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Control algorithms for active vibration isolation systems subject to random disturbances

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Abstract

Isolation from disturbances, particularly from foundations of high precision instruments, is achieved through either passive or active vibration control systems. Although a passive isolation system offers a simple and reliable means of protecting precision equipment from a vibration environment, it has performance limitations since its controllable frequency range is limited. An effective method for reducing an oscillation is by using an active vibration isolation system, which allows control of the dynamic rigidity of shock absorbers. In this paper, by considering the characteristics of the disturbing influences acting upon vibro-isolated objects, the dynamic characteristics of the AVIS device and control restriction, new optimal and quasi-optimal control algorithms are proposed. The characteristics of the new quasi-optimal active vibration isolation system proposed in the paper are investigated via experiments. It is shown that the adopted quasi-optimal active vibration isolation system can improve performance using in situ measurements.

1. Introduction

The intensity of vibration in present day instruments significantly affects the operational reliability and performance of systems, such as high precision equipment, machine-tools, measuring instruments and test facilities.

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The accuracy of measurement, validity of tests and quality of alignment are influenced by foundation disturbances. An effective method for reducing vibration is by using an active vibration isolation system (AVIS), which allows the control of the dynamic rigidity of shock absorbers [1–3].

A significant improvement in the vibro-isolation quality can be achieved by using a vibro-isolation system controlled by optimised algorithms. Despite a great amount of research work carried out in this field, there are a number of problems associated with the system. When developing AVIS optimised control algorithms, it is necessary to model the characteristics of the disturbing influences acting upon vibro-isolated objects with the dynamic characteristics of the AVIS device. This problem has not been previously solved.

It should be noted that the disturbances, which acts upon vibro-isolated objects, has random sequences. In recent studies [4], the optimal control of randomly disturbed objects was investigated by taking into account the characteristics of the disturbances.

The principal aim of this present work is to synthesise and to study the properties and beneficial effects of the new optimal AVIS. In general, an AVIS is used in combination with an ordinary uncontrollable actuator for wide band vibro-isolation, providing suppressions of the high-frequency oscillation [5,6]. Spring dampers, liquid dampers, pneumatic dampers and electro-magnetic dampers as well as others are used as actuating mechanisms. From a mathematical point of view, the represented device of AVIS could be considered as an actuator with an isolated object mass.

In this case, the schematic representation of the control object is shown in Fig. 1. The object is an equivalent mass of a vibro-isolated object placed on a platform with an actuator, the actuating mechanism being a controlled pneumosupport. Fig. 2 shows a picture of the controlled pneumosupport, which consists of two chambers. The upper chamber (2) is made of a rubberized tire fabric and is flexible; the lower chamber is a rigid one. The chambers are separated by a diaphragm (3) with throttle openings (4). Owing to the electromagnetic drive (6), rigidly connected to the diaphragm, the upper chamber pressure varies and, as a result, the stress acting upon the vibro-isolated object is relieved.

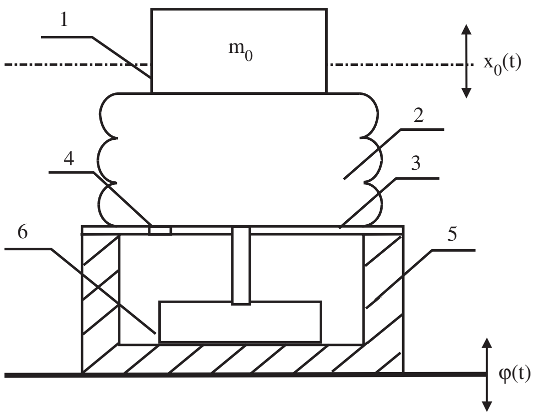


Fig. 1. Controlled pneumosupport.

Fig. 2. Controlled pneumosupport.

It will be shown that by using optimal control algorithms and the AVIS device, the AVIS may be designed to improve the operational reliability and performance of high precision instruments via measurement in situ.

The subsequent discussion concerns the probabilistic disturbing process that is classified as a normal Gaussian distribution. The hypothesis of a normal Gaussian distribution is based on real values of random processes, which are the effects of a large number of factors. The summation of these unconnected and weakly connected factors tends to be within the limits of the normal distribution.

The normal random process correlation functions can be approximated by the cosine exponential function as follows:

$$R_{\varphi}(\tau) = D_{\varphi}e^{-\alpha\tau}\cos(\beta\tau), \quad (1)$$

where D_{φ} is the variation, α is the damping constant and β is the time period of the function.

It is useful for optimisation purposes to take the minimum of the square $\langle z_0^2 \rangle$ from the deviation of the output z_0. This choice is based on the minimisation of the mean square $\langle z_0^2 \rangle$ and its variation, which are both specified for normal distribution, ensuring minimisation of the inequality:

$$|z_0(t)| > z_{\max}, \quad (2)$$

where $z_{\max}$ is the undesired deviation of z_0.

The adopted optimisation criterion must be completed by the requirement of automatic control system stability and control signal restriction.

The papers by Petrov [4] assumed the synthesis of the method of automatic control system by mean square deviation subject to control signal restriction on N_U, the control power:

$$\langle u_0^2 \rangle \leqslant N_u. \tag{3}$$

According to the variation evaluation, the minimisation of $\langle z_0 \rangle$ subject to the inequality condition (3) is equivalent to the function:

$$J = \lambda_0 \langle x_0^2 \rangle + \langle u_0^2 \rangle, \tag{4}$$

where λ_0 is the Lagrange coefficient, $\langle x_0^2 \rangle$ is the mean square value of disturbance and $\langle u_0^2 \rangle$ is the mean square value of the control signal.

The minimum criterion is obtained from positive values of the Lagrange coefficient, marked $\lambda_0 = m^2$, since it can be represented as follows:

$$J = m^2 \langle x_0^2 \rangle + \langle u_0^2 \rangle. \tag{5}$$

2. Optimal synthesis of active vibration isolation systems for stationary disturbances

Generally, to solve the task of optimisation by the use of linearisation models the system block-diagram shown in Fig. 3 is used.

The dynamic characteristics of the AVIS device by a control link (i.e. by a control input signal) are described with differential equations the nth order, which can be expressed in the form

$$A_0(\mathrm{d})x_0(t) = B_0(\mathrm{d})u(t), \tag{6}$$

where d is the symbol of differentiation $\mathrm{d} = \mathrm{d}/\mathrm{d}t$, and polynomials

$$A_0(\mathrm{d}) = a_0\mathrm{d}^n + a_1\mathrm{d}^{n-1} + \cdots + a_{n-1}\mathrm{d} + a_n,$$
$$B_0(\mathrm{d}) = b_0\mathrm{d}^m + b_1\mathrm{d}^{m-1} + \cdots + b_{m-1}\mathrm{d} + b_m$$

are differential operators, where $n \geqslant m$.

