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Hybrid identification method for multi-story buildings with unknown ground motion: theory

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Abstract

Simultaneous identification of both structural parameters and ground motion of an earthquake-excited structure by using measured structural response time histories only has received great interests in recent years. However, this paper demonstrates through a multi-story shear building that the structural parameters and the ground motion cannot be uniquely identified when the absolute forced structural response time histories are used directly. A hybrid identification method is thus proposed for the problem concerned. The hybrid identification method first identifies the structural parameters above the first floor of a multi-story shear building using the least-squares method after the corresponding parametric identification equation is established. The minimum modal information is then introduced to find the structural parameter of the first floor of the building to eliminate the non-uniqueness problem. After all the structural parameters are identified, the unknown earthquake-induced ground motion is finally constructed by solving a first-order differentiation equation. To enhance the capability of the hybrid identification method against measurement noise, an amplitude-selective filtering procedure is also proposed. Numerical example demonstrates the feasibility and efficiency of the hybrid identification method and the effectiveness of the amplitude-selective filtering procedure. The restriction of the proposed methodology in real application and future research on this topic are also pointed out.

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Nomenclature

$\mathbf{M}$	mass matrix
$\mathbf{K}$	stiffness matrix
$\mathbf{C}$	damping matrix
$\ddot{\mathbf{Y}}$, $\dot{\mathbf{Y}}$, $\mathbf{Y}$	acceleration, velocity and displacement response vector
$m_1, m_2, \ldots, m_n$	mass of each floor of shear building
$k_1, k_2, \ldots, k_n$	stiffness of each floor of shear building
$c_1, c_2, \ldots, c_n$	damping of each floor of shear building
$\mathbf{F}_I, \mathbf{F}_D, \mathbf{F}_E$	inertial, damping and elastic force vector
$\mathbf{H}$	coefficient matrix of identification equation
$\boldsymbol{\theta}$	parameter vector
a_0, a_1	damping constants
$\boldsymbol{\varphi}$	modal shape vector
ω	frequency
ζ	damping ratio

1. Introduction

System identification techniques using only measured structural responses to identify modal or structural parameters invoked great interests in the past few decades since external excitations such as wind forces or earthquake loads cannot be obtained or accurately measured under actual operating conditions. This is particularly true for large civil engineering structures such as tall buildings, long bridges, and offshore platforms. Great efforts have now been exerted to identify both structural parameters and unknown external excitations in the time domain.

Toki et al. [1] proposed a time-domain identification technique by which structural parameters and ground motion of an earthquake-excited structure could be identified using measured structural responses only. The coda of measured structural response time histories, which was treated as free vibration response without ground motion, was first utilized to identify the structural parameters in the time domain with the Kalman filter. The input ground motion was then estimated from the measured structural responses and the identified structural parameters. Wang and Haldar [2] developed an iterative least-squares method to identify simultaneously the structural parameters and ground motion of an earthquake-excited structure. Their method assumed that the ground motion at the first four time instances was zero so that the measured absolute structural responses at the first four time instances could be seen as the relative structural responses and the initial seismic forces could be evaluated. Using the initial seismic forces and the measured structural responses, the system parameters were then estimated using the least-squares method, and the seismic forces were re-evaluated from the estimated system parameters. The preceding identification steps were reiterated until the convergence of seismic forces was obtained. Wang and Haldar [3] also extended their identification method to the structures with limited observations. To reduce the sensitivity of the initial values of unknown structural parameters or ground motion to the identified results, Li and Chen [4] proposed a statistical average algorithm based on the equation of motion established in the relative coordinate system. The ground motion was first conjectured using the structural responses and the assumed initial values of structural parameters. The conjectured ground motions were then forced to be the same since the ground acceleration acting on a multi-story shear building in the relative coordinate system should be the same. The modified ground motion was further used to provide a new estimation of the structural parameters using the least-squares method. Repeating the iterative procedure until a present convergence criterion was reached then provided the simultaneous estimation of structural parameters and earthquake-induced ground motion.

The aforementioned identification procedures are all based on the equation of motion established in the relative coordinate system for an earthquake-excited structure. Since the absolute structural responses other than the relative structural responses are often measured in practice, the relative structural responses cannot be obtained if the ground motion is unknown. Thus, there is a limitation for the aforementioned procedures to be implemented in practice. In recognition of this restriction, Hoshiya and Sutoh [5] extended the Toki's identification procedure to account for absolute structural responses by using an extended Kalman filter. The basic steps in their procedure are, however, the same as the Toki's ones. That is, the coda of measured absolute structural response time histories was utilized to identify the structural parameters, and the input ground motion was then estimated from the measured absolute structural responses and the identified structural parameters.

It is noted that in practice, the separation of coda part from the entire structural response time history recorded during an earthquake event is quite difficult. Even though it is approximately obtained, the duration of the coda (so-called free vibration) of the structural response is very short and the amplitude of the free vibration is very small compared with the earthquake-induced structural vibration. Therefore, the accuracy of identified results may be significantly affected because of short duration and the measurement noise may become a serious problem. Furthermore, one may note that the damping parameters determined from low-amplitude free or ambient vibration tests differ greatly from the damping parameters obtained from earthquake-induced structural responses though it may not true for stiffness parameters [6].

This paper first discusses the uniqueness of solutions in the simultaneous identification of both structural parameters and ground motion using measured structural responses only. It then demonstrates through a multi-story shear building that the structural parameters and the ground motion cannot be uniquely identified when the absolute forced structural response time histories are used directly. A hybrid identification method is thus proposed for the problem concerned. The hybrid identification method first identifies the structural parameters above the first floor of a multi-story shear building using the least-squares method after the corresponding parametric identification equation is established. The minimum modal information is then introduced to find the structural parameter of the first floor of the building to eliminate the non-uniqueness problem. After all the structural parameters are identified, the unknown earthquake-induced ground motion is finally constructed by solving a first-order differentiation equation. To enhance the capability of the hybrid identification method against measurement noise, an amplitude-selective filtering procedure is also proposed. Numerical example is used to show the efficiency of the hybrid identification method and the effectiveness of the amplitude-selective filtering procedure.

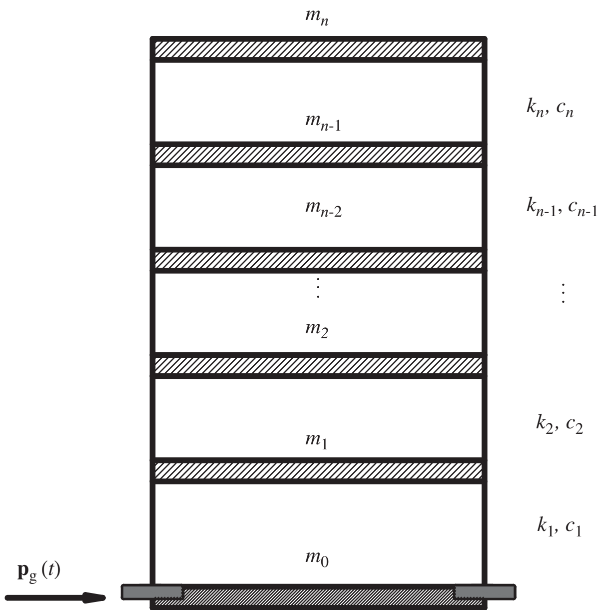

Fig. 1. Mechanical model of a shear building.

2. Uniqueness of identification

The equation of motion of an n-story shear building subject to earthquake-induced ground motion, as shown in Fig. 1 and established in the absolute coordinate system, can be expressed as

$$\begin{bmatrix} m_{gg} & \mathbf{M}_g \\ \mathbf{M}_g^{\mathrm{T}} & \mathbf{M} \end{bmatrix} \begin{Bmatrix} \ddot{Y}_0(t) \\ \ddot{\mathbf{Y}}(t) \end{Bmatrix} + \begin{bmatrix} c_{gg} & \mathbf{C}_g \\ \mathbf{C}_g^{\mathrm{T}} & \mathbf{C} \end{bmatrix} \begin{Bmatrix} \dot{Y}_0(t) \\ \dot{\mathbf{Y}}(t) \end{Bmatrix} + \begin{bmatrix} k_{gg} & \mathbf{K}_g \\ \mathbf{K}_g^{\mathrm{T}} & \mathbf{K} \end{bmatrix} \begin{Bmatrix} Y_0(t) \\ \mathbf{Y}(t) \end{Bmatrix} = \begin{Bmatrix} p_g(t) \\ \mathbf{0} \end{Bmatrix}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are, respectively, the mass, damping, and stiffness matrix of the building with the details as

$$\mathbf{M} = \mathrm{diag}(m_1, m_2, \ldots, m_n), \tag{2}$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & \ldots & 0 & 0 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 & 0 & 0 & 0 \\ 0 & \ldots & \ldots & \ldots & 0 & 0 & 0 \\ 0 & \ldots & -c_i & c_i + c_{i+1} & -c_{i+1} & \ldots & 0 \\ 0 & 0 & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & 0 & 0 & \ldots & -c_{n-1} & c_{n-1} + c_n & -c_n \\ 0 & 0 & 0 & 0 & \ldots & -c_n & c_n \end{bmatrix}, \tag{3}$$

