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Reflection of SV waves from the free surface of an elastic solid in generalized thermoelastic diffusion

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Abstract

The governing equations for two-dimensional generalized thermoelastic diffusion in an elastic solid are solved. There exist three compressional waves and a shear vertical (SV) wave. The reflection phenomena of SV wave from the free surface of an elastic solid with generalized thermoelastic diffusion is considered. The closed-form expressions for the reflection coefficients for various reflected waves are obtained. These reflection coefficients are found to depend upon the angle of incidence of SV wave, thermoelastic diffusion parameter and other material constants. The numerical values of modulus of the reflection coefficients are presented graphically for different thermal and diffusion parameters.

1. Introduction

Duhamel [1] and Neumann [2] introduced the theory of uncoupled thermoelasticity. There are two shortcomings of this theory. First, the fact that the mechanical state of the elastic body has no effect on the temperature, is not in accordance with true physical experiments. Second, the heat equation being parabolic predicts an infinite speed of propagation for the temperature, which is not physically admissible.

Biot [3] developed the coupled theory of thermoelasticity which eliminates the first defect, but shares the second defect of uncoupled theory. In the classical theory of thermoelasticity, when an

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elastic solid is subjected to a thermal disturbance, the effect is felt at a location far from the source, instantaneously. This implies that the thermal wave propagates with infinite speed, a physically impossible result. In contrast to conventional thermoelasticity, non-classical theories came into existence during the last part of 20th century. These theories, referred to as generalized thermoelasticity, were introduced in the literature in an attempt to eliminate the shortcomings of the classical dynamical thermoelasticity. For example, Lord and Shulman [4], by incorporating a flux-rate term into Fourier's law of heat conduction, formulated a generalized theory which involves a hyperbolic heat transport equation admitting finite speed for thermal signals. Green and Lindsay [5], by including temperature rate among the constitutive variables, developed a temperature-rate-dependent thermoelasticity that does not violate the classical Fourier law of heat conduction, when body under consideration has center of symmetry and this theory also predicts a finite speed for heat propagation. Chandresekharaiah [6] referred to this wavelike thermal disturbance as "second sound". The Lord and Shulman theory of generalized thermoelasticity was further extended by Sherief [7] and Dhaliwal and Sherief [8] to include the anisotropic case. A survey article of representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak [9].

Sinha and Sinha [10] and Sinha and Elsibai [11,12] studied the reflection of thermoelastic waves from the free surface of a solid half-space and at the interface of two semi-infinite media in welded contact, in the context of generalized thermoelasticity. Abd-Alla and Al-Dawy [13] studied the reflection phenomena of SV waves in a generalized thermoelastic medium. Recently, Sharma et al. [14] investigated the problem of thermoelastic wave reflection from the insulated and isothermal stress-free as well as rigidly fixed boundaries of a solid half-space in the context of different theories of generalized thermoelasticity.

Diffusion may be defined as the random walk, of an ensemble of particles, from regions of high concentration to regions of lower concentration. The study of this phenomenon is of great concern due to its many geophysical and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in metal oxide semiconductor (MOS) transistors and dope poly-silicon gates in MOS transistors. The phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from oil deposits.

Using the coupled thermoelastic model, Nowacki [15–17] developed the theory of thermoelastic diffusion. Using Lord–Shulman (L-S) model, Sherief et al. [18] generalized the theory of thermoelastic diffusion, which allows the finite speeds of propagation of waves. The present study is motivated by the importance of thermoelastic diffusion process in the field of oil extraction. Also, the development of generalized theory of thermoelastic diffusion by Sherief et al. [18] provide a chance to study the wave propagation in such an interesting media. The paper is organized as follows: in Section 2, the wave propagation in an isotropic, homogeneous model of elastic solid with generalized thermoelastic diffusion is studied for Lord–Shulman (L-S) as well as for Green–Lindsay (G-L) model. The governing equations for L-S and G-L models are solved in x–z plane to show the existence of three compressional waves and a SV wave. In Section 3, the closed-form expressions for reflection coefficients are obtained for the incidence of SV wave at a thermally insulated free surface. In the last section, a numerical example is given to discuss the dependence of reflection coefficients upon thermal relaxation times, diffusion relaxation times, angle of incidence of SV wave and other thermal and diffusion parameters. This dependence is shown graphically.

2. Governing equations and solution

Following, Lord and Shulman [4], Green and Lindsay [5] and Sherief et al. [18], the governing equations for an isotropic, homogeneous elastic solid with generalized thermoelastic diffusion with reference temperature T_0 in the absence of body forces are:

(i) the constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1(\Theta + \tau_1\dot{\Theta}) - \beta_2(C + \tau^1\dot{C})], \tag{1}$$

$$\rho T_0 S = \rho c_E(\Theta + \alpha\dot{\Theta}) + \beta_1 T_0 e_{kk} + aT_0(C + \beta\dot{C}), \tag{2}$$

$$P = -\beta_2 e_{kk} + b(C + \tau^1\dot{C}) - a(\Theta + \tau_1\dot{\Theta}). \tag{3}$$

Here $\alpha = \beta = \tau_1 = \tau^1 = 0$ for L-S model and $\alpha = \tau_0,\ \beta = \tau^0$ for G-L model.

