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Short Communication

A new derivation for journal bearing stiffness and damping coefficients in polar coordinates

J.K. Wang, M.M. Khonsari*

Department of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803, USA

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Abstract

A general derivation for obtaining bearing dynamic coefficients in polar coordinates is presented. It is shown that this derivation is simple and consistent with the commonly used formulae for the linearized stiffness and damping coefficients. What is more important is that the physical meaning of this derivation is clearer and simpler.

1. Introduction

The existing definitions of the linearized stiffness and damping coefficients in polar coordinates are derived from the Taylor series expansions of the radial component $\mathbf{f}_\varepsilon$ and tangential component $\mathbf{f}_\phi$ of the fluid force $\mathbf{f}$ (Fig. 1) and neglecting higher than first order terms. There are two commonly used approaches for arriving at definitions of these coefficients (see Section 2). However, they lead to inconsistencies in the final expressions for the stiffness and damping coefficients. Especially, when applied to infinitely short bearings, the inconsistencies become so obvious.

This paper presents a more general and straightforward derivation of bearing stiffness and damping coefficients in polar coordinates. The final expressions for the stiffness and damping coefficients are consistent with those in both Refs. [1,2].

*Corresponding author. Tel.: +1 225 578 9192; fax: +1 225 578 5924.
E-mail address: khonsari@me.lsu.edu (M.M. Khonsari).

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Nomenclature

b_{ij}	damping coefficients of fluid film bearing, N s/m, $(i,j=\varepsilon,\phi)$ (i is the direction of the force, j is the direction of the speed)
$\bar{b}_{ij}$	$\frac{(C/R)^3}{\mu L} b_{ij}\ (i,j=\varepsilon,\phi)$
C	radial clearance, m
f_ε	radial component of the fluid force, N
f_ϕ	tangential component of the fluid force, N
k_{ij}	stiffness coefficients of fluid film bearing, N/m, $(i,j=\varepsilon,\phi)$ (i is the direction of the force, j is the direction of the displacement)
$\bar{k}_{ij}$	$\frac{(C/R)^3}{\mu\omega L} k_{ij}\ (i,j=\varepsilon,\phi)$
L	bearing length, m
O_b	center of the journal bearing
O_j	center of the journal
R	journal radius, m
t	time, s
x	the coordinate in the horizontal direction
y	the coordinate in the vertical direction
μ	lubricant viscosity, Pa s
ω	running speed of the rotor, rad/s
ε	eccentricity ratio
ϕ	attitude angle

2. Existing inconsistencies between two kinds of existing definitions

Fig. 1 shows the sketch of the decomposition of the fluid force in journal bearing.

In Fig. 1, O_b is the center of the bearing; O_{js} is the steady-state equilibrium position of the journal center; O_j is the dynamic position of the journal center; ε and ϕ are the eccentricity ratio and attitude angle of the journal center, respectively; subscript s represents the steady-state equilibrium position; C is the radial clearance; $\mathbf{f}_\varepsilon$ and $\mathbf{f}_\phi$ are the radial and tangential components of the fluid forces $\mathbf{f}$ in journal bearing; x and y are the coordinates in the horizontal direction and vertical direction, respectively; $\mathbf{e}_\varepsilon$ and $\mathbf{e}_\phi$ are the unit vectors in radial and tangential directions, respectively. Referring to Fig. 1, the fluid force $\mathbf{f}$ is

$$\mathbf{f} = f_\varepsilon \mathbf{e}_\varepsilon + f_\phi \mathbf{e}_\phi = \begin{bmatrix} \mathbf{e}_\varepsilon & \mathbf{e}_\phi \end{bmatrix} \begin{bmatrix} f_\varepsilon \\ f_\phi \end{bmatrix}. \tag{1}$$