The AVIS undergoes stationary disturbances $\varphi(t)$ (Fig. 3), which are classified as a normal Gaussian distribution. The mathematical model of disturbances is approximated by the correlation function (1). The differential equations for the response of the AVIS device to a

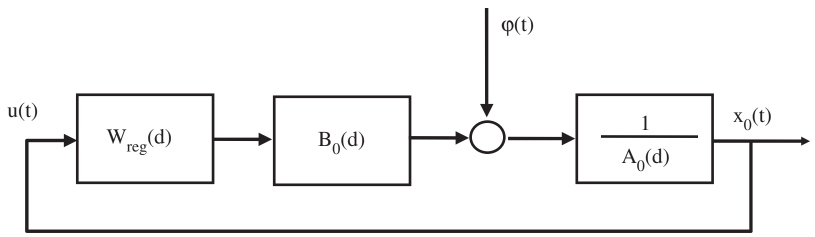


Fig. 3. Structure circuit of the optimal active vibration isolation system investigated.

disturbance can be represented in the following form:

$$A_0(\mathrm{d})x(t) = \varphi(t). \quad (7)$$

The control system is made into a closed-loop through a regulator, which has the dynamic characteristics and represented in symbolic form as follows:

$$W_{\mathrm{reg}}(\mathrm{d}) = \frac{B_{\mathrm{reg}}(\mathrm{d})}{A_{\mathrm{reg}}(\mathrm{d})} = \frac{b_{\mathrm{reg}0}\mathrm{d}^n + b_{\mathrm{reg}1}\mathrm{d}^{n-1} + \cdots + b_{\mathrm{reg}n-1}\mathrm{d} + b_{\mathrm{reg}n}}{a_{\mathrm{reg}0}\mathrm{d}^n + a_{\mathrm{reg}1}\mathrm{d}^{n-1} + \cdots + a_{\mathrm{reg}n-1}\mathrm{d} + a_{\mathrm{reg}n}}. \quad (8)$$

The structure and parameters of the regulator are to be defined while solving the AVIS optimization problem using the quality criterion (5). The closed-loop system equation, when the numerator polynomial of the AVIS device $B(\mathrm{d}) = 1$, takes the form:

$$[A_0(\mathrm{d}) - W_{\mathrm{reg}}(\mathrm{d})]x_0(t) = \varphi(t). \quad (9)$$

To solve this problem one can use Fourier transformation on the above equation to obtain:

$$[A_0(\mathrm{i}\omega) - W_{\mathrm{reg}}(\mathrm{i}\omega)]x_0(\mathrm{i}\omega) = \varphi(\mathrm{i}\omega). \quad (10)$$

The mean square value of the disturbance $\langle x_0^2 \rangle$ of the output of the closed-loop system is defined by the power spectral density (PSD) $S_\varphi(t)$ and amplitude of the frequency characteristics of the closed-loop system. It is described by

$$\langle x_0^2 \rangle = \int_0^\infty S_\varphi(\omega) \frac{1}{|A_0(\mathrm{i}\omega) - W_{\mathrm{reg}}(\mathrm{i}\omega)|^2}\, \mathrm{d}\omega. \quad (11)$$

In a similar way we define the mean square value of the control signal and the optimal criterion will be expressed in the following way:

$$J = m^2\langle x_0^2 \rangle + \langle u_0^2 \rangle = \int_0^\infty S_\varphi(\omega) \frac{m^2 + |W_{\mathrm{reg}}(\mathrm{i}\omega)|^2}{|A_0(\mathrm{i}\omega) - W_{\mathrm{reg}}(\mathrm{i}\omega)|^2}\, \mathrm{d}\omega. \quad (12)$$

Thus the optimal criterion depends on the function $W_{\mathrm{reg}}(\mathrm{i}\omega)$ and hence ω, the frequency. The task of the optimal regulator reduces to the variation problem of defining a function which gives a minimum value for Eq. (5). To take into account the requirement of system stability, in Eq. (5) a transition from variable ω to variable $p = \mathrm{i}\omega$ gives

$$J = \frac{1}{2j} \int_0^\infty S_\varphi(p) \frac{m^2 + W_{\mathrm{reg}}(p)W_{\mathrm{reg}}(-p)}{[A_0(p) - W_{\mathrm{reg}}(p)][A_0(-p) - W_{\mathrm{reg}}(-p)]}\, \mathrm{d}\omega. \quad (13)$$

Supposing that the power density spectrum $S_\varphi(p)$ is an even order fraction (rational) function of the variable p, then:

$$S_\varphi(p) = \frac{a_0p^{2\gamma} + a_1p^{2\gamma-2} + \cdots + a_\gamma}{b_0p^{2q} + b_1p^{2q-2} + \cdots + b_q}. \quad (14)$$

This function can be decompose into symmetrical multipliers

$$S_\varphi(p) = S_1(p)S_1(-p). \quad (15)$$

Substituting the expression for $S_\varphi(p)$ into formula (5), the optimal criterion, we solve this variation task. However, only those δJ values are admissible which do not invalidate the stability condition for closed-loop systems.

The optimal regulator structure and parameters can be determined on the bases of the following algorithm stages:

1. Factorisation of the PDS equation of disturbances:
$$S_\varphi(p) = S_1(p)S_1(-p). \tag{16}$$
2. Factorisation of the polynomial:
$$A_0(p)A_0(-p) + m^2 = G(p)G(-p). \tag{17}$$
3. Decomposition of fractions:
$$\frac{A_0(-p)}{G(-p)} S_1(p) = M_0(p) + M_+(p) + M_-(p), \tag{18}$$
where $M_0(p)$ is the integer polynomial; $M_+(p)$ is the fraction with positive poles in the left half-plane; $M_-(p)$ the fraction with negative poles in the right half-plane.
4. Building the auxiliary function:
$$Q(p) = \frac{M_0(p) + M_+(p)}{G(p)S(p)}. \tag{19}$$
5. Definition of the optimal regulator transfer function:
$$W_{\text{reg}}(p) = A_0(p) - \frac{1}{Q(p)}. \tag{20}$$

Using the above definitions of the algorithm, synthesis of the regulator is possible, in view of the AVIS device transfer function in the form:
$$W_{\text{dev}}(p) = \frac{B_0(p)}{A_0(p)} = \frac{1}{(T_1^2 + T_2p + 1)}. \tag{21}$$
It should be noted that Eq. (21) can be used to describe dynamic characteristics of AVIS devices, both in active and passive control systems. As mentioned above, the AVIS device can be represented as an actuator with a vibro-isolated object mass. Liquid dampers, pneumatic dampers, electromagnetic dampers, etc. can be used as actuating mechanisms. Amplitude–frequency characteristics of the mechanisms can be approximated by transfer function (18) as it is shown in Refs. [1,5–7].

The mathematical model of the disturbances assume a random process with a correlation function (1). This function corresponds with the following of PSD:
$$S_\varphi(p) = D_\varphi \frac{2\alpha}{\pi} \frac{\alpha^2 + \beta^2 + \omega^2}{(\alpha^2 + \beta^2 + \omega^2) - 4\beta^2\omega^2}. \tag{22}$$
The factorisation of the PSD will take a value of $D_\varphi = \pi/2\alpha$, since parameters of the synthesised regulator are independent of the disturbance value. In the PSD Eq. (22) by changing

variable $p = \mathrm{i}\omega$, we can modify the numerator as follows:

$$\alpha^2 + \beta^2 - p^2 = (b_{s0}p + b_{s1})(b_{s0}p - b_{s1}). \tag{23}$$

Equating coefficients the same order of p we have: $b_{s0} = 1$; $b_{s1} = \sqrt{\alpha^2 + \beta^2}$.