(ii) the equation of motion

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1(\Theta + \tau_1\dot{\Theta})_{,i} - \beta_2(C + \tau^1\dot{C})_{,i} = \rho\ddot{u}_i, \tag{4}$$

(iii) the equation of heat conduction

$$\rho c_E(\dot{\Theta} + \tau_0\ddot{\Theta}) + \beta_1 T_0(\dot{e} + \Omega\tau_0\ddot{e}) + aT_0(\dot{C} + \gamma\ddot{C}) = K\Theta_{,ii}, \tag{5}$$

(iv) the equation of mass diffusion

$$D\beta_2 e_{,ii} + Da(\Theta + \tau_1\dot{\Theta})_{,ii} + (\dot{C} + \Omega\tau^0\ddot{C}) - Db(C + \tau^1\dot{C})_{,ii} = 0, \tag{6}$$

where $\beta_1 = (3\lambda + 2\mu)\alpha_t$ and $\beta_2 = (3\lambda + 2\mu)\alpha_c$, λ, μ are Lame's constants, α_t is the coefficient of linear thermal expansion and α_c is the coefficient of linear diffusion expansion. $\Theta = T - T_0, T_0$ is the temperature of the medium in its natural state assumed to be such that $|\Theta/T_0| \ll 1$. σ_{ij} are the components of the stress tensor, u_i are the components of the displacement vector, ρ is the density assumed independent of time, e_{ij} are the components of the strain tensor, $e = e_{kk}$, T is the absolute temperature, S is the entropy per unit mass, P is the chemical potential per unit mass, C is the concentration, c_E is the specific heat at constant strain, K is the coefficient of thermal conductivity, D is thermoelastic diffusion constant. τ_0 is the thermal relaxation time, which will ensure that the heat conduction equation, satisfied by the temperature Θ will predict finite speeds of heat propagation. τ is the diffusion relaxation time, which will ensure that the equation, satisfied by the concentration C will also predict finite speeds of propagation of matter from one medium to the other. The constants a and b are the measures of thermoelastic diffusion effects and diffusive effects, respectively. The superposed dots denote derivative with respect to time.

For the L-S model, $\tau_1 = 0,\ \tau^1 = 0,\ \Omega = 1,\ \gamma = \tau_0$. The governing equations in L-S model are same as given by Sherief et al. [18].

For G-L model, $\tau_1 > 0,\ \tau^1 > 0,\ \Omega = 0,\ \gamma = \tau^0$. The thermal relaxation times τ_0 and τ_1 satisfy the inequality $\tau_1 \geqslant \tau_0 \geqslant 0$ for G-L model. The diffusion relaxation times τ^0 and τ^1 also satisfy the inequality $\tau^1 \geqslant \tau^0 \geqslant 0$ for G-L model.

For two-dimensional motion in x–z plane, Eqs. (4)–(6) are written as

$$(\lambda + 2\mu)u_{1,11} + (\mu + \lambda)u_{3,13} + \mu u_{1,33} - \beta_1 \tau_\theta^1 \Theta_{,1} - \beta_2 \tau_c^1 C_{,1} = \rho \ddot{u}_1, \tag{7}$$

$$\mu u_{3,11} + (\mu + \lambda)u_{1,13} + (\lambda + 2\mu)u_{3,33} - \beta_1 \tau_\theta^1 \Theta_{,3} - \beta_2 \tau_c^1 C_{,3} = \rho \ddot{u}_3, \tag{8}$$

$$K\nabla^2\Theta = \rho c_E \tau_\theta^0 \dot{\Theta} + \beta_1 T_0 \tau_e^0 \dot{e} + a T_0 \tau_c^0 \dot{C}, \tag{9}$$

$$D\beta_2 \nabla^2 e + Da\tau_\theta^1 \nabla^2 \Theta - Db\tau_c^1 \nabla^2 C + \tau_f^0 \dot{C} = 0, \tag{10}$$

where

$$\tau_\theta^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \quad \tau_\theta^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \quad \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t},$$

$$\tau_c^0 = 1 + \gamma \frac{\partial}{\partial t}, \quad \tau_e^0 = 1 + \Omega\tau_0 \frac{\partial}{\partial t}, \quad \tau_f^0 = 1 + \Omega\tau^0 \frac{\partial}{\partial t}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

The displacement components u_1 and u_3 may be written in terms of potential functions ϕ and ψ as

$$u_1 = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z}, \quad u_3 = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x}. \tag{11}$$

Using Eq. (11) into Eqs. (7)–(10), we obtain

$$\mu\nabla^2\psi = \rho \frac{\partial^2\psi}{\partial t^2}, \tag{12}$$

$$(\lambda + 2\mu)\nabla^2\phi - \beta_1 \tau_\theta^1 \Theta - \beta_2 \tau_c^1 C = \rho \frac{\partial^2\phi}{\partial t^2}, \tag{13}$$

$$K\nabla^2\Theta = \rho c_E \tau_\theta^0 \frac{\partial\Theta}{\partial t} + \beta_1 T_0 \tau_e^0 \frac{\partial}{\partial t}\nabla^2\phi + a T_0 \tau_c^0 \frac{\partial C}{\partial t}, \tag{14}$$

$$D\beta_2 \nabla^4\phi + Da\tau_\theta^1 \nabla^2\Theta - Db\tau_c^1 \nabla^2 C + \tau_f^0 \frac{\partial C}{\partial t} = 0. \tag{15}$$

Eq. (12) is uncoupled, whereas Eqs. (13)–(15) are coupled in ϕ, Θ and C. From Eqs. (12)–(15), we see that while the P-wave is affected due to the presence of thermal and diffusion fields, the SV remains unaffected. The solution of Eq. (12) corresponds to the propagation of SV wave with velocity $v_0 = \sqrt{\mu/\rho}$.

Solutions of Eqs. (13)–(15) are now sought in the form of the harmonic travelling wave

$$\{\phi, \Theta, C\} = \{\phi_0, \Theta_0, C_0\} e^{\imath k(x\sin\theta + z\cos\theta - vt)}, \tag{16}$$

where v is the phase speed, k is the wavenumber and $(\sin\theta, \cos\theta)$ denotes the projection of the wave normal onto x–z plane.