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & -k_2 & \dots & 0 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & \dots & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 0 & 0 \\ 0 & \dots & -k_i & k_i+k_{i+1} & -k_{i+1} & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & -k_{n-1} & k_{n-1}+k_n & -k_n \\ 0 & 0 & 0 & 0 & \dots & -k_n & k_n \end{bmatrix}, \tag{4}$$

$\mathbf{0}$ represents the zero vector; $\ddot{\mathbf{Y}}(t)$, $\dot{\mathbf{Y}}(t)$ and $\mathbf{Y}(t)$ are, respectively, the absolute acceleration, velocity, and displacement response vector of the building; $\ddot{Y}_0(t)$, $\dot{Y}_0(t)$ and $Y_0(t)$ are, respectively, the absolute acceleration, velocity, and displacement of the foundation or the ground; $p_g(t)$ is the seismic force exerted on the foundation; $k_1, k_2, \dots, k_n$ and $c_1, c_2, \dots, c_n$ are, respectively, the stiffness and damping coefficients of the building for each story, which are the structural parameters to be identified; m_{gg}, c_{gg} and k_{gg} are, respectively, the mass, damping, and stiffness related to the foundation with the following relations derived from the dynamic equilibrium conditions:

$$m_{gg} = m_0, \quad c_{gg} = c_1 + c_2, \quad k_{gg} = k_1 + k_2, \tag{5}$$

$\mathbf{M}_g$, $\mathbf{C}_g$ and $\mathbf{K}_g$ are the coupling terms between the ground and superstructure, which are given as follows for a shear building:

$$\mathbf{M}_g = \mathbf{0}^{\mathrm{T}}, \quad \mathbf{C}_g = \begin{bmatrix} -c_1 & 0 & \cdots & 0 \end{bmatrix}, \quad \mathbf{K}_g = \begin{bmatrix} -k_1 & 0 & \cdots & 0 \end{bmatrix}. \tag{6}$$

The second of the two partitioned equations in Eq. (1) yields

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + \mathbf{C}\dot{\mathbf{Y}}(t) + \mathbf{K}\mathbf{Y}(t) = -\mathbf{C}_g \dot{Y}_0(t) - \mathbf{K}_g Y_0(t), \tag{7}$$

which is actually the equation of motion of the superstructure (building) excited by the ground motion.

According to Eq. (7), the inertial, damping and elastic forces of the shear building are respectively

$$\mathbf{F}_I(t) = \mathbf{M}\ddot{\mathbf{Y}}(t), \quad \mathbf{F}_D(t) = \mathbf{C}\dot{\mathbf{Y}}(t), \quad \mathbf{F}_E(t) = \mathbf{K}\mathbf{Y}(t). \tag{8}$$

Given the assumption that mass parameters are known, the damping and stiffness parameters can be extracted from damping forces and elastic forces based on response vector sensitivities [7]. When the forces are linear functions of structural parameters, as is the case in this study, the forces can be expressed as

$$\begin{aligned} \mathbf{F}_D(t) &= \frac{\partial \mathbf{F}_D(t)}{\partial c_1} c_1 + \cdots \frac{\partial \mathbf{F}_D(t)}{\partial c_i} c_i + \cdots + \frac{\partial \mathbf{F}_D(t)}{\partial c_n} c_n, \\ \mathbf{F}_E(t) &= \frac{\partial \mathbf{F}_E(t)}{\partial k_1} k_1 + \cdots \frac{\partial \mathbf{F}_E(t)}{\partial k_i} k_i + \cdots + \frac{\partial \mathbf{F}_E(t)}{\partial k_n} k_n. \end{aligned} \tag{9}$$

For a multi-story shear building, when $i = 1$

$$\frac{\partial \mathbf{F}_D(t)}{\partial c_1} = \begin{bmatrix} \dot{Y}_1(1) & 0 & \cdots & 0 \end{bmatrix}^{\mathrm{T}},$$
$$\frac{\partial \mathbf{F}_E(t)}{\partial k_1} = \begin{bmatrix} Y_1(1) & 0 & \cdots & 0 \end{bmatrix}^{\mathrm{T}}, \tag{10}$$

when $1 < i \leqslant n$

$$\frac{\partial \mathbf{F}_D(t)}{\partial c_i} = \begin{bmatrix} 0 & \cdots & 0 & \underset{i-1}{\dot{Y}_{i-1}(t) - \dot{Y}_i(t)} & \underset{i}{\dot{Y}_i(t) - \dot{Y}_{i-1}(t)} & 0 & \cdots & 0 \end{bmatrix}^{\mathrm{T}},$$
$$\frac{\partial \mathbf{F}_E(t)}{\partial k_i} = \begin{bmatrix} 0 & \cdots & 0 & \underset{i-1}{Y_{i-1}(t) - Y_i(t)} & \underset{i}{Y_i(t) - Y_{i-1}(t)} & 0 & \cdots & 0 \end{bmatrix}^{\mathrm{T}}. \tag{11}$$

Thus, one may have

$$\begin{aligned} \mathbf{F}_D(t) + \mathbf{F}_E(t) &= \mathbf{H}_D(t)\boldsymbol{\theta}_D + \mathbf{H}_E(t)\boldsymbol{\theta}_E \\ &= \mathbf{H}(t)\boldsymbol{\theta}, \end{aligned} \tag{12}$$

where

$$\mathbf{H}_D(t) = \begin{bmatrix} \frac{\partial \mathbf{F}_D(t)}{\partial c_1} & \cdots & \frac{\partial \mathbf{F}_D(t)}{\partial c_n} \end{bmatrix}, \quad \boldsymbol{\theta}_D = \begin{bmatrix} c_1 & \cdots & c_n \end{bmatrix}^{\mathrm{T}},$$
$$\mathbf{H}_E(t) = \begin{bmatrix} \frac{\partial \mathbf{F}_E(t)}{\partial k_1} & \cdots & \frac{\partial \mathbf{F}_E(t)}{\partial k_n} \end{bmatrix}, \quad \boldsymbol{\theta}_E = \begin{bmatrix} k_1 & \cdots & k_n \end{bmatrix}^{\mathrm{T}},$$
$$\mathbf{H}(t) = \begin{bmatrix} \mathbf{H}_D(t) & \mathbf{H}_E(t) \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_D^{\mathrm{T}} & \boldsymbol{\theta}_E^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \tag{13}$$

The matrix $\mathbf{H}(t)$ consists of the velocity and displacement responses while the vector $\boldsymbol{\theta}$ contains the damping and stiffness parameters to be identified. By using Eq. (12), Eq. (7) can be rewritten as

$$\mathbf{F}_I(t) + \mathbf{H}(t)\boldsymbol{\theta} = \mathbf{F}(t), \tag{14}$$

where $\mathbf{F}(t)$ has the following form for the shear building:

$$\mathbf{F}(t) = \begin{bmatrix} c_1 Y_0(t) + k_1 Y_0(t) & 0 & \cdots & 0 \end{bmatrix}^{\mathrm{T}}. \tag{15}$$

Excluding the first equation in Eq. (14) then yields

$$\tilde{\mathbf{F}}_I(t) + \tilde{\mathbf{H}}(t)\tilde{\boldsymbol{\theta}} = \mathbf{0}, \tag{16}$$

where the terms with "$\sim$" contain the 2 to n entries of their corresponding terms in Eq. (14).

Assembling Eq. (16) at all sampling instants $(t_1, \ldots, t_m)$ together leads to

$$\tilde{\mathbf{H}}\tilde{\boldsymbol{\theta}} = -\tilde{\mathbf{F}}_I, \tag{17}$$

where

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{\mathbf{H}}(t_1) \\ \vdots \\ \tilde{\mathbf{H}}(t_m) \end{bmatrix}, \quad \tilde{\mathbf{F}}_I = \begin{bmatrix} \tilde{\mathbf{F}}_I(t_1) \\ \vdots \\ \tilde{\mathbf{F}}_I(t_m) \end{bmatrix}.$$

The least-squares method can then be used to solve Eq. (17), from which the structural parameters above the first floor of the building can be uniquely identified.

Now let us discuss the uniqueness of the identification problem. The first equation in Eq. (14) can be written as

$$\left[\dot{Y}_1(t) - \dot{Y}_0(t)\right]c_1 + [Y_1(t) - Y_0(t)] = f(t), \tag{18}$$

where

$$f(t) = -m_1\ddot{Y}_1 - \left[\dot{Y}_1(t) - \dot{Y}_2(t)\right]c_2 - [Y_1(t) - Y_0(t)]k_2. \tag{19}$$

Select a new set of the structural parameters of the first story of the building and the ground velocity and displacement as follows:

$$\bar{c}_1 = \alpha c_1, \;\; \bar{k}_1 = \alpha k_1, \tag{20}$$

$$\dot{\bar{Y}}_0(t) = \dot{Y}_1(t) - \frac{1}{\alpha}\left[\dot{Y}_1(t) - \dot{Y}_0(t)\right], \tag{21}$$

$$\bar{Y}_0(t) = Y_1(t) - \frac{1}{\alpha}[Y_1(t) - Y_0(t)]. \tag{22}$$

One can easily prove that the above structural parameters and the ground motion satisfy Eq. (18) with $f(t)$ remaining unchanged. Since the value of coefficient α is arbitrary, infinite combinations of first story parameters and seismic input, which will result in the same $f(t)$, can be constructed. Because $f(t)$ is actually the internal force of the first story, the same $f(t)$ means the same absolute response at all floors. In other words, since different sets of first story parameters and seismic input lead to the same responses, the first story parameters and seismic input cannot be uniquely determined by the forced structural responses. The numerical examples support this conclusion. Since only linear operations are involved in the above derivation, the velocity $\dot{\bar{Y}}_0(t)$ and displacement $\bar{Y}_0(t)$ are compatible to each other, and the compatible acceleration $\ddot{\bar{Y}}_0(t)$ is thus

$$\ddot{\bar{Y}}_0(t) = \ddot{Y}_1(t) - \frac{1}{\alpha}\left[\ddot{Y}_1(t) - \ddot{Y}_0(t)\right]. \tag{23}$$

3. Hybrid identification method

In consideration of practical application of the hybrid identification method, the damping ratios other than the damping coefficients should be identified as structural parameters. In this connection, the Rayleigh damping assumption is employed in this study and the damping matrix is assumed to be proportional to mass and stiffness matrices through two damping constants a_0 and a_1.

