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The response of a Duffing–van der Pol oscillator under delayed feedback control

Xinye Li[a,*], J.C. Ji[b], Colin H. Hansen[b], Chunxiao Tan[c]

[a] *a School of Mechanical Engineering, Hebei University of Technology, Tianjin 300130, PR China*
[b] *b School of Mechanical Engineering, University of Adelaide, SA 5005, Australia*
[c] *School of Science, Hebei University of Technology, Tianjin 300130, PR China*

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Abstract

The nonlinear dynamics of a Duffing–van der Pol oscillator under linear-plus-nonlinear state feedback control with a time delay are investigated. By means of the averaging method and Taylor expansion, two slow-flow equations for the amplitude and phase of the primary resonance response are derived, from which the relations between the amplitude and phase of the primary resonance response and all other parameters are obtained, respectively. The singularity analysis of the equation governing the amplitude of the primary resonance response shows that the bifurcation modes are perturbations of the pitchfork bifurcation. Conditions preventing multiple solutions, corresponding to two different kinds of bifurcation modes, are given, since cases for which multiple solutions are available should be avoided. The stable condition for steady-state response is also given by the Routh–Hurwitz criterion. It is also shown that coupled nonlinear state feedback control can be replaced by uncoupled nonlinear state feedback control.

1. Introduction

Nonlinear systems may exhibit considerably complex dynamic behaviour such as change in stability of response, quasiperiodic motion and chaotic motion. Both the predictability and stability of engineering systems are rather important, which explains the reason that research in

*Corresponding author.
E-mail address: xinyeli@eyou.com (X. Li).

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the area of dynamics and its control of nonlinear systems has received a great deal of attention in the past two decades.

Two classical nonlinear systems, the Duffing oscillator and the van der Pol oscillator can describe many kinds of practical systems. Therefore, there have been many studies of the behaviour of these systems and its control. For example, Hiroshi Yabuno [1] studied the bifurcation control of a parametrically excited Duffing system using combined linear-plus-nonlinear feedback control. Hu et al. [2] considered the primary resonance and the 1:3 subharmonic resonance of a Duffing oscillator under linear state feedback control with a time delay. Ji [3] investigated the saddle-node bifurcation control of a forced single degree of freedom Duffing oscillator with damping for the cases of primary and superharmonic resonances, by means of feedback control without time delay. In Refs. [4,5], Ji and Leung discussed the primary, subharmonic and superharmonic resonances of a Duffing system with damping under linear feedback control with two time-delays and bifurcation control of a parametrically excited Duffing system, respectively. Xu and Jiangjun [6] examined the global bifurcation characteristics of a forced van der Pol oscillator. Atay [7] investigated the effect of delayed position feedback on the response of a van der Pol oscillator. Maccari [8] dealt with the principal parametric resonance of a van der Pol oscillator with time delay linear state feedback. He also investigated the vibration control for the primary resonance of a forced van der Pol oscillator using time delay linear state feedback [9] and concluded that the suppression of quasiperiodic motion can be accomplished by appropriate choices for feedback gains and time delay.

As the combination of these two classical nonlinear systems, a Duffing–van der Pol oscillator can be used as a model in physics, engineering, electronics, biology, neurology and many other disciplines. It is therefore one of the most intensively studied systems in nonlinear dynamics [10]. Tsuda et al. [11] investigated the 1:2 subharmonic resonance of a Duffing–van der Pol system with a retarded argument under a harmonic excitation force. Zhu et al. [12] applied a new stochastic averaging method to predict the response of a Duffing–van der Pol oscillator under both external and parametric excitation of wide-band stationary random processes. Xu and Chung [13] discussed a Duffing–van der Pol oscillator with time delayed position feedback and found two routes to chaos, namely period-doubling bifurcation and torus breaking. Kakmeni et al. [10] studied the strange attractors and chaos control in a Duffing–Van der Pol oscillator with two external periodic forces.