The denominator of this equation will be

$$(\alpha^2 + \beta^2 + p^2)^2 - 4\beta^2 p^2 = (a_{s0}p^2 + a_{s1}p + a_{s2})(a_{s0}p^2 - a_{s1}p + a_{s2}).$$

Equations for a_{s0}, a_{s1}, a_{s2} are defined after the open brackets and equating the coefficients under the same order of p in the left and the right sides of the previous equation: $a_{s0} = 1$; $a_{s1} = 2\alpha$; $a_{s2} = \alpha^2 + \beta^2$. Subsequently, the decomposition of the PSD Eq. (22) will take the form:

$$S_1(p) = \frac{b_{s0}p + b_{s1}}{a_{s0}p^2 + a_{s1}p + a_{s2}}, \quad S_1(-p) = \frac{b_{s0}p + b_{s1}}{a_{s0}p^2 - a_{s1}p + a_{s2}}. \tag{24}$$

Having factorised polynomial (17) one can obtain:

$$\begin{aligned} G(p)G(-p) &= (T_1^2 + T_2 p + 1)(T_1^2 - T_2 p + 1) + m^2 \\ &= (a_{g0}p^2 + a_{g1}p + a_{g2})(a_{g0}p^2 - a_{g1}p + a_{g2}). \end{aligned} \tag{25}$$

Open brackets and equating the coefficients under the same order of p in the left and the right sides we obtain the coefficients:

$$a_{g0} = T_1^2, \quad a_{g2} = \sqrt{m^2 + 1}, \quad a_{g1} = \sqrt{2T_1^2 + \sqrt{m^2 + 1} - T_1^2 + T_2^2}. \tag{26}$$

Thus

$$G(p) = a_{g0}p^2 + a_{g1}p + a_{g2}, \quad G(-p) = a_{g0}p^2 - a_{g1}p + a_{g2}. \tag{27}$$

To rewrite the equation for Eq. (18) taking into account Eqs. (21), (24) and (25)

$$\frac{A_0(-p)}{G(-p)} S_1(p) = \frac{(T_1^2 p^2 + T_2 p + 1)(b_{s0}p + b_{s1})}{(a_{s0}p^2 + a_{s1}p + a_{s2})(a_{g0}p^2 - a_{g1}p + a_{g2})}. \tag{28}$$

Taking into account that the denominator in this equation has a higher order than that of the numerator, after decomposition of Eq. (18) in view of Eq. (28) we have: $M_0(p) = 0$.

In order to define $M_+(p)$ and $M_-(p)$ represent Eq. (28) as a summation of fractions with undetermined coefficients C_1, C_2, C_3 and C_4 it is deduced that:

$$M_+(p) + M_-(p) = \frac{C_1 + C_2 p}{a_{s0}p^2 + a_{s1}p + a_{s2}} + \frac{C_3 + C_4 p}{a_{g0}p^2 - a_{g1}p + a_{g2}}. \tag{29}$$

By reducing to a common denominator and by equating the numerators in the initial equation (28) and modifying Eq. (29), we obtain:

$$\begin{aligned} &(T_1^2 + T_2 p + 1)(b_{s0}p + b_{s1}p) \\ &\quad = (C_1 + C_2 p)(a_{g0}p^2 - a_{g1}p + a_{g2}) + (C_3 + C_4 p)(a_{s0}p^2 - a_{s1}p + a_{s2}). \end{aligned} \tag{30}$$

Coefficients C_1 and C_2 are necessary for the forthcoming computations since they are the elements of the equation for $M_+(p)$. These can be defined, based on the following consideration.

Supposing that operator p in Eq. (30) equates the root $\alpha_S + i\beta_S$ of polynomial $(a_{s0}p^2 + a_{s1}p + a_{s2})$, i.e. root of polynomial $S_1(p)$. Thus Eq. (30) is reduced due to the second component of Eq. (30) being equal to zero.

Substituting the coefficients a_{si} into Eq. (30) for computing the roots of second-order equation with known values α and β of PSD, we have: $a_s = \alpha$ and $\beta_s = \beta$.

Taking into account the above computations, after transformation (23), one can write:

$$\begin{aligned} &T_1^2(3\alpha\beta^2 - \alpha^3) + (b_{s1}T_1^2 - T_2)(\alpha^2 + \beta^2) - (1 - b_{s1}T_2)\alpha + b_{s1} + i[T_1^2(3\alpha\beta - \beta^2) \\ &\quad - 2\alpha\beta(b_{s1}T_1^2 - T_2) + \beta(1 - b_{s1}T_2)] = C_2a_{g0}(3\alpha\beta^2 - \alpha^3) + (C_1a_{g0} - C_2a_{g1})(\alpha^2 + \beta^2) \\ &\quad - (C_2a_{g2} - C_1a_{g1})\alpha + C_1a_{g2} + i[C_2a_{g0}(3\alpha\beta^2 - \alpha^2) - 2\alpha\beta(C_1a_{g0} - C_2a_{g1}) \\ &\quad + \beta(C_2a_{g2} - C_1a_{g1})]. \end{aligned}$$

Having equated real and imaginary parts of the latter identity and solving the simultaneous equation we can define coefficients C_1 and C_2. They are described as follows:

$$C_1 = \frac{L_1 - L_2}{L_4 - D_4}, \quad C_2 = \frac{L_1 - C_1D_2}{L_4 + D_4}, \tag{31}$$

where

$$L_1 = T_1^2(3\alpha\beta^2 - \alpha^3) + (b_{s1}T_1^2 - T_2)(\alpha^2 + \beta^2) - (1 - b_{s1}T_2)\alpha + b_{s1},$$

$$L_2 = T_1^2(3\alpha\beta^2 - \beta^2) - 2\alpha\beta(b_{s1}T_1^2 - T_2) + \beta(1 - b_{s1}T_2),$$

$$L_3 = \frac{L_1D_3}{D_1}, \quad L_4 = \frac{D_2D_3}{D_1},$$

$$D_1 = a_{g0}(3\alpha\beta^2 - \alpha^3) + a_{g1}(\alpha^2 + \beta^2) - a_{g2}\beta, \quad D_2 = a_{g0}(\alpha^2 - \beta^2) + a_{g2},$$

$$D_3 = a_{g0}(3\alpha\beta^2 - \beta^3) + a_{g1}2\alpha\beta + a_{g2}\beta, \quad D_4 = 4a_{g0}a_{g1}\alpha\beta^2.$$

Taking into account the defined equation for $M_+(p)$ and using Eqs. (24) and (25), auxiliary function (13) takes the form

$$Q(p) = \frac{C_1 + C_2p}{(a_{g0}p^2 + a_{g1}p + a_{g2})(b_{s0}p + b_{s1})}. \tag{32}$$

Based on Eq. (20), the transfer function of the optimal regulator can be modified as follows:

$$W_{\text{reg}}(p) = \frac{b_{r0}p^3 + b_{r1}p^2 + b_{r2}p + b_{r3}}{a_{r0}p + a_{r1}}, \tag{33}$$

where

$$b_{r0} = T_1^2C_1 - a_{g0}, \quad b_{r1} = C_1T_2 + C_1T_1^2 - a_{g1} - a_{g0}b_{s1}, \quad b_{r2} = C_2 + C_1T_2 - a_{g2} - a_{g1}b_{s1},$$
$$b_{r3} = C_1 - a_{g2}b_{s1}, \quad a_{r0} = C_2, \quad a_{r1} = C_1.$$

Software can be used that allows calculation of the parameters of the optimal regulator. Such a program was developed according to the above assumptions and computations.

