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Direct observations of non-stationary bridge deck aeroelastic vibration in wind tunnel

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Abstract

It is useful to study the ambient non-stationary response of a cable-supported bridge to monitor the "aeroelastic status" of the structure. In the absence of effective non-stationary analyzing tools, it is suitable to have signal processing techniques to pre-examine the measured ambient responses for the time scale at which the aeroelastic system varies to know if the recording can be treated as a quasi-stationary process or must be treated as non-stationary one. Among the factors contributing to the non-stationarity, the effect of signature turbulence is studied in this paper by joint time–frequency analysis on wind tunnel experiments with a partially streamlined box girder sectional model. Hilbert transform combined with empirical mode decomposition method is used. Instantaneous frequency domain properties of the aeroelastic vibration are obtained. Time-dependant features of the interactive system are partially revealed.

1. Introduction

With the increasing number of cable-supported bridges in service, it is of practical importance to have systematic methods to monitor the "aeroelastic status" of a bridge under its working

Abbreviations: dof, degree of freedom; EMD, empirical mode decomposition; IMF, intrinsic mode function; IF, instantaneous frequency; MBW, mean bandwidth; MIF, mean instantaneous frequency; PCIF, IF of principal component; WAIF, weighted averaging instantaneous frequency.

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Nomenclature

$a(t)$	amplitude of an analytical signal
A	rotational displacement
$A(\tau, n_i)$	amplitude of an intrinsic mode function
$C_i(t)$	an intrinsic mode function
$E(t)$	energy of a wave
$h_{ik}(t)$	variables
H	vertical displacement
$m_i(t)$	local mean of a signal
n_i, $\langle n \rangle t$	instantaneous frequency (IF) and weighted averaging IF
P	lateral displacement
$R_i(t)$	residue signal
SD	standard deviation
t, t_1, t_2, τ	time
$X(t)$, $Y(t)$, $Z(t)$	signal
$\theta(t)$	angular phase
Δ_t^2	bandwidth

condition, i.e. to monitor the interaction between the wind forces and the bridge deck motion by interpreting the buffeting response. Most of current analysis methods available, (e.g. Ref. [1]) are to calculate the response spectra when the flutter derivatives [2] and the oncoming flow turbulence intensity are known; they are not suitable for the back analysis as in the case of monitoring.

It is well known that the measured buffeting responses of a bridge are usually non-stationary. Although the velocity fluctuation in the oncoming flow is the major concern, the signature turbulence generated by the structure itself may also be one of the reasons for the non-stationarity. It is possible to study this effect in wind tunnel experiments if the velocity fluctuation in the oncoming flow is small.

The postulated differences in the flutter derivatives for the ambient vibration and exponentially modified sinusoidal motion, as suggested by Zhang and Brownjohn [3], indicates that the interactive terms may be affected by the ‘‘relative amplitude’’, which, in their experiments, mainly reflects the effects of signature turbulence. The relative amplitude was defined as the ratio of the amplitude of triggered exponentially modified sinusoidal motion to the magnitude of the ‘‘structural noise’’ in the vibration due to the signature turbulence. Because the flutter derivatives were designed for pure or exponentially modified sinusoidal motions with effective damping less than 20% [2], the effect of signature turbulence on flutter derivatives was always associated with a physically or nominally triggered motion, which decays in the ‘‘noisy’’ ambient vibration environment.

It is reasonable to have the following extension: in the experiments, due to the random nature of the signature turbulence, the aeroelastic parameters of the system may be time dependent due to the time varying properties of the signature turbulence. However, in Ref. [3] the statement of nonlinearity in the self-excited wind forces is based on output covariance analysis (e.g. Ref. [4]), which inevitably averages the parameters over so long a time interval that the ambient vibration was studied as a stationary phenomenon.

The instantaneous properties of the aeroelastic problem also need to be emphasized. Non-stationary processes, such as the ambient response of the sectional model, do not possess an ordinary spectral density because they have a time-dependent covariance structure. These covariance structures describe the evolution of the second-order properties of the process. In the very special case of weak stationary processes, the second-order properties can be successfully represented in the frequency domain by ordinary spectral densities. The definition of a time-varying spectrum can therefore be considered as a natural generalization of the idea of a spectrum, but additionally exhibiting the time-dependent changes of the second-order structure of the process. It should be noted that changes of this structure are not necessarily given by changes of the mean power of the process alone. In particular, if one deals with non-stationary processes possessing a slow evolution time of the second-order structure (quasi-stationary), the time-varying spectrum will exhibit different stationary processes within different time intervals and frequency bands [5]. An appropriate general methodology does not exist for analyzing the properties of all types of non-stationary random data from individual sample records [6]. This is due partly to the fact that a non-stationary conclusion is a negative statement specifying only a lack of stationary properties, rather than a positive statement defining the precise nature of the non-stationarity.

