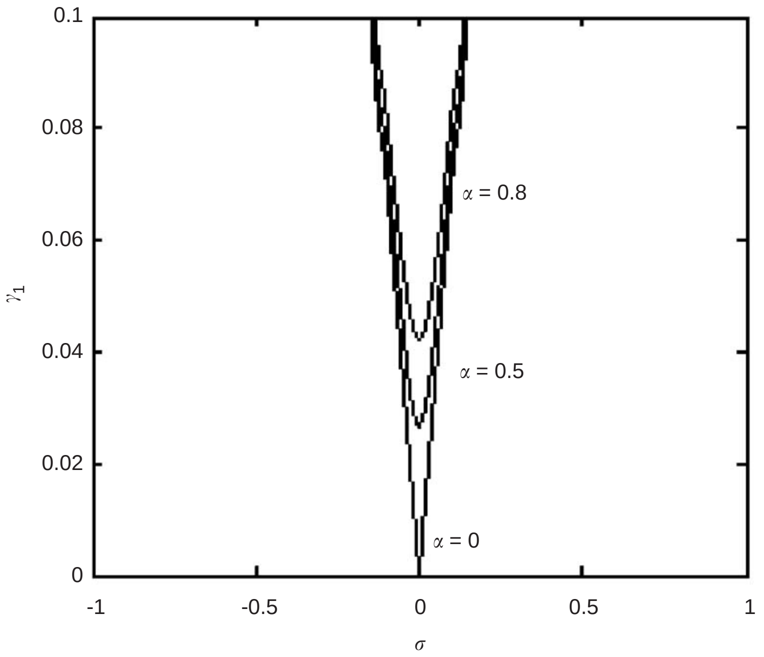

Fig. 1. The stability boundaries for the summation resonance of beams with simple supports.

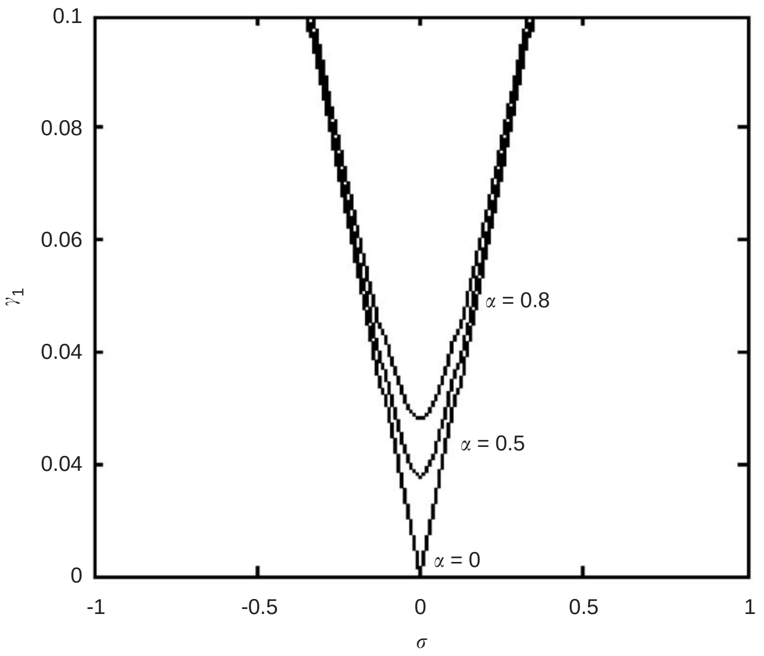

Fig. 2. The stability boundaries for the first principal resonance with simple supports.