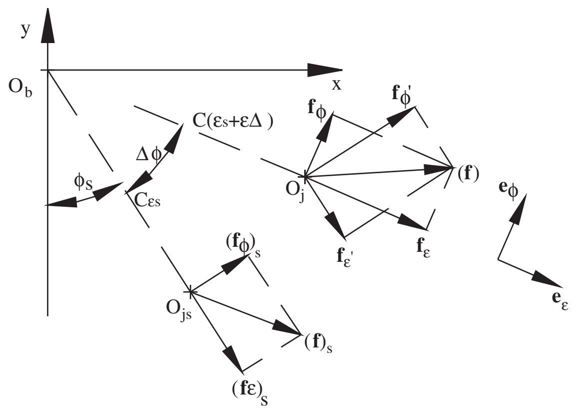

Fig. 1. Sketch of the decomposition of the fluid force in journal bearing.

The two commonly used definitions mentioned in Section 1 are both based on the same Taylor series expansions of the radial component and tangential component of the fluid force **f** separately while neglecting higher than first-order terms as follows [1–3].

$$f_\varepsilon = (f_\varepsilon)_s + \left[\frac{\partial f_\varepsilon}{\partial \varepsilon}\ \frac{\partial f_\varepsilon}{\partial \phi}\ \frac{\partial f_\varepsilon}{\partial \dot{\varepsilon}}\ \frac{\partial f_\varepsilon}{\partial \dot{\phi}}\right]\begin{bmatrix}\Delta\varepsilon\\ \Delta\phi\\ \Delta\dot{\varepsilon}\\ \Delta\dot{\phi}\end{bmatrix},$$

$$f_\phi = (f_\phi)_s + \left[\frac{\partial f_\phi}{\partial \varepsilon}\ \frac{\partial f_\phi}{\partial \phi}\ \frac{\partial f_\phi}{\partial \dot{\varepsilon}}\ \frac{\partial f_\phi}{\partial \dot{\phi}}\right]\begin{bmatrix}\Delta\varepsilon\\ \Delta\phi\\ \Delta\dot{\varepsilon}\\ \Delta\dot{\phi}\end{bmatrix}.$$

The above equations can be written in the following form:

$$\begin{pmatrix} f_\varepsilon - (f_\varepsilon)_s \\ f_\phi - (f_\phi)_s \end{pmatrix} = \begin{bmatrix} \dfrac{\partial f_\varepsilon}{C\partial\varepsilon} & \dfrac{\partial f_\varepsilon}{C\varepsilon\partial\phi} \\ \dfrac{\partial f_\phi}{C\partial\varepsilon} & \dfrac{\partial f_\phi}{C\varepsilon\partial\phi} \end{bmatrix} \begin{pmatrix} C\Delta\varepsilon \\ C\varepsilon\Delta\phi \end{pmatrix} + \begin{bmatrix} \dfrac{\partial f_\varepsilon}{C\partial\dot{\varepsilon}} & \dfrac{\partial f_\varepsilon}{C\varepsilon\partial\dot{\phi}} \\ \dfrac{\partial f_\phi}{C\partial\dot{\varepsilon}} & \dfrac{\partial f_\phi}{C\varepsilon\partial\dot{\phi}} \end{bmatrix} \begin{pmatrix} C\Delta\dot{\varepsilon} \\ C\varepsilon\Delta\dot{\phi} \end{pmatrix}, \tag{2}$$

where all the first-order derivatives are evaluated at the steady-state equilibrium position (ε_s, ϕ_s).

2.1. Direct deduction of the stiffness and damping coefficients

One of the approaches in the existing literature arrives at the definition of the stiffness coefficients (k_{ij}) and damping coefficients (b_{ij}) from the Taylor series expansions—Eq. (2)—directly [3]:

$$\begin{pmatrix} f_\varepsilon - (f_\varepsilon)_s \\ f_\phi - (f_\phi)_s \end{pmatrix} = -\begin{bmatrix} k_{\varepsilon\varepsilon} & k_{\varepsilon\phi} \\ k_{\phi\varepsilon} & k_{\phi\phi} \end{bmatrix} \begin{pmatrix} C\Delta\varepsilon \\ C\varepsilon\Delta\phi \end{pmatrix} - \begin{bmatrix} b_{\varepsilon\varepsilon} & b_{\varepsilon\phi} \\ b_{\phi\varepsilon} & b_{\phi\phi} \end{bmatrix} \begin{pmatrix} C\Delta\dot{\varepsilon} \\ C\varepsilon\Delta\dot{\phi} \end{pmatrix}, \tag{3}$$

where

$$\begin{bmatrix} k_{\varepsilon\varepsilon} & k_{\varepsilon\phi} \\ k_{\phi\varepsilon} & k_{\phi\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial f_\varepsilon}{C\partial\varepsilon} & -\dfrac{\partial f_\varepsilon}{C\varepsilon\partial\phi} \\ -\dfrac{\partial f_\phi}{C\partial\varepsilon} & -\dfrac{\partial f_\phi}{C\varepsilon\partial\phi} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} b_{\varepsilon\varepsilon} & b_{\varepsilon\phi} \\ b_{\phi\varepsilon} & b_{\phi\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial f_\varepsilon}{C\partial\dot{\varepsilon}} & -\dfrac{\partial f_\varepsilon}{C\varepsilon\partial\dot{\phi}} \\ -\dfrac{\partial f_\phi}{C\partial\dot{\varepsilon}} & -\dfrac{\partial f_\phi}{C\varepsilon\partial\dot{\phi}} \end{bmatrix}. \tag{4}$$

2.2. Transformation approach for deriving the stiffness and damping coefficients

From Fig. 1, the following transformation is used [1,2]:

$$\begin{pmatrix} f'_\varepsilon \\ f'_\phi \end{pmatrix} = \begin{pmatrix} \cos\Delta\phi & -\sin\Delta\phi \\ \sin\Delta\phi & \cos\Delta\phi \end{pmatrix} \begin{pmatrix} f_\varepsilon \\ f_\phi \end{pmatrix}, \tag{5}$$

where $\mathbf{f}'_\varepsilon$ is the component of the fluid force $\mathbf{f}$ in the same direction as the fluid force component $(\mathbf{f}_\varepsilon)_s$ at the steady-state equilibrium position, and $\mathbf{f}'_\phi$ is the component of the fluid force $\mathbf{f}$ in the same direction as the fluid force component $(\mathbf{f}_\phi)_s$ at the steady-state equilibrium position.

For a small perturbation, $\Delta\phi \ll 1$, so that $\cos\Delta\phi \approx 1$, $\sin\Delta\phi = \Delta\phi$.

Using these two approximations, Eq. (5) can be rewriten as

$$\begin{pmatrix} f'_\varepsilon \\ f'_\phi \end{pmatrix} = \begin{pmatrix} f_\varepsilon \\ f_\phi \end{pmatrix} + \Delta\phi \begin{pmatrix} -f_\phi \\ f_\varepsilon \end{pmatrix}.$$

Then, the following definitions of the stiffness and damping coefficients are developed [1,2].

$$\begin{pmatrix} f'_\varepsilon - (f_\varepsilon)_s \\ f'_\phi - (f_\phi)_s \end{pmatrix} = -\begin{bmatrix} k_{\varepsilon\varepsilon} & k_{\varepsilon\phi} \\ k_{\phi\varepsilon} & k_{\phi\phi} \end{bmatrix} \begin{pmatrix} C\Delta\varepsilon \\ C\varepsilon\Delta\phi \end{pmatrix} - \begin{bmatrix} b_{\varepsilon\varepsilon} & b_{\varepsilon\phi} \\ b_{\phi\varepsilon} & b_{\phi\phi} \end{bmatrix} \begin{pmatrix} C\Delta\dot{\varepsilon} \\ C\varepsilon\Delta\dot{\phi} \end{pmatrix}, \tag{6}$$