It follows that special techniques must be developed for non-stationary data that apply only to limited classes of these data. Classic works of Gabor [7], Ville [8] and Page [9] have been developed as alternatives for the varying spectra. The basic idea is to devise a joint function of time and frequency, a distribution that will describe the energy density or intensity of a signal simultaneously in time and frequency domain. Normally the basic feature behind these techniques is the time–frequency plane, which is the conjunction of the time and frequency domain signal representations.

In the case of the aeroelastic testing of a bridge, the time–frequency analysis may provide the information regarding to the time scale and magnitude at which the aeroelastic system varies. The former gives the time scale information about whether the response can be seen as quasi-stationary one or must be seen as non-stationary one; the latter helps to determine the errors if the non-stationary process is considered as stationary one. The former defines the objective of this paper. Although the major concern in bridge aeroelasticity is the aerodynamic damping, due to complexity in the non-stationary cases, the authors would like to focus on the frequency changes only at current stage and leave the aerodynamic damping issue for later studies.

This paper is only to identify some of the characteristic problems in the direct observation of the non-stationarity in ambient aeroelastic vibration due to the signature turbulence and explore some of the available tools for the possibility of applying them for the understanding of the phenomenon.

Considering the complexity of the problem, the discussions in this paper apply to the specific sectional type of the model under the experimental condition of this study only. Generalized remarks applicable to different sectional shapes and experimental conditions are beyond the scope of this paper.

2. Instantaneous frequency (IF) and empirical mode decomposition (EMD)

In order to study the instantaneous property of the aeroelastic system, instantaneous frequency should be defined. Among the different definitions, the most frequently used one is by the Hilbert transform. For an arbitrary time series, $X(t)$, its Hilbert transform, $Y(t)$, is

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} \, \mathrm{d}t', \tag{1}$$

where P is the Cauchy principle value.

$X(t)$ and $Y(t)$ form a complex conjugate pair, so we can have an analytical signal:

$$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)}, \tag{2}$$

where $a(t) = [X^2(t) + Y^2(t)]^{0.5}$, $\theta(t) = \arctan\left[Y(t)/X(t)\right]$.

Essentially, Eq. (1) defines the Hilbert transform as the convolution of $X(t)$ with $1/t$; therefore, it emphasizes the local properties of $X(t)$. In Eq. (2), the polar coordinate expression further clarifies the local nature of this representation: it is the best local fit of an amplitude and phase varying trigonometric function to $X(t)$. With the Hilbert transform, the instantaneous frequency is defined as

$$n(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}. \tag{3}$$

In principle, the instantaneous frequency given in Eq. (3) is a single value function of time. At any given time, there is only one frequency value; therefore, it can only represent one component, hence the signal must be 'mono-component' or narrow banded. In the sense of practical data processing, in order to obtain meaningful instantaneous frequency, additional restrictive conditions have to be imposed on the data: (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one to make sure the data is narrow banded; and (2) at any point, the local mean value of the signal is zero.

Generally, the recordings of bridge deck response to ambient wind excitations will not satisfy these two conditions. Therefore, the EMD procedure [10] is used to decompose the original recordings into series of intrinsic mode functions (IMF), which will meet these conditions individually.

The nature of the EMD is a series of processes shifting the signal $X(t)$ by its local mean $m_1(t)$, i.e.

$$X(t) - m_1(t) = h_1(t). \tag{4}$$

This operation repeats. In this process, the $k-1$th result, $h_{1(k-1)}(t)$, is used as the signal to calculate the kth result, $h_{1k}(t)$, i.e.

$$h_{1(k-1)}(t) - m_{1k}(t) = h_{1k}(t). \tag{5}$$

It is required by the EMD method that the shifting process should not be applied too many times to avoid obliterating the physically meaningful amplitude fluctuations. The standard deviation computed from two consecutive shifts, i.e.

$$SD = \sum_{t=0}^{T}\left[\frac{\left|h_{1(k-1)}(t) - h_{1k}(t)\right|^2}{h_{1(k-1)}^2(t)}\right] \tag{6}$$

should fall between 0.2 and 0.3 for 1024 data points. Therefore, in this paper, the standard deviation is set to n*0.2 to n*0.3 for n*1024 data points.

