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Short Communication

Boundary element analysis of a straight-through hybrid silencer

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Abstract

Combining the acoustic attenuation behaviors of reactive and dissipative silencers, a straight-through hybrid silencer consisting of a concentric folded resonator and a dissipative chamber is presented and the substructure boundary element approach is employed to predict and analyze the acoustic attenuation characteristics in absence of mean flow. The BEM predictions demonstrated the acoustic attenuation effectiveness of the hybrid silencer over a wide frequency range. The effects of internal geometry, porosity of perforation and flow-resistivity of sound-absorbing material on acoustic attenuation performance of the hybrid silencer are investigated in detail.

1. Introduction

Silencers are widely used to reduce the engine and blower intake and exhaust (discharge) noise. There are two basic types of silencers: reactive and dissipative [1]. Reactive silencers generally consist of a number of chambers and tubes. Those configurations can provide effective noise attenuation at lower frequencies, but fail to attenuate higher-frequency noise and usually produce a high pressure drop due to the presence of baffles and flow reversals. Dissipative silencers are usually composed of perforated tubes and chambers which are filled with sound-absorbing materials such as fiberglass and basalt wool. These materials absorb effectively high-frequency

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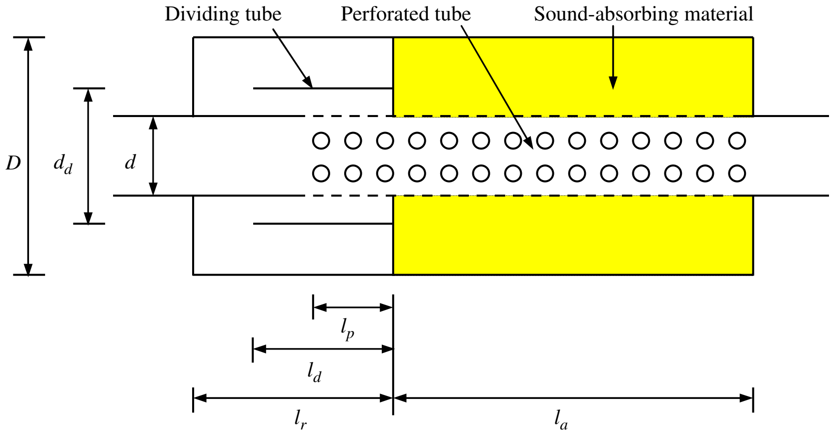

Fig. 1. Configuration of a straight-through hybrid silencer.

sound energy and transform it into thermal energy, but fail to absorb the low-frequency sound waves. It is expected that a combination of reactive and dissipative silencers can lead to desirable noise attenuation in the frequency range of interest. The objective of present study is to combine the acoustic attenuation behaviors of both types of silencers to develop a hybrid silencer with broadband high noise attenuation and low pressure drop. The presented hybrid silencer consists of a straight-through perforated tube, an empty chamber with an annular dividing tube, and an expansion chamber filled with sound absorbing material, as shown in Fig. 1. The perforated tube communicates with two chambers via the holes. The first chamber combined the annular dividing tube acts as a resonator, which is tuned to attenuate low-frequency sound waves, while the sound absorbing material in the second chamber absorbs effectively the higher-frequency sound energy. To optimize the silencer design, the acoustic attenuation performance prediction and analysis of silencers are essential. Studies on the reactive and dissipative silencers demonstrated that a multidimensional approach is required for the accurate prediction of acoustic attenuation performance [2–9]. The three-dimensional boundary element method (BEM) has been applied to predict the acoustic attenuation performance of the reactive and dissipative silencers, and BEM predictions agree well with experimental results for several configurations [6–9]. The present study employs the BEM to predict the transmission loss of the hybrid silencer and to examine the effects of internal geometry, porosity of perforation and flow-resistivity of sound absorbing material on acoustic attenuation performance of the hybrid silencer.

