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Short Communication

Adaptive algorithm for active control of impulsive noise

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Abstract

When active noise control (ANC) systems are applied for reducing impulsive noise, conventional adaptive algorithms for ANC may not be effective. In this paper, a new adaptive algorithm for active control of impulsive noise is presented. Compared to the filtered-x least-mean-square algorithm, the proposed algorithm has a much better convergence and stability. Illustrative simulations are conducted to verify the proposed algorithm.

1. Introduction

Active noise control (ANC) has received extensive research in the past two decades [1,2]. Although great progress has been made in ANC, practical applications are still limited. One important challenge comes from active control of impulsive noise. In practice, impulsive noises are often due to the occurrence of noise disturbance with low probability but large amplitude. Impulsive noises exist widely in practice [3], and it is of great meaning to study its control. One difficulty for active control of impulsive noises lies in the convergence and stability problems. The most popular algorithm in ANC is the filtered-x least-mean-square (FXLMS) algorithm. This algorithm may become unstable in cases where the primary noise is impulsive or impulsive disturbances present at the reference sensor. The reason is that the FXLMS algorithm is designed to minimize the mean-square error, however, which may not exist for impulsive noise. It is shown that the impulsive noise tends to have infinite variance [3].

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There are just few literatures addressing the active control of impulsive noises. It has been shown that minimizing the mean p-power of error instead of mean-square error can have a better robustness to impulsive noise. In Ref. [4], the filtered-x least mean p-power (FXLMP) algorithm is developed for active control of impulsive noise. However, the FXLMP algorithm requires computation of the fractional power and usually has a formidable computational load. Moreover, for the FXLMP algorithm, there is no guided way to select p when the primary noise is not α-stable. It is worthy to note that some modified version of the FXLMS algorithm, such as the normalized FXLMS algorithm, leakage FXLMS algorithm, and some other modified FXLMS algorithms based on constrained optimization [1,2], can also be helpful in some cases to maintain the stability of ANC systems. However, these algorithms cannot guarantee convergence and still have risk of instability when applied to impulsive noises. Moreover, operations required by constrained optimization may significantly increase the complexity of the algorithm, which may prohibit the use in many practical applications.

In this paper, a simple but very effective adaptive algorithm is proposed for active control of impulsive noises. Compared to the standard FXLMS algorithm, it has much better convergence and stability characteristics with similar computational load.

2. Algorithm derivation

2.1. Burst of the FXLMS algorithm

The block diagram of an adaptive algorithm for active control is shown in Fig. 1, where $P(z)$, $S(z)$, $e(n)$, and $x(n)$ represent the primary path, secondary path, residual noise, and reference signal, respectively. The FXLMS algorithm is expressed as [1]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)[s(n) * x(n)] \tag{1}$$

or

$$\begin{bmatrix} w_0(n+1) \\ w_1(n+1) \\ \vdots \\ w_{L-1}(n+1) \end{bmatrix} = \begin{bmatrix} w_0(n) \\ w_1(n) \\ \vdots \\ w_{L-1}(n) \end{bmatrix} + \mu e(n) s(n) * \begin{bmatrix} x(n) \\ x(n-1) \\ \vdots \\ x_{L-1}(n) \end{bmatrix}, \tag{2}$$

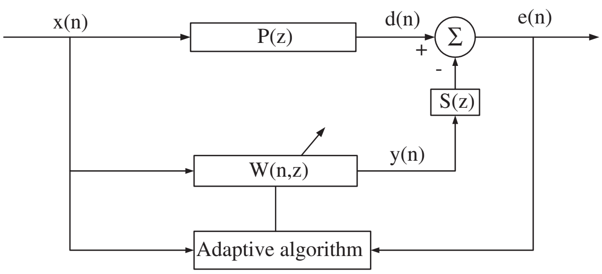

Fig. 1. Block diagram of adaptive algorithm for active noise control.

where n is the time index, μ is the step size, and $s(n)$ is the impulse response of $S(z)$. From the above equation, it can be seen that in the FXLMS algorithm, the reference signal $x(n)$ at different time are treated "equally". That is, all the reference signal samples $x(n)$ are used to update the adaptive filter coefficient vector $w(n)$ with the same weight. It may cause the FXLMS algorithm instable when the primary noise is impulsive.