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K}. \tag{24}$$

By using Eq. (24), Eq. (7) or Eq. (14) can be rewritten as

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + a_0\mathbf{M}\dot{\mathbf{Y}}(t) + a_1\mathbf{K}\dot{\mathbf{Y}}(t) + \mathbf{K}\mathbf{Y}(t) = \mathbf{F}(t). \tag{25}$$

After extracting the structural parameters related to elastic forces, Eq. (25) can be rewritten as:

$$\mathbf{F}_I(t) + \mathbf{H}_{D0}a_0 + \mathbf{H}_{D1}\boldsymbol{\theta}_E a_1 + \mathbf{H}_E\boldsymbol{\theta}_E = \mathbf{F}(t). \tag{26}$$

It can be seen from Eq. (26) that the coefficient a_1 is coupled with the stiffness parameter vector $\boldsymbol{\theta}_E$, leading to a nonlinear identification problem. By altering the form of the parameter vector, however, Eq. (26) can still be treated as a linear identification problem. That is, rewrite Eq. (26) as follows:

$$\mathbf{F}_I(t) + \mathbf{H}_{D0}a_0 + \mathbf{H}_{D1}\boldsymbol{\theta}_{D1} + \mathbf{H}_E\boldsymbol{\theta}_E = \mathbf{F}(t). \tag{27}$$

For the ith parameter in the parameter vector $\boldsymbol{\theta}_{D1}$, the following relation exists:

$$\theta_{D1,i} = \theta_{E,i}a_1. \tag{28}$$

Excluding the first equation and gathering the rest equations at all sampling instants in Eq. (27) result in

$$\tilde{\mathbf{H}}\tilde{\boldsymbol{\theta}} = -\tilde{\mathbf{F}}_I, \tag{29}$$

where $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{F}}_I$ have the same meanings as the counterparts in Eq. (17), and $\tilde{\boldsymbol{\theta}}$ has the following form:

$$\tilde{\boldsymbol{\theta}} = \begin{bmatrix} a_0 & \tilde{\boldsymbol{\theta}}_{D1}^{\mathrm{T}} & \tilde{\boldsymbol{\theta}}_E^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

where $\tilde{\boldsymbol{\theta}}_{D1}$ and $\tilde{\boldsymbol{\theta}}_E$ contain the 2 to n entries of their corresponding terms in Eq. (27). If the parameters are accurately obtained, the following equation holds exactly for arbitrary i:

$$a_1 = \tilde{\theta}_{D1,i}/\tilde{\theta}_{E,i}. \tag{30}$$

Due to the errors occur in practical application, the ith ratio $\tilde{\theta}_{D1,i}/\tilde{\theta}_{E,i}$ will be different from the others, the coefficient a_1 can then be calculated using the following equation:

$$a_1 = \frac{1}{n}\sum_{i=1}^{n}\left[\tilde{\theta}_{D1,i}/\tilde{\theta}_{E,i}\right]. \tag{31}$$

To reach unique identification of the structural parameters using the absolute forced structural response time histories, the minimum modal information is now introduced to the identification procedure to eliminate the non-uniqueness problem. Because not only time-domain information, but also certain modal information are used in the identification procedure, the method is termed the hybrid identification method.

For the ith mode shape $\boldsymbol{\varphi}_i$, there exists the following relationship:

$$\mathbf{K}\boldsymbol{\varphi}_i = \omega_i^2\mathbf{M}\boldsymbol{\varphi}_i. \tag{32}$$

When the stiffness parameters above the first floor have been estimated based on the time-domain identification method proposed in the preceding section, the stiffness of the first floor k_1 can be expressed as the function of ω_i and $\boldsymbol{\varphi}_i$

$$k_1 = \omega_i^2 m_1 + k_2\left(\frac{\phi_{2,i}}{\phi_{1,i}} - 1\right), \tag{33}$$

where $\phi_{1,i}$ and $\phi_{2,i}$ are, respectively, the first and second entry of the ith mode shape $\boldsymbol{\varphi}_i$ of the whole structure. It is implied by Eq. (33) that the stiffness of the first story can be calculated from the modal frequency and the first two entries of the modal shape of a certain mode. In this study, the first mode of vibration is used in the calculation of k_1 because they can be identified with a high precision in practice. The first natural frequency and the first two entries of the first mode shape are termed the minimum modal information for the subsequent study.

In engineering applications, once damping constants a_0 and a_1 are identified the damping ratios can be calculated from the following equation:

$$\zeta = \frac{a_0 + a_1\omega^2}{2\omega}. \tag{34}$$

The frequencies in the above equation can be obtained by solving the following eigenvalue problem based on the identified stiffness matrix.

$$\mathbf{K}\boldsymbol{\varphi} = \lambda\mathbf{M}\boldsymbol{\varphi}, \tag{35}$$

where $\lambda = \omega^2$. In the calculating of ζ_1, although ω_1 is already contained from the minimum modal information, the first natural frequency obtained from Eq. (35) can be used since it is indirectly associated with the measured structural responses. Iteration may be executed if necessary.

After all the structural parameters are obtained, the earthquake-induced ground motion (seismic input) can now be reconstructed. Considering the dynamic equilibrium of the first floor of the building in the absolute coordinate system leads to

$$a_1k_1Y_0 + k_1Y_0 = f_1, \tag{36}$$

where

$$f_1 = m_1\ddot{Y}_1 + [a_0m_1 + a_1(k_1 + k_2)]\dot{Y}_1 - a_1k_2\dot{Y}_2 + (k_1 + k_2)Y_1 - k_2Y_2. \tag{37}$$

The seismic input can be reconstructed by solving the first-order differential Eq. (36). The Newmark method is used in this study to solve Eq. (36).

4. Numerical example

4.1. Building model description and response simulation

A three-story shear building model, which will be tested to verify the proposed hybrid identification method in the later stage, is selected as a numerical example. The mass and horizontal stiffness of the building are, respectively, 230.2 kg and 5.46×10^5 N/m for the first story, and 230.4 kg and 5.04×10^5 N/m for the second and third story. The damping constants a_0 and a_1 are set to 0.62566 rad/s and 4.973×10^{-4} s/rad, respectively. The corresponding damping ratios are 2% for the first mode, 2% for the second mode and 2.475% for the third mode. The three natural frequencies of the shear building are, respectively, 3.384, 9.417, and 13.477 Hz. The unity-normalized modal shapes of the building are listed in Table 1. Two kinds of excitation are used in this study: simulated ambient ground motion and earthquake ground motion recorded in a shaking table test. The structural

Table 1
Modal shapes of the building model

	Mode 1	Mode 2	Mode 3
First floor	0.4227	1.0000	−0.8390
Second floor	0.7933	0.4843	1.0000
Third floor	1.0000	−0.8065	−0.4390

response from the ambient ground motion is used to find the minimum modal information, and the structural response from the earthquake ground motion is used to identify the structural parameters and reconstruct the seismic input. In actual applications, the ambient vibration responses of the structure just before an earthquake event can be used to find the minimum modal information.

The ambient ground motion is simulated from a band-limited stationary random white-noise spectrum distributed between 0.5 and 20 Hz with a peak ground acceleration of 0.05 m/s^2, and the time history duration of the ground motion is 200 s. An earthquake ground motion recorded in a shaking table test, which has peak ground acceleration about 1 m/s^2, is used in this example. The length and sampling frequency of the recorded earthquake ground motion are, respectively, 40 s and 500 Hz. To obtain the velocity and displacement time histories of the earthquake ground motion from the recorded ground acceleration time history, the unconstrained processing steps developed by the USGS National Strong-Motion Program Data Center [8] are adopted with the only alternation that the corner frequency is set to 0.7 Hz according to the energy content of the recorded earthquake acceleration in the frequency domain. With all the information available, the Newmark method is employed in this example to generate the absolute structural responses of the building. The proposed hybrid method is then applied to the absolute structural responses to identify the structural parameters and the earthquake-induced ground motion. The identified structural parameters and the ground motion are finally compared with the preset ones to verify the proposed hybrid identification method. No measurement noise is considered in this section.

