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Acoustic transfer admittance of cylindrical cavities

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Abstract

The reciprocity calibration method uses two microphones acoustically connected by a coupler, a cylindrical cavity closed at each end by the diaphragms of the transmitting and receiving microphones. The acoustic transfer admittance of the coupler, including the thermal conductivity effect of the fluid, must be modelled precisely to obtain the accurate sensitivity of the microphones from the electrical transfer impedance measurement. It appears that the analytical model quoted in the current standard [International Electrotechnical Commission IEC 61064-2, Measurement Microphones, Part 2: Primary Method for Pressure Calibration of Laboratory Standard Microphones by the Reciprocity Technique, 1992] is not the appropriate one and that it should be revised, as also suggested by a recent EUROMET project report [K. Rasmussen, Datafiles simulating a pressure reciprocity calibration of microphones, EUROMET Project 294 Report PL-13, 2001]. Thus, it is the aim of the paper to investigate analytically the acoustic field inside the coupler, revisiting the assumptions of the earlier work, leading to a coherent description and therefore providing clarity which should facilitate discussion of a possible revised standard.

1. Introduction

The usual primary method of determining the complex pressure sensitivity of laboratory standard microphones is the well-known pressure reciprocity calibration method using closed couplers. This reciprocity calibration of microphones (described in the IEC publication 61094-2 [1]) is carried out by means of three microphones, two of which are reciprocal. The general principle is to measure the electrical transfer impedance between two microphones acoustically connected by a coupler (an acoustic cavity), using one of them as a sound source (transmitter) and the other one as a sound receiver. If the acoustic transfer impedance of the cavity is known, then the product of the pressure sensitivities of the two coupled microphones can be determined. Using pair-wise combinations of microphones, three such mutually independent products are available from which an expression of the pressure sensitivity of each of the three microphones can be derived [1]. The typical configuration is shown in Fig. 1.

During recent decades, there has been strong motivation both for modelling each part of the system (microphones, electrical circuits, and acoustic coupler) and for determining the parameters that govern the

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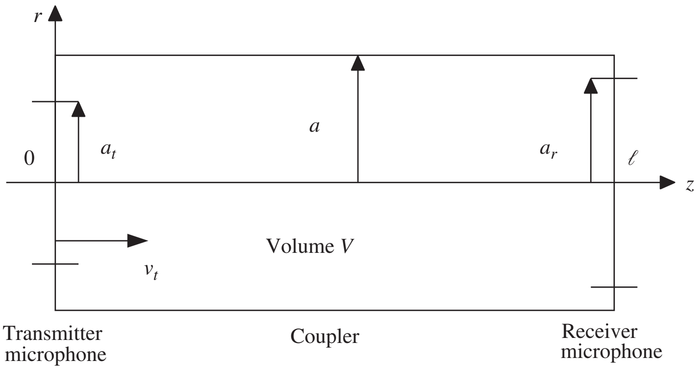

Fig. 1. Geometry of coupler and microphones.

behaviour of each component of the system (physical properties of the gases, parameters of the microphones, shape of the couplers, and so on). Moreover, research efforts have concentrated on improving the measurement uncertainty of the calibration method, by developing an understanding of the physical processes involved, and evaluating associated correction factors (or eliminating the need of some correction factors [2]), or by evaluating the effects of the ambient environment conditions. Especially in the closed coupler, the correction due to the heat conduction between the fluid and the walls, inside the thermal boundary layers, has been investigated by several authors [3–7]. It has been considered as an ''apparent increase in the coupler volume'' expressed by a complex correction factor to the geometrical volume (in the lower-frequency range, when the dimensions of the coupler are much lower than the wavelength) or to the cross-sectional area of the cylindrical coupler (at higher frequencies).

As mentioned above, the acoustic transfer admittance of the coupler, including an accurate analytical modelling for the pressure fluctuations inside the cavity which takes into account this thermal conductivity effect of the fluid, must be known precisely to obtain the accurate sensitivity of the microphones from the electrical transfer impedance measurement. In the latter period of the pioneering works in this field [8–11], Gerber [5] provided his theory for the heat conduction process in the lower-frequency range. He gave two different expressions for the correction to the geometric volume of the cavity (to obtain the effective complex volume) depending on the acoustic impedance of the driver (though not stated explicitly), one for a low acoustic impedance of the driver, the other one for a high acoustic impedance of the driver. In fact, the results of Gerber's modelling and his own interpretation of these results are three-fold: (i) the lower acoustic impedance driver is assumed to be an ideal source of acoustic pressure (the volume change being caused by the pressure change), (ii) the converse is assumed for the higher acoustic impedance driver, (iii) the real driver gives an intermediate result (expressed using the previous ones).