2. Theory

There are two media inside the hybrid silencer: the air and the sound absorbing material. Assuming homogeneous sound absorbing material in the expansion chamber and harmonic wave propagation in both media, and treating the sound absorbing material as an equivalent fluid with complex dynamic density and speed of sound, the continuity and momentum equations yield [7–9]

$$\nabla^2 p + k_0^2 p = 0 \text{ in the air,} \tag{1}$$

$$\nabla^2 p + \tilde{k}^2 p = 0 \text{ in the sound absorbing material,} \tag{2}$$

where p is the acoustic pressure, k_0 is the wavenumber in the air and $\tilde{k}$ is the complex wavenumber in the sound absorbing material.

Eqs. (1) and (2) may be represented in the form of the boundary integral expression [3–5] as

$$C(X)p(X) = -\int_{\Gamma} \left[jk_0 z_0 v(Y) G(X, Y) + p(Y) \frac{\partial G}{\partial n}(X, Y) \right] \mathrm{d}\Gamma(Y), \tag{3}$$

$$C(X)p(X) = -\int_{\Gamma} \left[j\tilde{k}\tilde{z} v(Y) \tilde{G}(X, Y) + p(Y) \frac{\partial \tilde{G}}{\partial n}(X, Y) \right] \mathrm{d}\Gamma(Y), \tag{4}$$

where Γ is the boundary surface of the acoustic domain, n is the unit normal vector on Γ directed away from the domain, j is the imaginary unit, v is the outward normal particle velocity, $G(X, Y) = \exp(-jk_0 R)/4\pi R$ and $\tilde{G}(X, Y) = \exp(-j\tilde{k}R)/4\pi R$ are the Green's functions of free space, R being the distance between any two points X and Y in the domain or on the surface, and $C(X)$ is a coefficient which depends on the position of point X, z_0 and $\tilde{z}$ are the characteristic impedances of the air and the sound absorbing material, respectively. The complex acoustic impedance $\tilde{z}$ and the complex wavenumber $\tilde{k}$ of the sound absorbing material can be measured by the two-cavity method [10], or calculated by empirical expressions if the flow resistivity of the material is known. For example, Delany and Bazley [11] suggest

$$\frac{\tilde{z}}{z_0} = 1.0 + 9.08 \left(\frac{f}{\sigma/10^3} \right)^{-0.75} - j11.9 \left(\frac{f}{\sigma/10^3} \right)^{-0.73}, \tag{5}$$

$$\frac{\tilde{k}}{k_0} = 1.0 + 10.8 \left(\frac{f}{\sigma/10^3} \right)^{-0.70} - j10.3 \left(\frac{f}{\sigma/10^3} \right)^{-0.59}, \tag{6}$$

where f is the frequency and σ is the flow resistivity of the material in MKS Rayls/m.

Numerical solutions of the boundary integral equations (3) and (4) can be achieved by discretizing the boundary surface of the domain into a number of elements. For each boundary integral equation, a set of algebraic system of equations may be obtained by using discretization and numerical integration. These can be written in matrix form as

$$[H]\{P\} = z_0[G]\{V\} \text{ in the air,} \tag{7}$$

$$[H]\{P\} = \tilde{z}[G]\{V\} \text{ in the sound absorbing material,} \tag{8}$$

where $[H]$ and $[G]$ are the coefficient matrices, and $\{P\}$ and $\{V\}$ are the vectors whose elements are the sound pressure p and outward normal particle velocity v on the boundary nodes, respectively. The detailed treatment of the BEM numerical solution procedure for silencer analysis is provided elsewhere [3–6].

