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Sensitivity analysis of brake squeal tendency to substructures' modal parameters

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Abstract

Sensitivity analysis methods are explored to determine the dominant modal parameters of substructures of a brake system for brake squeal suppression analysis. The related formulae of sensitivities of the positive real part of the eigenvalue of squeal mode (RES) to substructures' modal parameters (SMP) are derived. The sensitivity analysis method can determine the dominant modal parameters influencing the squeal occurrence in a regular way, and the dominant modal parameters will be set to be the target of structural modification attempted to eliminate the squeal mode. Sensitivity analysis of a typical squealing disc brake is performed. The analysis results show that a modified rotor or a modified bracket can be used to eliminate the squealing and that the latter is taken in practice and verified experimentally.

1. Introduction

Brake squealing noise is a key element of city environmental pollution and reduced ride comfort. Brake squeal has been an intractable problem in automotive industry for a long time. Replacement of asbestos with the newer materials which have low damping, high thermal resistance for the friction lining and the need for lightweight vehicles are making the problem more severe than ever [1]. So it is necessary as well as profitable to eliminate it.

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In recent years, many researchers [2–5] through linear model analysis, consider that the brake system whose characteristic equation has complex eigenvalues with positive real parts has the unstable mode, and will vibrate divergently under an unavoidable initial random disturb, while the noise occurs. Furthermore, Ref. [5] puts forward the idea that the stability of the brake system is determined by the match of the dynamic characteristics of the brake components.

A lot of successful instances indicate that structural modification of the brake components is an effective approach to suppress the brake squeal [3,6]. The approach of Ref. [6], mainly based on experimental investigation, although effective in resolving some practical cases with brake squeal problems, needs a lot of tests in general, and leads to high cost in both time and finance. While, Ref. [3], mainly based on theoretical model analysis, directly derives an effective scheme for suppressing brake squeal, which has been verified by experiment. In the paper, the analysis method of coefficient of substructure's modal contribution is used to determine the dominant substructures' modal parameters (SMP) influencing occurrence of the unstable mode. Furthermore, aiming at the flaw of the method of substructure's modal contribution analysis, Ref. [7] introduces the feed-in energy analysis method, through which the assumption that feed-in energy, as well as the real part of the eigenvalue of squeal mode (RES) can indicate the squeal tendency is verified. Comparing to the method in Ref. [3], the feed-in energy analysis method in Ref. [7] can be used for brake squeal analysis much more precisely.

In this paper, as a different approach to determine the dominant SMP (including the eigenvalue and eigenvector), the sensitivity analysis of the positive RES to the SMP is put forward. Since the positive RES can be used to indicate the tendency of brake squeal occurrence, the sensitivities of the RES to the SMP can disclose the influence of the SMP on tendency of squeal occurrence. Based on this, the formulae of sensitivities of the RES to the SMP are derived. The sensitivity analysis method is then applied to a squealing problem of a typical disc brake, which shows that the sensitivity analysis method can be used to analyze the influences of the SMP on the squeal tendency in a regular, comprehensive way and can be easily integrated into CAE design for brake squeal suppression.

2. The closed-loop coupling model for brake squeal analysis

The characteristic equation of the brake closed-loop coupling model is [3]

$$[M]\{\ddot{u}\} + ([K] - [K_f])\{u\} = 0, \tag{1}$$

where $\{u\}$ denotes the displacement vector of all nodes of the brake FE model, $[M]$ and $[K]$ are the mass and stiffness matrices respectively, $[K_f]$ is the unsymmetric friction coupling stiffness matrix which represents the frictional and elastic coupling relationships between substructure's interface nodes.

Convert the displacement vector $\{u\}$ into the substructure's modal coordinates $\{q\}$

$$\{u\} = [\Phi]\{q\}, \tag{2}$$

where $[\Phi]$ is the matrix composed of respective mass normalized modal shape matrix of substructures.

Then substituting Eq. (2) into Eq. (1), we have

$$\{\ddot{q}\} + [K_{\text{sys}}]\{q\} = 0, \tag{3a}$$

where

$$[K_{\text{sys}}] = [\Lambda] - [K_\Phi] = [\Lambda] - [\Phi]^{\mathrm{T}}[K_f][\Phi], \tag{3b}$$

$[\Lambda]$ is a diagonal matrix composed of respective eigenvalue matrix of brake substructures.

