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Active control of a structure with continuously closely spaced natural frequencies

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Abstract

In this paper, active control of a structure with continuously closely spaced natural frequencies is discussed. To simplify the analysis, a fundamental case is examined for a structure with three degrees of freedom and a single control input, and the control algorithm is velocity feedback. Perturbation method is used to analyze the relationship between modal damping ratios and control gain. Through the analysis of the effect of different spacing of natural frequencies, it is found that a single input can only control one of the three modes of the closely spaced natural frequencies and the modal damping ratios of the rest close modes will approach zero. The influence of uncertainty parameters on control effect is also analyzed and it shows that the sum of modal damping is independent of uncertainty parameters of the system, but depends only on the control gain.

1. Introduction

Many kinds of space structures trend to become larger and more flexible, so vibration control of large space flexible structures is very important [1]. Large space flexible structures are characterized by lower and closely spaced natural frequencies, small structure damping, high degree of mode coupling, etc. and therefore their vibration control is difficult. For the structures with identical natural frequencies, control of these structures needs the same number of inputs with the number of identical frequencies [2]. However, for the structures with closely spaced but not identical natural frequencies, according to the control theory, it is possible to control these structures using smaller number of actuators than the number of closely spaced frequencies. But, Forward and Swigert [3] studied a cylindrical mast with two closely spaced natural frequencies and found that when a single control input was used, one mode was highly damped and the other was nearly undamped. Xu et al. [4,5] applied perturbation method to investigate the vibration control of a structure with two closely spaced natural frequencies and found that the system became less robust as the spacing of natural frequencies became closer. Abe [6] proposed a direct velocity feedback algorithm with variable gain to control vibrations in structures with two closely spaced natural frequencies using a single actuator. Williams and Cheng [7]

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discussed degrees of controllability and observability for close modes of flexible space structures. Chen et al. [8–10] treated the control problem of defective and near-defective system well using the generalized modal theory based on the invariant subspace recursive method, but the number of actuators cannot less than that of closed eigenvalues.

So far, the control problems of structures with closed spaced frequencies using smaller number of actuators than the number of closely spaced frequencies have been limited to two closed spaced frequencies. In practice, large space flexible structures usually have a cluster of closely spaced natural frequencies which are continuously distributed in low frequency range [11]. Hence it is necessary to research the active control problem of structures with continuously closely spaced frequencies. The purpose of this paper is to show how control affects a structure with continuously closely spaced frequencies. To make the results simple and clear, the simplest and typical case of structures with continuously closely spaced frequencies, i.e. a structure with three closely spaced frequencies, is analyzed for the vibration control using direct velocity feedback algorithm. For more than three closely spaced frequencies, similar analysis can be done without new phenomenon, but at the expense of additional complexity; therefore, the problem of three closely spaced frequencies has its generality.

In Section 2, the relationship between modal damping and control gain is investigated based on perturbation method. It demonstrates the influence of the spacing of natural frequencies on control effects. In Section 3, the influence of uncertainty parameters on control effect is analyzed. Conclusions are given in Section 4.

2. Influence of the spacing of frequencies on control effect

2.1. Equations of motion

Equations of motion of a structure with three degrees of freedom controlled by a single actuator expanded by open-loop vibration modes are

$$\mathbf{I}\ddot{\mathbf{x}}(t) + \mathbf{\Lambda}\mathbf{x}(t) = \mathbf{B}u(t), \tag{1}$$

$$u(t) = -\mathbf{H}\dot{\mathbf{x}}(t), \tag{2}$$

where $\mathbf{I}$ is the identity matrix; $\mathbf{B} = \{b_1, b_2, b_3\}^{\mathrm{T}}$ is the actuator position vector; $\mathbf{H} = 2\omega_2\{h_1, h_2, h_3\}$ is the gain vector of direct velocity feedback; and $\mathbf{x}$ is the vector of modal displacements. Matrix $\mathbf{\Lambda} = \mathrm{diag}(\omega_1^2, \omega_2^2, \omega_3^2)$, is the open-loop normalized stiffness matrix and ω_i is the ith open-loop natural frequency. To measure the spacing of frequencies, two tuning parameters are defined as: $\alpha = (\omega_2 - \omega_1)/\omega_2$, $\beta = (\omega_3 - \omega_2)/\omega_2$. If frequencies are closely spaced, $\alpha, \beta = O(\varepsilon)$, where ε is small relative to unity. Without loss of generality, it is assumed that $\alpha, \beta > 0$. The eigenvalue problem for the closed-loop system is

$$[\mathbf{I}s_j^2 + \mathbf{BH}s_j + \mathbf{\Lambda}]\boldsymbol{\phi}_j = 0. \tag{3}$$