The homogeneous system of equations in ϕ_0, Θ_0, and C_0, obtained by inserting Eq. (16) into Eqs. (13)–(15), admits non-trivial solutions and enables to conclude that ξ satisfies the cubic equation

$$\xi^3 + L\xi^2 + M\xi + N = 0, \tag{17}$$

where

$$\xi = \rho v^2,$$

$$L = -(\varepsilon + \varepsilon\varepsilon_1\varepsilon_2\varepsilon_3 + d_1 + d_2 + \lambda + 2\mu),$$

$$M = (\lambda + 2\mu)(d_1 + d_2 + \varepsilon\varepsilon_1\varepsilon_2\varepsilon_3) + d_1 d_2 + d_2\varepsilon - \varepsilon\varepsilon_2(\varepsilon_1 + \varepsilon_3) - \varepsilon_2,$$

$$N = -d_1 d_2(\lambda + 2\mu) + \varepsilon_2 d_1,$$

$$d_1 = \frac{K}{c_E\tau_\theta}, \quad d_2 = \frac{\rho D b \tau_c^{11}}{\tau_f},$$

$$\varepsilon = \frac{\beta_1^2 T_0 \tau_e \tau_\theta^{11}}{\rho c_E \tau_\theta}, \quad \varepsilon_1 = -\frac{a}{\beta_1\beta_2}, \quad \varepsilon_2 = \frac{\rho D \beta_2^2 \tau_c^{11}}{\tau_f}, \quad \varepsilon_3 = -\frac{a\tau_c}{\beta_1\beta_2\tau_e\tau_c^{11}},$$

$$\tau_\theta = \tau_0 + \frac{\iota}{\omega}, \quad \tau_c = \gamma + \frac{\iota}{\omega}, \quad \tau_e = \Omega\tau_0 + \frac{\iota}{\omega}, \quad \tau_f = \Omega\tau^0 + \frac{\iota}{\omega},$$

$$\tau_\theta^{11} = 1 - \iota\omega\tau_1, \quad \tau_c^{11} = 1 - \iota\omega\tau^1.$$

Eq. (17) is cubic in ξ. The roots of this equation give three values of ξ. Each value of ξ corresponds to a wave if v^2 is real and positive. Hence, three positive values of v will be the velocities of propagation of three possible waves. Cardan's method is used to solve Eq. (17). Using Cardan's method, Eq. (17) is written as

$$\Lambda^3 + 3H\Lambda + G = 0, \tag{18}$$

where

$$\Lambda = \xi + \frac{L}{3}, \quad H = \frac{3M - L^2}{9}, \quad G = \frac{27N - 9LM + 2L^3}{27}. \tag{19}$$

For all the three roots of Eq. (18) to be real, $\Delta_0 (= G^2 + 4H^3)$ should be negative. Assuming the Δ_0 to be negative, we obtain the three roots of Eq. (18) as

$$\Lambda_n = 2\sqrt{-H}\cos\left(\frac{\phi + 2\pi(n-1)}{3}\right) \quad (n = 1, 2, 3), \tag{20}$$

where

$$\phi = \tan^{-1}\left(\frac{\sqrt{|\Delta_0|}}{-G}\right). \tag{21}$$

Hence,

$$v_n = \sqrt{\left(\Lambda_n - \frac{L}{3}\right) \Big/ \rho} \quad (n = 1, 2, 3) \tag{22}$$

are velocities of propagation of the three possible coupled dilatational waves. The waves with velocities v_1, v_2 and v_3 correspond to P wave, mass diffusion (MD) wave and thermal (T) wave, respectively. This fact may be verified, when we solve Eq. (17), using a computer program of Cardan's method. If we neglect thermal effects, i.e. for $\varepsilon = 0$, $d_1 = 0$, the cubic equation (17) reduces to a quadratic equation whose roots are as

$$2\rho v^2 = [d_2 + (\lambda + 2\mu)] \pm \sqrt{[d_2 - (\lambda + 2\mu)]^2 + 4\varepsilon_2}, \tag{23}$$

where the positive and negative signs correspond to P wave and MD wave, respectively. Moreover, the MD wave exists if $\beta_2^2 < b(\lambda + 2\mu)$. Similarly, if we neglect the diffusion effects, i.e. for $\varepsilon_1 = \varepsilon_2 = d_2 = 0$, Eq. (17) reduces to a quadratic equation whose roots are as

$$2\rho v^2 = [\{d_1 + (\lambda + 2\mu)\} + \varepsilon] \pm \sqrt{[\{d_1 - (\lambda + 2\mu)\} - \varepsilon]^2 + 4\varepsilon d_1}, \tag{24}$$

where the positive and negative signs correspond to P wave and T waves, respectively. The T wave exists, if $d_1 > 0$, which is true. These two-dimensional roots are in agreement with the non-dimensional roots obtained by Abd-Alla and Al-Dawy [13] for Lord and Shulman theory. In absence of thermoelastic diffusion effects, Eq. (17) corresponds to P wave with velocity $v_1 = \sqrt{(\lambda + 2\mu)/\rho}$.

3. Reflection coefficients

In previous section, it has been discussed that there exists three compressional waves and a SV wave in an isotropic elastic solid with generalized thermoelastic diffusion. Any incident wave at the interface of two elastic solid bodies, in general, produce compressional and distortional waves in both media (see for example, Refs. [19,20]). Let us consider an incident SV wave (Fig. 1). The boundary conditions at the free surface $z = 0$ are satisfied, if the incident SV wave gives rise to a reflected SV and three reflected compressional waves (i.e. P, MD and T waves). The surface $z = 0$ is assumed to be traction free and thermally insulated so that there is no variation of temperature and concentration on it. Therefore, the boundary conditions on $z = 0$ are written as

$$\sigma_{zz} = 0, \quad \sigma_{zx} = 0, \quad \frac{\partial \Theta}{\partial z} = 0, \quad \frac{\partial C}{\partial z} = 0, \quad \text{on } z = 0. \tag{25}$$