When $h_{1k}(t)$ meets the aforementioned two conditions, it is named the first IMF, i.e.

$$C_1(t) = h_{1k}(t). \tag{7}$$

The first IMF is then removed from the original signal to produce the first residue signal $r_1(t)$, i.e.

$$X(t) - C_1(t) = r_1(t). \tag{8}$$

The first residue signal is then used as the signal for second round of shifting to obtain the second IMF: $C_2(t)$. This process continues until nIMFs are found and the residue signal is the mean trend or constant. The original signal is then expressed as

$$X(t) = \sum_{i=1}^{n} C_i(t) + r_n(t). \tag{9}$$

Instantaneous frequencies can be calculated for each IMF by using Eq. (1)–(3).

According to Huang et al. [10], the IMFs identified in such way form a complete and orthogonal basis for the signal decomposition. Because the procedure is empirical, it is adaptive to individual data sets.

The intrinsic mode functions in the EMD method may not always have physical meaning. They may just appear as numerical phenomena. Therefore, for a recording full of uncertainties, such as a general motion of the bridge deck due to ambient wind excitations, not all the IFs corresponding to the IMFs may represent a physical aeroelastic state. This study will use Hilbert transform and EMD in a ‘‘conservative’’ way, not looking into all the details that can be obtained.

3. Direct observation of general aeroelastic motions

The general aeroelastic motions in this study are the free vibrations of an elastically supported rigid sectional bridge deck model due to ambient wind excitations. The typical turbulence intensity was less than 2%, therefore, the oncoming flow was considered smooth. Even so, the bridge started to response when the wind speed increased. This is because not only the small fluctuations in the oncoming flow, but also the signature turbulences generated by the bluff model itself serve as sources of excitation. Under this condition, the non-stationarity in the response reflects the effect of signature turbulence [3].

In the experiment, a partially streamlined box girder section (Fig. 1) was suspended on a three-dimensional suspension system (Fig. 2). The natural frequencies of the suspension system under no-wind condition are 4.18, 5.66 and 3.20 Hz in vertical, rotational and lateral (parallel to wind) directions, respectively. Wind was then applied. Laser displacement sensors were used to record the displacement time histories of vertical rotational and lateral motions. The sampling rate was 200 Hz, to allow reasonable averaging of instantaneous frequencies over a period of time without losing the concentration of the identified frequencies in time domain.

Three recordings of the general vibration due to ambient wind excitations were made at wind speeds of 17.5, 14 and 10 m/s. Each recording lasted for 5 min. Because the EMD procedure is a

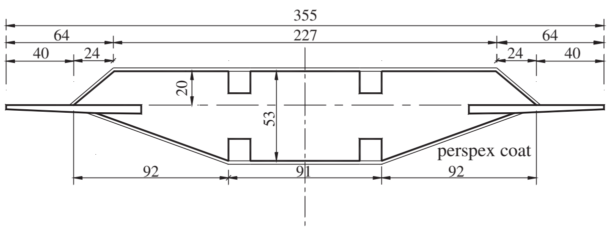

Fig. 1. Streamlined box girder model (dimension mm).