The first IEC publication on reciprocity calibration (IEC 327, 1970) [12] quoted a formula derived from published research pre-dating Gerber's paper [9–11]. In fact, this was equivalent to Gerber's formula corresponding, in his paper, to a low acoustic impedance of the driver. However, first the formula quoted in the IEC 327 was difficult to use in practice (slow convergence of modal expansion), and second Jarvis [6] subsequently pointed out that the heat conduction correction calculated according to Gerber's formulation when a real driver is assumed, for a source impedance typical of a Brüel and Kjær-type 4160 microphone, ''differs so little from the correction calculated on the assumption of infinite impedance that the effect on calibration in IEC 3 cm^3 coupler is less than 0.001 dB at frequencies above 63 Hz'' (''infinite impedance'' meaning ''infinite acoustic impedance driver'' according to Gerber's formulation). Therefore, the second IEC publication (number 61094-2, 1992) [1] quoted the Gerber's formula corresponding, in his paper, to a high acoustic impedance of the driver. Recently, in the EUROMET project 294 [13], ''to test the performance of the calculation procedure and software used to determine the sensitivity of the microphones, by construction of a set of artificial measurement data'', Rasmussen shows that several results indicate that ''the calculation of the influence of heat conduction in a pressure reciprocity calibration as given in the IEC standard 61094-2 (1992) needs to be revised.''

In our opinion, this revision must start from the results given by Gerber, in particular revisiting the interpretation of these results with the benefit of current understanding. For example the thermo-acoustical problem presented at the beginning of the Gerber's paper does not appear to be appropriate to real acoustic situation: the author presents two different mathematical problems whereas the acoustical problem for the cavity is unique, provided it takes into account the thermal (and viscous) effects inside the boundary layers on the whole surface of the cavity. Moreover, in his first formulation (which corresponds in his paper to an infinite impedance driver), Gerber assumes that the time varying density of the fluid does not depend on the coordinates inside the cavity; this hypothesis is inconsistent with the assumption that the temperature field is non-uniform inside the cavity (when the pressure field is uniform in the lower frequency range), according to the basic thermodynamic laws for gases (see Eq. (A.7) below).

Thus, it is the aim of the paper to investigate analytically the acoustic field inside the cylindrical cavity, in order to produce a consistent approach across the whole frequency range, and not restricted by any assumption about the relative size of the wavelength and coupler dimensions. Actually, this approach is done using revisited and coherent description which is built on the analytical results available in the literature, essentially the Kirchhoff theory [9,11,14–16]. Then, no claims are made here as to novelty. However, the present effort toward clarity of presentation, should be of some interest because it is confirmed that the results summarized by Ballagh in a short paper [4] are the right ones and consequently that the current IEC standard (IEC 61094-2, 1992) should be modified (in fact the formula quoted in the previous one (IEC 327, 1970) was the right one).

2. The fundamental problem and its solution

The system considered is a cylindrical cavity (length ℓ, radius a), filled with a non-ideal, but uniform fluid at rest, and closed at its ends by the diaphragms of two microphones assumed to have the same radius as the cavity, one used as a sound source (transmitter) set at $z = 0$, the other one as a receiver set at $z = \ell$ (Fig. 1). For analysing source-driven cavity excitation, with application relating to cylindrical cavity shapes, the standard analytic procedure wherein the viscosity and heat conduction effects are expressed, beyond the usually considered inertia and compressibility of the gas, is reviewed. Realistic boundary conditions, i.e. no slip condition and no temperature variation, are considered, and both the coupling of the diaphragms of the microphones with the acoustic field and the acoustic impedances of the microphones are accounted for.