The appropriate displacement potentials ϕ and ψ, temperature Θ and concentration C are taken in the form

$$\psi = B_0 \exp[\iota k_0(x \sin\theta_0 + z\cos\theta_0) - \iota\omega t)] + B_1 \exp[\iota k_0(x\sin\theta_0 - z\cos\theta_0) - \iota\omega t)], \tag{26}$$

$$\begin{aligned}\phi = {} & A_1 \exp[\iota k_1(x\sin\theta_1 - z\cos\theta_1) - \iota\omega t)] + A_2 \exp[\iota k_2(x\sin\theta_2 - z\cos\theta_2) - \iota\omega t)] \\ & + A_3 \exp[\iota k_3(x\sin\theta_3 - z\cos\theta_3) - \iota\omega t],\end{aligned} \tag{27}$$

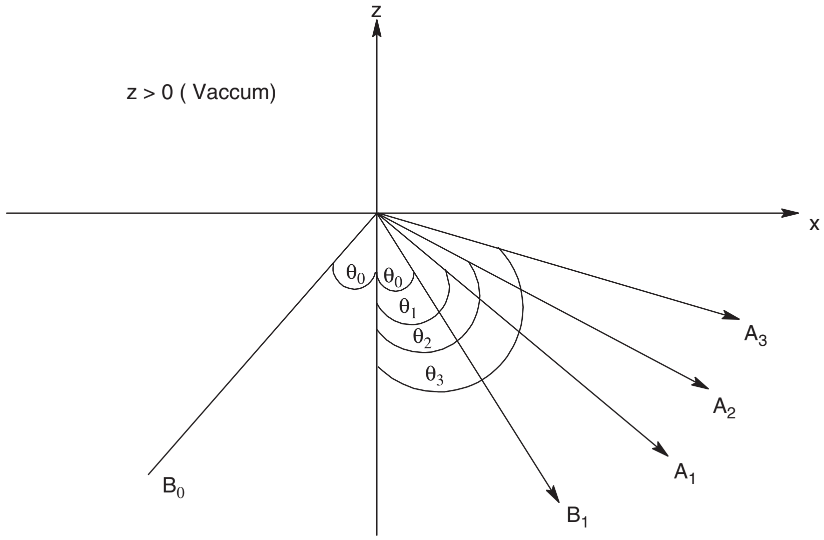

Fig. 1. Schematic diagram for the problem.

$$\Theta = \zeta_1 A_1 \exp[\imath k_1(x\sin\theta_1 - z\cos\theta_1) - \imath\omega t)] + \zeta_2 A_2 \exp[\imath k_2(x\sin\theta_2 - z\cos\theta_2) - \imath\omega t)] + \zeta_3 A_3 \exp[\imath k_3(x\sin\theta_3 - z\cos\theta_3) - \imath\omega t], \tag{28}$$

$$C = \eta_1 A_1 \exp[\imath k_1(x\sin\theta_1 - z\cos\theta_1) - \imath\omega t)] + \eta_2 A_2 \exp[\imath k_2(x\sin\theta_2 - z\cos\theta_2) - \imath\omega t)] + \eta_3 A_3 \exp[\imath k_3(x\sin\theta_3 - z\cos\theta_3) - \imath\omega t], \tag{29}$$

where the wave normal of the incident SV wave makes angle θ_0 with the positive direction of the z-axis, and those of reflected P, MD and T waves make θ_1, θ_2 and θ_3 with the same direction, and

$$\zeta_i = k_i^2 G_i(\rho v_i^2 - \lambda - 2\mu), \quad \eta_i = k_i^2 H_i(\rho v_i^2 - \lambda - 2\mu) \quad (i = 1, 2, 3) \tag{30}$$

and

$$G_i = \frac{\varepsilon\rho v_i^2(\varepsilon_1\varepsilon_2 - d_2 + \rho v_i^2)}{d_1\varepsilon_2 + \rho v_i^2[\varepsilon(d_2 - \rho v_i^2) - \varepsilon_2 - \varepsilon\varepsilon_2(\varepsilon_1 + \varepsilon_3)]}, \tag{31}$$

$$H_i = \frac{\varepsilon_2[\rho v_i^2(\varepsilon\varepsilon_1 + 1) - d_1]}{d_1\varepsilon_2 + \rho v_i^2[\varepsilon(d_2 - \rho v_i^2) - \varepsilon_2 - \varepsilon\varepsilon_2(\varepsilon_1 + \varepsilon_3)]}. \tag{32}$$

The ratios of the amplitudes of the reflected waves to the amplitude of the incident wave, namely B_1/B_0, A_1/B_0, A_2/B_0 and A_3/B_0 give the reflection coefficients for reflected SV, reflected P, reflected MD and reflected T waves, respectively. The wavenumber k_0, k_1, k_2, k_3 and the angles $\theta_0, \theta_1, \theta_2, \theta_3$ are connected by the relation

$$k_0\sin\theta_0 = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3 \tag{33}$$

at surface $z = 0$. Relation (33) may also be written in order to satisfy the boundary conditions (25) as

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3}, \tag{34}$$

where $v_0 = \sqrt{\mu/\rho}$ is the velocity of SV wave and v_i, $(i = 1, 2, 3)$ are the velocities of three sets of reflected compressional waves. Using the potentials given by Eqs. (26)–(29) in boundary conditions (25), the reflection coefficients may be expressed as

$$\frac{B_1}{B_0} = \frac{\Delta_1}{\Delta}, \quad \frac{A_1}{B_0} = \frac{\Delta_2}{\Delta}, \quad \frac{A_2}{B_0} = \frac{\Delta_3}{\Delta}, \quad \frac{A_3}{B_0} = \frac{\Delta_4}{\Delta}, \tag{35}$$

where

$$\Delta = \frac{x_{43}x_{31} - x_{33}x_{41}}{x_{42}x_{31} - x_{32}x_{41}} - \frac{x_{33}(x_{21} - x_{11}) - x_{31}(x_{23} - x_{13})}{x_{32}(x_{21} - x_{11}) - x_{31}(x_{22} - x_{12})},$$