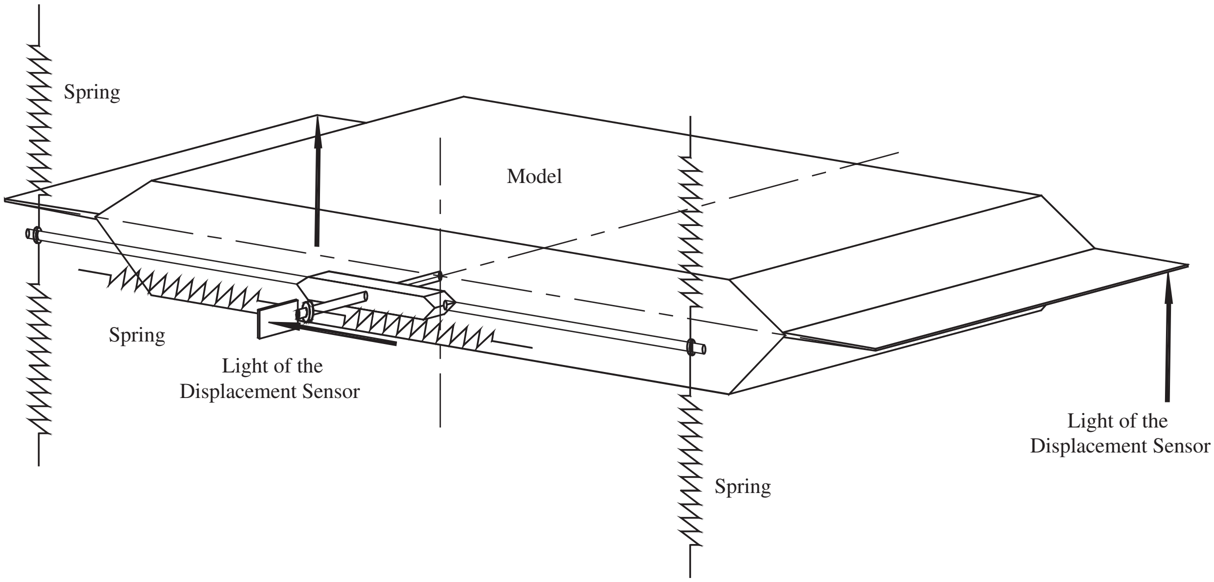

Fig. 2. Setup for vibration test (one end).

little time consuming, 5000 data, spanning for 25 s, were selected from the 5 min recordings for analysis with EMD method.

Because the EMD procedure would normally pick up noise as first several IMFs, which are apparently not meaningful signals for this study, the original recordings were de-noised using wavelet packets before being subjected to EMD.

A record of the vertical motion at wind speed of 17.5 m/s and its IMFs are shown in Fig. 3. The EMD procedure produces six IMFs and a residue signal. The residual is shown in the upper plot in darker color together with the original displacement time history. The main body of the recording is contained in IMF Nos. 2 and 3.

It can be observed that the IMF No. 1 is a pulse like signal. Between the pulses, the small amplitude parts are actually noise components which have not been removed totally by the de-noise procedure. If an IMF contains only noise, it can be safely deleted. This IMF, because of the meaningful components in it, needs to be included. The noise components so introduced will not be a problem as they are small in amplitude and can be dealt with in the following procedures.

The residue signal in the upper plot of Fig. 3 is the "left over" of the EMD process. It equals the difference between the displacement time history and the summation of the IMFs. If the EMD process continues, it will generate more IMFs and a less fluctuating residue. As a matter of fact, if the residue fluctuates in considerably lower frequency than the main frequency, it should be considered as the time-dependent equilibrium position of the oscillation. In this case, the EMD process should be stopped otherwise we will lose some useful information about the changes of equilibrium position, which is an indication of non-stationarity.

The residues of vertical, rotational and lateral displacement at wind speed of 17.5 m/s are shown in Fig. 4 together with respective time history and the instantaneous average wind speed. From this figure, it can be observed that the vertical and rotational movements change their equilibrium