The assumption is that the control gain is small, $b_i h_j = O(\varepsilon)$, and the natural choice for the gains are $b_1 h_1 = b_2 h_2 = b_3 h_3 = h$. The complex frequencies are written as $s_j = \omega_2(i + \delta_j)$, where δ_j is a perturbation parameter of order ε. Substituting the expression of s_j into Eq. (3) and retaining first-order terms yields the third-order equation for δ_j

$$\delta_j^3 + [3h + (\alpha - \beta)i]\delta_j^2 + [2h(\alpha - \beta)i + \alpha\beta]\delta_j + h\alpha\beta = 0. \tag{4}$$

The closed-loop natural frequencies and modal damping can be expressed as

$$\omega_j = |s_j| \doteq \omega_2(1 + \mathrm{Im}\delta_j), \xi_j = -Re\,\delta_j. \tag{5}$$

The above deductions are similar with the work of Xu [4], but for a structure with three degrees of freedom here.

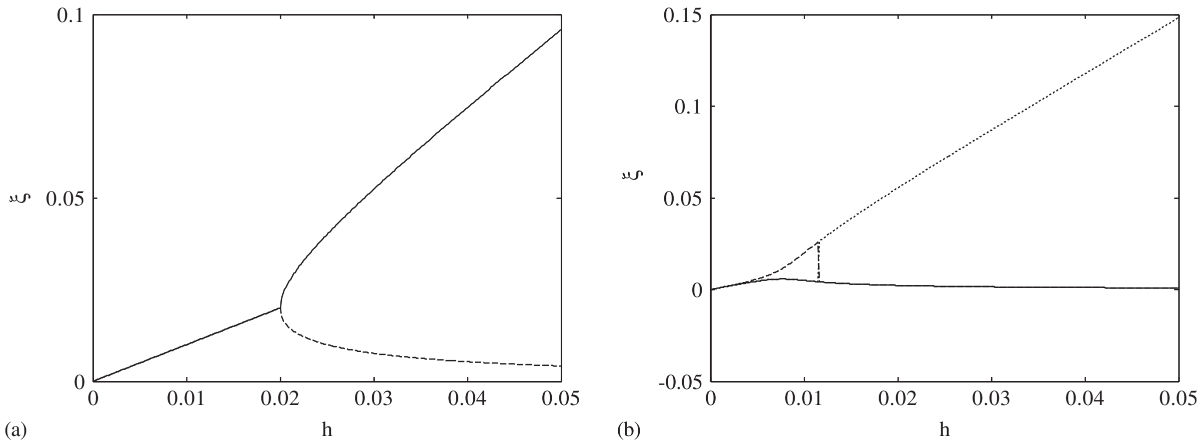

Fig. 1. Modal damping ratios versus control gain: (a) case of the system with two degrees of freedom ($\beta = 0.02$). Solid line, ξ_1; dashed line, ξ_2. (b) Case of the system with three degrees of freedom. Solid line, ξ_1; dashed line, ξ_2; dotted line, ξ_3.

2.2. Computational results

2.2.1. Case $\alpha = \beta$

For $\alpha = \beta = 0.02$, Eq. (4) is simplified as

$$\delta_j^3 + 3h\delta_j^2 + \alpha^2\delta_j + h\alpha^2 = 0. \tag{6}$$

According to Eq. (5), the modal damping ratios can be plotted versus the velocity control gain h. Modal damping ratios of the systems with two and three closely spaced natural frequencies are shown in Fig. 1 where Fig. 1(a) is based on the results of the literature [4]. When the control gain h is small ($h<\beta$ in Fig. 1(a), $h<(\sqrt{3}/6)\alpha$ in Fig. 1(b)), the damping ratios ξ are all linear with respect to h for the two cases. However, when the control gain h is large, one damping ratio increases and the rest asymptotically approaches zero as the control gain h increases. That is, only one mode can be controlled when a single input is used. The comparison of Fig. 1(a) and (b) shows that the low damping ratios of the structure with continuously closely spaced natural frequencies decrease earlier than that of the structure with two closely spaced natural frequencies. Furthermore, the maximum value of the low damping ratios in the three frequencies structure is smaller than that in the two frequencies structure; in other words, the control of the former one is more difficult.