$$\Delta_1 = \frac{x_{43}x_{31} - x_{33}x_{41}}{x_{42}x_{31} - x_{32}x_{41}} - \frac{x_{33}(x_{21} + x_{11}) - x_{31}(x_{23} + x_{13})}{x_{32}(x_{21} + x_{11}) - x_{31}(x_{22} + x_{12})},$$

$$\Delta_2 = \frac{-2(x_{43}x_{32} - x_{42}x_{33})}{x_{42}x_{32}},$$

$$\Delta_3 = \frac{2(x_{43}x_{31} - x_{41}x_{33})}{x_{41}x_{31}},$$

$$\Delta_4 = \frac{-2(x_{42}x_{31} - x_{41}x_{32})}{x_{41}x_{31}},$$

$$x_{1i} = -\frac{[\lambda + 2\mu\cos^2\theta_i + \tau_\theta^{11}(\zeta_i/k_i^2)\beta_1 + \tau_c^{11}(\eta_i/k_i^2)\beta_2](k_i/k_0)^2}{\mu\sin 2\theta_0},$$

$$x_{2i} = \frac{\sin 2\theta_i(k_i/k_0)^2}{\cos 2\theta_0},$$

$$x_{3i} = \cos\theta_i(\zeta_i/k_i^2)(k_i/k_0)^3,$$

$$x_{4i} = \cos\theta_i(\eta_i/k_i^2)(k_i/k_0)^3.$$

In the absence of thermoelastic diffusion, these reflection coefficients reduce to

$$\frac{B_1}{B_0} = \frac{\sin 2\theta_1 \sin 2\theta_0 - (v_1/v_0)^2\cos 2\theta_0}{\sin 2\theta_1 \sin 2\theta_0 + (v_1/v_0)^2\cos 2\theta_0}, \tag{36}$$

$$\frac{A_1}{B_0} = \frac{(v_1/v_0)\sin 4\theta_0}{\sin 2\theta_1 \sin 2\theta_0 + (v_1/v_0)^2\cos 2\theta_0}, \tag{37}$$

which are the same as those given by Ben-Menahem and Singh [20], if θ_1, θ_0, v_1 and v_0 are replaced by e, f, α and β, respectively. It may also be mentioned that the MD and T waves will disappear in absence of thermoelastic diffusion.

4. Numerical results

For computational work, the following material constants at $T_0 = 27\,^\circ\mathrm{C}$ are considered for an elastic solid with generalized thermoelastic diffusion

$$\lambda = 5.775 \times 10^{11}\,\mathrm{dyn/cm^2}, \quad \mu = 2.646 \times 10^{11}\,\mathrm{dyn/cm^2}, \quad \rho = 2.7\,\mathrm{g/cm^3},$$

$$c_E = 2.361\,\mathrm{cal/g\,^\circ C}, \quad K = 0.492\,\mathrm{cal/cm\,s\,^\circ C}, \quad \alpha_t = 0.05\,\mathrm{cm^3/g},$$

$$\alpha_c = 0.06\,\mathrm{cm^3/g}, \quad \omega = 10\,\mathrm{s^{-1}}, \quad a = 0.005\,\mathrm{cm^2/s^2\,^\circ C}, \quad b = 0.05\,\mathrm{cm^5/g\,s},$$

$$D = 0.5\,\mathrm{g\,s/cm^3}.$$

The numerical values of reflection coefficients of various reflected waves are computed for angle of incidence varying from 1° to 45° for L-S (Lord–Shulman) and G-L (Green–Lindsay) models when $\tau_1 = \tau_0 = 0.05\,\mathrm{s}$ and $\tau^1 = \tau^0 = 0.04\,\mathrm{s}$. These numerical values of reflection coefficients are shown graphically in Figs. 2–5 where solid and dotted lines correspond to L-S and G-L models, respectively.

Fig. 2 shows the reflection coefficients of reflected SV waves for L-S and G-L models. The reflection coefficient for each model, first increases and then decreases to its minima, thereafter, it attains its value one at $\theta_0 = 45^\circ$. Figs. 3–5 show the variations for reflected P, reflected MD and

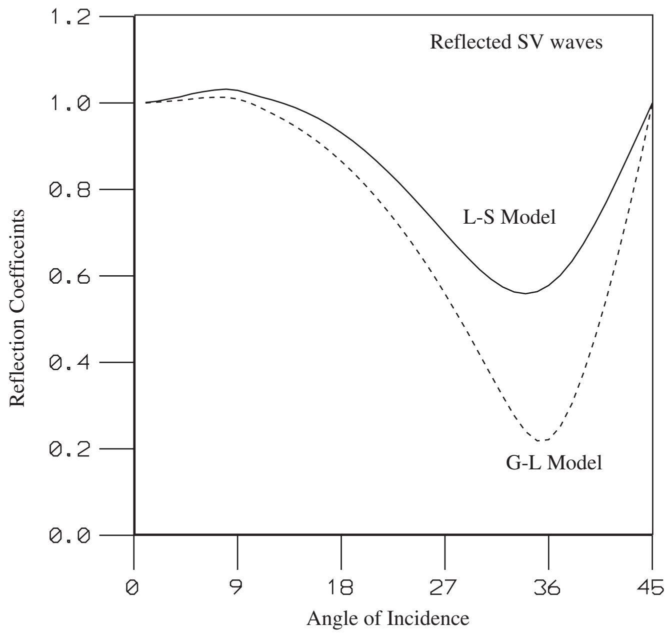

Fig. 2. Variations of reflection coefficients of SV waves with the angle of incidence for L-S and G-L models.