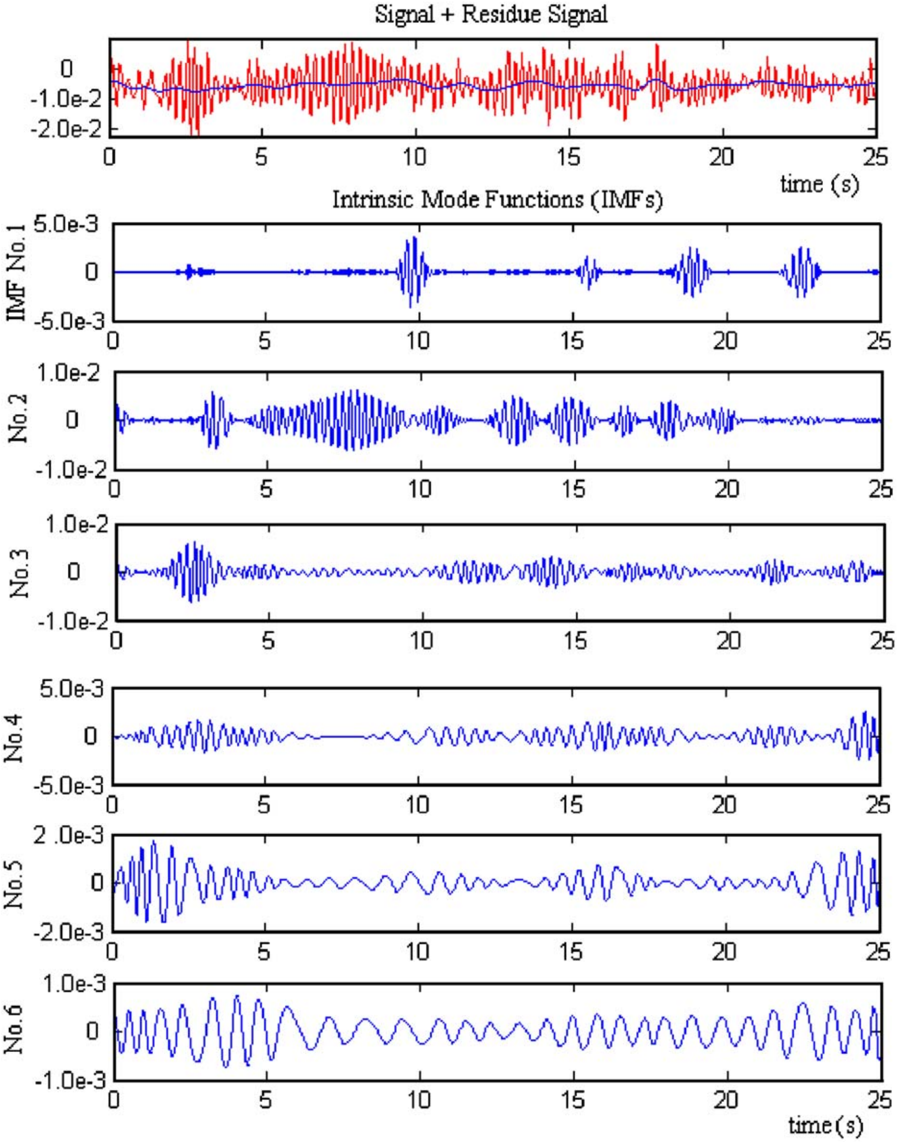

Fig. 3. Vertical displacements, intrinsic mode functions and residue signal.

position more frequently than the lateral movement. This is an indication that the non-stationarity is more active in the vertical and rotational motion than in the lateral motion.

The IMFs are then subjected to Hilbert transform for instantaneous frequencies. The Hilbert transform of the IMF Nos. 2 and 3 of the vertical displacement time history is shown in Fig. 5. The role of the Hilbert transform may be explained as follows: while sampled data are comprised of pure real numbers, it is treated as a set of complex numbers for the purpose of signal processing. For the imaginary component of the signal to be zero, according to signal processing theory, each positive frequency must be canceled by a corresponding negative frequency. The

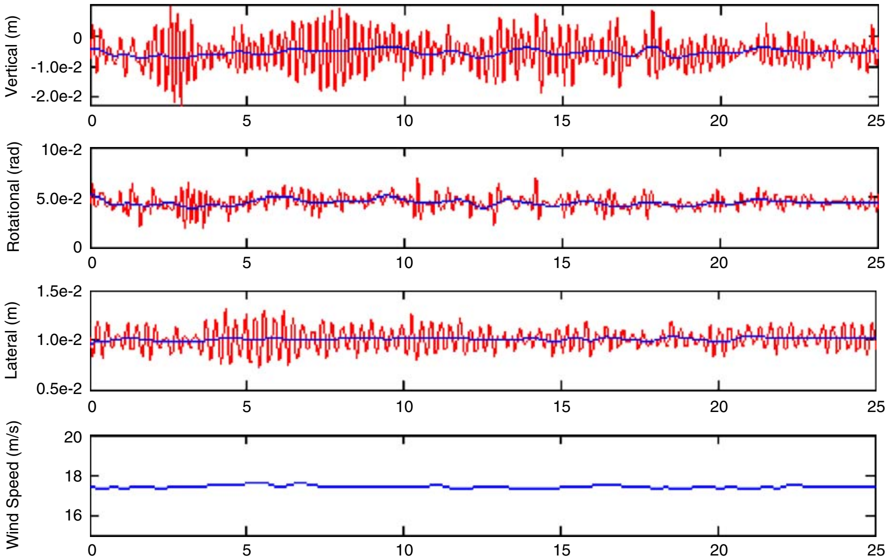

Fig. 4. Changing equilibrium position.

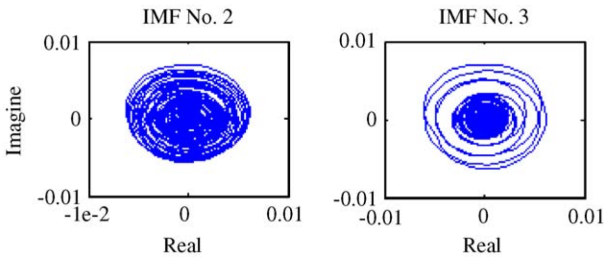

Fig. 5. Hilbert transform of IMF No. 2, 3 of vertical displacement.

Hilbert transform essentially subtracts out just negative frequencies, which means the resulting signal becomes complex. When graphed in a complex plane, the resulting signal will tend to follow a locally circular path as shown in Fig. 5. The first derivative of the polar phase angle with respect to time is instantaneous frequency.