The eigenvalues s_i and corresponding right eigenvectors $\{\psi_i\}$ can be obtained by the eigenanalysis of Eq. (3a), $i = 1, 2, \ldots, n$, where n is the number of the total orders of the system. The mode whose eigenvalue has positive real part is an unstable mode, and may result in squeal occurrence. The imaginary part of the mode represents the mode frequency.

3. Sensitivity analysis of the RES to the SMP

The tendency of brake squeal occurrence can be indicated by the RES; therefore, the sensitivity of the RES to the SMP can be used to show the influence of substructures' modal parameters on squealing tendency. The formulae of the sensitivity of the RES to the substructures' eigenvalues and eigenvectors are derived as follows.

3.1. The concepts of sensitivity and relative sensitivity

The sensitivity, represented by partial derivates, denotes the influence of changes of independent variables on the function. In some instance, the concept of relative sensitivity, which represents the influence of relative changes of independent variables on relative variation of function, is more reasonable to express the influence of parameters. Ref. [8] gives the formula of relative sensitivity η as

$$\eta(F/v) = \lim_{\Delta v \to 0} \frac{\Delta F/F}{\Delta v/v} = \frac{v}{F}\frac{\partial F}{\partial v}, \tag{4}$$

where F and v are function and independent variables, respectively. There are no strict regulations on adopting which concept as the criterion weighting the sensitivity of parameters. In this paper, we assume when there are large differences on scale among the independent variables, the concept of relative sensitivity is adopted. When the independent variables are of the same scale, the concept of sensitivity is adopted. Thus, in the latter discussions, the concept of relative sensitivity is used to analyze the sensitivity of the RES to the substructures' eigenvalues, and the concept of sensitivity is used to analyze the sensitivity of the RES to the substructures' eigenvectors.

3.2. Sensitivity analysis of RES to the substructure's eigenvalues

Through performing eigenanalysis of Eq. (3a), the right and left eigenvector matrices $[Y]$ and $[\Psi]$ are obtained. Assume that the norms of the right eigenvectors are unity. And the right and left eigenvectors have the biorthogonal relationship as follows:

$$[Y]^{\mathrm{T}}[\Psi] = [I]. \tag{5}$$

The right eigenvectors satisfy the following expression:

$$[K_{\text{sys}}]\{\Psi\} = \{\lambda_{\text{sys}}\}\{\Psi\}, \tag{6a}$$

where

$$\lambda_{i\text{sys}} = -s_i^2. \tag{6b}$$

Considering the eigenvalue of jth mode of rth substructure $\lambda_j^{(r)}$ as the independent variable which influences the squeal mode i, we can obtain the partial derivative of the ith eigenpair to $\lambda_j^{(r)}$ from Eq. (6a),

$$(\partial/\partial\lambda_j^{(r)})([K_{\text{sys}} - \lambda_{i\text{sys}}I]\{\psi_i\}) = 0. \tag{7}$$

It can be expressed as

$$\frac{\partial\lambda_{i\text{sys}}}{\partial\lambda_j^{(r)}} = \{y_i\}^{\mathrm{T}}\frac{\partial[K_{\text{sys}}]}{\partial\lambda_j^{(r)}}\{\psi_i\}. \tag{8}$$

The sensitivity S of the RES to jth eigenvalue of rth substructure can be derived from Eqs. (6b) and (8),

$$S_{ij} = \frac{\partial\mathrm{Re}(s_i)}{\partial\lambda_j^{(r)}} = -\frac{1}{2}\,\mathrm{Re}\left(\frac{1}{s_i}\frac{\partial\lambda_{i\text{sys}}}{\partial\lambda_j^{(r)}}\right), \tag{9}$$

where $\mathrm{Re}(\cdot)$ denotes the real part of the variable in the bracket. From Eqs. (4) and (9), the relative sensitivity S_r of the RES to substructure's eigenvalue is

$$S_{rij} = \frac{\partial\mathrm{Re}(s_i)}{\partial\lambda_j^{(r)}}\frac{\lambda_j^{(r)}}{\mathrm{Re}(s_i)}. \tag{10}$$

3.3. Sensitivity analysis of the RES to the substructure's eigenvector

Considering the substructure's eigenvector as the independent variables which influence the squeal mode, we can derive the sensitivity of the RES to $\{\phi_j^{(r)}\}$ by a similar method as did for the $\partial\mathrm{Re}(s_i)/\partial\lambda_j^{(r)}$.