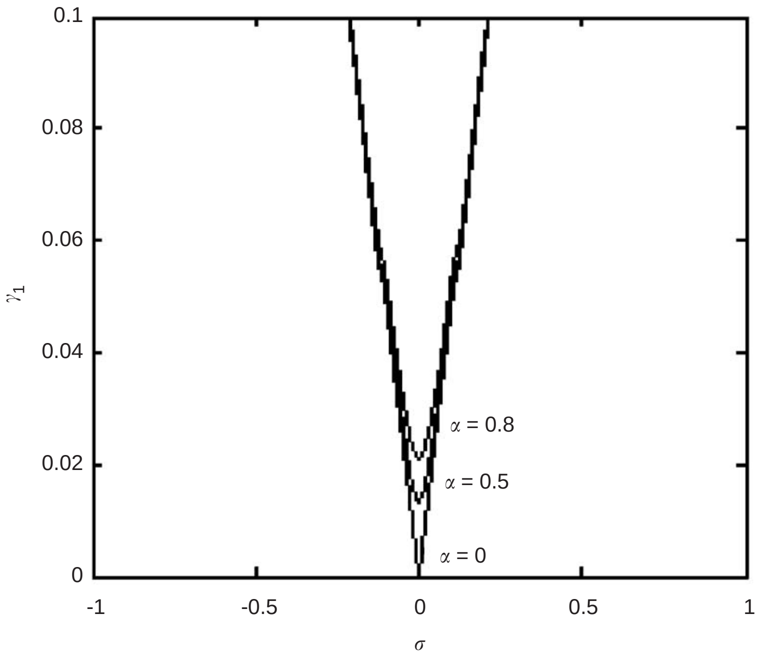

Fig. 3. The stability boundaries for the second principal resonance with simple supports.

5. Stability boundaries of beams with fixed supports

For an axially moving beam with simple supports, the boundary conditions in dimensionless form are

$$u(0,t) = u(1,t) = 0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=0} = \left.\frac{\partial u}{\partial x}\right|_{x=1} = 0. \tag{39}$$

And the eigenfunction of the boundary conditions equation (39) corresponding to the kth natural frequency ω_k is given by Eq. (17) in Ref. [7]

$$\begin{aligned}\phi_k(x) = {} & e^{i\beta_{1k}x} - \frac{(\beta_{4k}-\beta_{1k})(e^{i\beta_{3k}}-e^{i\beta_{1k}})}{(\beta_{4k}-\beta_{2k})(e^{i\beta_{3k}}-e^{i\beta_{2k}})} e^{i\beta_{2k}x} - \frac{(\beta_{4k}-\beta_{1k})(e^{i\beta_{3k}}-e^{i\beta_{1k}})}{(\beta_{4n}-\beta_{3n})(e^{i\beta_{2k}}-e^{i\beta_{3k}})} e^{i\beta_{3k}x} \\ & - \left[1 - \frac{(\beta_{4k}-\beta_{1k})(e^{i\beta_{3n}}-e^{i\beta_{1k}})}{(\beta_{4k}-\beta_{2k})(e^{i\beta_{3k}}-e^{i\beta_{2k}})} - \frac{(\beta_{4k}-\beta_{1k})(e^{i\beta_{2k}}-e^{i\beta_{1k}})}{(\beta_{4k}-\beta_{3k})(e^{i\beta_{2k}}-e^{i\beta_{3k}})}\right] e^{i\beta_{4k}x},\end{aligned} \tag{40}$$

where β_{jk} ($j = 1,2,3,4$) are eigenvalues.

Consider an axially moving beam on fixed supports with $D_0 = 2.0$, $D_1 = 0.1$ and $\gamma = 8.0$. Eq. (29) gives $c_{11} = 0.1391$, $c_{22} = 0.0356$, $d_{12} = -0.2067 - 0.2028i$, and $d_{21} = -0.0594 - 0.0583i$, $d_{11} = 2.2594 - 1.2201i$, $d_{22} = 0.5021 - 0.9733i$. The stability boundaries in the summation resonance of first two modes in plane σ–γ_1 are illustrated in Fig. 4 for $\alpha = 0, 0.1, 0.2$. The stability boundaries for the first and second principal resonance in plane σ–γ_1 are illustrated,