A segment of instantaneous frequencies so identified for all the IMFs of the vertical displacement time history are shown in Fig. 6. It can be observed that there are independent frequencies for each IMF. The first IMF shows the largest dispersal among the IMFs. This is because the first IMF still carries some noises although the signal has already been de-noised before hand. Other IMFs show clear frequency trends changing with time.

4. Weighted average instantaneous frequency

Normally, other signal analysis methods would treat a changing instantaneous frequency as a series of waves with constant frequencies. One wave begins where another wave ends. This is

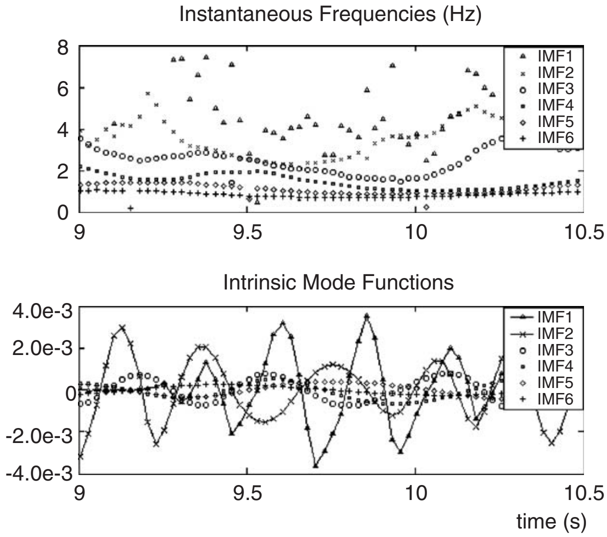

Fig. 6. Instantaneous frequencies of each IMF from vertical motion.

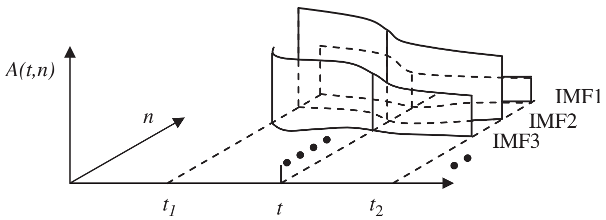

Fig. 7. Definition of WAIF.

referred to as inter-wave frequency modulation. The Hilbert transform also allows this, through having multiple IMFs, but by having instantaneous frequency value for each point, it allows for a new interpretation: changing frequencies within one IMF represents an intra-wave frequency modulation.

However, due to the highly uncertain factors in the turbulence and the interaction mechanism under investigation, also due to the unclear physical meaning of individual IMF, it is not safe to run into such a delicate conclusion. At current stage, we would rather focus on the overall frequency for all IMFs in one direction, instead of one IMF. For the vibration in individual directions: vertical, rotational and lateral directions, there should be mainly one predominant frequency, which may be changing from time to time. Therefore we define the weighted average instantaneous frequency (WAIF) using the instantaneous energy in each IMF as a weight.

As shown in Fig. 7, for all the IMFs in the given time window with width of $2w$ centered around time t, $t_1 = t - w$, $t_2 = t + w$, $t = (t_2 + t_1)/2$, the total energy in the window is the integration over time $\tau \in [t_1, t_2]$ of the summation of the square of all the IMF amplitude $A(t,\ n_i)$ with instantaneous frequency of n_i, i.e.

$$E(t) = \int_{t_1}^{t_2} \sum_i \left|A(\tau, n_i)\right|^2 \mathrm{d}\tau. \tag{10}$$

The WAIF at time t is the first-order moment of instantaneous frequency

$$\langle n \rangle_t = \frac{\int_{t_1}^{t_2} \sum_i n_i \left|A(\tau, n_i)\right|^2 \, \mathrm{d}\tau}{E(t)}. \tag{11}$$

The WAIF bandwidth is defined as

$$\Delta_t^2 = \int_{t_1}^{t_2} \sum_i (n_i - \langle n \rangle)^2 \frac{\left|A(\tau, n_i)\right|^2}{E(t)} \, \mathrm{d}\tau. \tag{12}$$

Therefore, we assume there is only one predominant frequency in each direction and the effect of the existence of other frequencies in smaller amplitude is taken into consideration by the computation of the bandwidth. The averaging window width in this study corresponds to five times of the sampling interval. The computed WAIFs of the general motion in three directions of the bridge deck model at wind speed of 17.5 m/s are shown in Fig. 8. The bandwidth is also shown in the figure by dotted symbols.