It can be expressed as

$$S = \frac{\partial\mathrm{Re}(s_i)}{\partial\{\phi_j^{(r)}\}} = -\frac{1}{2}\mathrm{Re}\left(\frac{1}{s_i}\frac{\partial\lambda_{i\text{sys}}}{\partial\{\phi_j^{(r)}\}}\right). \tag{11}$$

And the sensitivity of $\lambda_{i\text{sys}}$ to the kth element $\phi_{kj}^{(r)}$ of jth modal vector of substructure r can be expressed as

$$\frac{\partial\lambda_{i\text{sys}}}{\partial\phi_{kj}^{(r)}} = \{y_i\}^{\mathrm{T}}\frac{\partial[K_{\text{sys}}]}{\partial\phi_{kj}^{(r)}}\{\psi_i\}, \tag{12a}$$

where

$$\frac{\partial [K_{\rm sys}]}{\partial \phi_{kj}^{(r)}} = \frac{\partial ([\Lambda] - [\Phi]^{\rm T}[K_f][\Phi])}{\partial \phi_{kj}^{(r)}} = -\left(\frac{\partial [\Phi]^{\rm T}}{\partial \phi_{kj}^{(r)}}[K_f][\Phi] + [\Phi]^{\rm T}[K_f]\frac{\partial [\Phi]}{\partial \phi_{kj}^{(r)}}\right). \tag{12b}$$

4. The example sensitivity analysis of a squealing disc brake

The closed-loop coupling model is used for a typical squealing disc brake [3]. The eigenvalue of squeal mode of the brake is 21.4 + 2222.5i (note that the unit of imaginary part has been converted into Hz). The squealing frequency of this brake is identified also by the field experiment [9], which is 2112 Hz. The difference between the calculated and experimental values is about 5%, which shows that the analysis model has sufficient accuracy.

The brake model is composed of five components: an outer pad 1, a rotor 2, an inner pad 3, clip 4 and bracket 5. Their FE modes are shown in Fig. 1. For convenience, the substructure, the substructure's mode, and the element number of substructure's vector are represented by letter r, j and k, respectively. For instance, the 3466th element of 9th mode eigenvector of substructure 5 can be represented as $r5j9k3466$.

4.1. The sensitivity analysis of the RES to substructure's eigenvalues

According to Eq. (10), the relative sensitivity of the RES to substructure's eigenvalues, S_r on the disc brake, is computed. Here only the first 10 largest calculated S_r are listed in Table 1, sorted by ascending absolute values of relative sensitivity.

Comparing the sensitivity values in Table 1, we can find the most dominant substructure eigenvalue is the eigenvalue of $r2j7$, i.e. the 7th eigenvalue of the rotor (−14.5846). It is far larger than those of the other modes. Therefore, an eigenvalue of $r2j7$ is the key substructure eigenvalue, which can be set as the target modal parameter for substructure modification.

For further verification, the substructure's eigenvalues listed in Table 1 with positive sensitivities are decreased by 1%, while the substructure's eigenvalues with negative sensitivities

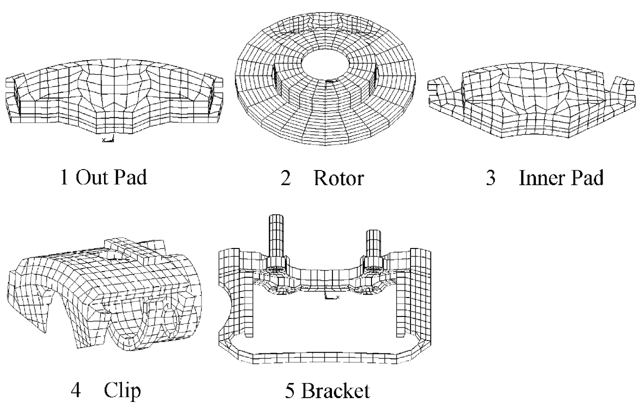

Fig. 1. The FE model of the disc brake components.