For defining the mean square value of disturbance $\langle x_0^2 \rangle$, in view of Fig. 3, it can be stated that transfer function of the closed-loop system under the disturbance link as

$$W_{\mathrm{dis}}(p) = \frac{x_0(p)}{\varphi(p)} = \frac{A_r(p)}{A_r(p)A_0(p) - B_r(p)}. \tag{34}$$

Having done the transformations, one can have

$$W_{\mathrm{dis}}(p) = \frac{x_0(p)}{\varphi(p)} = K_B \frac{b_{c0}p + 1}{a_{c0}p^3 + a_{c1}p^2 + a_{c2}p + a_{c3}}, \tag{35}$$

where

$$K_B = \frac{a_{r1}}{a_{r1} - b_{r3}}, \quad b_{c0} = \frac{a_{r0}}{a_{r2}}, \quad a_{c0} = \frac{a_{r0}T_1^2 - b_{r0}}{a_{r0} - b_{r3}},$$

$$a_{c1} = \frac{a_{r1}T_1^2 + a_{r0}T_2 - b_{r1}}{a_{r1} - b_{r3}}, \quad a_{c2} = \frac{a_{r1}T_2 + a_{r2} - b_{r2}}{a_{r1} - b_{r3}}, \quad a_{c3} = 1.$$

As for the disturbance, when going to the frequency domain for the square of the closed-loop system amplitude–frequency characteristics obtain:

$$|W_{\mathrm{dis}}(p)|^2 = \frac{K_B^2(b_{c0}\omega^2 + 1)}{(1 - a_{c1}\omega^2)^2 + (a_{c2}\omega - a_{c0}\omega^3)^2}. \tag{36}$$

The mean square value of disturbance $\langle x_0^2 \rangle$ is only calculated for certain values AVIS as follows:

$$\langle x_0^2 \rangle = \int_0^\infty S_\varphi(\omega)|W_{\mathrm{dis}}(\mathrm{i}\omega)|^2\,\mathrm{d}\omega. \tag{37}$$

The transfer function of the closed-loop system by the control link is

$$W_{\mathrm{con}}(p) = \frac{U_0(p)}{\varphi(p)} = \frac{B_r(p)}{A_r(p)A_0(p) - B_r(p)}. \tag{38}$$

After substituting the transfer function (38) using appropriate equations and changing to the frequency domain, the equation for the square amplitude–frequency characteristics can be represented as

$$|W_{\mathrm{con}}(p)|^2 = \frac{(b_{r1}\omega^2 - b_{r3}\omega)^2 + (b_{r0}\omega^3 + b_{r2}\omega)^2}{(a_{r1} - b_{r3})[(1 - a_{c1}\omega^2)^2 + (a_{c2}\omega - a_{c0}\omega^3)^2]}. \tag{39}$$

The mean square value of control signal $\langle u_0^2 \rangle$ is calculated only for certain values of AVIS by equation:

$$\langle u_0^2 \rangle = \int_0^\infty S_\varphi(\omega)|W_{\mathrm{con}}(\mathrm{i}\omega)|^2\,\mathrm{d}\omega. \tag{40}$$

According to the above equations, the parameters of the optimal regulator depend on the characteristics of the disturbances and AVIS device, and also of the values of indefinite Lagrange coefficient m^2.

For defining the Lagrange coefficient m_0^2 it is necessary to calculate the optimal regulator parameters as well as the appropriate mean square values of the control signal $\langle u_0^2 \rangle$ and then we obtain the dependence $\langle u_0^2 \rangle m^2$. In view of the control signal restriction, the Lagrange coefficient is determined due to this dependence. Hence optimal regulator characteristics are defined.

The choice of the control signal restriction is based on the following. The synthesised optimal regulator operates in a linear zone, if the following is observed: $U_{\min} < U < U_{\max}$.

By transition to relative values, these conditions reduce to the control signal restriction magnitude: $|U_0| \leqslant 1$.

In normal cases, the algorithm of the optimal regulator is not linear and can be represented as

$$U(t) = \begin{cases} +1 & \text{by } U > 1, \\ U_L & \text{by } |U_0| \leqslant 1, \\ -1 & \text{by } U < 1. \end{cases}$$

Naturally, the control signal value is different from the optimal value, but there is the tendency to the optimal value if the regulator time period in the saturation zone is reduced and the less the time period the stronger the tendency. When the disturbing process is classified as normal Gaussian distribution $\varphi(t)$ and the regulator operates in the linear zone, then output signal $Z_0(t)$ is also classified in the linear zone. When the regulator operates in the saturation zone for a short period of time there is insignificant distortion of the Gaussian distribution. The time period, when $|U_0| \leqslant 1$ is observed, can be defined, in view of the normal distribution law, by the Laplace integral:

$$F(K_U) = \frac{2}{\sqrt{2\pi}} \int_0^{k_u} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x, \tag{41}$$

where coefficient K_U is defined from the condition

$$K_u^2 < u^2 < 1. \tag{42}$$

Using Laplace integral tables, it is easy to define, for example, $F(K_U) = 0.9$, if $K_U = 1.645$.

Therefore, if $K_U = 1.645$, then the regulator is operated 90% of the complete time as linear, and only 10% of the time as nonlinear. According to an analysis of real time operating influence on the optimal criterion, the time operation of regulator is recommended to between the following values: $K_U \geqslant (1.5 \ldots 1.645)$. This is the appropriate condition for nonlinear operation of the regulator (10...15%) of the time. This assumption permits a linear system with little distortion to be obtained.

Additional attention should be given to the problem of the system stability under parameter variations. It is known that there are cases when optimal, in mean square values, systems can lose their stability at insignificant parameter variations. Such a class of optimal closed-loop systems will be unstable if the denominator of auxiliary function (19) is equal to the order of equation $A_0(p)$ and also if the coefficients values coincide with the oldest order of operator p. Moreover, the order of the numerator polynomial $W_{\text{reg}}(p)$ in Eq. (20) is decreased by one compared to the polynomials $A_0(p)$ and $F(p)$. Even at infinitesimal parameter variations the polynomial coefficients with oldest orders of operator p become unequal. So the characteristic equation for the closed-loop system can have roots with a positive imaginary part, i.e. the system will be unstable. For the synthesised optimal system such a crucial situation does not occur, i.e. the denominator order of $Q(p)$ and characteristic equation $A_0(p)$ are different.

For the AVIS stability, under parameter variations with the given PSD, there is a general condition to stick to. This condition is represented as follows:

$$\gamma \geqslant m + q - 1, \tag{43}$$

where m is the order of the AVIS denominator polynomial.