Let the primary noise $d(n)$ be expressed as

$$d(n) = \mathbf{w}^{o\mathrm{T}}[s(n) * x(n)] + \varepsilon(n), \tag{3}$$

where $\mathbf{w}^o$ represents the optimal filter. The error signal $e(n)$ can be expressed as

$$\begin{aligned} e(n) &= d(n) - s(n) * [\mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n)] \approx d(n) - \mathbf{w}^{\mathrm{T}}(n)[s(n) * x(n)] \\ &= \varepsilon(k) - [\mathbf{w}(n) - \mathbf{w}^o]^{\mathrm{T}}[s(n) * \mathbf{x}(n)]. \end{aligned} \tag{4}$$

In the above equation, it is assumed that

$$s(n) * [\mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n)] \approx \mathbf{w}^{\mathrm{T}}(n)[s(n) * \mathbf{x}(n)]. \tag{5}$$

This assumption is extensively employed in ANC [1]. Substituting Eq. (4) into Eq. (1) yields

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\varepsilon(n)[s(n) * \mathbf{x}(n)] - \mu[s(n) * \mathbf{x}(n)][s(n) * \mathbf{x}(n)]^{\mathrm{T}}[\mathbf{w}(n) - \mathbf{w}^o]. \tag{6}$$

Suppose the primary noise is impulsive, and there is an impulsive disturbance at time k where $n - L + 1 < k < n$. Let $x(k) = M$, where M is a very larger value. Since $\mathbf{w}(n) - \mathbf{w}^o$ will not be zero in practice, it can be seen from Eq. (6) that the adaptive filter coefficient will have a magnitude of order M^2. So the FXLMS algorithm may burst and become unstable when impulsive noise presents.

2.2. Derivation of the proposed algorithm

To overcome the above problem, the samples of the reference signal in the signal vector $x(n)$ should be treated probabilistically instead of equally, i.e., the algorithm should depend more on samples with larger probability than the samples with smaller probability. As a result, the reference signal samples with small probabilistic mass values will only have very slight effects on the algorithm.

One natural way to implement the above goal is to weight the reference signal $x(n)$ by using its probabilistic density (mass) function. Thus Eq. (2) becomes

$$\begin{bmatrix} w_0(n+1) \\ w_1(n+1) \\ \vdots \\ w_{L-1}(n+1) \end{bmatrix} = \begin{bmatrix} w_0(n) \\ w_1(n) \\ \vdots \\ w_{L-1}(n) \end{bmatrix} + \mu e(n) s(n) * \begin{bmatrix} p_{x(n)} x(n) \\ p_{x(n-1)} x(n-1) \\ \vdots \\ p_{x(n-L+1)} x_{L-1}(n-L+1) \end{bmatrix} \tag{7}$$

or

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)s(n) * [\mathbf{P}(n)\mathbf{x}(n)], \tag{8}$$

where $p_{x(n-i)}$ represents the probabilistic density function of the sample $x(n-i)$, and the matrix $\mathbf{P}(n)$ is expressed as

$$\mathbf{P}(n) = \begin{bmatrix} p_{x(n)} & 0 & 0 & 0 \\ 0 & p_{x(n-1)} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & p_{x(n-L+1)} \end{bmatrix}. \tag{9}$$

It is interesting to see that if

$$p_{x(n)} = p_{x(n-1)} = \cdots = p_{x(n-L+1)}, \tag{10}$$

Eq. (8) becomes Eq. (1), the FXLMS algorithm. Therefore, uniform distribution is assumed in the FXLMS algorithm for the primary noise. Obviously, this assumption is not valid for impulsive noise.