reflected T, respectively. Comparing the solid and dotted lines in these figures, the effects of second thermal relaxation time and second diffusion relaxation time are observed on the reflection coefficients. The further numerical study is restricted to L-S model only. The numerical values of

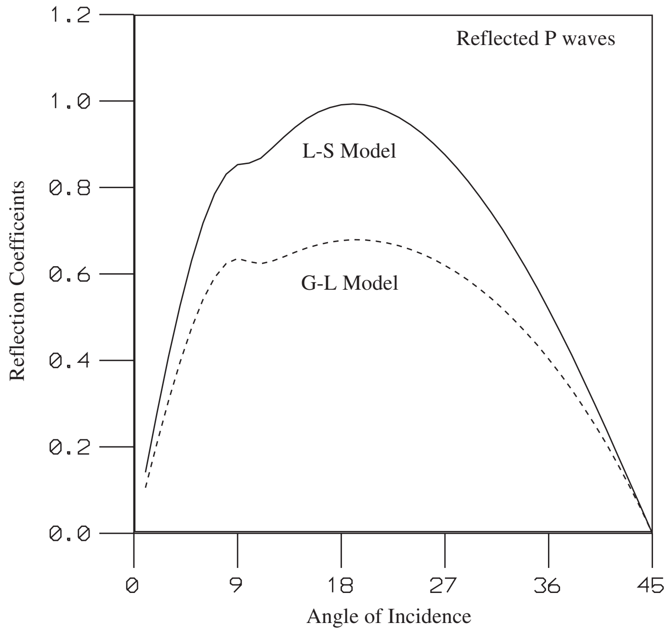

Fig. 3. Variations of reflection coefficients of P waves with the angle of incidence for L-S and G-L models.

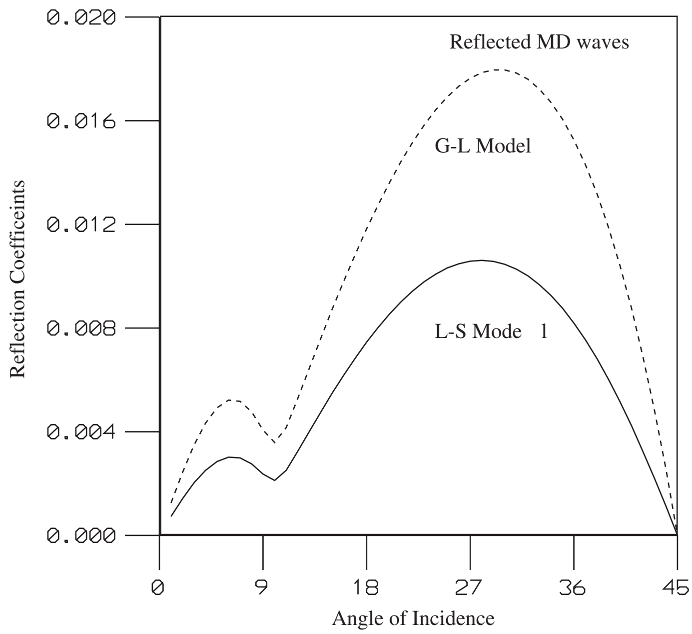

Fig. 4. Variations of reflection coefficients of MD waves with the angle of incidence for L-S and G-L models.

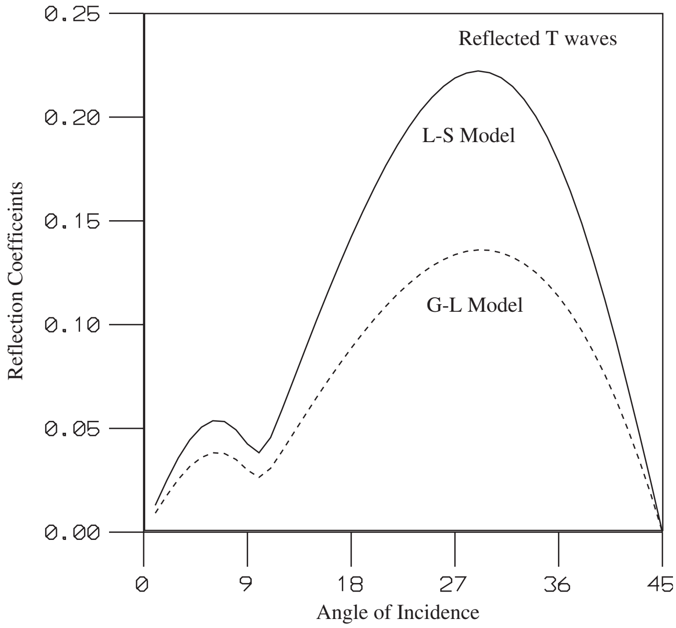

Fig. 5. Variations of reflection coefficients of T waves with the angle of incidence for L-S and G-L models.

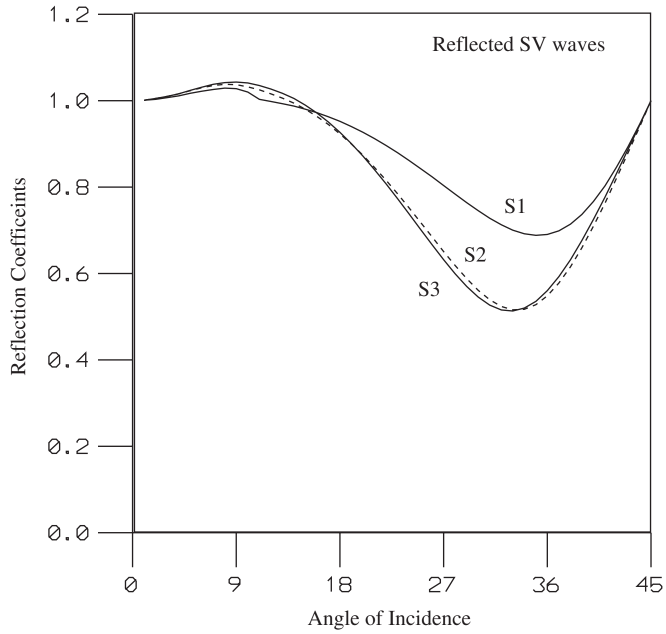

Fig. 6. Variations of reflection coefficients of SV waves with the angle of incidence for sets $S1$, $S2$ and $S3$ of L-S model.