Table 1
The relative sensitivity of the RES to substructure's eigenvalue (S_r)

Substructure r	Substructure mode j	The relative sensitivity of the RES to substructure's eigenvalue S_r
2	12	−0.4832
4	11	−0.5246
5	23	0.8798
5	6	0.973
5	26	0.9935
5	18	1.4923
5	10	2.822
5	11	3.5067
5	9	4.521
2	7	−14.5846

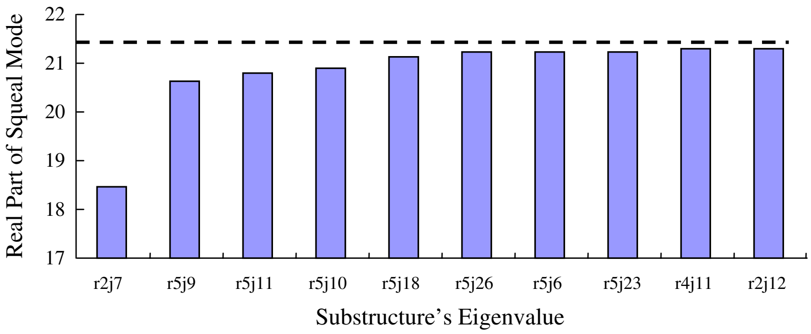

Fig. 2. The decreasing magnitudes of RESs due to the 1% modifications of substructure's eigenvalue.

increased by 1%. Then eigenanalysis is performed for the modified system. The corresponding RES are shown in Fig. 2. The dashed line represents the RES of original brake system. It can be seen that the effectiveness of the *r*2*j*7 is the most remarkable, and the sequence of the effectiveness of the substructure's eigenvalue modifications is the same as indicated in Table 1. The comparison shows that the sensitivity analysis can accurately reflect the influence of substructure mode parameters on squeal mode.

Based on above analysis, in order to eliminate the squeal mode, an optimization problem is constructed, in which the eigenvalue is the design variable; the objective function is the modification of eigenvalue of *r*2*j*7 to its original value; the constraint condition is that the real parts of all modes in band 0–3500 Hz are equal to or less than 0. The optimization result shows that when the frequency of *r*2*j*7 increases by 1.67% from 2180.2 to 2216.2 Hz, the original squeal mode is eliminated and there is no squeal mode below 3500 Hz.

By a slight change in the thickness of the rotor, a target frequency increase of *r*2*j*7 can be achieved. And the eigenanalysis of the brake system with a modified rotor is performed. The distribution of complex eigenvalues of both original ($\times$) and modified ($\diamond$) brake systems is shown

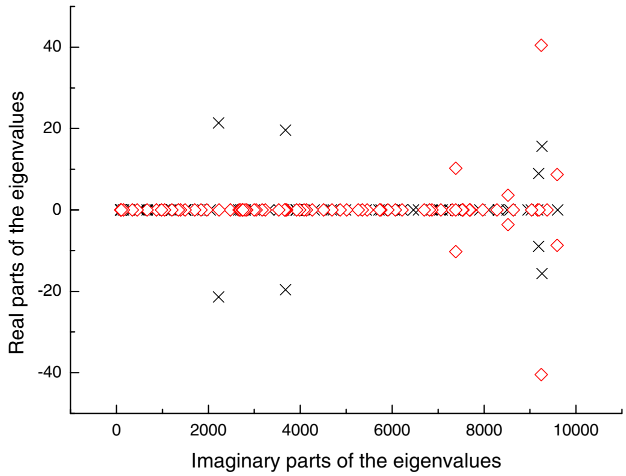

Fig. 3. The distribution of the complex eigenvalues of the brake system (the original '×' and the modified '◇' brake system).

Table 2
The sensitivity of the RES to the elements of substructure's eigenvectors (r)

Substructure r	Substructure mode j	The sequence numbers of elements of substructure's eigenvector k	The values of the elements of substructure's eigenvectors ϕ_{kj}	The sensitivities of the RES to the elements of substructure's eigenvectors S
3	1	1135 (x)	−1.1266	−201.3525
3	3	1123 (x)	−0.0365	208.2412
3	1	506 (y)	1.4625	−210.5422
3	6	506 (y)	1.2746	219.7791
5	6	3466 (x)	0.1457	−237.9519
5	9	3382 (x)	−0.6323	241.9015
5	10	3466 (x)	−1.2051	−260.5269
5	11	3466 (x)	0.5116	263.9265
5	9	3466 (x)	−0.0113	−329.6303
3	1	1123 (x)	−1.1536	−383.1908

in Fig. 3. We can see that the modification is effective in eliminating the squeal mode below 3500 Hz.