4.2. Application of hybrid identification method

The first natural frequency and the first two entries of the first modal shape of the building are identified from the ambient vibration response using the output-only system identification method [9]. The resulting first natural frequency is 3.369 Hz, which is slightly smaller than the preset one of 3.384 Hz. The identified first two entries of the first modal shape are 0.4239 and 0.7937 compared with the original values of 0.4227 and 0.7933.

Two cases are conducted to assess the accuracy of the proposed hybrid identification method without measurement noise. In the first case, the preset (actual) minimum modal information is used in the parameter estimation and seismic input reconstruction procedure. The identified values of the damping constants and stiffness coefficients are listed in Table 2 and compared with the actual values. The reconstructed seismic ground motions are plotted in Fig. 2 and compared with the preset seismic ground motions. The power spectrum of the reconstructed ground

Table 2
Identified structural parameters using actual minimum modal information

Parameter	Identified value	Actual value	Relative error (%)
First damping ratio	2.0%	2.0%	0.00
Second damping ratio	2.0%	2.0%	0.00
Third damping ratio	2.5%	2.5%	0.00
First stiffness (N/m)	5.46×10^5	5.46×10^5	0.00
Second stiffness (N/m)	5.04×10^5	5.04×10^5	0.00
Third stiffness (N/m)	5.04×10^5	5.04×10^5	0.00

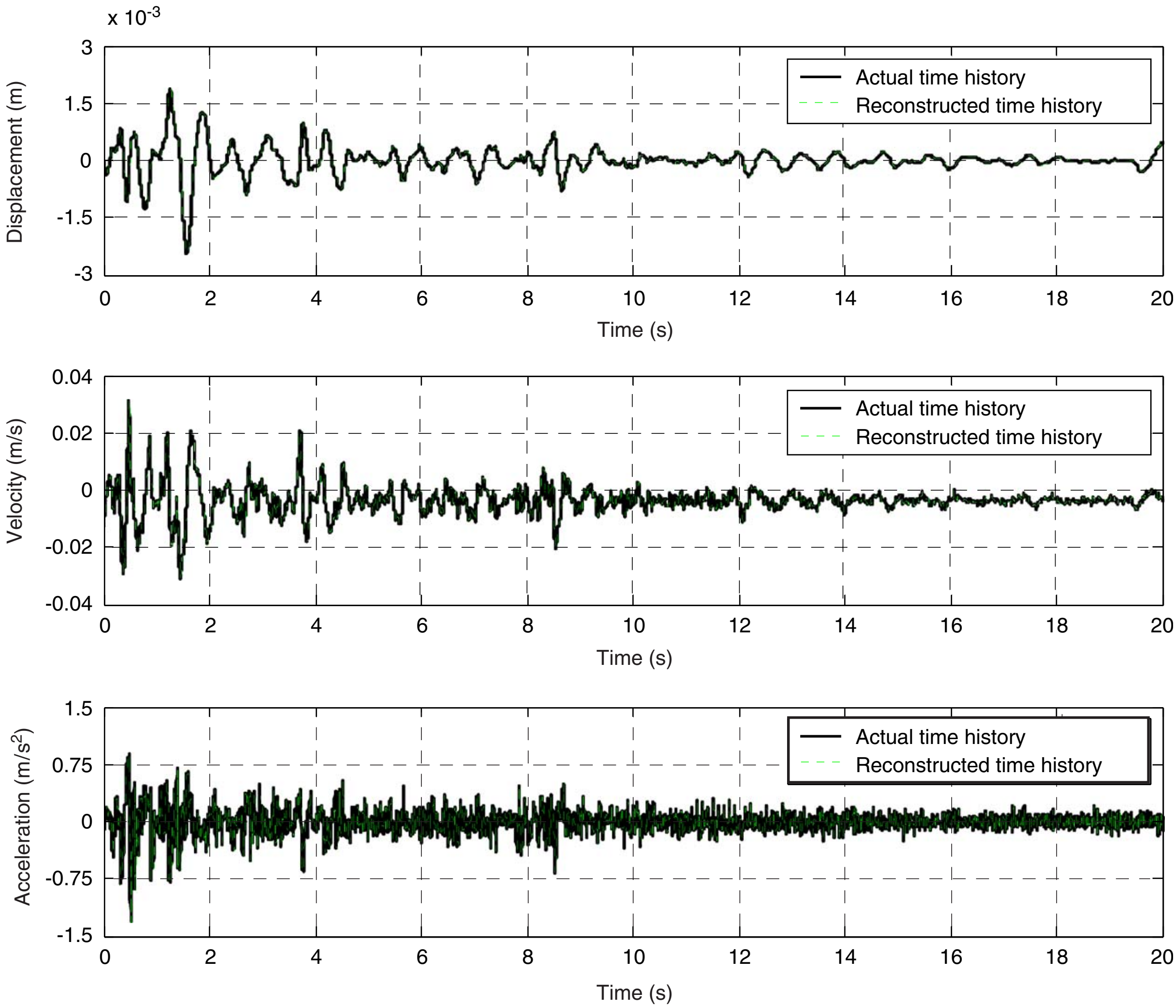

Fig. 2. Comparison of ground motion time histories using actual modal information.

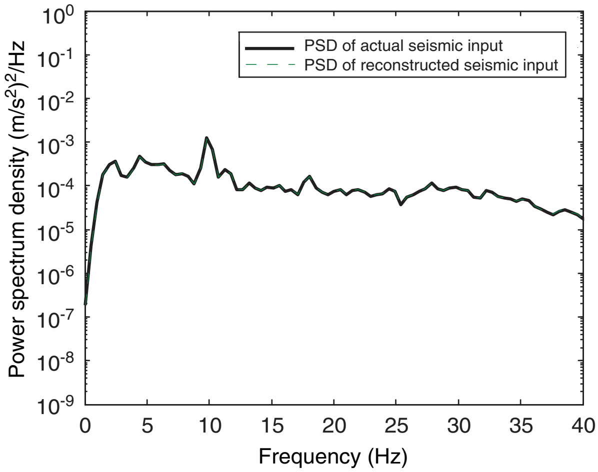

Fig. 3. Comparison of ground acceleration spectrum using actual modal information.

Table 3
Identification structural parameters using identified minimum modal information

Parameter	Identified value	Accurate value	Relative error (%)
First damping ratio	2.0%	2.0%	0.05
Second damping ratio	2.0%	2.0%	0.03
Third damping ratio	2.5%	2.5%	0.01
First stiffness (N/m)	5.44×10^5	5.46×10^5	0.41
Second stiffness (N/m)	5.04×10^5	5.04×10^5	0.00
Third stiffness (N/m)	5.04×10^5	5.04×10^5	0.00

acceleration is also computed and compared with that of the actual ground acceleration, as shown in Fig. 3. It is seen that the structural parameters and the ground motion identified by the proposed method are the same as the actual ones.

In the second case, the minimum modal information identified from the ambient ground motion is used. The identified structural parameters are listed in Table 3 and compared with the actual values. It is seen that due to the small errors in the identified first natural frequency and the first two entries of the first mode shape, the first story stiffness is not exactly the same as the actual value. The damping constants and the stiffness coefficients of the upper stories, however, are not affected by the errors in the identified minimum modal information. By using the identified structural parameters and the absolute structural responses, the seismic input is reconstructed and shown in Fig. 4 for the ground displacement, velocity, and acceleration and in Fig. 5 for the power spectrum of the ground acceleration. It is seen that the reconstructed seismic acceleration, velocity and displacement time histories match the actual time histories of the ground motion very well.