In this paper, the dynamic analysis of a forced Duffing–van der Pol oscillator under weak linear-plus-nonlinear state feedback control with a time delay is presented. The governing equation of motion is

$$\ddot{x} - (\mu - \beta x^2)\dot{x} + \omega_0^2 x + \alpha x^3 = u(t) + f\ \cos(\Omega t), \tag{1}$$

where the dot denotes differentiation with respect to time, ω_0 is the natural frequency, μ, β and α are positive constants, and f and Ω are the amplitude and frequency of the external excitation, respectively. The linear-plus-nonlinear delayed feedback control is of the form

$$\begin{aligned} u(t) = {} & px(t-\tau) + q\dot{x}(t-\tau) + k_1 x^3(t-\tau) + k_2 \dot{x}^3(t-\tau) \\ & + k_3 \dot{x}(t-\tau)x^2(t-\tau) + k_4 \dot{x}^2(t-\tau)x(t-\tau), \end{aligned} \tag{2}$$

where p and q are scalar linear feedback gains, and k_i $(i = 1, \ldots, 4)$ are nonlinear feedback gains.

The focus of this paper is mainly on the nonlinear dynamics of the system considered and its control by the appropriate choice of gains and time delay of feedback when the amplitude and frequency of the external excitation vary. The design of feedback control is presented making the assumption that number of control channels, feedback gains and time delay are three main control measures.

The remainder of this paper is arranged as follows: in Section 2, the modulation equations governing the amplitudes and phases of the steady-state primary resonance responses, are derived by the averaging method as well as by a Taylor expansion. Equations determining the amplitudes and phases of responses are also given. In Section 3, bifurcation analysis and stability analysis are performed using singularity theory and the Routh–Hurwitz criterion, respectively. Conditions preventing multiple solutions as well as conditions to determine the stability of a solution are given, since the cases for which multiple solutions are available should be avoided. In Section 4, the design of different kinds of feedback control, including linear and nonlinear feedback control, is discussed and in Section 5, conclusions are summarized.

2. Derivation of the modulation equations by the perturbation technique

Here, small damping, weak nonlinearity, weak feedback and soft excitation are assumed for the purpose of analysing the primary resonance response using the perturbation technique, namely, Eq. (1) is rewritten as

$$\ddot{x} - \varepsilon(\mu - \beta x^2)\dot{x} + \omega_0^2 x + \varepsilon\alpha x^3 = \varepsilon[u(t) + f\cos(\Omega t)], \tag{3}$$

where ε is a small positive parameter. For the case of primary resonance for which $\Omega \approx \omega_0$, a detuning parameter σ is introduced, such that

$$\Omega^2 \approx \omega_0^2(1 + \varepsilon\sigma). \tag{4}$$

According to the averaging method [14,15], the approximate solution of Eq. (1) in the primary resonant frequency region is assumed to take the following form:

$$x = a\cos(\Omega t - \theta), \tag{5}$$

where the amplitude a and phase θ are time dependent and given by

$$\begin{aligned} \dot{a} &= -\varepsilon\frac{1}{2\pi}\int_0^{2\pi/\Omega}\sin(\Omega t - \theta)F\,\mathrm{d}t, \\ a\,\dot{\theta} &= \varepsilon\frac{1}{2\pi}\int_0^{2\pi/\Omega}\cos(\Omega t - \theta)F\,\mathrm{d}t, \end{aligned} \tag{6}$$

where F denotes all the terms with ε in Eq. (3), namely,

$$F = (\mu - \beta x^2)\dot{x} - \alpha x^3 + u(t) + f\cos(\Omega t).$$

After integrating and using a Taylor expansion for those terms containing a time delay [16], Eq. (6) can be expressed in the following form, known as slow-flow equations:

$$\begin{aligned} 8\dot{a} &= \varepsilon(4f\sin\theta + af_{10} + a^3 f_{11}), \\ 8a\dot{\theta} &= \varepsilon(4f\cos\theta + af_{20} + a^3 f_{21}), \end{aligned} \tag{7}$$

where

$$f_{10} = -4p\sin(\Omega\tau) + 4q\Omega\cos(\Omega\tau) + 4\mu\Omega, \quad f_{20} = 4q\Omega\sin(\Omega\tau) + 4p\cos(\Omega\tau) + 4\omega_0^2\sigma,$$
$$f_{11} = (k_3 + 3k_2\Omega^2)\Omega\cos(\Omega\tau) - (3k_1 + k_4\Omega^2)\sin(\Omega\tau) - \beta\Omega$$
$$f_{21} = (3k_1 + k_4\Omega^2)\cos(\Omega\tau) + (k_3 + 3k_2\Omega^2)\Omega\sin(\Omega\tau) - 3\alpha.$$

Eq. (7) is an autonomous dynamic system, the fixed points of which govern the amplitudes and phases of the periodic solutions of the original system (3). It can be clearly seen that the structure of the slow flow equations describing an externally excited controlled and uncontrolled van der Pol–Duffing oscillator are essentially the same. If all the feedback gains are equal to zero, Eq. (7) corresponds to the modulation equations for the uncontrolled system. Nevertheless, the addition of feedback control makes the coefficients in the modulation equations more complex. Consequently, it is possible to change the nonlinear dynamic characteristics associated with a system, to perform or avoid some kinds of dynamic behaviour.