4.2. The sensitivity analysis of the RES to elements of substructure's eigenvectors

According to Eq. (11), the sensitivity of the RES to elements of substructure's eigenvectors, S, is computed. Here only the elements of substructure's eigenvectors with the first 10 largest absolute sensitivities are listed in Table 2, sorted by ascending absolute sensitivity. In Table 2, the values of

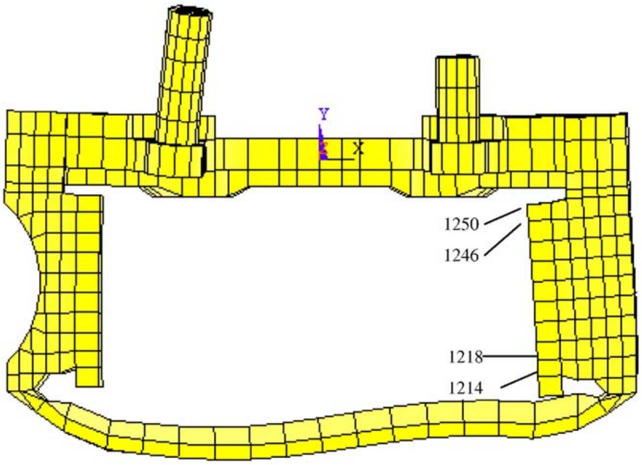

Fig. 4. The nodes on the bracket.

elements of substructure's eigenvectors ϕ_{kj} and the corresponding degrees of freedom (x, y or z as shown in Fig. 4) are also indicated.

From Table 2, the 4 elements of substructure's eigenvector with the relatively large absolute sensitivities are $r3j1k1123$, $r5j9k3466$, $r5j11k3466$ and $r5j10k3466$, and among which the elements $k3466(x)$ of bracket and $k1123(x)$ of inner pad are coupled corresponding to the position of bracket node 1246 (Fig. 4). The mode of the inner pad is the rigid mode in the x direction, whose movement is constrained by the corresponding vibration of the bracket node 1246. So the elastic vibration of the bracket node 1246 in the x direction which concerns the elements $r5j9k3466(x)$, $r5j11k3466(x)$, and $r5j10k3466(x)$ has a big influence on the squeal occurrence. It can be concluded that the decrease in the vibration magnitude of 9–11th mode of node 1246 in the x direction is beneficial to eliminate the squeal mode. This is completely consistent with the analysis of the feed-in energy method in Ref. [7].

According to the above analysis, an optimization problem is constructed, in which the design variables are the bracket design parameters; the objective function is the sum of the magnitudes of $r5j11k3466$, $r5j10k3466$, and $r5j9k3466$. The optimization result shows that an increase in the width of the beam a (Fig. 4) is required, which is consistent with the modification scheme verified by experiment [9]. The modified scheme is then synthesized in the brake model. The distribution of complex eigenvalues of the new system with a modified bracket is shown in Fig. 5. In the figure, the diamonds ($\diamond$) represent new brake system, while the X's ($\times$) represent the original brake system. We can see the squeal mode below 7000 Hz has been eliminated.

5. Conclusion

To determine the dominant modal parameters of substructures influencing the brake squeal occurrence, the sensitivity analysis of the RES to the SMP is explored and the corresponding formulae are derived. The sensitivity analysis method can effectively determine the influence of substructures' modal parameters on the squeal mode in a regular way.

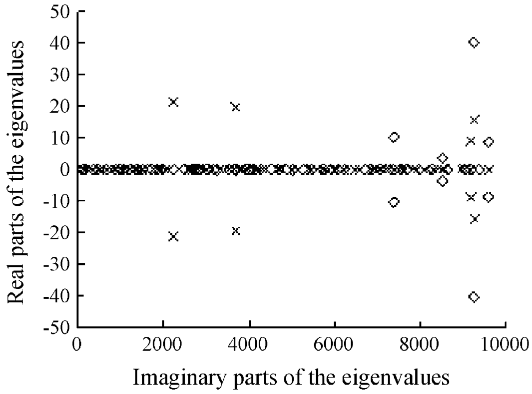

Fig. 5. The distributions of the complex eigenvalues of the brake system (the original '×' and the modified '◇' brake system).

Sensitivity analysis of the RES to the SMP is applied to a typical squealing disc brake to determine the dominant substructure frequency and the dominant elements of substructures' eigenvectors influencing the RES. These modal parameters were then set to be the modification targets and the optimization schemes were applied, respectively. The modification scheme of the bracket has been verified by experimental investigation in Ref. [9].

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