To employ the BEM for the prediction of acoustic attenuation performance of the hybrid silencer in Fig. 1, a multidomain approach is needed. The silencer is divided into a number of substructures and then the BEM is applied to each one of these substructures leading to a system of equations. Continuity of sound pressure and normal particle velocity is then enforced at the interface between any two neighboring substructures. At the perforation, the specific acoustic impedance ζ_p is introduced and the boundary conditions may be expressed as

$$v_p^t = -v_p^c \quad \text{and} \quad p_p^t - p_p^c = z_0 \zeta_p v_p^t, \tag{9,10}$$

where the superscripts t and c represent the tube and chamber, respectively. Sullivan and Crocker presented an empirical expression for perforate specific acoustic impedance considering hole interactions as [12]

$$\zeta_p = [0.006 + jk_0(t + 0.75d_h)]/\phi, \tag{11}$$

where t is the perforated tube wall thickness, d_h the perforate diameter, and ϕ the porosity. However, expression (11) has been developed in the absence of filling material. For perforations facing absorbing material, such an expression needs to be modified in view of the work by Kirby and Cummings [13] as

$$\zeta_p = \left[0.006 + jk_0 \left\{ t + 0.375 d_h \left(1 + \frac{\tilde{z}\tilde{k}}{z_0 k_0} \right) \right\} \right] \Big/ \phi. \tag{12}$$

Some detailed descriptions of the multidomain BEM approach for silencer analysis can be found elsewhere [4–6].

3. Results and discussion

For all configurations, the present study considers $D = 15.0\,\text{cm}$ for the silencer inner diameter, $l_r = 15.0\,\text{cm}$ for the resonator length, $l_a = 25.0\,\text{cm}$ for the absorptive chamber length, $d = 4.90\,\text{cm}$

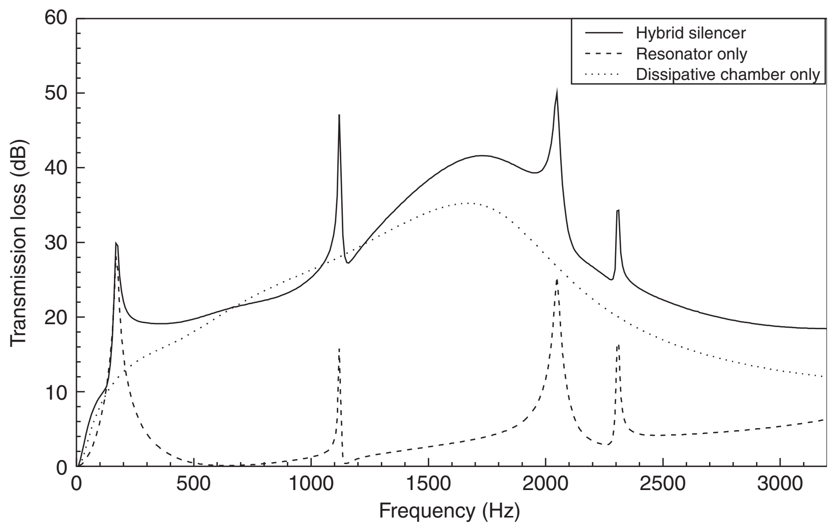

Fig. 2. Transmission loss comparison of the hybrid silencer, resonator only and absorptive chamber only ($l_p = 6.0\,\text{cm}$, $l_d = 12.0\,\text{cm}$, $d_d = 7.0\,\text{cm}$, porosity = 8%, flow-resistivity = 5000 MKS Rayls/m).