By setting $\dot{a} = \dot{\theta} = 0$, the equations determining the amplitude and phase of the steady-state response are obtained as follows:

$$a^6(f_{11}^2 + f_{21}^2) + 2a^4(f_{10}f_{11} + f_{20}f_{21}) + a^2(f_{10}^2 + f_{20}^2) - 16f^2 = 0, \tag{8}$$

$$\tan\theta = \frac{a^2 f_{11} + f_{10}}{a^2 f_{21} + f_{20}} \tag{9}$$

from which it can be determined how the amplitude a and phase θ of the steady-state primary resonance response vary as a function of gains, time delay, amplitude and frequency of excitation, as well as with other coefficients in the original Eq. (1). Thus Eq. (8) is the focus for vibration control.

3. Singularity and stability analyses

By means of singularity analysis [17], it is possible to gain a comprehensive understanding of all the possible bifurcations of the response of the system considered; for example, change in number and stability of solutions. Letting $\hat{a} = a^2$, Eq. (8) can be rewritten as

$$\hat{a}^3 + \Theta_1\hat{a}^2 - \Theta_2\hat{a} + \Theta_3 = 0, \tag{10}$$

where $\Theta_1 = 2(f_{10}\ \ f_{11} + f_{20}\ \ f_{21})/(f_{11}^2 + f_{21}^2)$, $\Theta_2 = (f_{10}^2 + f_{20}^2)/(f_{11}^2 f_{21}^2)$, $\Theta_3 = -16f^2/(f_{11}^2 + f_{21}^2)$. Regarding Θ_2 as a bifurcation parameter and Θ_1, Θ_2, as unfolding parameters, the transition sets of Eq. (10) are as follows:

$$\textit{Bifurcation set}: \quad \Theta_3 = 0.$$
$$\textit{Hysteresis set}: \quad \Theta_3 = \Theta_1^3/27.$$

The plane determined by Θ_3 and Θ_3 is divided into four regions by transition sets as shown in Fig. 1 and the corresponding persistent bifurcation diagrams are given in Fig. 2. Our interest is only in Cases (iii) and (iv) in Fig. 2 considering the fact that Θ_2 and Θ_3 are both negative. There

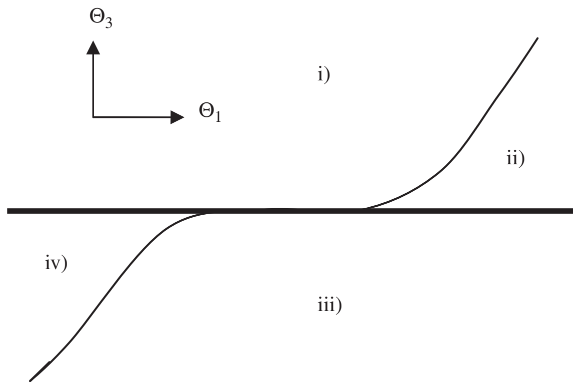

Fig. 1. Transition sets of Eq. (10).

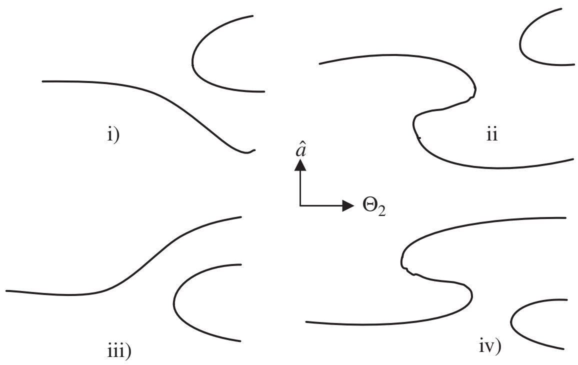

Fig. 2. Persistent bifurcation diagrams corresponding to different regions in Fig. 1.