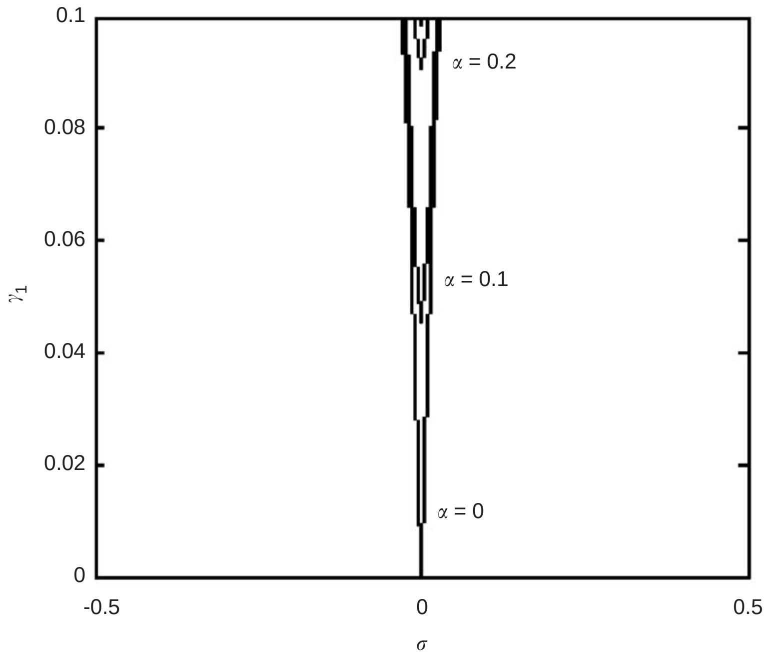

Fig. 4. The stability boundaries for the summation resonance of beams with fixed supports.

respectively, in Figs. 5 and 6 for $\alpha = 0, 0.5, 0.8$. In all figures, the instability regions draft towards the increasing direction of the amplitude with the increase of the viscosity coefficient. Like the results obtained for the simple supports case, the stability boundary in the first principal resonance is the least sensitive to the change of the viscosity coefficient and that in summation resonance the most. And also there is no instability region found in the difference resonance.

6. Conclusions

Transverse stability of axially moving viscoelastic beams is studied in this paper. The viscoelastic beam is constituted by Boltzmann's superposition principle. The axially moving speed is assumed as harmonically fluctuating about a constant mean value. The method of multiple scales is applied to the partial-differential equation governing the transverse parametric vibration. The stability boundary is derived from the solvability conditions. Axially accelerating beams with simple supports and fixed supports are numerically investigated. The results show that instability occurs when the axial speed fluctuation frequency is close to the sum of any two natural frequencies or two times of a natural frequency of the unperturbed system. A detuning parameter is used to quantify the deviation between the speed fluctuation frequency and the sum of two natural frequencies or the multiple of a natural frequency. The stability boundaries are numerically determined in the axial speed fluctuation detuning parameter–amplitude plane for varying viscosity coefficient. With the increase of the viscosity coefficient, the lager instability

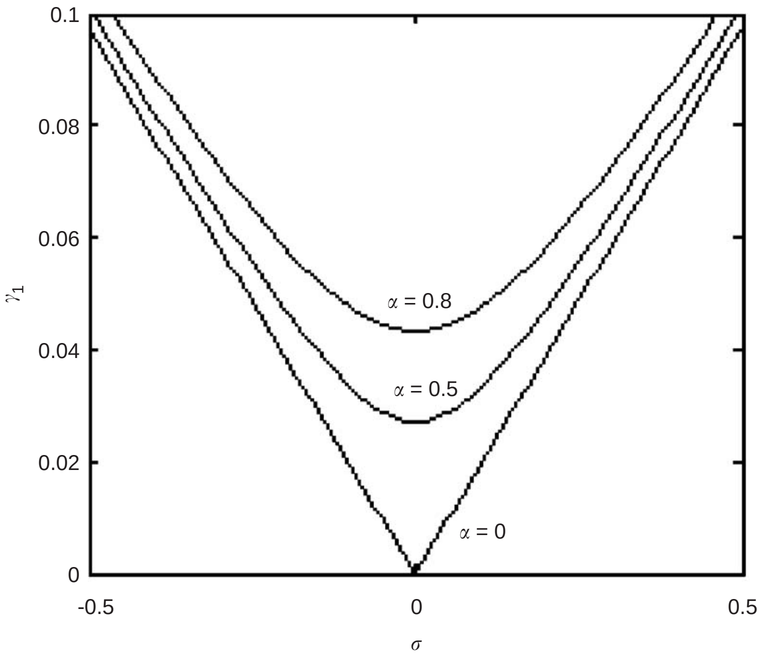

Fig. 5. The stability boundaries for the first principal resonance with fixed supports.