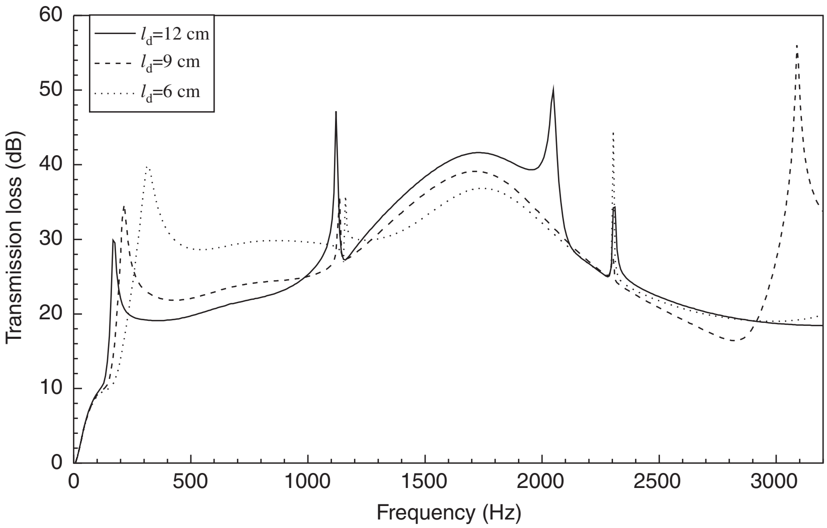

Fig. 3. Effect of the dividing tube length on transmission loss of the hybrid silencer ($l_p = 6.0\,\text{cm}$, $d_d = 7.0\,\text{cm}$, porosity = 8%, flow-resistivity = 5000 MKS Rayls/m).

for the perforated tube inner diameter, $t = 0.09\,\text{cm}$ for the perforated tube wall thickness, $d_h = 0.25\,\text{cm}$ for the perforate diameter, and speed of sound in the air is $c_0 = 344\,\text{m/s}$.

The design of hybrid silencer requires the understanding of acoustic attenuation characteristics of individual reactive and dissipative elements as well as their interactions. Fig. 2 compares the transmission loss predictions of the hybrid silencer, resonator only and absorptive chamber only. It is clear that the resonator contributes a primary low frequency (around 180 Hz for this configuration) resonance and several narrow peaks at higher frequencies, while the absorptive chamber provides an effective acoustic attenuation at higher frequencies. The superimposition of low frequency resonance of the resonator and higher frequency acoustic attenuation of the absorptive chamber leads to a desirable broadband acoustic attenuation of the hybrid silencer. The trends are similar to the two-chamber hybrid silencer presented by Lee et al. [14].

The effects of the dividing tube length and diameter in the resonator are examined next by fixing all other parameters. Fig. 3 shows the transmission loss comparison for three different dividing tube lengths. Decreasing the length of dividing tube shifts the primary resonance to higher frequency. It is well known that the resonance frequency of a lumped Helmholtz resonator is inversely proportional to square root of the neck length, which is $l_d - l_p + \delta$ (δ being the end correction) in the present configuration. In addition, the decrease in the length of dividing tube increases the acoustic attenuation between the first and second resonance frequencies, while the acoustic behavior is complex beyond the second resonance frequency for this specific configuration. The effect of the dividing tube diameter on the acoustic attenuation performance of the hybrid silencer is illustrated in Fig. 4. Increasing the diameter of dividing tube moves the primary resonance to higher frequency, which is attributed to the enlarged neck cross-sectional area of the resonator (the resonance frequency of Helmholtz resonator is directly proportional to square root of the neck cross-sectional area).