where

$$\begin{bmatrix} k_{\varepsilon\varepsilon} & k_{\varepsilon\phi} \\ k_{\phi\varepsilon} & k_{\phi\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial f_\varepsilon}{C\partial\varepsilon} & -\dfrac{\partial f_\varepsilon}{C\varepsilon\partial\phi} + \dfrac{f_\phi}{C\varepsilon} \\ -\dfrac{\partial f_\phi}{C\partial\varepsilon} & -\dfrac{\partial f_\phi}{C\varepsilon\partial\phi} - \dfrac{f_\varepsilon}{C\varepsilon} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} b_{\varepsilon\varepsilon} & b_{\varepsilon\phi} \\ b_{\phi\varepsilon} & b_{\phi\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial f_\varepsilon}{C\partial\dot{\varepsilon}} & -\dfrac{\partial f_\varepsilon}{C\varepsilon\partial\dot{\phi}} \\ -\dfrac{\partial f_\phi}{C\partial\dot{\varepsilon}} & -\dfrac{\partial f_\phi}{C\varepsilon\partial\dot{\phi}} \end{bmatrix}. \tag{7}$$

2.3. Inconsistencies between two kinds of existing definitions

From Eqs. (4) and (7), it is very clear that some inconsistencies between the definitions of $k_{\varepsilon\phi}$ and $k_{\phi\phi}$ exist. The second definitions of $k_{\varepsilon\phi}$ and $k_{\phi\phi}$ each has one more coupled term than does the first set of equations presented in Section 2.1.

3. A new derivation of the definitions of stiffness and damping coefficients

Taking the derivative of both sides of Eq. (1) with respect to time t yields:

$$\frac{\mathrm{d}\mathbf{f}}{\mathrm{d}t} = \frac{\mathrm{d}f_\varepsilon}{\mathrm{d}t}\mathbf{e}_\varepsilon + \frac{\mathrm{d}f_\phi}{\mathrm{d}t}\mathbf{e}_\phi + f_\varepsilon\frac{\mathrm{d}\mathbf{e}_\varepsilon}{\mathrm{d}t} + f_\phi\frac{\mathrm{d}\mathbf{e}_\phi}{\mathrm{d}t}, \tag{8}$$

where

$$\begin{aligned} \frac{\mathrm{d}f_\varepsilon}{\mathrm{d}t} &= \frac{\partial f_\varepsilon}{\partial\varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \frac{\partial f_\varepsilon}{\partial\phi}\frac{\mathrm{d}\phi}{\mathrm{d}t} + \frac{\partial f_\varepsilon}{\partial\dot{\varepsilon}}\frac{\mathrm{d}\dot{\varepsilon}}{\mathrm{d}t} + \frac{\partial f_\varepsilon}{\partial\dot{\phi}}\frac{\mathrm{d}\dot{\phi}}{\mathrm{d}t}, \\ \frac{\mathrm{d}f_\phi}{\mathrm{d}t} &= \frac{\partial f_\phi}{\partial\varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \frac{\partial f_\phi}{\partial\phi}\frac{\mathrm{d}\phi}{\mathrm{d}t} + \frac{\partial f_\phi}{\partial\dot{\varepsilon}}\frac{\mathrm{d}\dot{\varepsilon}}{\mathrm{d}t} + \frac{\partial f_\phi}{\partial\dot{\phi}}\frac{\mathrm{d}\dot{\phi}}{\mathrm{d}t}. \end{aligned} \tag{9}$$

The derivatives of unit vectors $\mathbf{e}_\varepsilon$ and $\mathbf{e}_\phi$ are [4]

$$\begin{aligned} \frac{\mathrm{d}\mathbf{e}_\varepsilon}{\mathrm{d}t} &= \frac{\mathrm{d}\phi}{\mathrm{d}t}\mathbf{e}_\phi, \\ \frac{\mathrm{d}\mathbf{e}_\phi}{\mathrm{d}t} &= -\frac{\mathrm{d}\phi}{\mathrm{d}t}\mathbf{e}_\varepsilon. \end{aligned} \tag{10}$$