exists only one solution of $\hat{a}$ for Case (iii) in Fig. 2 where the following inequality holds:

$$\Theta_3 < \Theta_1^3/27. \tag{11}$$

However, for Case (iv) in Fig. 2 there may exist three solutions of $\hat{a}$, which means that there exist multiple-solution intervals in frequency–amplitude or amplitude–amplitude curves. Therefore, vibration control is necessary when $\Theta_{21} < \Theta_2 < \Theta_{22}$ to maintain smaller amplitudes. The quantities Θ_{21} and Θ_{22}, the two threshold values of Θ_2 and also the boundaries of the interval $[\Theta_{21}, \Theta_{22}]$ in which there exists multiple solutions of $\hat{a}$, can be computed using the following set of algebraic equations:

$$\hat{a}^3 + \Theta_1\hat{a}^2 - \Theta_2\hat{a} + \Theta_3 = 0,$$
$$\frac{\mathrm{d}\Theta_2}{\mathrm{d}\hat{a}} = \frac{3\hat{a}^2 + 2\Theta_1\hat{a} - \Theta_2}{\hat{a}} = 0. \tag{12}$$

Or in the form of solutions of an algebraic polynomial determined by Θ_{21} and Θ_3 considering $\hat{a} \neq 0$, namely

$$\Theta_{2i} = \mathrm{Root\ of}(2Z^3 + \Theta_1 Z^2 - \Theta_3)(3\mathrm{Root\ of}(2Z^3 + \Theta_1 Z^2 - \Theta_3) + 2\Theta_1). \tag{13}$$

Figs. 3 and 4 are a set of curves of the amplitude of the primary resonance response versus the excitation frequency, corresponding to different time delays and gains, respectively. Figs. 5 and 6 are a set of curves of the amplitude of the primary resonance response versus the excitation amplitude. It can be seen that frequency–amplitude curves and amplitude–amplitude curves can be greatly different when the feedback gains and time delay vary.

Fig. 3 shows that there exist three amplitude solutions when $\Omega = 1.2$ and $\tau = 3$. The corresponding values of Θ_1 and Θ_3 are -3.1 and -0.17, respectively, which confirms that the corresponding value of Θ_1 is negative and $\Theta_3 > \Theta_1^3/27$. Therefore, (Θ_3, Θ_1) is in Region (iv) of Fig. 1. In addition, the values of Θ_2, Θ_{21} and Θ_{22} are -2.44, -2.51 and -1.40, which means that Θ_2 is between Θ_{21} and Θ_{22}.

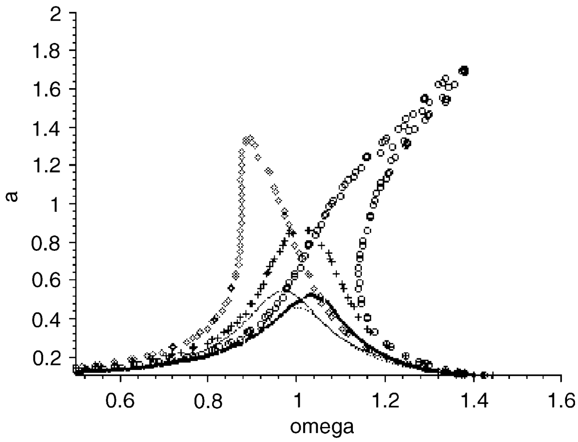

Fig. 3. Frequency–amplitude relations ($\beta = \mu = 1$, $\alpha = 0.5$, $p = 1$, $q = 0.1$, $k_1 = k_2 = k_3 = k_4 = 1$, $p = 1$, $q = 0.1$, $k_1 = k_2 = k_3 = k_4 = 1$, $f = 1$, $\omega_0 = 1$): diamond-line: $\tau = 1$, cross-line: $\tau = 2$, circle-line: $\tau = 3$, bold solid line: $\tau = 4$, dot-line: $\tau = 5$, solid line: $\tau = 6$.