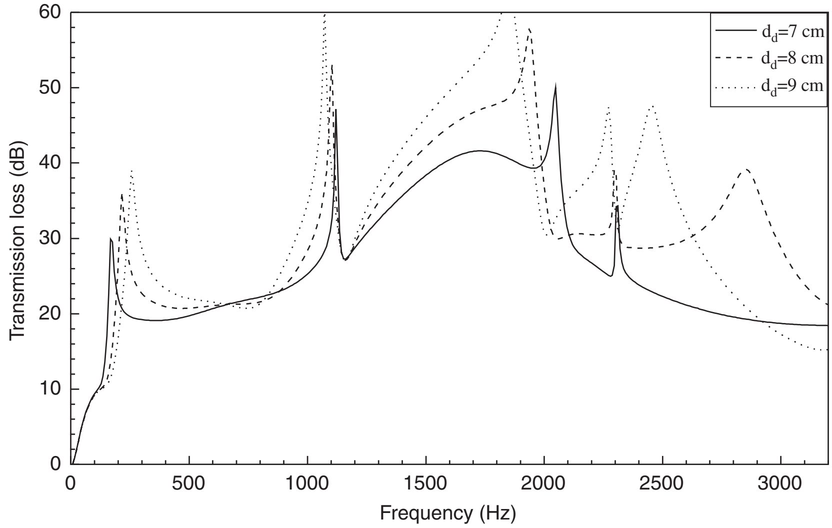

Fig. 4. Effect of the dividing tube diameter on transmission loss of the hybrid silencer ($l_p = 6.0\,\text{cm}$, $l_d = 12.0\,\text{cm}$, porosity = 8%, flow-resistivity = 5000 MKS Rayls/m).

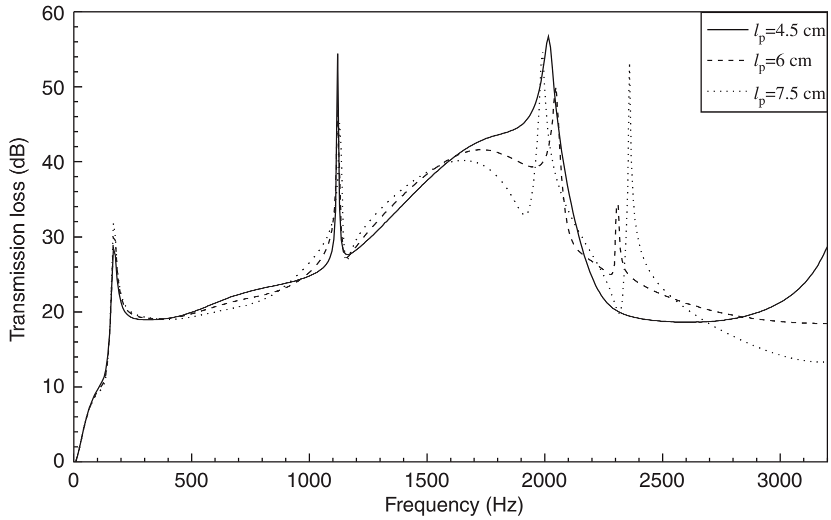

Fig. 5. Effect of the perforated length on transmission loss of the hybrid silencer ($l_d - l_p = 6.0\,\text{cm}$, $d_d = 7.0\,\text{cm}$, porosity = 8%, flow-resistivity = 5000 MKS Rayls/m).

Fig. 5 compares the transmission loss predictions for three different perforated lengths by fixing the length $l_d - l_p$. It is clear that the primary resonance frequency is independent on l_p, while the perforated length affects the high-frequency acoustic attenuation.

The sensitivity of the hybrid silencer to variation of porosity is shown in Fig. 6. Increasing the porosity of perforation improves, in general, the acoustic attenuation performance at high frequencies and slightly shifts the primary resonance peak to higher frequency. The primary