Consider the situation that the AVIS device has $m = 0$. According to the mathematical model (22), the coefficients $\gamma = 1$ and $q = 2$. Therefore, the unequal equation (43) is true and this guarantees stability of the system. This conclusion is confirmed by the following analysis of sensitivity analysis to variation of AVIS device and disturbance characteristics.

3. Appreciation of AVIS'S efficiency operation

For the AVIS device transfer function (21) and the mathematical model of the disturbances (22), using specially developed software, the optimal regulator will be synthesised. The computation will be carried out for several possible characteristics of the active vibration isolation system AVIS device and disturbances. Base values take the following form: $\alpha = 2$; $\beta = 20$; $T = T_1 = 0.1$ s; $T_2 = 0.09$ s.

In order to obtain the structure and parameters of the optimal regulator, we use the previous algorithm to find the parameters of the closed-loop system, and characteristics of the mean square values of control signal $\langle u^2 \rangle$ and disturbances $\langle x_0^2 \rangle$, and values of criterion (5).

The correlation function is normalised by D_φ, which allows us to receive parameters of $\langle x_0^2 \rangle$ and $\langle u^2 \rangle$ in the relative range. Appropriate dependencies of $\langle u^2 \rangle$ from m^2, and $\langle x_0^2 \rangle$ from m^2 and also criterion J (Eq. (5)) from m^2 are represented in Figs. 4–6. As shown in Figs. 4 and 5 the point $m^2 = 0$ corresponds to the absence of control signal $\langle u^2 \rangle$, but at this time $\langle x_0^2 \rangle$ is a maximum. If the coefficient m^2 increases, the gain of the optimal regulator and control power increases as well. When $\langle u^2 \rangle$ tends to one, $\langle x_0^2 \rangle$ tends to zero.

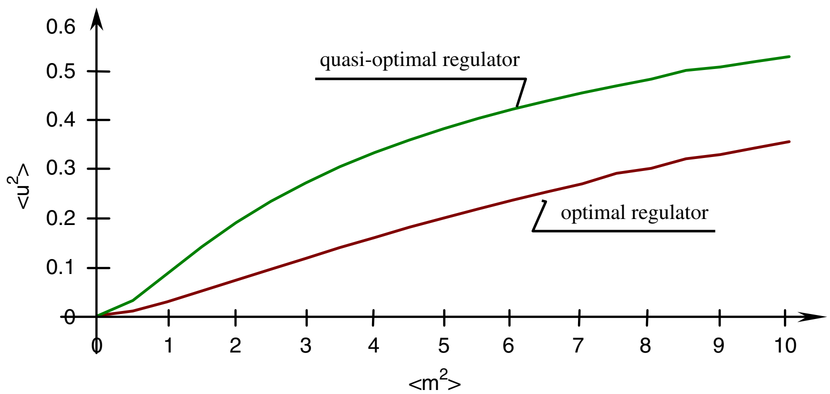


Fig. 4. Dependence of mean square value $\langle u^2 \rangle$ of control signal from Lagrange coefficient m^2.

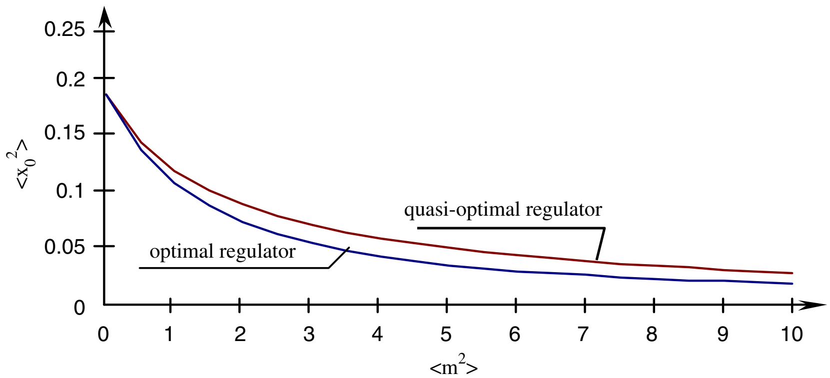


Fig. 5. Dependence of mean square value $\langle x_0^2\rangle$ of disturbance from Lagrange coefficient m^2.

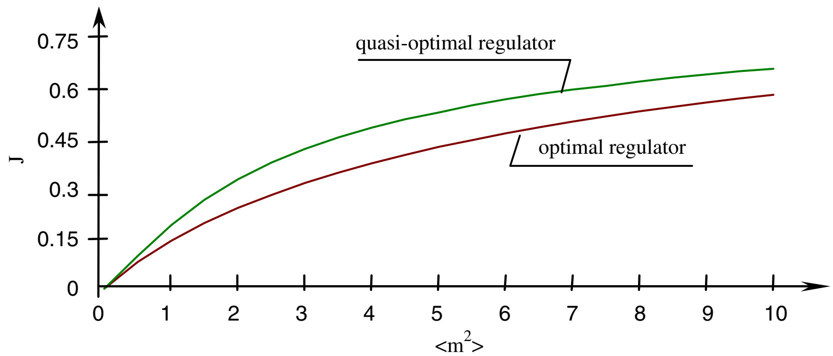


Fig. 6. Dependence of adopted criterion J from Lagrange coefficient m^2.

According to the above control considering signal restriction, values of K_U^2 Eq. (42), can take the form:

$$\langle u^2\rangle \leqslant \frac{1}{(2.25\ldots 2.71)} = 0.37\ldots 0.45. \tag{44}$$

The optimised system permits, by minimum control power, the suppression of maximum disturbances subject to the restriction on control signal. The further increase of power does not result in efficient suppression of disturbances. The increasing of power influences environmental conditions. This can lead to overheating, as a result changing of time constant, and decreasing of the efficiency of the vibro-isolation system can occur.

We can see from the computation, that values of Lagrange coefficient corresponds to $m^2 \leqslant 10$. If $m^2 = 10$, which corresponds to a control restriction of 0.4105, we have $\langle x_0^2\rangle = 0.1225$. Thus, the mean square value of the output of the closed-loop system with the optimal regulator is about 12% from disturbances (relative disturbances power is 1).

According to the experimental data [8] foundation displacements are about 25–30 μm and the vibro-isolated object displacements are reduced to 2–3 μm. The optimal AVIS provides 8–10 time disturbance suppression and this ensures the required operational environment for high precision instruments. Theodolite error, for example, when making a space point measurement, is about 25 μm and accurate results cannot be obtained when the foundation disturbances are about 30 μm. However, the designed optimal AVIS provides accurate operation of such measuring facilities as theodolite within the instrument error range. It is, then, possible to receive more accurate results.

The computations show that for the adopted $m^2 = 10$ the optimal regulator transfer function will be represented as follows:

$$W_{\text{reg}}(p) = -\frac{0.833p^3 + 1.804p^2 + 4.820p + 3.26}{0.167p + 2.056}. \tag{45}$$

The minus sign in the transfer function shows that there is negative feedback in the closed-loop system.

It should be noted, however, that for the adopted disturbance model, the defined optimal regulator requires output deviation derivatives up to the third-order inclusive. With all this in mind the optimal regulator practical feasibility, especially its re-tuning, presents a lot of technical problems. Therefore, the quality characteristics attained under the optimal control are to be considered as the best ones. On the basis of these best characteristics it is possible to assess the properties of the quasi-optimal system with a simplified regulator.