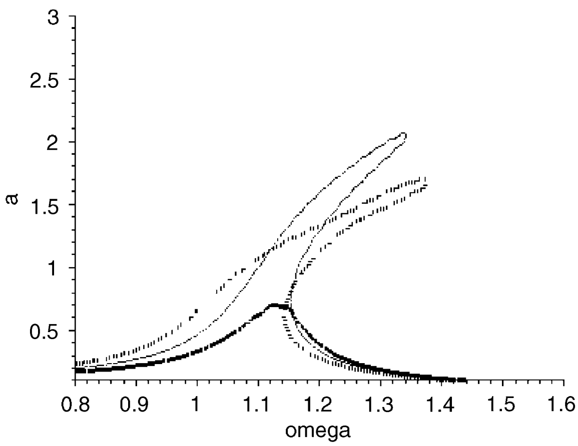

Fig. 4. Frequency–amplitude relations ($f = 1$, $\omega_0 = 1$, $\tau = 3$, $\mu = 1$, $\beta = 1$, $\alpha = 0.5$): dash-dot-line—$p = 1$, $q = 0.1$, $k_1 = k_2 = k_3 = k_4 = 1$, dot-line—$p = 2$, $q = 0.2$, $k_1 = k_2 = k_3 = k_4 = 0.5$, bold solid line—$p = 3$, $q = 0.3$, $k_1 = k_2 = k_3 = k_4 = 0.3$.

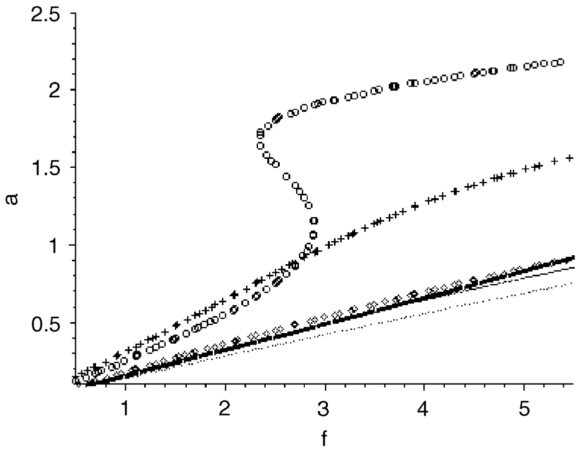

Fig. 5. Response amplitude as a function of excitation amplitude ($\Omega = 1.2$, $\omega_0 = 1.0$, $p = 2$, $q = 0.2$, $\mu = 2$, $\beta = 0.5$, $\alpha = 0.25$, $k_1 = k_2 = k_3 = k_4 = 0.5$): diamond-line: $\tau = 1$, cross-line: $\tau = 2$, circle-line: $\tau = 3$, bold solid line: $\tau = 4$, dot-line: $\tau = 5$, solid line: $\tau = 6$.

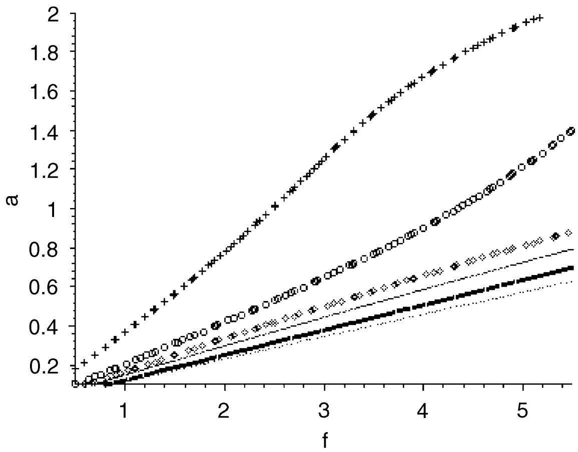

Fig. 6. Response amplitude as a function of excitation amplitude ($\Omega = 1.2$, $\omega_0 = 1.0$, $p = 3$, $q = 0.3$, $\mu = 3$, $\beta = 0.25$, $\alpha = 0.15$, $k_1 = k_2 = k_3 = k_4 = 0.3$): diamond-line: $\tau = 1$, cross-line: $\tau = 2$, circle-line: $\tau = 3$, bold solid line: $\tau = 4$, dot-line: $\tau = 5$, solid line: $\tau = 6$.