The variables describing the dynamic and thermodynamic states of the fluid are the pressure variation p, the particle velocity $\mathbf{v}$, the density variation ρ', the entropy variation σ, and the temperature variation τ. The parameters which specify the properties and the nature of the fluid are the ambient values of the density ρ_0, the static pressure P_0, the shear viscosity coefficient μ, the bulk viscosity coefficient η, the coefficient of thermal conductivity λ, the specific heat coefficient at constant pressure and constant volume per unit of mass C_P and C_V, respectively, the specific heat ratio γ, and the increase in pressure per unit increase in temperature at constant density β ($\beta\gamma = \alpha\rho_0 c_0^2$ with α the volume thermal expansivity and c_0 the adiabatic speed of sound). A complete set of linearized homogeneous equations governing small amplitude disturbances of the fluid includes the following:

- the Navier–Stokes equation

$$\frac{1}{c_0}\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho_0 c_0}\nabla p = \ell_v \nabla(\nabla \cdot \mathbf{v}) - \ell'_v \nabla \times (\nabla \times \mathbf{v}), \tag{1}$$

  where the characteristic lengths ℓ_v and ℓ'_v are defined as follows:

$$\ell_v = \frac{1}{\rho_0 c_0}\left(\frac{4}{3}\mu + \eta\right) \quad \text{and} \quad \ell'_v = \frac{\mu}{\rho_0 c_0},$$

- the conservation of mass equation, taking into account the thermodynamic law expressing the density variation as function of the independent variables p and τ,

$$\rho_0 c_0 \nabla \cdot \mathbf{v} + \frac{\gamma}{c_0}\frac{\partial}{\partial t}(p - \beta\tau) = 0, \tag{2}$$

- the Fourier equation for heat conduction, taking into account the thermodynamic law expressing the entropy variation as function of the independent variables p and τ,

$$\left(\frac{1}{c_0}\frac{\partial}{\partial t} - \ell_h \nabla^2\right)\tau = \frac{\gamma - 1}{\beta\gamma}\frac{1}{c_0}\frac{\partial p}{\partial t}, \tag{3}$$

the operator ∇^2 being the laplacian, where the characteristic length ℓ_h is defined as

$$\ell_h = \frac{\lambda}{\rho_0 c_0 C_P}.$$

The acoustic pressure inside the cavity should be the solution of this set of three equations with the requirement of regular behaviour at the cylinder centre, this being

$$p, \mathbf{v} \text{ and } \tau \text{ finite at } r = 0 \tag{4}$$

and with the boundary conditions at the outer surface of the cavity

$$\mathbf{v} = \mathbf{0} \quad \text{at } r = a, \tag{5}$$

$$\tau = 0 \quad \text{at } r = a, \tag{6}$$

$$S v_z = S v_t - Y_b p \quad \text{at } z = 0, \tag{7}$$

$$S v_z = (Y_b + Y_r) p \quad \text{at } z = \ell, \tag{8}$$

where Y_r is the acoustic admittance of the receiver microphone, v_t is the velocity field of the transmitter diaphragm, with $S = \pi a^2$, and Y_b is the thermal boundary layer admittance (to the first-order approximation with respect to $\sqrt{k_0 \ell_h}$) [15,16]

$$Y_b = \frac{S}{\rho_0 c_0}\frac{1 + \mathrm{j}}{\sqrt{2}}\sqrt{k_0}(\gamma - 1)\sqrt{\ell_h}, \tag{9}$$

$k_0 = \omega / c_0$ being the "adiabatic" wavenumber.

Several hypotheses can be made here in order to avoid overly intricate formulation which would overshadow the purpose of this paper, mentioned above. These assumptions can be summarized as follows:

- The solutions do not depend on the azimuthal coordinate (the problem is assumed axisymmetric).
- The radial component of the particle velocity v_r vanishes, and thus the acoustic pressure does not depend on the radial coordinate r (this plane wave approximation can be replaced by a more appropriate approximation when considering the calibration precision obtainable today at high frequencies [17]).
- The spatial derivative of the z-component of the particle velocity v_z with respect to the coordinate z in the Navier–Stokes equation is much smaller than the derivative with respect to the radial coordinate r.
- The temperature variation τ and consequently the density variation ρ', and the z-component of the particle velocity v_z depend on both axial z and radial r coordinates because v_z and τ vanish at the lateral wall of the cavity, so they are replaced by their mean value across the section of the cylindrical cavity (plane wave approximation).