reflection coefficients are computed for three different sets of thermal and diffusion relaxation times, namely $S1(\tau_0 = 0.2,\ \tau^0 = 0.1)$; $S2(\tau_0 = 0.02,\ \tau^0 = 0.01)$ and $S3(\tau_0 = 0.002,\ \tau^0 = 0.001)$. These coefficients are shown graphically in Figs. 6–9. The reflection coefficients of various

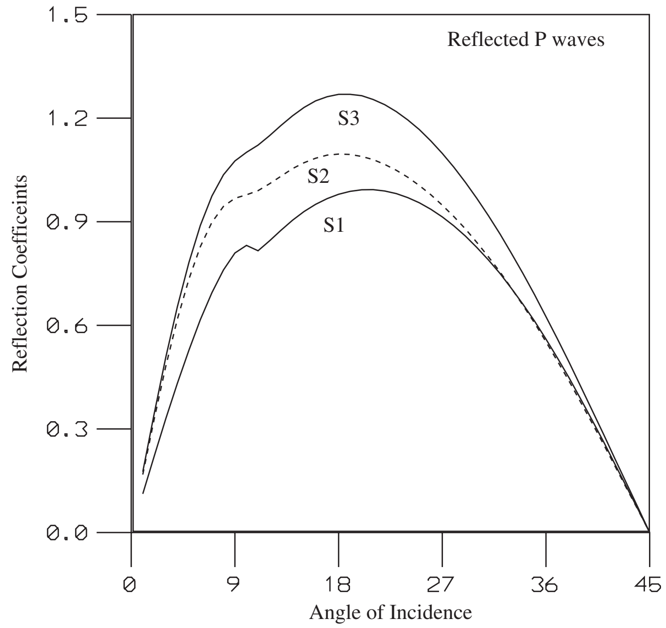

Fig. 7. Variations of reflection coefficients of P waves with the angle of incidence for sets $S1$, $S2$ and $S3$ of L-S model.

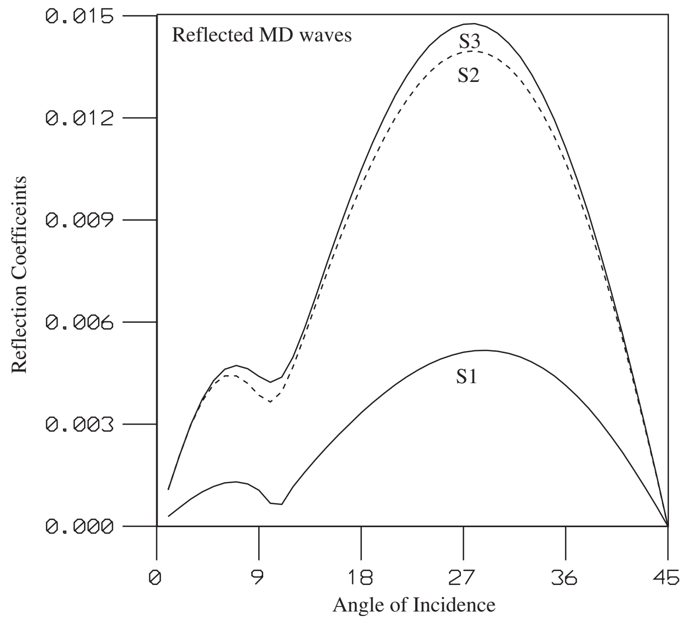

Fig. 8. Variations of reflection coefficients of MD waves with the angle of incidence for sets $S1$, $S2$ and $S3$ of L-S model.

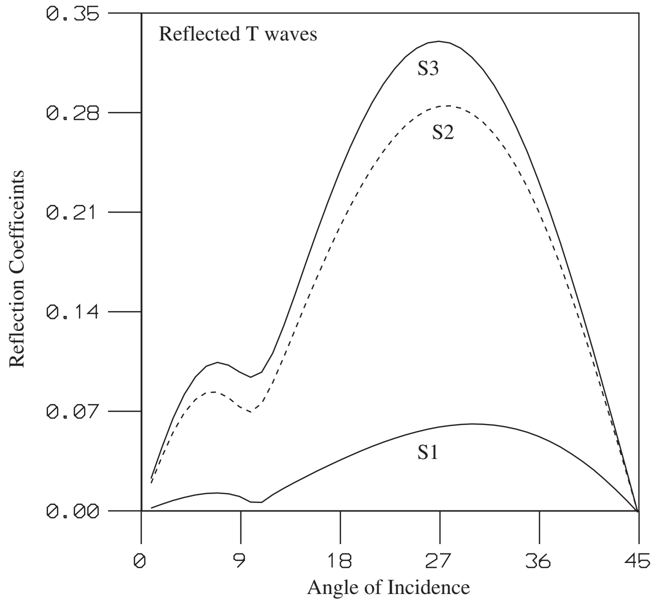

Fig. 9. Variations of reflection coefficients of T waves with the angle of incidence for sets $S1$, $S2$ and $S3$ of L-S model.