2.3. Simplification

When using the algorithm expressed in Eq. (7) or (8), two problems may arise:

(1) In Eq. (7) or (8), an analytical expression for the probability density function (PDF) of the reference signal is required, which may not be available in practice. It is shown in Ref. [3] that in many cases, the impulsive noise can be modeled by α-stable process with $0<\alpha<2$, and the smaller the α is, the more impulsive the noise is. Some PDFs for symmetric α-stable processes with different α are shown in Fig. 2. It can be seen from Fig. 2 that the PDF with smaller α has heavier tail, which is the consequence of the fact that a smaller α represents a more impulsive noise process. However, there are still many type of impulsive noise that cannot be modeled by α-stable process. Generally speaking, it is very difficult and sometimes even impossible to get analytical expression of the PDF for a given noise in practice.
(2) Even if the exact analytical expression of the PDF is available, computing the PDF of the noise often involves the operation of power and exponent [3], which will significantly increase the complexity of the algorithm.

To overcome the above two problems, the PDF as shown in Fig. 3 is used for the proposed algorithm. It is based on the following two approximations:

(1) The probability of the sample larger than $c1$ or less than $c2$ are assumed to be 0, which is consistent with the fact that the tail of PDF for practical noise always tends to 0 when the noise value is approaching $\pm\infty$. In practice, $c1$ and $c2$ can be estimated by statistical method.
(2) The noise has a uniform distribution within the range $[c1, c2]$.

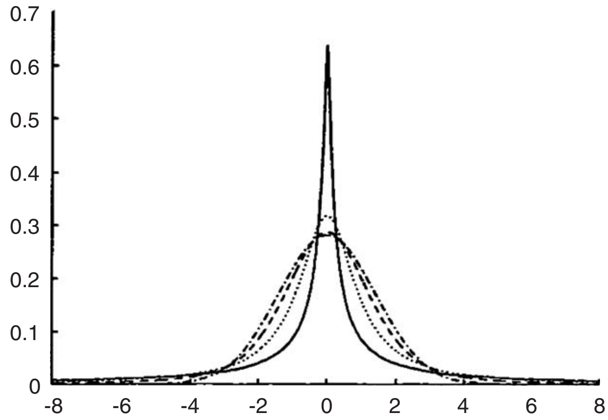

Fig. 2. PDFs of α-stable process for 0.5 (solid), 1.0 (dotted), 1.5 (dashed) and 2.0 (dash–dotted).

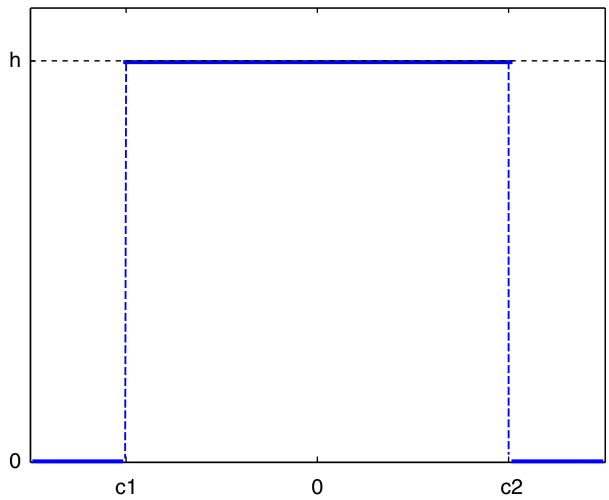

Fig. 3. PDF employed in the proposed algorithm.

Using the PDF shown in Fig. 3, the algorithm expressed in Eq. (8) can be greatly reduced. Replace $\mathbf{P}(n)$ in Eq. (8) with the PDF shown in Fig. 3, the simplified algorithm can be expressed as

$$w_i(n+1) = w_i(n) + \mu e(n)s(n) * x'(n), \tag{11}$$

where $x'(n)$ is defined as

$$x'(n) = \begin{cases} x(n) & \text{if } x(n) \in [c1, c2], \\ 0 & \text{if } x(n) \notin [c1, c2]. \end{cases} \tag{12}$$

Actually, the idea of the algorithm expressed in Eq. (11) is to ignore the samples of reference signal $x(n)$ if its magnitude is above a certain value set by its statistics. This statistics can be obtained off-line in practice.