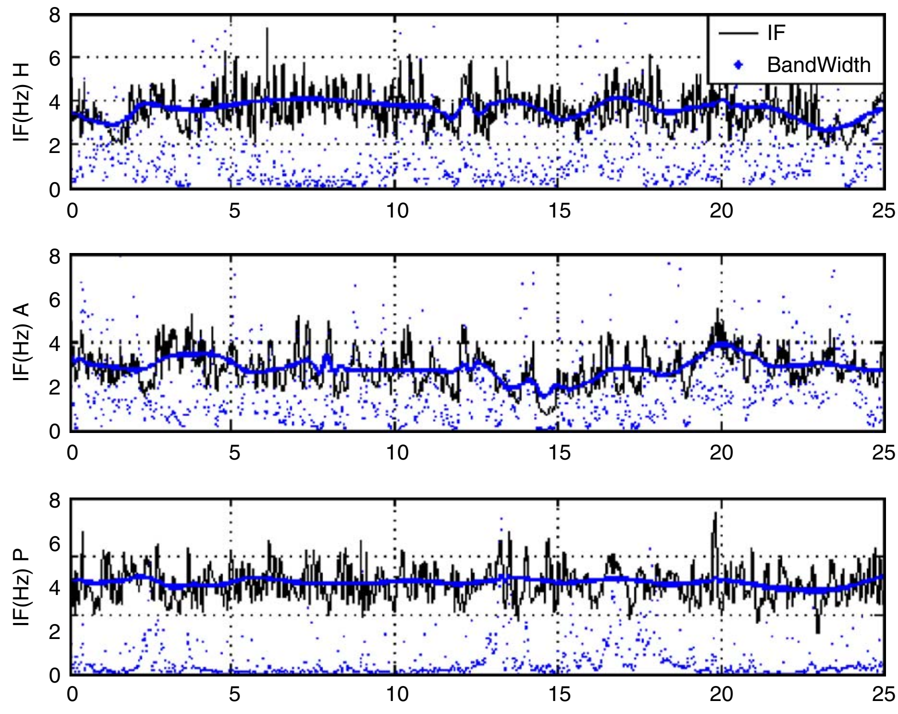

Fig. 8. Instantaneous frequencies and bandwidth (wind speed: 17.5 m/s).

The highly fluctuating part or the random part indicated by the thin continuous line does not necessarily indicate the non-stationarity of the interactive system as it does not seem to be correlated to the model vibration. The changing trend lines indicated by the thick continuous lines of the identified WAIF may be more reliable than the highly fluctuating part in observing the non-stationarity. These trend lines are computed by wavelet regression estimation tools. The basic wavelet used is db4 and the decomposition level is four. Because the WAIF is computed by using all the IMFs in the respective directions, it reflects the both intra-wave and inter-wave amplitude changes with respect to time.

From the figures, we have several observations:

1. The vertical and rotation WAIF fluctuate at times scales of the order of several seconds, i.e. the aeroelastic system may exhibit a noticeable change in the parameters within several circles of vibration of the rigid sectional model. In this case, it would be unreliable to treat the ambient vibration as a single frequency stationary process. The loss of concentration of energy in the frequency domain is the major reason for the fluctuation of the WAIF. When IMFs of instantaneous frequency other than the principle frequency exist, the value of WAIF will certainly be affected as a result of inter-wave energy transferring. This may indicate the weakness of the assumption of single predominant frequency in each direction, which has been widely accepted for the aeroelastic testing of sectional models. Generally speaking, the changes in the vertical and rotational motion are more frequent than in the lateral motion, which is parallel to wind direction. This phenomenon also indicates, as do the changing equilibrium position, that the non-stationarity is more apparent in the vertical and rotational direction than in the lateral motion.
2. The changes in frequency are also observed to correlate, to some extent, with the changes in the equilibrium position shown in Fig. 4. For example, between the time of 5 and 10 s, the equilibrium position of the vertical motion is relatively stable; accordingly, the vertical WAIF is also stable during this period of time. On the other hand, between the time of 10 and 20 s, the vertical equilibrium position changes more frequently; the vertical WAIF also experiences a bumping trend line. This might suggest one of the major factors affecting the instantaneous frequency is the unsteady equilibrium. However, more detailed research is needed to investigate the problem.
3. The bandwidth in the lateral direction is generally smaller than those in the other two directions. A wide bandwidth indicates the signal is multi-component while a narrow bandwidth indicates the signal is mono-component. In the former case, there is relatively more energy existing in the frequency other than the main frequency. It is still not clear what are the factors affecting the distribution of the energy among IMFs and why there are low frequency IMFs as has been shown in Fig. 6. Generally speaking, the bandwidth is smaller in the lateral direction than in the vertical and rotational directions.