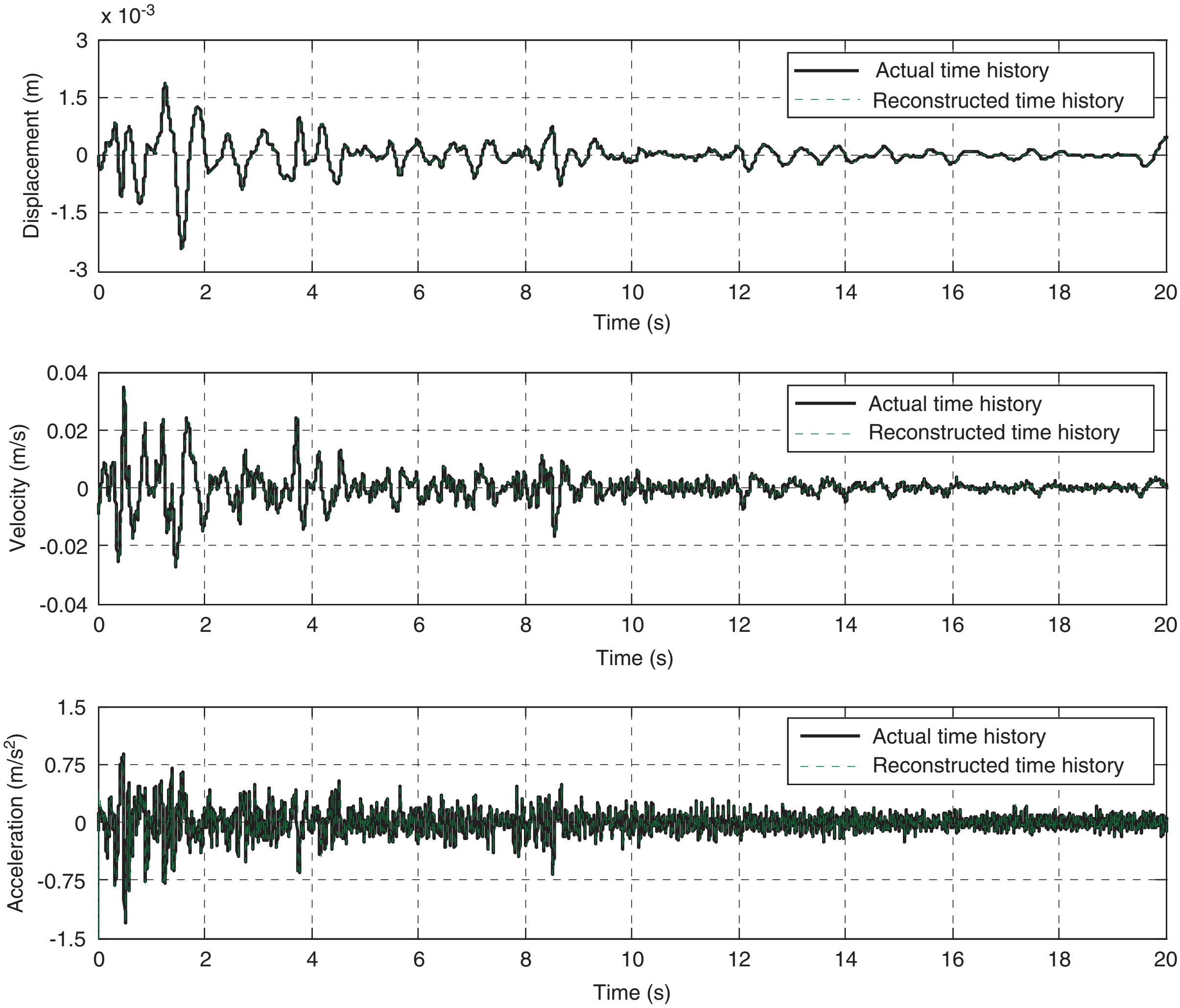

Fig. 4. Comparison of ground motion time histories using identified modal information.

The power spectrum of the identified ground acceleration is also in good agreement with the actual one.

5. Effect of measurement noise

The hybrid identification procedure described in the preceding sections is straightforward in theory, but several practical problems may occur in its application. One of the problems is that for weak ground motions, the responses of a multi-story shear building, such as inter-story drifts, may be too small to be used for the identification. The other problem is the effect of measurement noise on the identification quality. This section thus first examines the sensitivity of identified

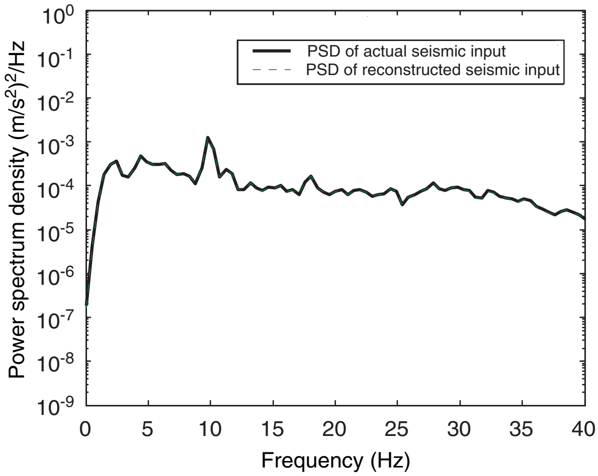

Fig. 5. Comparison of ground acceleration spectrum with identified modal information.

results to measurement noise and then presents an amplitude-selective filtering procedure to overcome the two problems mentioned above.

5.1. Identification with measurement noise

To investigate the effect of measurement noise on the identification quality, several levels of measurement noise are simulated and added to the ideal structural response time histories obtained in the preceding section with the identified minimum modal information but without considering the measurement noise. The measurement noise is assumed to be normally distributed white noise, and the noise level is defined as the ratio of root-mean-square (rms) of measurement noise to the rms of the ideal structural response.

$$r = \frac{\mathrm{RMS}(\varepsilon)}{\mathrm{RMS}(x)} \times 100\% \tag{38}$$

where r represents the noise level; ε represents the noise time history; and x acts for the ideal structural response time history. For a given noise level, the noise time history is generated, respectively, for the ideal acceleration, velocity, and displacement response at each floor of the building. The noise time histories generated are assumed to be independent of each other.

Two cases are examined with the noise level of 1% and 5%, respectively. The minimum modal information identified from the ambient vibration in the preceding section is used in both cases. The identified structural parameters of 1% noise level using the hybrid identification method are listed in Table 4, and those of 5% noise level are listed in Table 5. The identified structural parameters listed in Tables 4 and 5 show that when the measurement noise level is less than 1%, the relative identification errors in the stiffness parameters are less than 1.2% and the relative identification errors in the damping ratios are less than 8%. When the noise level is increased to 5%, the maximum relative error in the stiffness parameter reaches 20% and the maximum relative error in the damping ratio comes to 60%. Thus, an amplitude-selective filtering procedure is proposed in this study to reduce the effect of measurement noise on identification quality.

Table 4
Identification structural parameters with 1% noise level

Parameter	Identified value	Actual value	Relative error (%)
First damping ratio	1.99%	2.00%	0.75
Second damping ratio	2.12%	2.00%	5.89
Third damping ratio	2.66%	2.48%	7.55
First stiffness (N/m)	5.40×10^5	5.46×10^5	1.01
Second stiffness (N/m)	5.00×10^5	5.04×10^5	0.75
Third stiffness (N/m)	4.99×10^5	5.04×10^5	1.01

Table 5
Identification structural parameters with 5% noise level

Parameter	Identified value	Actual value	Relative error (%)
First damping ratio	1.73%	2.00%	13.62
Second damping ratio	2.91%	2.00%	45.45
Third damping ratio	3.98%	2.48%	60.75
First stiffness (N/m)	4.77×10^5	5.46×10^5	12.64
Second stiffness (N/m)	4.27×10^5	5.04×10^5	15.19
Third stiffness (N/m)	3.99×10^5	5.04×10^5	20.88

5.2. Amplitude-selective filtering procedure (ASF)

The measurement noise considered in this study is assumed to be normally distributed white noise. Thus, in the frequency domain, the amplitude of measurement noise at one frequency is the same as that at the other frequency. In the time domain, the amplitude of measurement noise at most time instants is below a certain value. For instance, for a normally distributed white noise the noise amplitude at 95% of the sampling points of measurement noise time history is below 1.645 times its rms. Since the amplitude of structural response at each time instant is different, the relative error induced by measurement noise would be different from time instant to time instant. According to the noise level defined in Eq. (38), the relative noise level will be small for the structural response of large amplitude in general. Furthermore, it is not necessary to take all the time instants (the sampling points) in one time history into full consideration when using the hybrid identification method.

An amplitude-selective filtering procedure is thus proposed based on the above concept to filter the structural responses below a preset threshold. Only the structural responses above the given threshold are retained in the subsequent identification so as to reduce the effect of measurement noise and at the same time to improve the quality of identification of the building under weak ground motions. The criterion used to determine the threshold is the number of the retained sampling points of structural responses. The number of retained sampling points should be smaller compared with the total number of sampling points, but it should be larger enough to satisfy the requirement of the least-squares method.

For a single-response time history $\{x_i\}$ of dimension n, if the number of retained sampling points is set to m the retained structural response time history after the filtering can be expressed as

$$\{\tilde{x}_j\} = \{x_i | x_i > x^*\}, \tag{39}$$

where x^* is the $(n-m)$ largest value in the original time history $\{x_i\}$. For more than one structural response time history, in consideration that different types of structural responses are involved and the same noise level is assigned to each structural response, the sampling points are discarded one by one starting from the point with the smallest response until the retained number reaches the given number but this procedure should be applied to all the response time histories with an even chance. The amplitude-selective filtering procedure can be schematically illustrated using the two response time histories $\{x_i\}$ and $\{y_i\}$ as shown in Fig. 6a. The retained number of sampling points is set to 5. Firstly, the smallest value in the time history $\{x_i\}$ is identified at the time instant 1. The sampling point at the time instant 1 is thus removed from both time histories $\{x_i\}$ and $\{y_i\}$ (Fig. 6b). Then, the smallest value in the remaining time history $\{y_i\}$ is identified at the time instant 3. The sampling point at the time instant 3 is thus removed from both the remaining time histories $\{x_i\}$ and $\{y_i\}$ (Fig. 6c). Such a procedure continues until the retained number of sampling points is equal to 5 (Fig. 6d).

5.3. Parameter identification with ASF

To assess the effectiveness of amplitude-selective filtering procedure, the procedure is applied to the structural response time histories with the noise level of 1% and 5%, respectively, which have been obtained in Section 5.1 already. The retained number of sampling points is set as 500. The identified structural parameters are listed in Table 6 for the noise level of 1% and in Table 7 for the noise level of 5%. They are also compared with the identified structural parameters obtained in Section 5.1 using the hybrid identification method without ASF. It is seen that the amplitude-selective filtering procedure greatly enhances the identification quality. Even with the noise level of 5%, the maximum error in the identified stiffness parameters is less than 2.0% and the maximum error in the identified damping ratios is less than 14.0%.