According to the analytical expression, when $h = (\sqrt{3}/3)\alpha$, there is a spurious damping jump in Fig. 1(b) which is meaningless in physics. In order to demonstrate the efficiency of the perturbation method, the perturbation solutions are compared with the direct numerical solutions of Eq. (3), as shown in Fig. 2 where the exact solutions are the numerical solutions. It is found that the exact solutions have no damping jump. However, both damping ratios and frequencies based on perturbation method have jumps. Actually, the perturbation damping ratio of the mode with middle frequency is always large compared to the other two and it is in accordance with the exact solutions, so the perturbation method is efficient.

2.2.2. Case $\alpha \neq \beta$

When $\alpha \neq \beta$, let $\beta = g\alpha$, where g is the ratio of the spacing of natural frequencies. Fig. 3(a) shows how the modal damping ratios vary with the control gain when $g = 5$. The control gain at damping jump is defined as h_L. The plots show that when $h<h_L$, three damping ratios are not equal and only one damping ratio quickly approaches zero. It is different from Fig. 1(b) where one damping is larger and the rest are equal on the whole. When $h>h_L$, the middle modal damping ratio also decreases as the control gain h increases. According to the analytical expressions of Eq. (4), the relationship $h_L = 1/150\sqrt{1+g+g^2}$ is found, as shown in Fig. 3(b). On the whole, h_L increases linearly with respect to g, that is, damping ratio ξ_3 can get to a larger value. As a matter of fact, when g is very large, the system has only two closely spaced natural frequencies and the closed-loop system has only one modal damping ratio approaching zero. In the discussion of the case $\alpha = \beta$, the system has

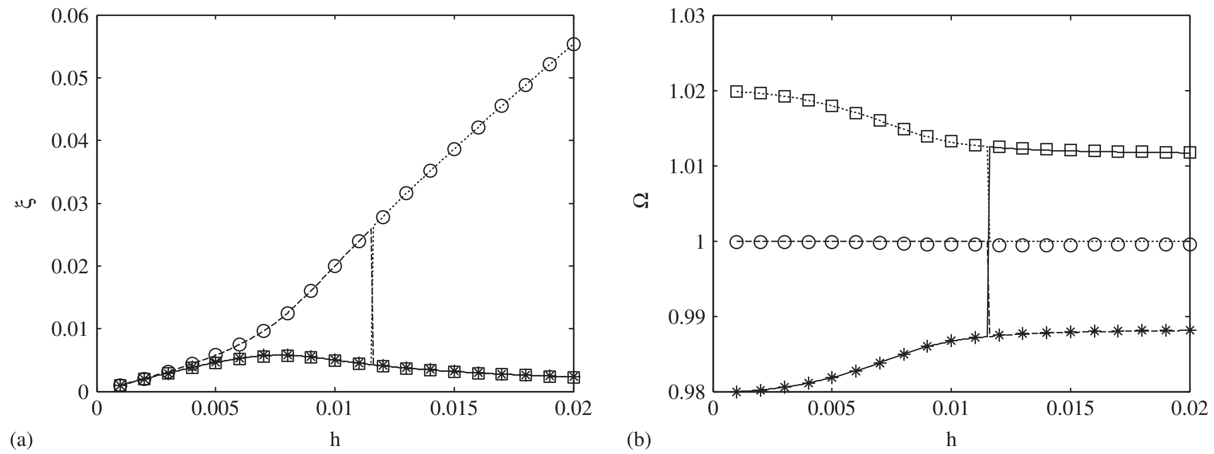

Fig. 2. Comparison of exact and perturbation solutions: (a) modal damping ratios versus control gain. Square, exact ξ_1; circle, exact ξ_2; asterisk, exact ξ_3; Solid line, perturbation ξ_1; dashed line, perturbation ξ_2; dotted line, perturbation ξ_3. (b) Normalized natural frequencies ($\Omega_j = \omega_j/\omega_2$) versus control gain. Square, exact Ω_1; circle, exact Ω_2; asterisk, exact Ω_3; Solid line, perturbation Ω_1; dashed line, perturbation Ω_2; dotted line, perturbation Ω_3.