Table 1
Results of computation the quality indices of quasi-optimal system in view of power spectrum density parameter β variation

β/β_0	$\langle x_0^2 \rangle$	$\langle u^2 \rangle$	J
0.5	0.0983	0.5147	1.105
0.55	0.0875	0.5146	1.040
0.6	0.0772	0.5102	0.974
0.65	0.0677	0.5021	0.908
0.7	0.0591	0.4913	0.846
0.75	0.0514	0.4787	0.787
0.8	0.0447	0.475	0.733
0.85	0.0388	0.469	0.684
0.9	0.0336	0.467	0.639
0.95	0.0294	0.4533	0.6
1	0.1267	0.4504	0.545
1.05	0.0226	0.4281	0.534
1.1	0.0199	0.4267	0.566
1.15	0.0176	0.426	0.481
1.2	0.0156	0.4161	0.46
1.25	0.0139	0.4157	0.44
1.3	0.0124	0.4086	0.423
1.35	0.0112	0.4009	0.408
1.4	0.0101	0.3937	0.394
1.45	0.0091	0.3871	0.382
1.5	0.0083	0.371	0.371

Table 2
Results of computation the quality indices of quasi-optimal system in view of time constant T variation

T/T_0	$\langle x_0^2 \rangle$	$\langle u^2 \rangle$	J
0.5	0.1375	0.7228	0.948
0.55	0.1363	0.6911	0.909
0.6	0.1352	0.6592	0.87
0.65	0.134	0.6271	0.831
0.7	0.1328	0.595	0.792
0.75	0.1316	0.5629	0.753
0.8	0.1304	0.5312	0.714
0.85	0.1294	0.4998	0.675
0.9	0.1281	0.4791	0.637
0.95	0.1279	0.4693	0.601
1	0.1267	0.4504	0.656
1.05	0.1246	0.4226	0.53
1.1	0.1235	0.396	0.497
1.15	0.1225	0.3708	0.466
1.2	0.1215	0.3470	0.436
1.25	0.1205	0.3245	0.408
1.3	0.1196	0.3035	0.381
1.35	0.1187	0.284	0.356
1.4	0.1179	0.2658	0.333
1.45	0.1171	0.2489	0.312
1.5	0.1164	0.2333	0.292

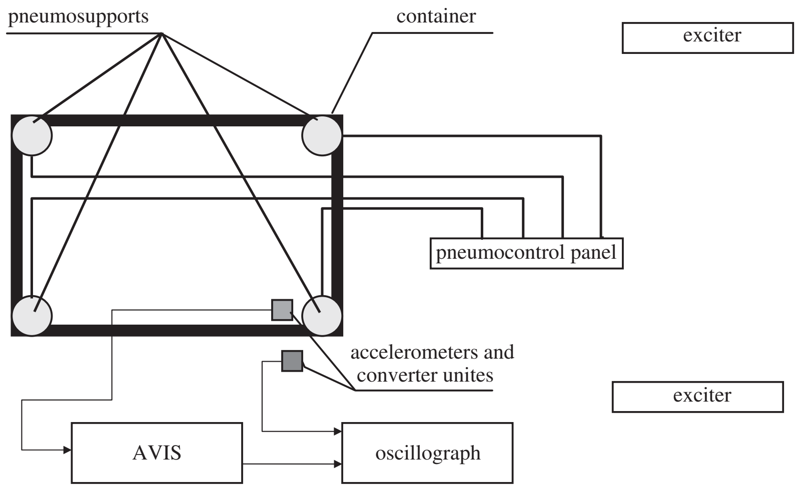
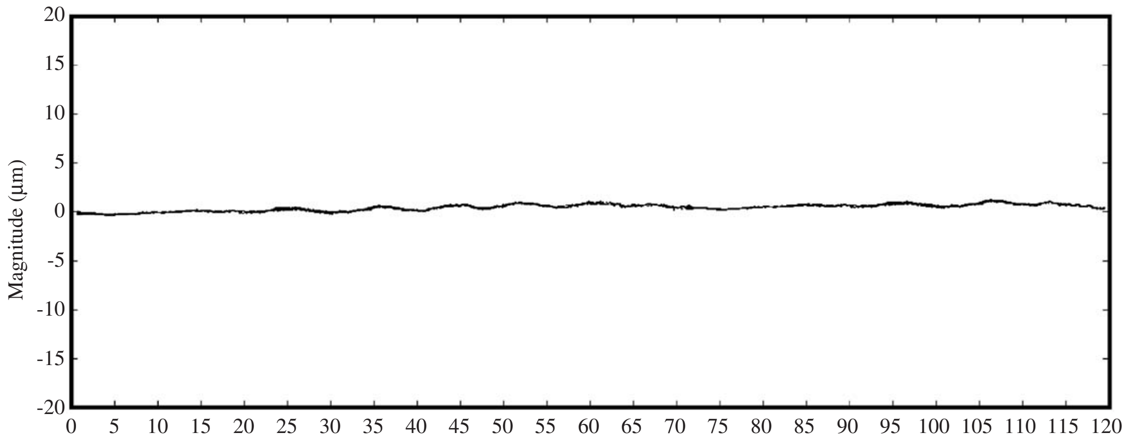


Fig. 7. The scheme of the experimental equipment.

Since the third derivative coefficient magnitude in the regulator transfer function $W_{\text{reg}}(p)$ is one order lower than that of the second derivative, a supposition can be made that the regulator with no third derivative shall not considerably worsen the control quality. In fact, in the system with a quasi-optimal regulator, calculations of $\langle x_0^2 \rangle$ and $\langle u^2 \rangle$ with the truncated numeration polynomial, show that the regulator modification does not worsen optimal criterion J which is within 11%...20% margin. The corresponding dependence is shown in Figs. 4–6. Thus, such regulator modification can be considered as acceptable and advisable from the engineering point of view.

It is then necessary to investigate how sensitive is the AVIS with a quasi-optimal regulator to the vibro-isolated object parameter variations and to the disturbance characteristics. Consider the situation when time constant $T = 1/\omega_c$ of the AVIS device either increases or decreases by 50%, while spectral power parameter β remains constant and also the case when parameter β increases or decreases by 50%, while parameter T remains constant. The results of the analysis are represented in Tables 1 and 2.