First, these approximations enable us to simplify greatly the expressions of Eqs. (1) and (3), giving

$$\left(\frac{1}{c_0}\frac{\partial}{\partial t} - \ell'_v \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\right) v_z(r, z) = -\frac{1}{\rho_0 c_0}\frac{\partial}{\partial z} p(z), \tag{10}$$

$$\left(\frac{1}{c_0}\frac{\partial}{\partial t} - \ell_h \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\right) \tau(r, z) = \frac{\gamma - 1}{\beta\gamma}\frac{1}{c_0}\frac{\partial}{\partial t} p(z). \tag{11}$$

Therefore, it is a simple matter to solve separately these equations, according to boundary conditions (5) and (6), respectively, yielding

$$v_z(r,z) = \frac{\mathrm{j}}{\omega\rho_0}\frac{\partial}{\partial z}p(z)\left[1 - \frac{J_0(k_v r)}{J_0(k_v a)}\right], \tag{12}$$

$$\tau(r,z) = \frac{\gamma - 1}{\beta\gamma}\, p(z)\left[1 - \frac{J_0(k_h r)}{J_0(k_h a)}\right], \tag{13}$$

where the expressions of the wavenumbers k_v (associated with the vortical movement due to viscosity effects) and k_h (associated with entropy diffusion due to heat conduction) are given by

$$k_v = \frac{1-\mathrm{j}}{\sqrt{2}}\sqrt{k_0/\ell_v'} \quad \text{and} \quad k_h = \frac{1-\mathrm{j}}{\sqrt{2}}\sqrt{k_0/\ell_h}.$$

Taking mean values across the section of the cylindrical cavity of Eqs. (12), (13), and (2) gives

$$\langle v_z(z)\rangle = \frac{2\pi}{S}\int_0^a v_z(r,z) r\,\mathrm{d}r = \frac{\mathrm{j}}{k_0\rho_0 c_0}\frac{\partial p(z)}{\partial z}\left[1 - \frac{2}{k_v a}\frac{J_1(k_v a)}{J_0(k_v a)}\right], \tag{14}$$

$$\langle\tau(z)\rangle = \frac{\gamma - 1}{\beta\gamma}\, p(z)\left[1 - \frac{2}{k_h a}\frac{J_1(k_h a)}{J_0(k_h a)}\right], \tag{15}$$

$$\frac{\partial}{\partial z}\langle v_z(z)\rangle + \frac{\mathrm{j}\omega}{\rho_0}\frac{\gamma}{c_0^2}\left(p(z) - \beta\langle\tau(z)\rangle\right) = 0. \tag{16}$$

Combining Eqs. (14) and (15) with Eq. (16) to remove the variable τ, denoting $w(z) = S\langle v_z(z)\rangle$ the axial acoustic volume velocity, leads to the usual pair of transmission line equations:

$$\frac{\partial}{\partial z}p(z) + Z_v w(z) = 0, \tag{17}$$

$$\frac{\partial}{\partial z}w(z) + Y_h p(z) = 0, \tag{18}$$

where the impedance Z_v and the admittance Y_h are given by

$$Z_v = \frac{1}{S}\frac{\mathrm{j}k_0\rho_0 c_0}{1 - K_v}, \tag{19}$$

$$Y_h = S\frac{\mathrm{j}k_0}{\rho_0 c_0}\left[1 + (\gamma - 1)K_h\right], \tag{20}$$

the expressions for $K_{h,v}$ (i.e. respectively, K_h and K_v) being given by

$$K_{h,v} = \frac{2}{k_{h,v}}\frac{J_1(k_{h,v}a)}{J_0(k_{h,v}a)}. \tag{21}$$

Finally, combining Eqs. (17) and (18) to remove the variable $w(z)$ leads to the usual propagation equation

$$\left(\frac{\partial^2}{\partial z^2} + k_z^2\right)p(z) = 0, \quad 0 \leqslant z \leqslant \ell, \tag{22}$$

the axial wavenumber k_z is given by

$$k_z^2 = -Z_v Y_h = k_0^2\frac{1 + (\gamma - 1)K_h}{1 - K_v}, \quad \text{where } \mathrm{Re}(k_z) > 0 \text{ and } \mathrm{Im}(k_z) < 0. \tag{23}$$