Although the singularity analysis of Eq. (10) can give a prediction of all possible bifurcation modes, it is more convenient to analyse the stability of the solutions by applying the classical method of linearization. The eigenvalues of the Jacobian matrix satisfy the following equation:

$$\lambda^2 + 2m\lambda + n = 0, \tag{14}$$

where

$$m = -f_{10} - 2a^2 f_{11},$$
$$n = (f_{10} + a^2 f_{11})(f_{10} + 3a^2 f_{11}) + (f_{20} + a^2 f_{21})(f_{20} + 3a^2 f_{21}).$$

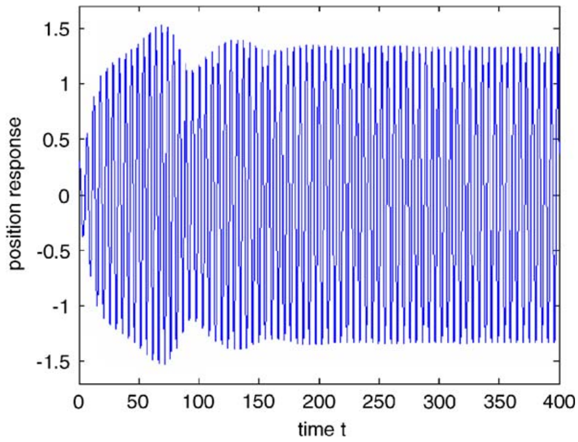

Fig. 7. Steady-state response with the same parameters corresponding to Fig. 3 ($\tau = 3$, $\Omega = 1.2$, initial value $(x_0, \dot{x}_0) = (0.3, 0)$).

From the Routh–Hurwitz criterion [18], the steady-state response is asymptotically stable if and only if the following two inequalities hold simultaneously, which keep the real parts of the eigenvalues negative.

$$m = -f_{10} - 2a^2 f_{11} > 0,$$
$$n = (f_{10} + a^2 f_{11})(f_{10} + 3a^2 f_{11}) + (f_{20} + a^2 f_{21})(f_{20} + 3a^2 f_{21}) > 0. \tag{15}$$

According to Eq. (8), the three values of the response amplitude corresponding to $\Omega = 1.2$ and $\tau = 3$ in Fig. 3 are 0.279, 1.108 and 1.340. The values of m and n are (−5.37, 184.81), (6.03, –61.22) and (11.68, 91.55), respectively, according to Eq. (14). Figs. 7 and 8 are derived using numerical integration of Eq. (3) with identical parameters as in Fig. 3 and $\varepsilon = 0.1$, which demonstrates the above stability prediction; that is, only the response corresponding to $a = 1.340$ is stable.

4. The design of feedback control

Singularity analysis can also explain why feedback control is necessary. Considering the case when feedback control is deleted, then

$$f_{10} = 4\mu\Omega, \quad f_{20} = 4\omega_0^2\sigma,$$
$$f_{11} = -\beta\Omega, \quad f_{21} = -3\alpha.$$

Θ_1 is consequently negative according to its definition.

Substituting $f_{10}, f_{11}, f_{20}, f_{21}$ into expressions following Eq. (10) for Θ_1 and Θ_3, Eq. (15) can be rewritten as

$$f^2(\Omega^2\beta^2 + 9\alpha^2)^2 > \frac{32}{27}(\mu\Omega^2\beta + 3\omega_0^2\sigma\alpha)^3. \tag{16}$$

Eq. (16) ensures that the bifurcation does not occur since Θ_2 is negative. Otherwise, the bifurcation corresponding to Case (iv) may take place for some parameters, which means there

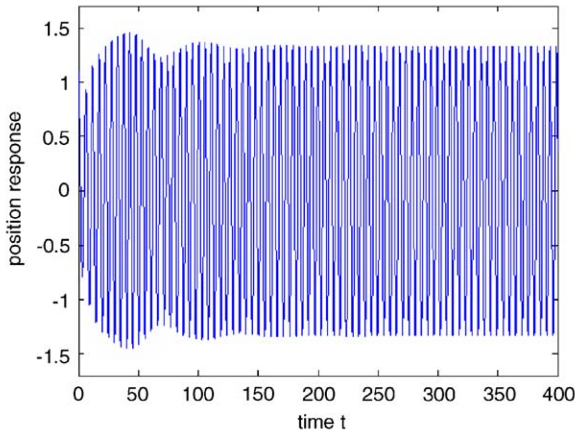

Fig. 8. Steady-state response with the same parameters corresponding to Fig. 3 ($\tau = 3$, $\Omega = 1.2$, initial values $(x_0, \dot{x}_0) = (1.1, 0)$).