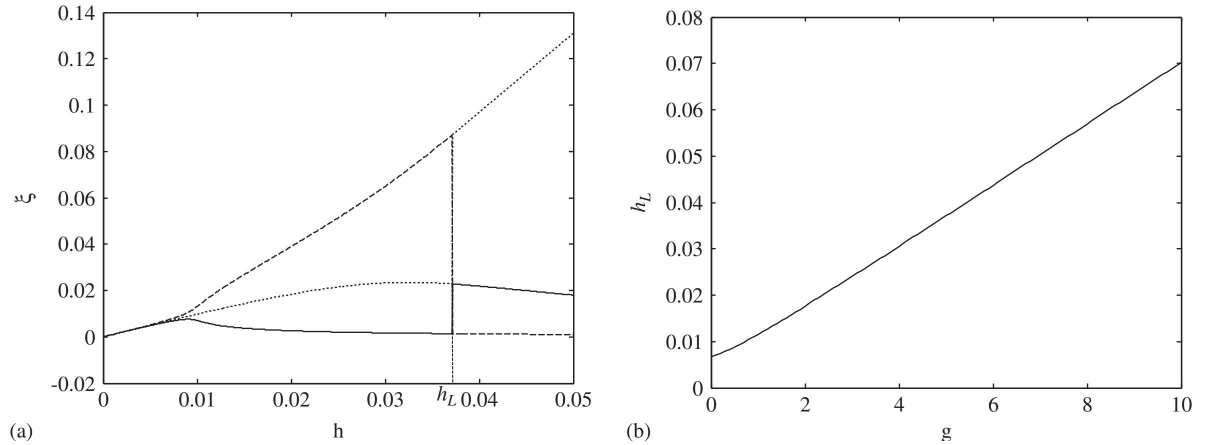

Fig. 3. The case of different spacing ($\alpha = g\beta$): (a) modal damping ratios versus control gain ($g = 5$). Solid line, ξ_1; dashed line, ξ_2; dotted line, ξ_3. (b) Control gain in damping jump versus spacing ratio.

three closely spaced natural frequencies and the closed-loop system has two modal damping ratio approaching zero. It comes to the conclusion that a single input can only control one mode of the closely spaced natural frequencies and the modal damping ratios of the rest close modes will approach zero.

3. Influence of uncertainty parameters on control effect

3.1. Equations of motion

Following the work of Xu [5], two sets of motion equations are considered: (1) the equation of the postulated structures, which contains errors and on which the control is based, (2) the equations of actual structures. At first, for an N-degree of freedom postulated structure with a single control force, the equations of motion in physical coordinates are

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}_0 + \mathbf{B_0}u, \tag{7}$$

$$u = -\mathbf{H_0}\dot{\mathbf{x}}, \tag{8}$$

where $\mathbf{x}$ and $\mathbf{F_0}$ are the N-component vectors for displacement response and external forces, respectively; $\mathbf{M}$, $\mathbf{K}$ are the $N \times N$ matrices of mass and stiffness, respectively; u is a single control force; $\mathbf{B_0}$ is an N-component vector whose elements depend on the placement of control force; $\mathbf{H_0}$ is an N-component gain vector. Let $\mathbf{\Phi_0}$ be the $N \times N$ matrix of postulated mode shapes which are normalized such that $\mathbf{\Phi}_0^{\mathrm{T}}\mathbf{M}\mathbf{\Phi_0} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. By using the transformation $\mathbf{x}=\mathbf{\Phi_0}\mathbf{y}$, the close-loop system is transformed to N modal equations:

$$\mathbf{I}\ddot{\mathbf{y}} + \mathbf{BH}\dot{\mathbf{y}} + \mathbf{\Lambda_0}\mathbf{y} = \mathbf{F}, \tag{9}$$

where, $\mathbf{\Lambda_0} = \mathrm{diag}[\omega_{10}^2, \ldots, \omega_{N0}^2]$, $\mathbf{F}=\mathbf{\Phi_0^T}\mathbf{F_0}$, $\mathbf{B}=\mathbf{\Phi_0^T}\mathbf{B_0}$, $\mathbf{H} = \mathbf{H_0}\mathbf{\Phi_0}$.

In practice, the actual parameters of mass and stiffness are known only approximately. Assuming that in actual system these parameters are denoted by $\mathbf{M}^{\boldsymbol{\alpha}}$ and $\mathbf{K}^{\boldsymbol{\alpha}}$, respectively, the equatons of the actual system are

$$\mathbf{M}^{\boldsymbol{\alpha}}\ddot{\mathbf{x}} + \mathbf{K}^{\boldsymbol{\alpha}}\mathbf{x} = \mathbf{F_0} + \mathbf{B_0}u. \tag{10}$$