According to these observations, it can be concluded that in the analysis of the ambient response due to signature turbulence, especially the rotational and vertical motions, non-stationarity should not be neglected. The aeroelastic system may change its parameters noticeably within several circles of oscillation. Current observations are not enough for detailed analysis of the non-stationary aeroelastic motion but it reveals the potential of the time–frequency analysis in tracking the time-dependent features of the general aeroelastic motion.

5. Comparison of stationary and non-stationary cases

Above observations show the time varying nature of the aeroelastic system under general motion even when the oncoming flow is considered as smooth flow. In order to compare these results with those from time invariant methods, frequencies from output covariance of the ambient response are also identified. The output covariance corresponds to the averaging scenario over the data length. Therefore, there is no time varying feature can be captured by it.

From Table 1, it can be observed that the mean instantaneous frequencies (WAIF), especially the rotational and vertical frequencies, are consistently lower than the corresponding frequencies identified from the output covariance for all the wind speed in the experiment. This is due to the presence of low frequency IMFs, e.g. IMF no. 4, 5, 6 in the vertical displacement as can be seen in Fig. 6. The lateral motion is affected very weakly.

The origin of these low frequency IMFs is not clear at current stage and needs to be investigated in future studies. They do not affect the identification by output covariance for the reason that during the identification, the system order, i.e. the degree of the freedom of the rigid model, is fixed to 3, (or 6 in the case of a state space representation), which is equivalent to assuming only one predominant frequency exists in each direction of motion. Information other than these 3 dofs are screened out by the fixed order, which is the usual practice in identifying the flutter derivatives. The measurement of vertical and rotational displacement might carry some information that has been overlooked.

It is also noticed in Fig. 3 that between the times of 5 and 10 s, the signal is relatively mono-component. The second IMF is the major part and other IMFs account for a very small fraction of the signal energy. This part of the second IMF is the principal component of the signal within this slot of time. Similar principal components can be observed in vertical displacement time history at different wind speed as well. Rotational displacements seem very transient and it is very hard to find such stable principal components, suggesting they are not mono-component at most of the time. On the other hand, the displacements in lateral direction are relatively more mono-component than vertical movement and the principle components are easier to find. Some of the IFs of the principal components (PCIF) are also given in Table 1. From these numbers, it can be seen the PCIFs are close to the frequency obtained by stationary method, indicating the importance of the principal component of the IMFs in analyzing the non-stationary aeroelastic

Table 1
Comparison of the output covariance and instantaneous cases

Wind speed		10 m/s			14 m/s			17.5 m/s			0 m/s		
Directions		H	A	P	H	A	P	H	A	P	H	A	P
OC	Frequency (Hz)	4.26	5.50	3.22	4.19	5.39	3.24	4.09	5.11	3.22	4.18	5.66	3.2
HT	WAIF (Hz)	3.9	4.3	3.1	3.9	3.7	3.1	3.6	3.5	3.1			
	MBW (Hz*Hz)	1.6	3.2	0.8	1.6	3.0	1.0	1.6	2.4	0.4			
	PCIF (Hz)	4.07	–	3.1	4.03	–	3.1	4.1	–	3.1	Not found		

OC: Output covariance; HT: Hilbert transform; WAIF: Weighted averaging instant frequency; MBW: Mean bandwidth; PCIF: Principal component IF.

vibration. However, at current stage, it seems impossible to control the decomposition process to have stable principal components.

Despite the unsolved problems, the comparison of stationary and non-stationary methods mentioned in this paper reveals the shortcomings of both cases. The former is lack of the ability to track the time-dependent properties, the latter needs to be improved to have more physically meaningful IMFs.

6. Conclusions

The analysis in this paper shows the non-stationarity in the ambient response of a partially streamlined box girder model due to the signature turbulence is not negligible. The time scale of the instantaneous frequency fluctuation in the vertical and rotational motion is of the order of several seconds. The parameters of the aeroelastic system may change noticeably within several circles of oscillation. To some extent, the fluctuation of the instantaneous frequency seems to correlate with the changes of the unsteady equilibrium positions.

The comparison of the non-stationary and stationary analysis results indicates the instantaneous frequency of the principle component of the IMF of the former case may correlate the result of the latter, which is the fixed-order realization of the aeroelastic system. However, such principle IMF may be hard to come by when the non-stationarity is strong as in the case of the rotational vibration.

Hilbert transform supplemented by empirical mode decomposition produces good time frequency representation of the ambient vibration.

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