exists a multiple-solution phenomenon. It is obvious that not all parameters of the system satisfy Eq. (16). If feedback control is considered, for example, linear feedback control, the condition corresponding to Eq. (16) is as follows:

$$f^2(\Omega^2\beta^2 + 9\alpha^2)^2 > \frac{32}{27}\left\{\beta\Omega[\mu\Omega + k_{pq}\sin(\phi_{pq} - \Omega\tau)] + 3\alpha[\omega_0^2\sigma + k_{pq}\cos(\phi_{pq} - \Omega\tau)]\right\}^3, \tag{17}$$

where

$$k_{pq} = \sqrt{p^2 + (q\Omega)^2}, \quad \cos\phi_{pq} = \frac{p}{k_{pq}}, \quad \cos\phi = \frac{q\Omega}{k_{pq}}.$$

Comparing Eqs. (16) and (17), it may be shown that parameters not satisfying Eq. (16) can satisfy Eq. (17) by adjusting the gains and time delay in feedback. Fig. 9 shows that the multiple-solution phenomenon can be avoided by using linear feedback control. In other words, the linear feedback component can delay the occurrence of undesired bifurcation.

Unfortunately there may exist a case when only linear feedback on its own is unable to give a satisfactory control result. Then nonlinear feedback control should be considered and the condition corresponding to Eq. (17) becomes more complex and it is likely that Case (iv) instead of Case (iii) in Fig. 2 takes place.

It is clear that the condition $\Theta_2 < \Theta_{21}$ is desired to suppress the amplitude of the steady-state response for Case (iv) in Fig. 2. Therefore, the absolute value of Θ_{21} should be as small as possible, although it is difficult to give an expression for Θ_{21}. However, it is easy to verify Eq. (11) or compute Θ_{21} by using Eq. (12). Fig. 10 demonstrates that the bifurcation does not take place and the amplitude of response is much smaller when nonlinear feedback is applied.

By examining the conditions in Eq. (15), it may also be confirmed that only the largest of the three amplitude solutions in Figs. 9 and 10 is stable. However, it is readily found that the shape of the dot-line curves in Figs. 9 and 10 is different from that of the curves in Figs. 3–5 to some extent.

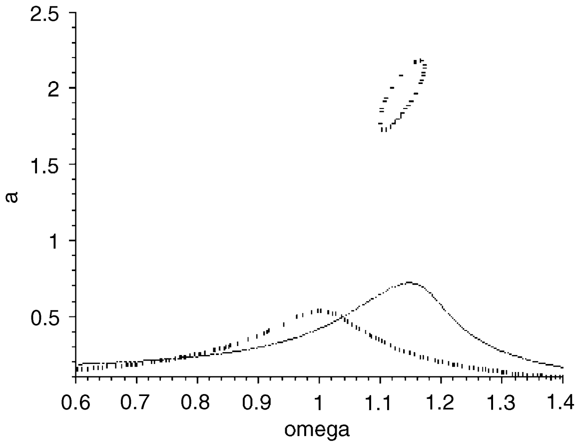

Fig. 9. Frequency—amplitude relations ($\mu = 2$, $\beta = 2$, $\alpha = 1$, $f = 1$, $\omega_0 = 1$): dot-line: no feedback control: and solid line: linear feedback control ($p = 4$, $q = 0.4$, $\tau = 2$).

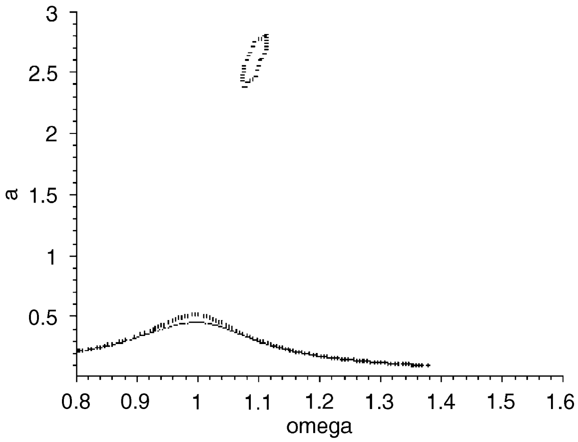

Fig. 10. Frequency–amplitude relations ($\mu = 1$, $\beta = 1$, $\alpha = 0.5$, $f = 1$, $\omega_0 = 1$, $p = 1$, $q = 0.1$, $\tau = 5$): dot-line: linear feedback control and solid line: linear-plus-nonlinear feedback control ($k_1 = k_2 = k_3 = k_4 = 1$).