Substituting Eqs. (9) and (10) into Eq. (8) yields

$$\begin{aligned} \frac{\mathrm{d}\mathbf{f}}{\mathrm{d}t} = & \left(\frac{\partial f_\varepsilon}{\partial \varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \frac{\partial f_\varepsilon}{\partial \phi}\frac{\mathrm{d}\phi}{\mathrm{d}t} + \frac{\partial f_\varepsilon}{\partial \dot{\varepsilon}}\frac{\mathrm{d}\dot{\varepsilon}}{\mathrm{d}t} + \frac{\partial f_\varepsilon}{\partial \dot{\phi}}\frac{\mathrm{d}\dot{\phi}}{\mathrm{d}t} - \frac{\mathrm{d}\phi}{\mathrm{d}t}f_\phi\right)\mathbf{e}_\varepsilon \\ & + \left(\frac{\partial f_\phi}{\partial \varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \frac{\partial f_\phi}{\partial \phi}\frac{\mathrm{d}\phi}{\mathrm{d}t} + \frac{\partial f_\phi}{\partial \dot{\varepsilon}}\frac{\mathrm{d}\dot{\varepsilon}}{\mathrm{d}t} + \frac{\partial f_\phi}{\partial \dot{\phi}}\frac{\mathrm{d}\dot{\phi}}{\mathrm{d}t} + \frac{\mathrm{d}\phi}{\mathrm{d}t}f_\varepsilon\right)\mathbf{e}_\phi. \end{aligned}$$

Simplifying further, we obtain

$$\frac{\mathrm{d}\mathbf{f}}{\mathrm{d}t} = \begin{bmatrix} \mathbf{e}_\varepsilon & \mathbf{e}_\phi \end{bmatrix} \begin{bmatrix} \dfrac{\partial f_\varepsilon}{\partial \varepsilon}\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \left(\dfrac{\partial f_\varepsilon}{\partial \phi} - f_\phi\right)\dfrac{\mathrm{d}\phi}{\mathrm{d}t} + \dfrac{\partial f_\varepsilon}{\partial \dot{\varepsilon}}\dfrac{\mathrm{d}\dot{\varepsilon}}{\mathrm{d}t} + \dfrac{\partial f_\varepsilon}{\partial \dot{\phi}}\dfrac{\mathrm{d}\dot{\phi}}{\mathrm{d}t} \\ \dfrac{\partial f_\phi}{\partial \varepsilon}\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \left(\dfrac{\partial f_\phi}{\partial \phi} + f_\varepsilon\right)\dfrac{\mathrm{d}\phi}{\mathrm{d}t} + \dfrac{\partial f_\phi}{\partial \dot{\varepsilon}}\dfrac{\mathrm{d}\dot{\varepsilon}}{\mathrm{d}t} + \dfrac{\partial f_\phi}{\partial \dot{\phi}}\dfrac{\mathrm{d}\dot{\phi}}{\mathrm{d}t} \end{bmatrix}. \tag{11}$$

For a very short-time interval Δt when a small perturbation is applied to the journal that was in steady-state equilibrium position (ε_s, ϕ_s), Eq. (11) can be approximated as follows through multiplying the two sides by Δt.

$$\mathbf{f} - \mathbf{f}_s = \begin{bmatrix} \mathbf{e}_\varepsilon & \mathbf{e}_\phi \end{bmatrix} \begin{bmatrix} \dfrac{\partial f_\varepsilon}{\partial \varepsilon}\Delta\varepsilon + \left(\dfrac{\partial f_\varepsilon}{\partial \phi} - f_\phi\right)\Delta\phi + \dfrac{\partial f_\varepsilon}{\partial \dot{\varepsilon}}\Delta\dot{\varepsilon} + \dfrac{\partial f_\varepsilon}{\partial \dot{\phi}}\Delta\dot{\phi} \\ \dfrac{\partial f_\phi}{\partial \varepsilon}\Delta\varepsilon + \left(\dfrac{\partial f_\phi}{\partial \phi} + f_\varepsilon\right)\Delta\phi + \dfrac{\partial f_\phi}{\partial \dot{\varepsilon}}\Delta\dot{\varepsilon} + \dfrac{\partial f_\phi}{\partial \dot{\phi}}\Delta\dot{\phi} \end{bmatrix}. \tag{12}$$