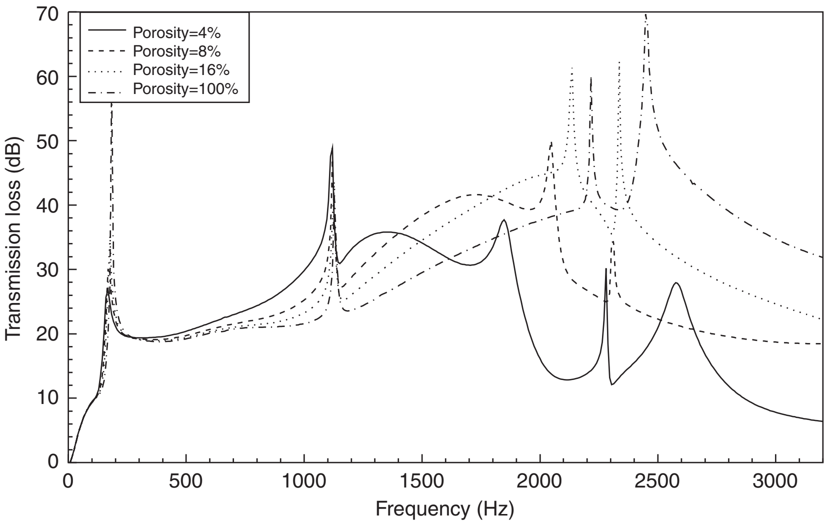

Fig. 6. Effect of the porosity of perforation on transmission loss of the hybrid silencer ($l_p = 6.0\,\text{cm}$, $l_d = 12.0\,\text{cm}$, $d_d = 7.0\,\text{cm}$, flow-resistivity = 5000 MKS Rayls/m).

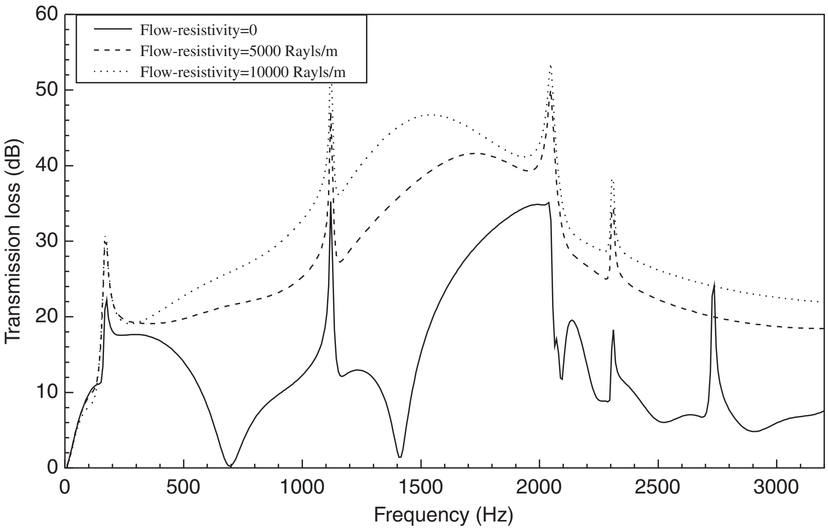

Fig. 7. Effect of the flow-resistivity of sound absorbing material on transmission loss of the hybrid silencer ($l_p = 6.0\,\text{cm}$, $l_d = 12.0\,\text{cm}$, $d_d = 7.0\,\text{cm}$, porosity = 8%).

resonance frequency shift may be attributed to the fact that the effective length of resonator neck is decreased.

Finally, the effect of flow-resistivity of the filling sound absorbing material on the acoustic attenuation performance of the hybrid silencer is examined. Fig. 7 compares the transmission loss for three different flow-resistivities. The zero flow resistivity corresponds to the case without any sound absorbing material in the chamber, therefore the silencer becomes reactive. Obviously, the filling sound absorbing material in the expansion chamber improves greatly the acoustic attenuation and removes the troughs in transmission loss of the reactive silencer. Increase in the flow-resistivity (corresponding to the increase in filling density) of the sound absorbing material leads to higher noise attenuation at higher frequencies, however higher filling density may actually deteriorate the attenuation at low frequencies.

4. Conclusions

A straight-through perforated tube hybrid silencer has been presented to reduce the broadband noise in ducts, and the substructure BEM is employed to predict and analyze the acoustic attenuation performance of the hybrid silencer. The presented silencer consists of a straight-through perforated tube resonator and a perforated tube absorptive chamber. The resonator is used to suppress the low-frequency noise, while the absorptive chamber absorbs effectively the high-frequency sound energy. Therefore, a desirable broadband noise attenuation performance may be obtained. The use of straight-through perforated tube ensures a low pressure drop of the hybrid silencer. For the different noise attenuation requirements, an optimal design of silencer may be achieved by choosing reasonably the dimensions of silencer, the porosity of perforation and the flow-resistivity (or filling density) of the sound absorbing material.

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