According to the studies outcomes the following conclusions can be reached: the system is rather sensitive to variations of the time constant T and to the disturbance influence β. When the



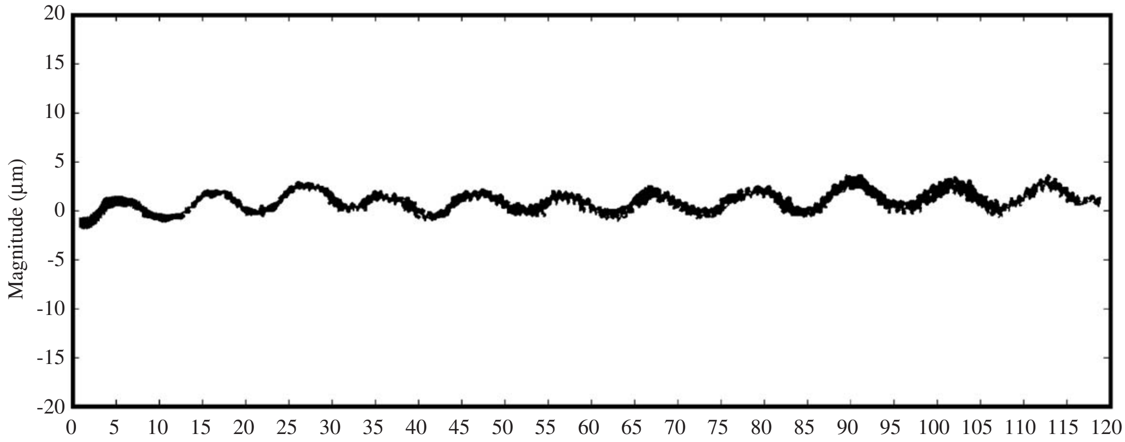


Fig. 8. Experimental results of the quasi-optimal AVIS: (a) output deviation of AVIS; (b) disturbance from the foundation.

AVIS device natural frequency ω_c decreases and β frequency decreases, then $\langle x_0^2 \rangle$ increases and, consequently, greater control power is required. When β frequency and ω_c show at least 2.5–3 fold difference then there is sharp decrease of the AVIS sensitivity to the parameter variations.

When parameter β remains constant, T and α influences on the AVIS operation quality were shown to be insignificant.

Therefore, the synthesized quasi-optimal regulator provides the regulation quality parameters close to the adopted optimal criterion.

4. Experimental results

Before conducting the real experiment, some preparations and preliminary experiments were carried out. Vibro-isolated object was imitated as a rectangular container with the 1000 kg mass.

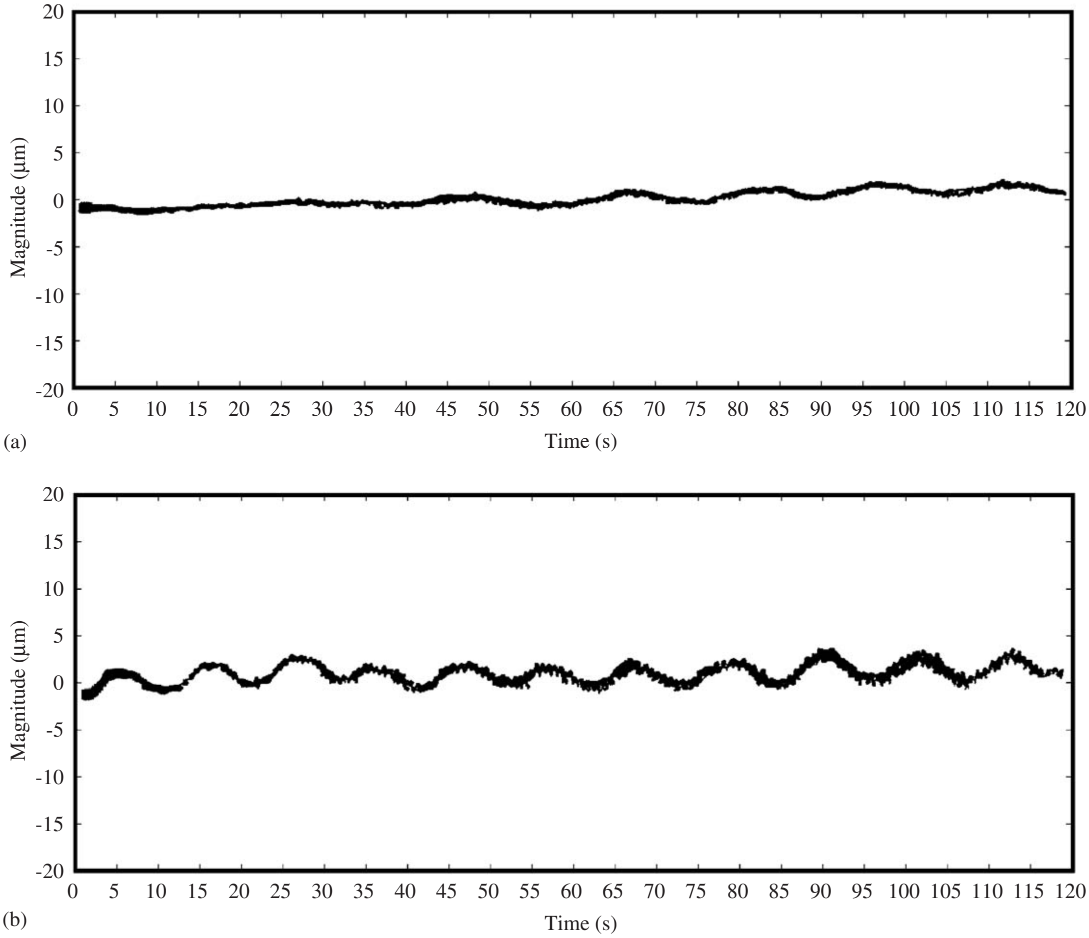


Fig. 9. Experimental results of AVIS with PID controller: (a) output deviation of AVIS; (b) disturbance from the foundation.

Four controlled pneumosupports were located between the ground and the container at the container's corners (one at each corner). The controlled pneumosupport is represented in Fig. 2. The scheme of the experimental equipment is shown in Fig. 7. An air pressure in the controlled pneumosupport up to 5.9 MPa was controlled with the pneumocontrol panel. The recording equipment registered disturbances from foundation and displacement of the vibro-isolated object. The recording equipment consists in the Bruel&Kjaer accelerometers and converter unites. The exciter generating 2–15 Hz harmonic motion frequencies disturbed the base. This frequency band matches a real disturbance frequency.

Experimental results showed the following. The quasi-optimal AVIS provides 5–7 times suppression of the disturbance (see Fig. 8). Fig. 9 shows the system's dynamic behaviour when a PID controller is applied. Vibration isolation performance is improved by 4–6 times. The transfer function of the PID controller is

$$W_{\text{PID}}(p) = \frac{b_{r1}p^2 + b_{r2}p + b_{r3}}{a_{r0}p + a_{r1}}. \tag{46}$$