The fluid force at the steady-state equilibrium position (ε_s, ϕ_s) is

$$\mathbf{f}_s = \begin{bmatrix} \mathbf{e}_\varepsilon & \mathbf{e}_\phi \end{bmatrix} \begin{bmatrix} (f_\varepsilon)_s \\ (f_\phi)_s \end{bmatrix}. \tag{13}$$

Substituting Eqs. (1) and (13) into Eq. (12), yields

$$\begin{bmatrix} \mathbf{e}_\varepsilon & \mathbf{e}_\phi \end{bmatrix} \begin{bmatrix} f_\varepsilon - (f_\varepsilon)_s \\ f_\phi - (f_\phi)_s \end{bmatrix} = \begin{bmatrix} \mathbf{e}_\varepsilon & \mathbf{e}_\phi \end{bmatrix} \begin{bmatrix} \dfrac{\partial f_\varepsilon}{\partial \varepsilon}\Delta\varepsilon + \left(\dfrac{\partial f_\varepsilon}{\partial \phi} - f_\phi\right)\Delta\phi + \dfrac{\partial f_\varepsilon}{\partial \dot{\varepsilon}}\Delta\dot{\varepsilon} + \dfrac{\partial f_\varepsilon}{\partial \dot{\phi}}\Delta\dot{\phi} \\ \dfrac{\partial f_\phi}{\partial \varepsilon}\Delta\varepsilon + \left(\dfrac{\partial f_\phi}{\partial \phi} + f_\varepsilon\right)\Delta\phi + \dfrac{\partial f_\phi}{\partial \dot{\varepsilon}}\Delta\dot{\varepsilon} + \dfrac{\partial f_\phi}{\partial \dot{\phi}}\Delta\dot{\phi} \end{bmatrix},$$

i.e.,

$$\begin{bmatrix} f_\varepsilon - (f_\varepsilon)_s \\ f_\phi - \left(f_\phi\right)_s \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_\varepsilon}{\partial\varepsilon}\Delta\varepsilon + \left(\dfrac{\partial f_\varepsilon}{\partial\phi} - f_\phi\right)\Delta\phi + \dfrac{\partial f_\varepsilon}{\partial\dot{\varepsilon}}\Delta\dot{\varepsilon} + \dfrac{\partial f_\varepsilon}{\partial\dot{\phi}}\Delta\dot{\phi} \\ \dfrac{\partial f_\phi}{\partial\varepsilon}\Delta\varepsilon + \left(\dfrac{\partial f_\phi}{\partial\phi} + f_\varepsilon\right)\Delta\phi + \dfrac{\partial f_\phi}{\partial\dot{\varepsilon}}\Delta\dot{\varepsilon} + \dfrac{\partial f_\phi}{\partial\dot{\phi}}\Delta\dot{\phi} \end{bmatrix}. \tag{14}$$

Rewriting Eq. (14), one arrives at the following equation:

$$\begin{bmatrix} f_\varepsilon - (f_\varepsilon)_s \\ f_\phi - \left(f_\phi\right)_s \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_\varepsilon}{C\partial\varepsilon} & \dfrac{\partial f_\varepsilon}{C\varepsilon\partial\phi} - \dfrac{f_\phi}{C\varepsilon} \\ \dfrac{\partial f_\phi}{C\partial\varepsilon} & \dfrac{\partial f_\phi}{C\varepsilon\partial\phi} - \dfrac{f_\varepsilon}{C\partial} \end{bmatrix} \begin{bmatrix} C\Delta\varepsilon \\ C\varepsilon\Delta\phi \end{bmatrix} + \begin{bmatrix} \dfrac{\partial f_\varepsilon}{C\partial\dot{\varepsilon}} & \dfrac{\partial f_\varepsilon}{C\varepsilon\partial\dot{\phi}} \\ \dfrac{\partial f_\phi}{C\partial\dot{\varepsilon}} & \dfrac{\partial f_\phi}{C\varepsilon\partial\dot{\phi}} \end{bmatrix} \begin{bmatrix} C\Delta\dot{\varepsilon} \\ C\varepsilon\Delta\dot{\phi} \end{bmatrix}. \tag{15}$$