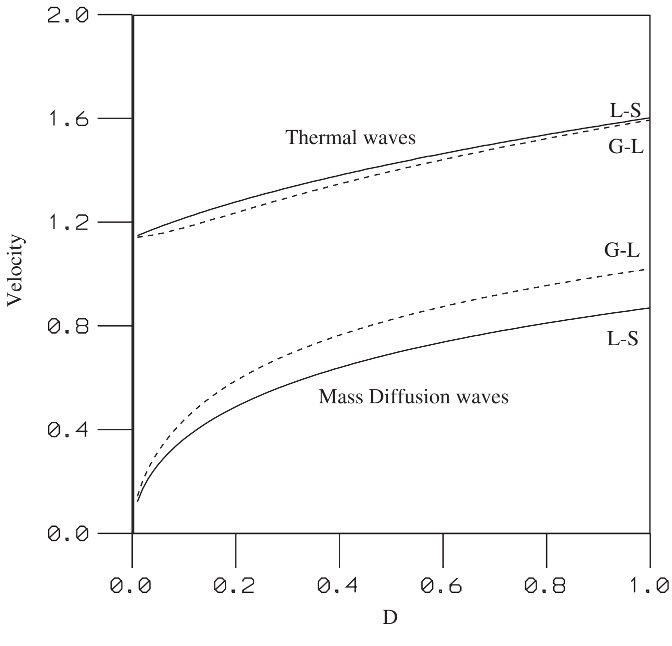

Fig. 10. Variations of velocities of MD and T waves with thermoelastic diffusion constant D for L-S and G-L models.

reflected waves are affected considerably by thermal as well as diffusion relaxation times. On comparing the curves for sets $S1$, $S2$ and $S3$, in Figs. 6–9, it may me remarked that the rate of change of reflection coefficients decreases as we take values of thermal relaxation time τ_0 and diffusion relaxation time τ^0 less than those given in set $S3$.

The variations of the velocities of MD and T waves with D are also shown graphically in Fig. 10 for L-S and G-L models. The effects of second thermal relaxation time and second diffusion relaxation time are noted on velocities of MD and T waves. The velocities P and SV waves are same at each value of D and are not affected by second thermal and diffusion relaxation times. Therefore, the graphs of these two waves are not included in Fig. 10.

5. Conclusions

From theory and numerical computation, the following points are concluded.

1. The theory of generalized thermoelastic diffusion is extended in the frame of G-L model. The solutions of governing equations lead to the existence of one shear and three compressional waves travelling with distinct speeds for two-dimensional motion in a solid with thermoelastic diffusion.
2. The reflection of SV wave is studied. The reflection coefficients of various waves are expressed in closed-form and computed numerically for both L-S and G-L models. The reflection coefficients are also computed for different values of thermal and diffusion relaxation times for L-S model only.
3. The velocities as well as reflection coefficients of various plane waves depend on various thermal and diffusion parameters.

The present model of elastic solid with thermoelastic diffusion becomes more realistic due to the existence of these new compressional waves. The present theoretical results may provide interesting information for experimental scientists/researchers/seismologists working on subjects of wave propagation.

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References

[1] J. Duhamel, Some memoire sur les phenomenes thermo-mechanique, *Journal de l' Ecole Polytechnique* 15 (1885).
[2] F. Neumann, *Vorlesungen Über die Theorie der Elasticität*, Brestau, Meyer, 1885.
[3] M. Biot, Thermoelasticity and irreversible thermo-dynamics, *Journal of Applied Physics* 27 (1956) 249–253.
[4] H. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity, *Journal of the Mechanics and Physics of Solids* 15 (1967) 299–309.
[5] A.E. Green, K.A. Lindsay, Thermoelasticity, *Journal of Elasticity* 2 (1972) 1–7.

[6] D.S. Chandresekharaiah, Thermoelasticity with second sound: a review, *Applied Mechanical Review* 39 (1986) 355–376.
[7] H.H. Sherief, On Generalized Thermoelasticity, PhD Thesis, University of Calgary, Canada, 1980.
[8] R.S. Dhaliwal, H.H. Sherief, Generalized thermoelasticity for anisotropic media, *Quarterly of Applied Mathematics* 33 (1980) 1–8.
[9] R.B. Hetnarski, J. Ignaczak, Generalized thermoelasticity, *Journal of Thermal Stresses* 22 (1999) 451–476.
[10] A.N. Sinha, S.B. Sinha, Reflection of thermoelastic waves at a solid half-space with thermal relaxation, *Journal of Physics of the Earth* 22 (1974) 237–244.
[11] S.B. Sinha, K.A. Elsibai, Reflection of thermoelastic waves at a solid half-space with two thermal relaxation times, *Journal of Thermal Stresses* 19 (1996) 763–777.
[12] S.B. Sinha, K.A. Elsibai, Reflection and refraction of thermoelastic waves at an interface of two semi-infinite media with two thermal relaxation times, *Journal of Thermal Stresses* 20 (1997) 129–146.
[13] A.N. Abd-Alla, A.A.S. Al-Dawy, The reflection phenomena of SV waves in a generalized thermoelastic medium, *International Journal of Mathematics and Mathematical Sciences* 23 (2000) 529–546.
[14] J.N. Sharma, V. Kumar, D. Chand, Reflection of generalized thermoelastic waves from the boundary of a half-space, *Journal of Thermal Stresses* 26 (2003) 925–942.
[15] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids I, *Bulletin de l' Academie Polonaise des Sciences Serie des Sciences Techniques* 22 (1974) 55–64.
[16] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids II, *Bulletin de l' Academie Polonaise des Sciences Serie des Sciences Techniques* 22 (1974) 129–135.
[17] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids III, *Bulletin de l' Academie Polonaise des Sciences Serie des Sciences Techniques* 22 (1974) 266–275.
[18] H.H. Sherief, F. Hamza, H. Saleh, The theory of generalized thermoelastic diffusion, *International Journal of Engineering Science* 42 (2004) 591–608.
[19] W.B. Ewing, W.S. Jardetzky, F. Press, *Elastic Waves in Layered Media*, McGraw-Hill, New York, 1957, p. 76.
[20] A. Ben-Menahem, S.J. Singh, *Seismic Waves and Sources*, Springer, New York, 1981, pp. 89–95.