5.4. Seismic input reconstruction with measurement noise

Although the amplitude-selective filtering procedure can significantly improve the quality of structural parameter identification, it may not considerably improve the reconstruction of seismic input because the measurement noise component in the structural response time histories cannot be disregarded as done in the ASF. Fig. 7 shows the reconstructed seismic input using Eqs. (36) and (37), the structural responses with 1% measurement noise, and the structural parameters identified using the ASF. It can be seen that although the reconstructed ground displacement matches the actual one well, there is obvious high-level noise in the reconstructed ground velocity. The situation of the reconstructed ground acceleration becomes even worse. The actual ground acceleration signals are totally annihilated by the noise signals.

To understand the reason behind this phenomenon and overcome the measurement noise disturbance, the spectral analysis is carried out on both the seismic input and the structural

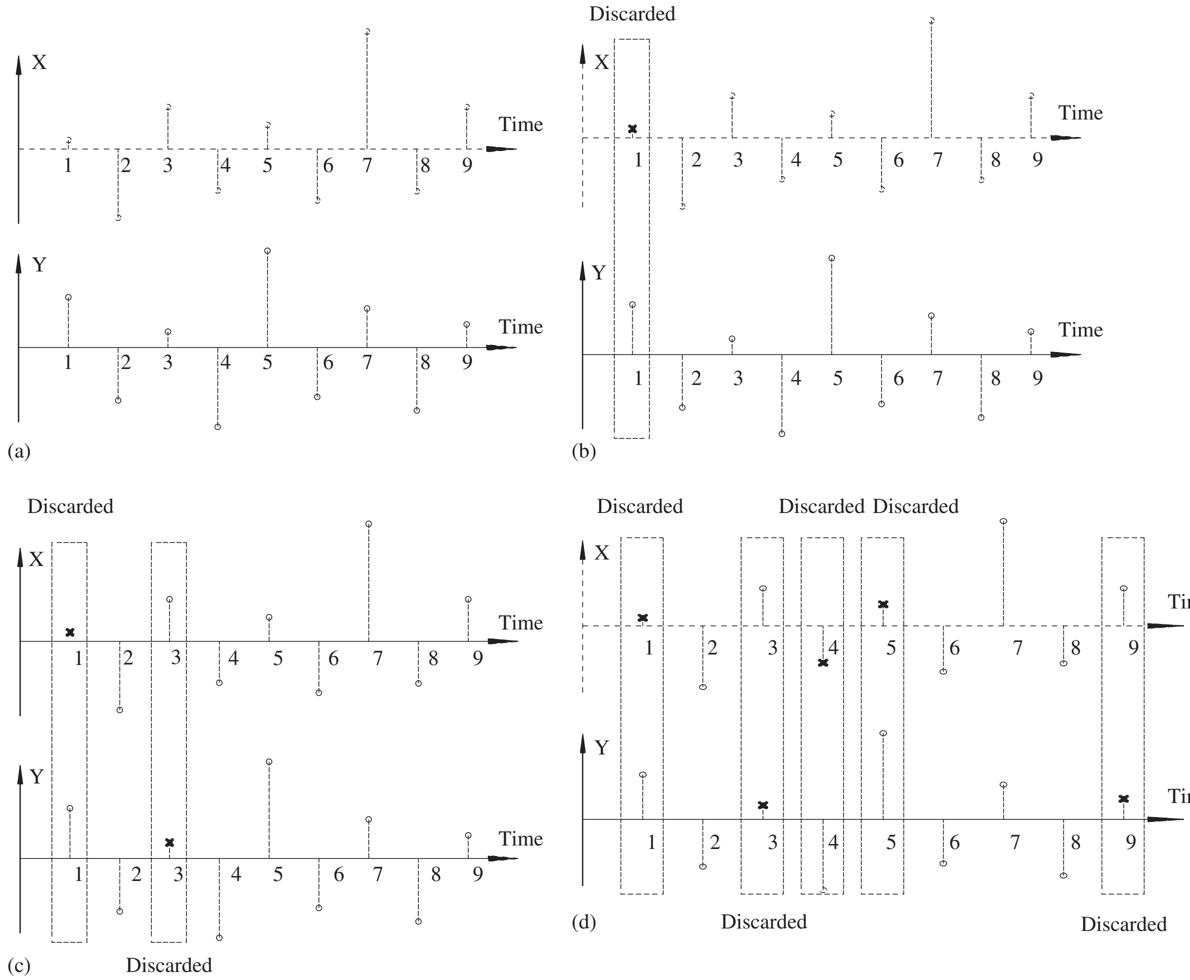

Fig. 6. Illustration of amplitude-selective filtering procedure. (a) Original time history, (b) the first discarded sampling point, (c) the first two discarded sampling points and (d) all discarded sampling points.

Table 6
Identification structural parameters with 1% noise level and ASF

Parameter	HI* method	RE (%)	HI-ASF	RE (%)
First damping ratio	1.99%	0.75	2.00%	0.25
Second damping ratio	2.12%	5.89	2.00%	0.00
Third damping ratio	2.66%	7.55	2.48%	0.02
First stiffness (N/m)	5.40×10^5	1.01	5.43×10^5	0.47
Second stiffness (N/m)	5.00×10^5	0.75	5.04×10^5	0.07
Third stiffness (N/m)	4.99×10^5	1.01	5.03×10^5	0.10

HI-ASF: hybrid identification with amplitude-selective filtering procedure.
RE: relative error.
*HI: hybrid identification.

Table 7
Identification structural parameters with 5% noise level and ASF

Parameter	HI method	RE (%)	HI-ASF	RE (%)
First damping ratio	1.73%	13.62	2.07%	3.27
Second damping ratio	2.91%	45.45	2.24%	11.78
Third damping ratio	3.98%	60.75	2.82%	13.87
First stiffness (N/m)	4.76×10^5	12.64	5.36×10^5	1.79
Second stiffness (N/m)	4.27×10^5	15.19	4.95×10^5	1.72
Third stiffness (N/m)	3.99×10^5	20.88	4.95×10^5	1.80

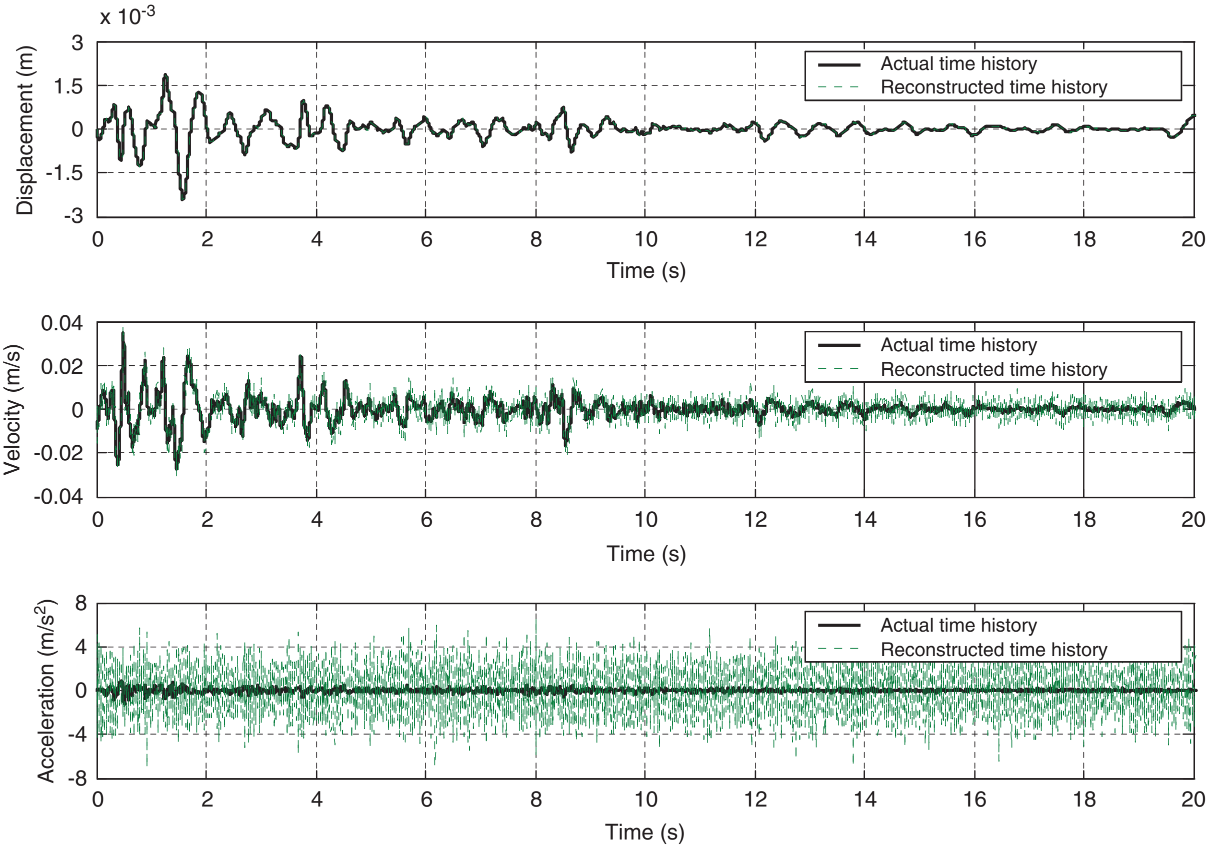

Fig. 7. Comparison of ground motion time histories with 1% measurement noise.