The plane wave solution of Eq. (22), subject to boundary conditions (7) and (8) on the diaphragms of the microphones set at $z = 0$ and ℓ respectively, invoking expression (17) to remove the volume velocity $w(z) = S\langle v_z(z)\rangle$, is the appropriate result needed to express the acoustic pressure inside the cavity. Therefore, when the time-periodic source activity is given by the harmonic ($e^{j\omega t}$) volume velocity (Sv_t) of the transmitting microphone ($z = 0$), the expression for the complex amplitude of the acoustic pressure is given by

$$p(z) = A(e^{-jk_z z} + Be^{jk_z z}), \tag{24a}$$

where the integration constants are given by

$$A = \frac{Sv_t}{Y_i(1 - B) + Y_b(1 + B)}, \tag{24b}$$

$$B = e^{-2jk_z\ell}\,\frac{Y_i - (Y_r + Y_b)}{Y_i + (Y_r + Y_b)}, \tag{24c}$$

with

$$Y_i = \sqrt{\frac{Y_h}{Z_v}} = \frac{jk_z}{Z_v} = \frac{S}{\rho_0 c_0}\sqrt{[1 + (\gamma - 1)K_h](1 - K_v)}. \tag{24d}$$

The radial dimension of the cavity justifies approximating k_z and Y_i asymptotically, to the first order of approximation, as follows:

$$k_z^2 \approx k_0^2\left[1 + \frac{1 - j}{\sqrt{2}}\,\frac{2}{a\sqrt{k_0}}\left(\sqrt{\ell_v'} + (\gamma - 1)\sqrt{\ell_h}\right)\right], \tag{25}$$

$$Y_i \approx \frac{S}{\rho_0 c_0}\left[1 + \frac{1 - j}{\sqrt{2}}\,\frac{2}{a\sqrt{k_0}}\left(-\sqrt{\ell_v'} + (\gamma - 1)\sqrt{\ell_h}\right)\right]. \tag{26}$$

As mentioned in the beginning of the introduction, the quantity of interest for the pressure reciprocity calibration method is the acoustic transfer admittance Y_T, the "quotient of the short-circuit volume velocity produced by the microphone used as a transmitter by the sound pressure acting on the diaphragm of the microphone used as a receiver" [1], namely

$$Y_T = \frac{Sv_t + Y_t p(0)}{p(\ell)}, \tag{27}$$

where Y_t is the acoustic admittance of the transmitting microphone. Invoking expressions (24a–c) for the acoustic pressure yields straightforwardly

$$Y_T = j\,\sin k_z\ell\left[Y_i + \frac{(Y_t + Y_b)(Y_r + Y_b)}{Y_i}\right] + \cos k_z\ell(2Y_b + Y_t + Y_r). \tag{28}$$

In the lower-frequency range, when the dimensions of the cavity are much lower than the acoustic wavelength ($|k_z\ell| \ll 1$), it is justified to retain the first order in the factor $k_z\ell$ in expression (28) of Y_T and to use the approximate expressions (9), (25) and (26) for Y_b, k_z, and Y_i, respectively, yielding

$$Y_T \approx j\omega\frac{V}{\gamma P_0}[1 + (\gamma - 1)X] + Y_t + Y_r, \tag{29a}$$

where

$$X = \frac{\mathscr{A}}{V}\,\frac{1 - j}{\sqrt{2}}\sqrt{\ell_h/k_0} \tag{29b}$$

and where the ratio of the total area of the surface of the cavity and its volume is $\mathscr{A}/V = 2/a + 2/\ell$.

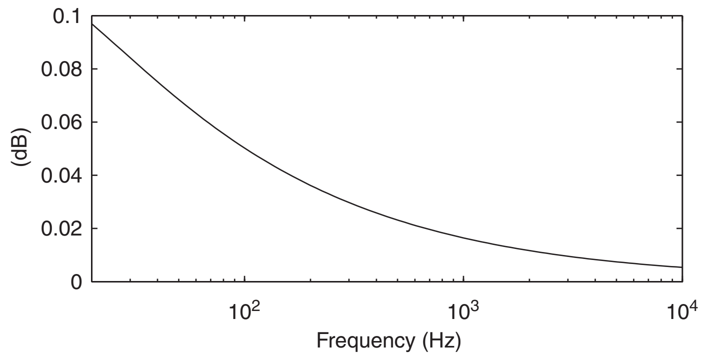

Fig. 2. Difference between acoustic transfer admittances derived from the two solutions of Gerber for LS2P microphones.