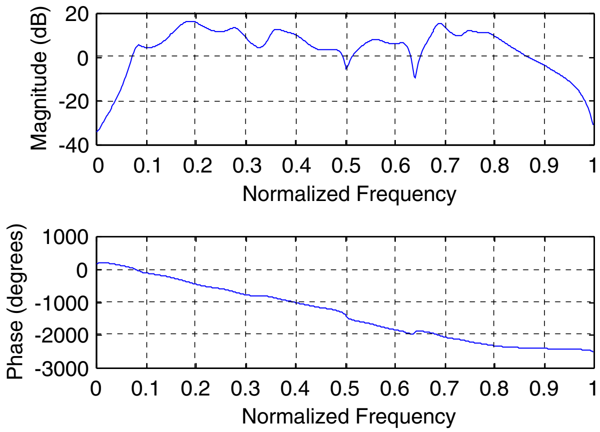

Fig. 4. Frequency response of the primary path.

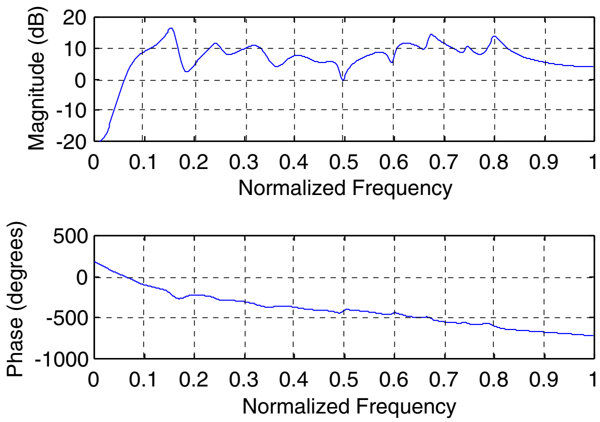

Fig. 5. Frequency response of the secondary path.

3. Simulations

The proposed algorithm has very fast convergence and good stability for active control of impulsive noises. In the simulation, the primary path $P(z)$ and the secondary path $S(z)$ are identified from experimental setup, which can be found in the disk attached to Ref. [1]. The frequency response of the primary path and secondary path are shown in Figs. 4 and 5, respectively. The reference noise is modeled as an α-stable process with α equal to 1.5. In the simulation, the length of the adaptive filter is selected as 256, the step size is 4.0×10^{-8}. The parameter $c1$ is selected as the 99.99 percentile of the noise, and $c2$ is as the 0.01 percentile of the noise. Trends of the primary noise and the residual noise after ANC are shown in Fig. 6, from which it can be seen that the proposed algorithm can convergence well despite of the larger impulsive disturbance. However, when the FXLMS algorithm is used for the simulation, convergence cannot be obtained even when a variety of step sizes have been tried.

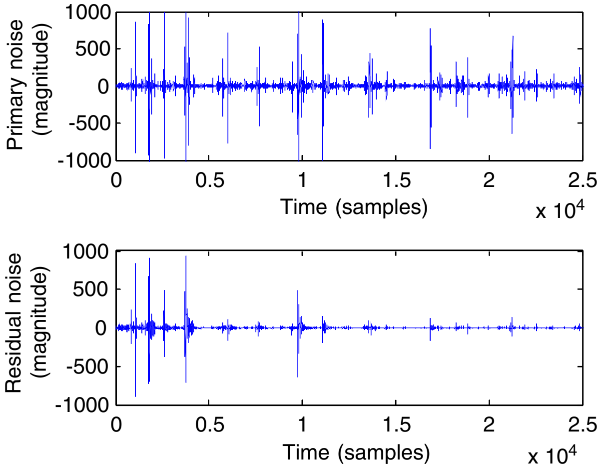

Fig. 6. Trends of the primary noise and residual noise.

4. Further discussion and the summary

Active control of impulsive noise is a challenging task because the conventional algorithms for ANC systems have poor convergence and stability. The proposed algorithm has improved performance for active control of impulsive noises.

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