responses. Figs. 8–10 exhibit the power spectra of the reconstructed ground acceleration, velocity, and displacement from the structural responses with 1% measurement noise together with the power spectra of the actual ground acceleration, velocity and displacement. Figs. 11a and b

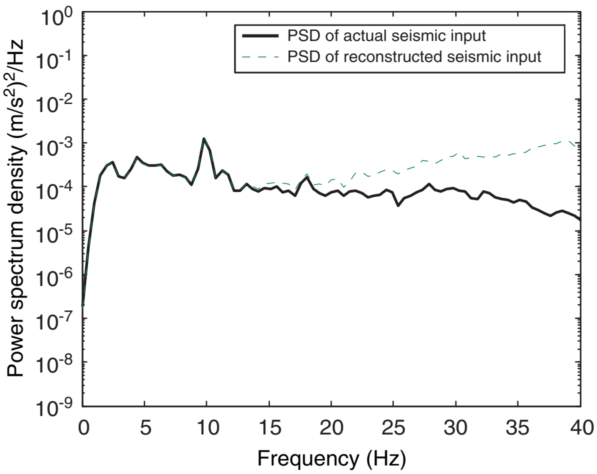

Fig. 8. Comparison of ground acceleration spectra with 1% measurement noise.

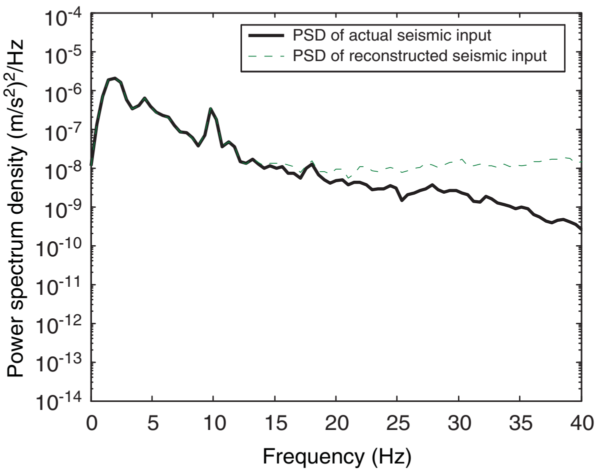

Fig. 9. Comparison of ground velocity spectra with 1% measurement noise.

display the power spectra of the 1st and 2nd floor acceleration responses, respectively, without and with 1% and 5% measurement noise levels. It is seen from Figs. 8–10 that the difference in spectral amplitude between the reconstructed and actual ground spectra mainly occurs within the high-frequency range (say 10 Hz above in this example). Since the amplitude of ground acceleration spectrum in the high-frequency range is compatible with that in the low-frequency range, the difference in spectral amplitude in the high-frequency range causes significant errors in the reconstructed ground acceleration time history. On the other hand, the amplitude of ground displacement spectrum in the high frequency range is much smaller than that in the low frequency range, the difference in spectral amplitude in the high-frequency range does not cause

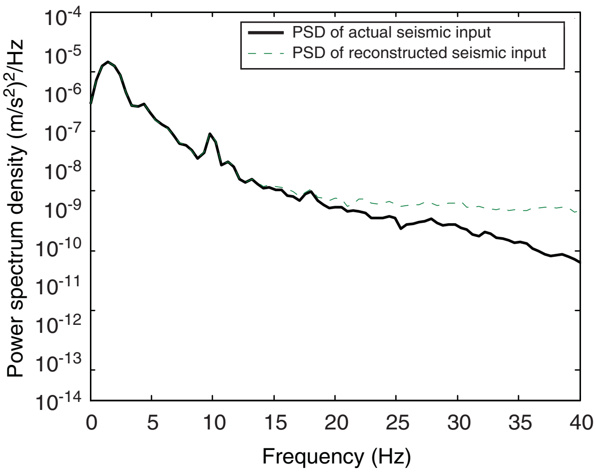

Fig. 10. Comparison of ground displacement spectra with 1% measurement noise.

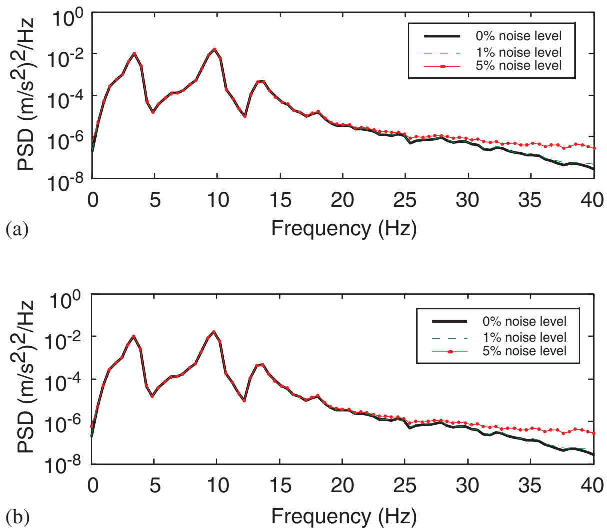

Fig. 11. Comparison of structural acceleration response spectra. (a) First floor acceleration and (b) second floor acceleration.

considerable errors in the reconstructed ground displacement time history. Figs. 11a and b demonstrate that the spectral amplitude of structural acceleration response in the low-frequency range is much larger than that in the high-frequency range and the measurement noise affects mainly the spectral amplitude of structural response in the high-frequency range. Therefore, a low pass filter is proposed in this study and applied to the structural responses with measurement noise. The structural response time histories after being low-pass filtered are then used to reconstruct the seismic input. In consideration that the highest natural frequency of the concerned building is 13.477 Hz and that the structural response energy is mainly distributed before 20 Hz, the cut-off frequency is selected as 20 Hz in this study.

Fig. 12 shows the reconstructed ground displacement, velocity, and acceleration time histories using the structural responses with 1% measurement noise and low-pass filtered at 20 Hz. Fig. 13 displays the power spectra of the reconstructed and actual ground accelerations. Clearly, after using a low-pass filter, not only the reconstructed ground displacement time history matches the actual one very well but also the reconstructed ground velocity and acceleration time histories match the actual ones very well. The good agreement is also seen from the comparison of the power spectra between the reconstructed and actual ground acceleration, as shown in Fig. 13. The low-pass filter is also applied to the structural responses with 5% measurement noise,

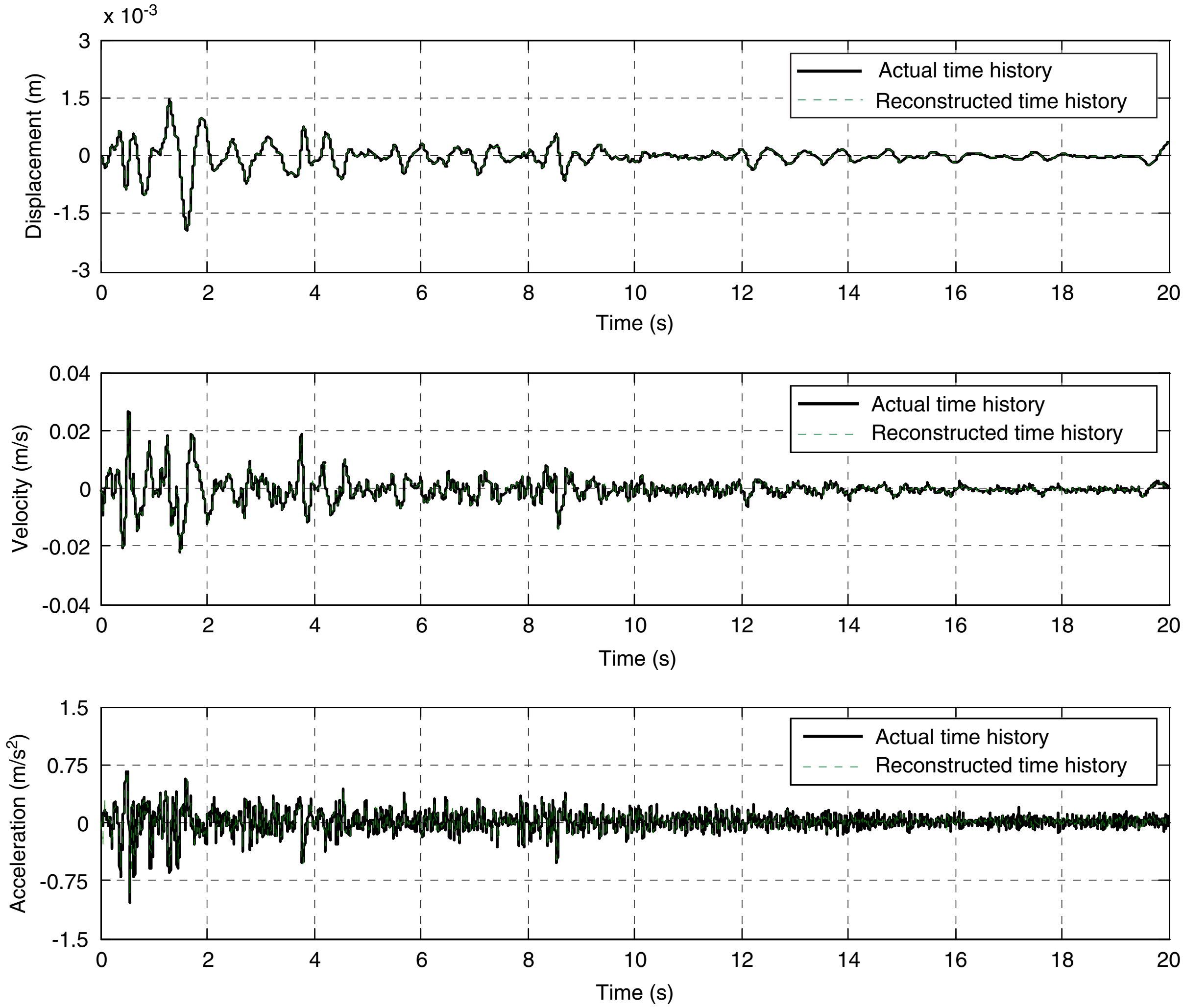

Fig. 12. Comparison of ground motion time histories with 1% measurement noise and low-pass filter.