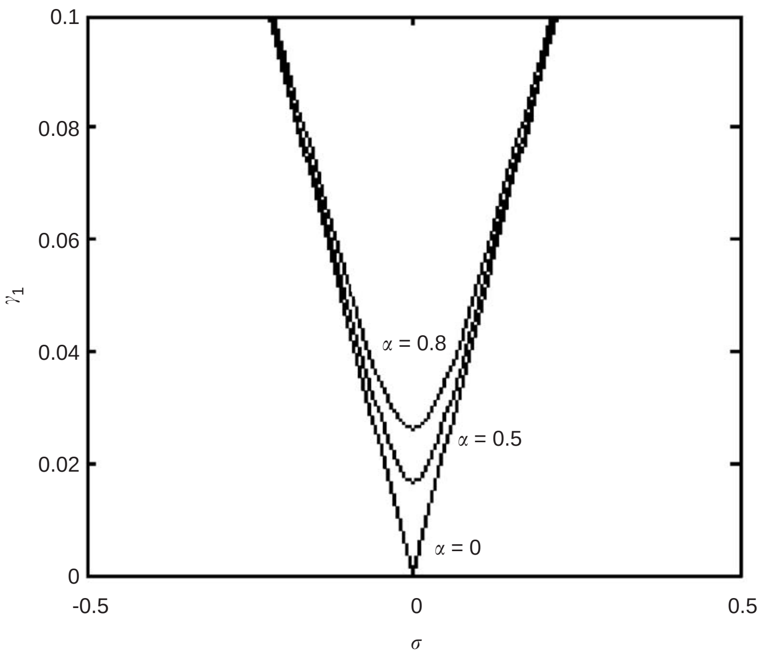

Fig. 6. The stability boundaries for the second principal resonance with fixed supports.

threshold of speed fluctuation amplitude becomes large for given detuning parameter, and the instability range of the detuning parameter becomes small for given speed fluctuation amplitude. The instability region is much bigger in first principal resonance than that in the higher order principal resonance and the summation resonance. In addition, the viscosity coefficient has more significance on the stability boundary in summation resonance than that in the principal parametric resonance.

Acknowledgements

The research is supported by the Natural Science Foundation of China (Project No. 10472060) and Natural Science Foundation of Shanghai Municipality (Project No. 04ZR14058).

References

[1] J.A. Wickert, C.D. Mote Jr., Current research on the vibration and stability of axially-moving materials, *The Shock and Vibration Digest* 20 (1988) 3–13.
[2] A.S. Abrate, Vibration of belts and belt drives, *Mechanism and Machine Theory* 27 (1992) 645–659.
[3] F. Pasin, Üeber die Stabilität der Beigeschwingungen von in längsrichtung periodisch hin und herbewegten Stäben, *Ingenieur-Archiv* 41 (1972) 387–393.
[4] H.R. Öz, M. Pakdemirli, E. Özkaya, Transition behaviour from string to beam for an axially accelerating material, *Journal of Sound and Vibration* 215 (1998) 571–576.
[5] E. Özkaya, M. Pakdemirli, Vibrations of an axially accelerating beam with small flexural stiffness, *Journal of Sound and Vibration* 234 (2000) 521–535.
[6] H.R. Öz, M. Pakdemirli, Vibrations of an axially moving beam with time dependent velocity, *Journal of Sound and Vibration* 227 (1999) 239–257.
[7] H.R. Öz, On the vibrations of an axially traveling beam on fixed supports with variable velocity, *Journal of Sound and Vibration* 239 (2001) 556–564.
[8] R.G. Parker, Y. Lin, Parametric instability of axially moving media subjected to multifrequency tension and speed fluctuations, *Journal of Applied Mechanics* 68 (2001) 49–57.
[9] E. Özkaya, H.R. Öz, Determination of natural frequencies and stability regions of axially moving beams using artificial neural networks method, *Journal of Sound and Vibration* 252 (2002) 782–789.
[10] X.D. Yang, L.Q. Chen, Dynamic stability of axially moving viscoelastic beams with pulsating speed, *Applied Mathematics and Mechanics*, in press.
[11] L.Q. Chen, X.D. Yang, C.J. Cheng, Dynamic stability of an axially accelerating viscoelastic beam, *European Journal of Mechanics A/Solid* 23 (2004) 659–666.
[12] X.D. Yang, L.Q. Chen, Bifurcation and chaos of an axially accelerating viscoelastic beam, *Chaos, Solitons and Fractals* 23 (2005) 249–258.
[13] L.Q. Chen, X.D. Yang, Stability in parametric resonance of axially moving viscoelastic beams with time-dependent speed, *Journal of Sound and Vibration*, in press; doi:10.1016/j.jsv.2004.07.024.
[14] J.A. Wickert, C.D. Mote Jr., Classical vibration analysis of axially moving continua, *Journal of Applied Mechanics* 57 (1990) 738–744.