Now, the stiffness and damping coefficients can be defined as follows:

$$\begin{bmatrix} f_\varepsilon - (f_\varepsilon)_s \\ f_\phi - \left(f_\phi\right)_s \end{bmatrix} = -\begin{bmatrix} k_{\varepsilon\varepsilon} & k_{\varepsilon\phi} \\ k_{\phi\varepsilon} & k_{\phi\phi} \end{bmatrix} \begin{bmatrix} C\Delta\varepsilon \\ C\varepsilon\Delta\phi \end{bmatrix} - \begin{bmatrix} b_{\varepsilon\varepsilon} & b_{\varepsilon\phi} \\ b_{\phi\varepsilon} & b_{\phi\phi} \end{bmatrix} \begin{bmatrix} C\Delta\dot{\varepsilon} \\ C\varepsilon\Delta\dot{\phi} \end{bmatrix}, \tag{16}$$

where

$$\begin{bmatrix} k_{\varepsilon\varepsilon} & k_{\varepsilon\phi} \\ k_{\phi\varepsilon} & k_{\phi\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial f_\varepsilon}{C\partial\varepsilon} & -\dfrac{\partial f_\varepsilon}{C\varepsilon\partial\phi} + \dfrac{f_\phi}{C\varepsilon} \\ -\dfrac{\partial f_\phi}{C\partial\varepsilon} & -\dfrac{\partial f_\phi}{C\varepsilon\partial\phi} - \dfrac{f_\varepsilon}{C\varepsilon} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} b_{\varepsilon\varepsilon} & b_{\varepsilon\phi} \\ b_{\phi\varepsilon} & b_{\phi\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial f_\varepsilon}{C\partial\dot{\varepsilon}} & -\dfrac{\partial f_\varepsilon}{C\varepsilon\partial\dot{\phi}} \\ -\dfrac{\partial f_\phi}{C\partial\dot{\varepsilon}} & -\dfrac{\partial f_\phi}{C\varepsilon\partial\dot{\phi}} \end{bmatrix} \tag{17}$$

The final expressions for the stiffness and damping coefficients described by Eq. (17) are consistent with the commonly used expressions described by Eq. (7) in Section 2.2. However, the physical meaning in this derivation is clearer and simpler. In Eq. (16), the change of the fluid force component $\mathbf{f}_\varepsilon$ is exactly the relative force change in the direction of $\mathbf{e}_\varepsilon$ due to the small perturbation, and the change of the fluid force component $\mathbf{f}_\phi$ is exactly the relative force change in the direction of $\mathbf{e}_\phi$ due to the small perturbation.

4. Verification and discussion

In this section, as an example, the definitions of stiffness and damping coefficients described by Eqs. (16) and (17) are applied to infinitely short bearing. Assuming that the short bearing theory with half-Sommerfeld boundary conditions applies [5], the fluid forces in the journal bearing are [6]

$$f_\varepsilon = -\frac{RL^3\mu}{2C^2}\left[\frac{2\varepsilon^2\left(\omega - 2\dot{\phi}\right)}{(1-\varepsilon^2)^2} + \frac{\pi\left(1+2\varepsilon^2\right)\dot{\varepsilon}}{(1-\varepsilon^2)^{5/2}}\right], \tag{18}$$

$$f_\phi = \frac{RL^3\mu}{2C^2}\left[\frac{\pi(\omega - 2\dot{\phi})\varepsilon}{2(1-\varepsilon^2)^{3/2}} + \frac{4\varepsilon\dot{\varepsilon}}{(1-\varepsilon^2)^2}\right]. \tag{19}$$