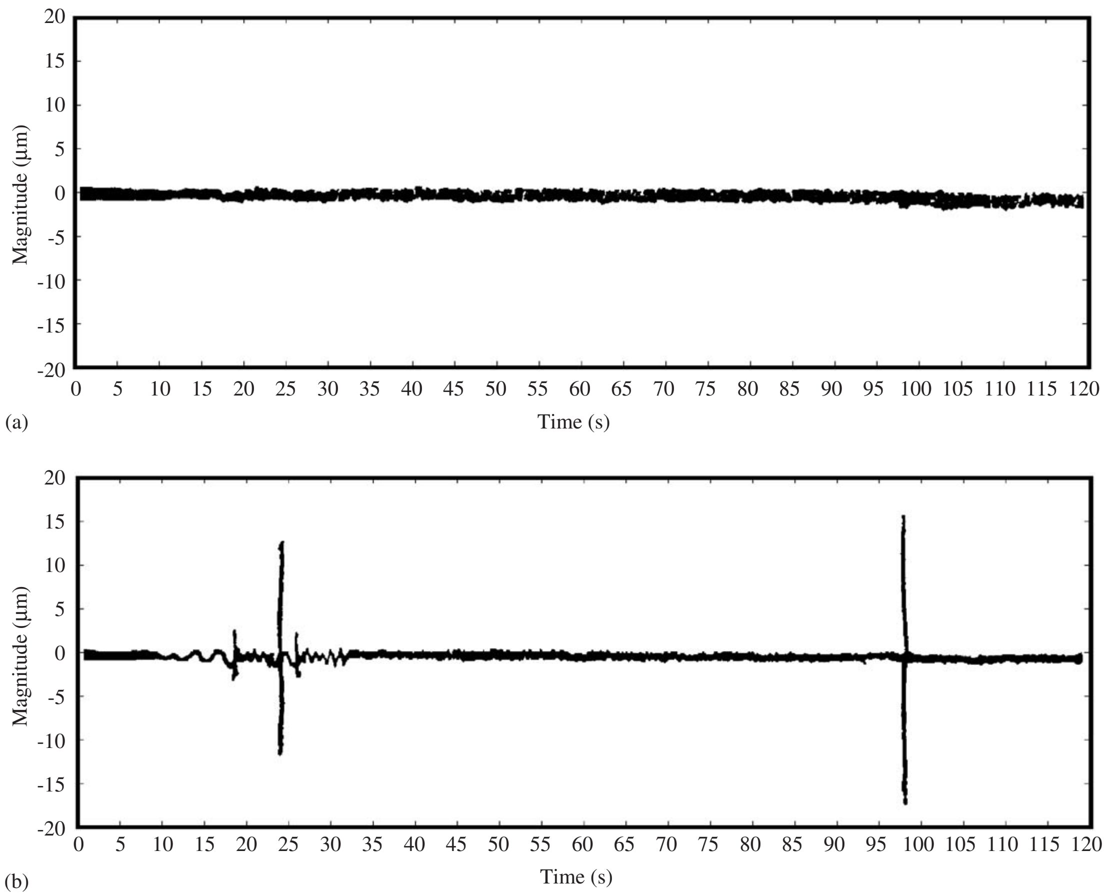


Fig. 10. Experimental results of the quasi-optimal AVIS: (a) output deviation of AVIS; (b) impulse disturbance from the foundation.

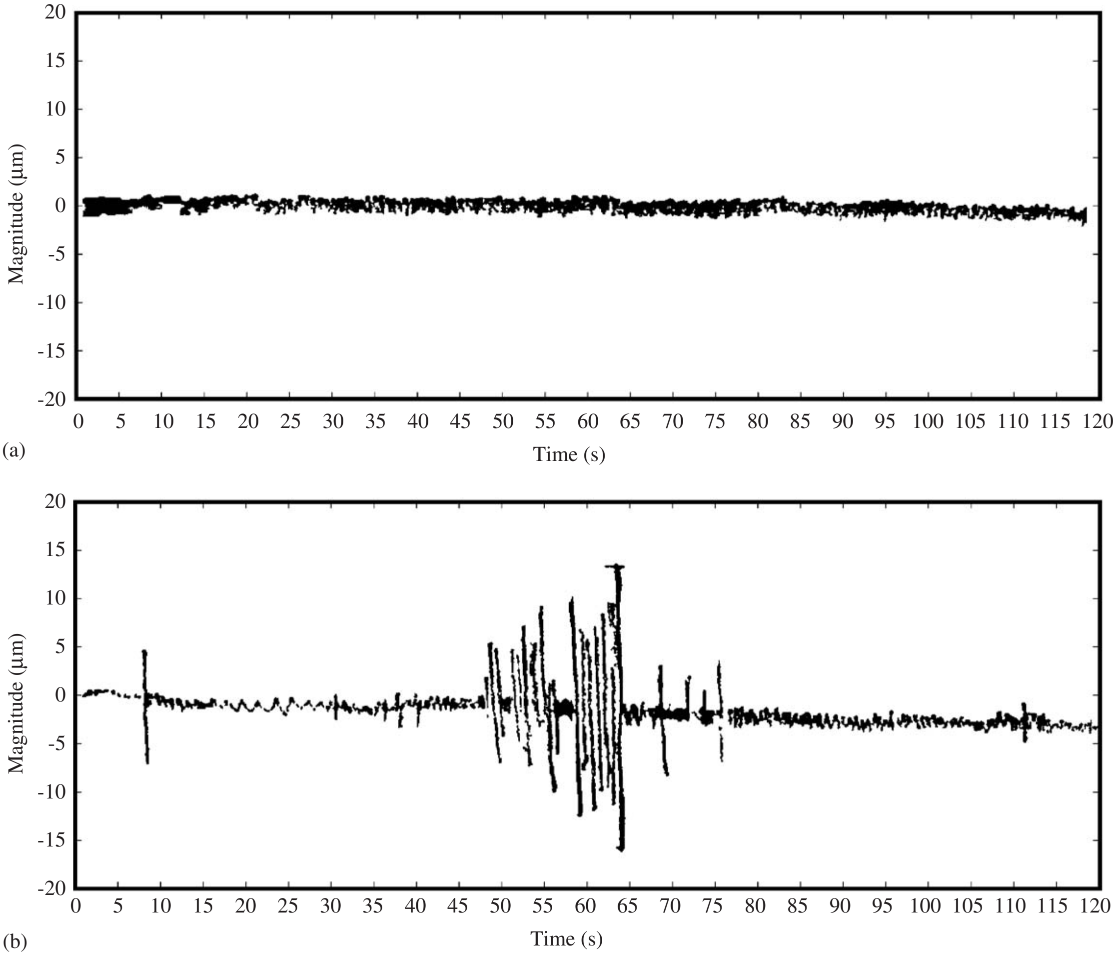


Fig. 11. Experimental results of AVIS with PID controller: (a) output deviation of AVIS; (b) impulse disturbance from the foundation.

To compare the quality indices of the above-mentioned systems make use of the mean square value from the output deviation. The results show that the quasi-optimal system allows 1.5–2 times better characteristics than a system with PID controller. When disturbance is an impulse signal, both systems suppress vibration effectively (see Figs. 10 and 11). Vibro-isolated object displacements are 15–20 times less in amplitude than those of the base. The impulse signal is simulated feasible switching of adjacent devices.

According to these experimental data the following conclusions can be reached. The disagreement between theoretical and experimental characteristics is quite acceptable, the discrepancy being about 10–15%. Thus the quasi-optimal algorithm guarantees better suppression disturbances than the PID control algorithm.

5. Conclusions

This paper proposes an optimal and quasi-optimal control algorithm for the purposes of protecting high precision facilities from foundation disturbances using AVIS. The algorithm allows, during synthesis, the characteristics of disturbances to be taken into account. In addition to the control signal restriction by the minimum control power, the maximum disturbance suppression is achieved. The minimum control power provides minimum energy losses and attains the lowest overheating condition. As a result the optimal AVIS operation is far more efficient.

Computation and experimental results confirm that there is a possibility to essentially reduce the displacement of the vibro-isolated object.

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