From the definitions of f_{11} and f_{21} which appear in the slow-flow equation (8), it may be shown that the linear combination of k_1 and k_4 as well as the linear combination of k_2 and k_3 appear as the coefficients of $\sin(\Omega\tau)$ and $\cos(\Omega\tau)$, respectively. In addition, k_1 and k_2 have coefficients 3 and $3\Omega^2$, respectively, compared with k_3 and k_4 having coefficients 1 and Ω^2, respectively. Therefore, for the case of near primary resonance, k_1 and k_2 must have a greater effect on the behaviour of the system considered than k_3 and k_4 when $\omega_0 \approx 1$. Therefore, the nonlinear feedback gains k_4 and k_3 can be removed by enlarging k_1 and k_2. Fig. 11 shows a comparison of two frequency–amplitude relations corresponding to two different kinds of nonlinear feedback control one of which has non-coupled feedback terms.

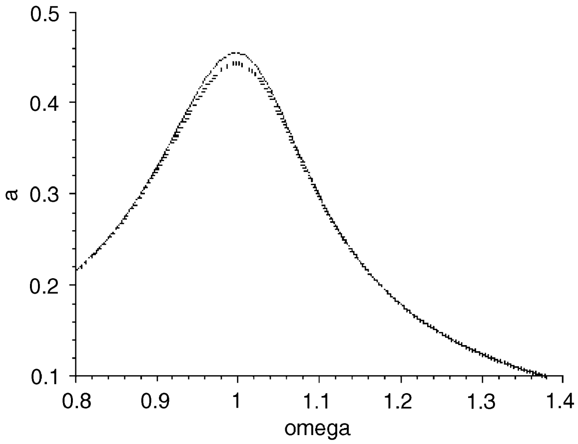

Fig. 11. Frequency–amplitude relations ($\mu = 1,\ \beta = 1,\ \alpha = 0.5,\ f = 1,\ \omega_0 = 1,\ p = 1,\ q = 0.1,\ \tau = 5$): dot-line: $k_1 = k_2 = 1.7$, $k_3 = k_4 = 0$ and solid line: $k_1 = k_2 = 1.7$, $k_3 = k_4 = 1$.

5. Discussion and conclusion

In this paper, the primary resonance response of a Duffing–van der Pol oscillator under feedback control with a time delay has been studied by means of an asymptotic perturbation technique, singularity theory and the Routh–Hurwitz criterion under the assumption that the amplitude of excitation is small. Although the structure of the slow-flow equations, governing the amplitude and phase of the steady-state response, is similar to that of uncontrolled system, the coefficients, associated with all the feedback gains, time delay and parameters of the system considered, in the slow-flow equations are much more complex, which imply more abundant dynamic behaviour.

The singularity analysis shows that the bifurcation mode of the algebraic equation determining the amplitude of the primary resonance response is a perturbation of pitchfork bifurcation, which means that there may exist three solutions of the response amplitude in some cases. Therefore, it is necessary to give some critical conditions, which are associated with feedback gains, time delay, amplitude and frequency of excitation and parameters in the system considered, to perform vibration control. These conditions are not in a simple form, but they are easy to compute.

Stability analysis is also given with the intention of identifying whether one of the multiple solutions is stable or not. In contrast to the traditional frequency–amplitude curve, where the middle of the three solutions is unstable and the smallest and largest ones are stable, it is shown that only the largest amplitude of response is stable, which is also demonstrated by numerical integration and found in Ref. [9] as well.

It is well known that, for some feedback gains and time delays, the response amplitude of a controlled system may be greater than that of an uncontrolled system. In addition, a combination of linear and nonlinear feedback may be not performed as well as linear feedback control in some cases. Fortunately it is theoretically easy to choose appropriate gains and time delay of a feedback controller for a practical system to delay the occurrence of undesired bifurcation and hence suppress the amplitude of the response.

This paper is mainly focused on the effect of feedback gain and time delay when the amplitude and frequency of an external excitation vary. Such a problem often arises in a practical application. The model is presented under the consideration that, generally speaking, three factors should be taken into account in the design of an active feedback vibration controller; namely number of control channels, feedback gains and feedback time delay.

Another conclusion is that coupled nonlinear state feedback can be replaced by uncoupled nonlinear state feedback.

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