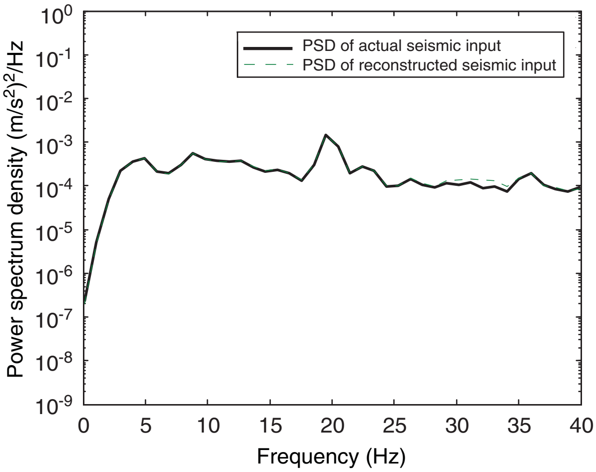

Fig. 13. Comparison of ground acceleration spectra with 1% measurement noise and low-pass filter.

and the reconstructed ground motion time histories and the comparison of ground acceleration spectra are plotted in Figs. 14 and 15, respectively. The reconstructed ground displacement and velocity time histories are again in good agreement with the actual ones. The reconstructed ground acceleration in the time domain is slightly higher than the actual one, and the amplitude of the reconstructed ground acceleration spectrum is also moderately higher than that of the actual one.

6. Concluding remarks

This paper has demonstrated that the structural parameters and ground motion of an earthquake-excited multi-story shear building cannot be uniquely identified when the absolute forced structural response time histories are used directly. A hybrid identification method, by combining the time-domain information with the modal information, has therefore been proposed in this study to identify the structural parameters and reconstruct the seismic input. The required minimum modal information includes only the first natural frequency and the first two entries of the first mode shape, which can be obtained from ambient vibration tests with high precision in practice. To enhance the capability of the hybrid identification method against measurement noise, the amplitude-selective filtering procedure has been proposed. Numerical example of a three-story shear building showed that the proposed hybrid identification method with the amplitude-selective filtering procedure could accurately estimate the structural parameters even for all the structural responses with 5% measurement noise disturbance. After all the structural parameters are identified, the unknown seismic input can be constructed by solving a first-order differentiation equation only. The numerical example showed that the seismic input could be accurately reconstructed for the structural responses without measurement noise. With the measurement noise in the structural

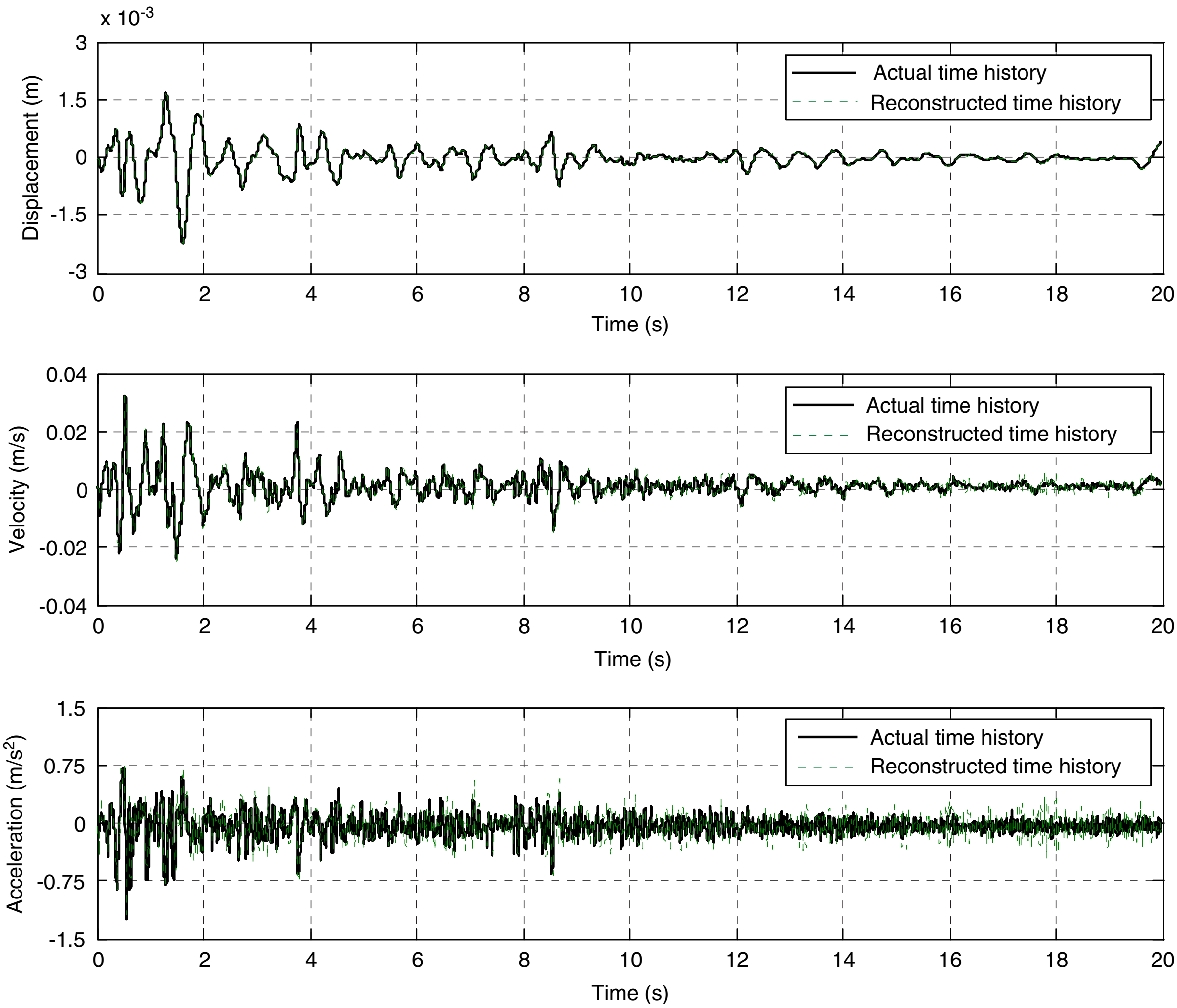

Fig. 14. Comparison of ground motion time histories with 5% measurement noise and low-pass filter.

responses, a low-pass filter has to be used to attenuate the influence of high-frequency noise on the reconstruction of seismic input.

It should be pointed out that this study is based on the assumptions of linear elastic materials and linear structural response. This is true for multi-story shear buildings under ambient type of excitations or under weak and moderate seismic-induced ground motion. For multi-story shear buildings under strong seismic-induced ground motion, the material nonlinearities (yield of steel bars and cracking of concrete) and geometric nonlinearities (P-delta and soft-story effect) of the building could be significant, and the dynamic parameters of the building, such as its stiffness and damping ratio, could change with respect to time. The methodology proposed in this study would then be hardly applied, and the new methodology should be sought to tackle nonlinear

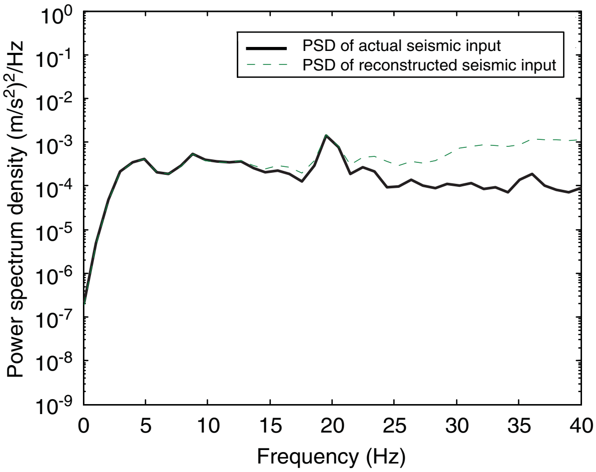

Fig. 15. Comparison of ground acceleration spectra with 5% measurement noise and low-pass filter.

identification problem. Furthermore, experimental study with consideration of various nonlinearities is also essential for checking the reliability and accuracy of the proposed methodologies, which deserves investigation in future.

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