This result is also demonstrated in the appendix by starting from the assumption that the acoustic pressure p is uniform all over the bulk of the cavity and integrating the basic equations over the whole volume of the cavity.

3. Discussion

In the upper frequency range, the expression of the acoustic transfer admittance quoted in the IEC standard 61064-2 [1] is the same as Eq. (28). But in this standard two approximations are assumed: first, the complex wavenumber k_z is reduced to the "adiabatic" one $k_0 = \omega/c_0$, and second, the iterative admittance Y_i is simplified according to the heat correction factor C_{TH} expressed by Gerber as the correction which corresponds to an infinite impedance driver: $Y_i = SC_{TH}/(\rho_0 c_0)$. Thus, the viscous effects are not accounted for in the IEC standard although they give a correction of the same order of magnitude as the thermal one (which, moreover, is not the right one in the standard).

Expression (29) for the acoustic transfer admittance, accurate in the lower-frequency range, was proposed by Riéty and Lecollinet [3] and Ballagh [4]. It is also the expression of the admittance $1/Z_P$ given by Gerber [5, Eq. (34)], when using the short time solution in the Laplace transform for $E_P = 1 - S_P$ [5, Eq. (25.b)] (omitting the factors S_p^n, $n > 1$, in the series). This expression, which is the right one, is quoted in the IEC standard 327 (1970) [12], but is not the one suggested in the IEC standard 61094-2 (1992) [1], which therefore needs, in our opinion, to be revised.

In order to show the discrepancies between this correct expression and the other one proposed by Gerber and suggested by the IEC standard 61094-2, the difference between the modulus of the acoustic transfer admittances derived from these two solutions, for LS2P microphones[1] with a 4.7 mm length and 9.3 mm in diameter coupling cavity, is given in Fig. 2. The solution which assumes that the time varying density does not depend on the spatial coordinates (high impedance source in Gerber's paper) gives higher values than the solution which assumes that the pressure variation is uniform in the cavity (low impedance source in his paper).

The sensitivity level of a microphone determined by the reciprocity calibration method being proportional to the square root of the acoustic transfer admittance Y_T, the discrepancies between the sensitivity levels reach 0.05 dB at the lowest-frequency range of interest (Fig. 2). Today the reproducibility on the determination of the pressure sensitivity using the closed coupler reciprocity technique is the order of 0.01–0.02 dB [7] and the uncertainties are evaluated to the order of 0.03 dB (0.1 dB at frequencies up to 50 Hz) [18] for LS2P microphones. The discrepancies shown in Fig. 2, which are systematic errors additive with the uncertainties, are greater than the experimental reproducibility and then are non-negligible. Therefore, the expression for the acoustic transfer admittance quoted in the IEC standard 61094-2 (1992) should be replaced by the expression

[1]The letters LS mean Laboratory Standard, the number denotes the mechanical configuration of the microphone (1 for 1 in, and 2 for $\frac{1}{2}$ in microphone), and the letter P refers to a microphone having a nominally flat pressure sensitivity (F for a free-field sensitivity).

given by Eq. (29). The upper frequency considered is limited to 10^4 Hz because in the upper frequency range (from about 5×10^3 Hz) the difference is not significant (i.e. lower than the experimental reproducibility).

4. Conclusion

To sum up, the present paper starts essentially from the result given by Gerber and discussed later on by several authors [3,4,6,7]

- Gerber presents two different mathematical problems whereas the acoustical problem for the cavity is unique.
- In the currently specified formulation, that is, in the first formulation given by Gerber, it is assumed that the time varying density of the fluid does not depend on the coordinates inside the cavity (this hypothesis is inconsistent with the assumption that the temperature field is non-uniform inside the cavity, when the pressure field is uniform in the lower-frequency range).