Substituting Eqs. (18) and (19) into Eqs. (17) and simplifying the resulting expressions, it can be shown that elements of the linearized stiffness matrix k_{ij} $(i,j = \varepsilon, \phi)$ are

$$\begin{aligned}
k_{\varepsilon\varepsilon} &= \frac{2\omega\mu RL^3\varepsilon(1+\varepsilon^2)}{C^3(1-\varepsilon^2)^3},\\
k_{\varepsilon\phi} &= \frac{\pi\omega\mu RL^3}{4C^3(1-\varepsilon^2)^{3/2}},\\
k_{\phi\varepsilon} &= -\frac{\pi\omega\mu RL^3(1+2\varepsilon^2)}{4C^3(1-\varepsilon^2)^{5/2}},\\
k_{\phi\phi} &= \frac{\varepsilon\omega\mu RL^3}{C^3(1-\varepsilon^2)^2}.
\end{aligned} \tag{20}$$

The elements of the damping matrix b_{ij} $(i,j = \varepsilon, \phi)$ are

$$\begin{aligned}
b_{\varepsilon\varepsilon} &= \frac{\pi\mu RL^3(1+2\varepsilon^2)}{2C^3(1-\varepsilon^2)^{5/2}},\\
b_{\varepsilon\phi} &= -\frac{2\varepsilon\mu RL^3}{C^3(1-\varepsilon^2)^2},\\
b_{\phi\varepsilon} &= -\frac{2\varepsilon\mu RL^3}{C^3(1-\varepsilon^2)^2},\\
b_{\phi\phi} &= \frac{\pi\mu RL^3}{2C^3(1-\varepsilon^2)^{3/2}}.
\end{aligned} \tag{21}$$

Using $\bar{k}_{ij} = \left((C/R)^3/\mu\omega L\right) k_{ij}$, $(i,j = \varepsilon, \phi)$ and $\bar{b}_{ij} = \left((C/R)^3/\mu L\right) b_{ij}$, $(i,j = \varepsilon, \phi)$, the linearized stiffness coefficients and damping coefficients can be normalized. It has been shown that the normalized expressions for the stiffness and damping coefficients are consistent with those in Refs. [1,2]. However, if the definitions described by Eqs. (3) and (4) are applied to the same infinitely short bearing theory, $k_{\varepsilon\phi} \equiv 0$ and $k_{\phi\phi} \equiv 0$ since both of the fluid force components f_ε and f_ϕ are not an explicit function of ϕ.

5. Conclusion

The derivations presented in this paper offer a simple procedure for arriving at the definitions of the stiffness and damping coefficients in polar coordinates. The method also offers a clear physical meaning of the dynamic coefficient and yields consistent results when applied to infinitely short bearings.

References

[1] R. Holmes, The vibration of a rigid rotor on short journal bearings, *Journal of Mechanical Engineering Science* 2 (1960) 337–341.
[2] A. Szeri, Linearized force coefficients of a 110° partial journal bearing, *Proceedings of the Institution of Mechanical Engineers* 181 (Part 3B) (1966–67) 130–133.
[3] D. Childs, H. Moes, H.V. Leeuwen, Journal bearing impedance description for rotor dynamic applications, *ASME Journal of Lubrication Technology* 99 (1977) 198–214.
[4] D.T. Greenwood, *Intermediate Dynamics*, second ed., Prentice-Hall, Englewood Cliffs, NJ, 1988.
[5] M.M. Khonsari, E.R. Booser, *Applied Tribology: Bearing Design and Lubrication*, Wiley, New York, 2001.
[6] M.M. Khonsari, Y.J. Chang, Stability boundary of non-linear orbits within clearance circle of journal bearings, *ASME Journal of Vibration and Acoustics* 115 (1993) 303–307.