Then the present approach is built on an analytical theory which starts from the Kirchhoff theory which includes the effect of the thermal conductivity of the gas, because it plays an important role in the acoustic field inside small cavities in the fundamental equations.

As a conclusion, it is clear (Fig. 2) that the model quoted in the IEC standard 61094-2 is not suitable in the lower-frequency range (up to 50–1000 Hz) when an accuracy better than 0.03 dB is needed. Consequently, the expression for the acoustic transfer admittance quoted in the IEC standard 61094-2 (1992) should be replaced by the expression given by Eq. (29).

Appendix A

In the lower-frequency range, i.e. when the wavelength is much greater than the dimensions of the cavity, relatively simple analytical solutions for the excitation of acoustic fields in cavities with lossy walls can be obtained for a broad class of such cavities. The suitable solution can be expressed directly as in Eq. (29), assuming appropriate approximations in Eqs. (1) and (2), in addition to those mentioned above (see, for example, [16, pp. 131–133]).

The acoustic pressure field can be assumed to be uniform everywhere inside the cavity even inside the viscous and thermal boundary layers (this is used in evaluating Eq. (A.1)).

The conservation of mass equation can be integrated all over the volume of the cavity, leading to (in using Gauss theorem):

$$\iiint_V \frac{\partial \rho'}{\partial t}\,\mathrm{d}V + \rho_0 \iint_{\mathscr{A}} \mathbf{v}\cdot \mathrm{d}\mathscr{A} = 0,$$

which can be written as

$$\frac{1}{\rho_0}\iiint_V \rho'\,\mathrm{d}V + \delta V + \frac{p\mathscr{A}}{\mathrm{j}\omega\overline{Z}} = 0, \tag{A.1}$$

wherein the mean admittance $1/\overline{Z}$ is defined as

$$\iint_{\mathscr{A}} \frac{p}{Z}\,\mathrm{d}\mathscr{A} = p\iint_{\mathscr{A}} \frac{1}{Z}\,\mathrm{d}\mathscr{A} = \frac{p\mathscr{A}}{\overline{Z}} \tag{A.2}$$

and where δV represents the variation of the volume of the cavity due to the displacement field of the transmitting diaphragm

$$\delta V = \iint_S \frac{v_t}{\mathrm{j}\omega}\,\mathrm{d}S. \tag{A.3}$$

Eq. (3) written as

$$\tau = \frac{c_0 \ell_h}{\mathrm{j}\omega} \nabla^2 \tau + \frac{\gamma - 1}{\beta\gamma} p, \tag{A.4}$$

can also be integrated all over the volume of the cavity, leading to (in using Gauss theorem)

$$\iiint_V \tau \, \mathrm{d}V = \frac{c_0 \ell_h}{\mathrm{j}\omega} \iint_{\mathscr{A}} (\nabla\tau) \cdot \mathrm{d}\mathscr{A} + \frac{\gamma - 1}{\beta\gamma} Vp, \tag{A.5}$$

showing that the mean value of the temperature variation τ depends on its evolution $(\nabla\tau) \cdot \mathrm{d}\mathscr{A}$ inside the thermal boundary layers.

Then, the solution of Eq. (A.4), subject to the boundary condition (6), for the harmonic motion considered here ($\mathrm{e}^{\mathrm{j}\omega t}$), can be written as follows:

$$\tau = \frac{\gamma - 1}{\beta\gamma} p[1 - \mathrm{e}^{-\mathrm{j}k_h u}], \tag{A.6}$$

where the first term on the right-hand side represents the "adiabatic" temperature variation (which is the temperature variation outside the thermal boundary layers) and the second term represents the evolution of the temperature variation inside the thermal boundary layers, the u-coordinates representing the local normal to the boundaries inwardly directed (the wavenumber k_h is given under Eq. (13)). Actually, here, the thickness of the boundary layer is much smaller than the dimensions of the cavity.

Using results (A.1), (A.5), (A.6), and the linearized equation of state

$$\rho' = \rho_0 \chi_T (p - \beta\tau) \tag{A.7}$$

to remove ρ', with $\chi_T = \gamma/(\rho_0 c_0^2)$, leads straightforwardly to the result given in Eq. (29), noting that $\mathscr{A}/\overline{Z} = Y_r$.

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