By using the same transformation, the close-loop modal equations of the actual system are

$$\mathbf{I}^*\ddot{\mathbf{y}} + \mathbf{BH}\dot{\mathbf{y}} + \mathbf{\Lambda}^*\mathbf{y} = \mathbf{F}, \tag{11}$$

where $\mathbf{I}^*$ and $\mathbf{\Lambda}^*$ are the disturbed matrices of matrices $\mathbf{I}$ and $\mathbf{\Lambda_0}$ in Eq. (9), respectively. For a system with three degrees of freedom, the disturbed matrices can be expressed as

$$\mathbf{I}^* = \begin{bmatrix} 1+2\mu_{11} & 2\mu_{12} & 2\mu_{13} \\ 2\mu_{12} & 1+2\mu_{22} & 2\mu_{23} \\ 2\mu_{13} & 2\mu_{23} & 1+2\mu_{33} \end{bmatrix},$$

$$\mathbf{K}^* = \omega_2^2 \begin{bmatrix} (1-\alpha)^2+2\gamma_{11} & 2\gamma_{12} & 2\gamma_{13} \\ 2\gamma_{12} & 1+2\gamma_{22} & 2\gamma_{23} \\ 2\gamma_{13} & 2\gamma_{23} & (1+\beta)^2+2\gamma_{33} \end{bmatrix}, \tag{12}$$

where $\mu_{ij}, \gamma_{ij} = O(\varepsilon)$, are disturbances of the postulated modal mass and stiffness matrices, respectively; α,β are the tuning parameters as defined in Section 2.1. By pre-multiplying $\mathbf{I}^{*-1}$ to Eq. (11) and keeping first-order terms in Eq. (9) yields

$$\mathbf{I}\ddot{\mathbf{y}} + \mathbf{BH}\dot{\mathbf{y}} + \mathbf{\Lambda}\mathbf{y} = \mathbf{F}, \tag{13}$$

where $\mathbf{I}$ is the identity matrix

$$\mathbf{\Lambda} = \omega_2^2 \begin{bmatrix} 1-2\alpha^* & \rho_1 & \rho_2 \\ \rho_1 & 1+2\gamma^* & \rho_3 \\ \rho_2 & \rho_3 & 1+2\beta^* \end{bmatrix} \tag{14}$$

and

$$\begin{gathered} \alpha^* = \alpha - (\gamma_{11} - \mu_{11}),\ \gamma^* = \gamma_{22} - \mu_{22},\ \beta^* = \beta + (\gamma_{33} - \mu_{33}) \\ \rho_1 = \gamma_{12} - \mu_{12},\ \rho_2 = \gamma_{13} - \mu_{13},\ \rho_3 = \gamma_{23} - \mu_{23}, \end{gathered} \tag{15}$$

where α^*, β^* and γ^* are the diagonal uncertainty parameters which are involved with uncertainty of the system and the tuning parameters of frequencies. ρ_1, ρ_2 and ρ_3 are off-diagonal uncertainty parameters which are only involved with uncertainty of the system. By solving directly the corresponding characteristic equation in Eq. (13), the relationship between modal damping ratios and uncertainty parameters can be obtained.

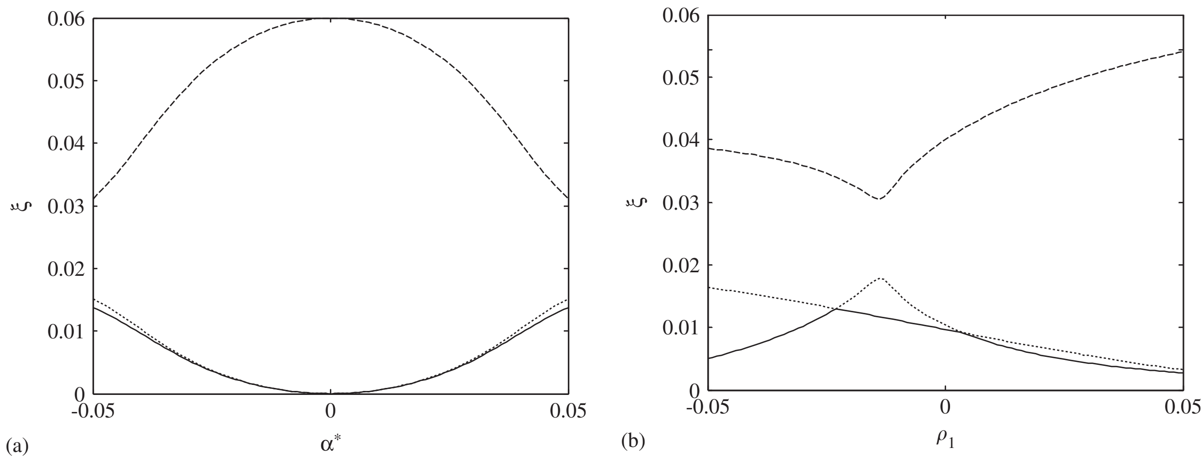

Fig. 4. Modal damping ratios versus uncertainty parameters: (a) modal damping ratios versus diagonal parameter α^*. (b) Modal damping ratios versus off-diagonal parameter ρ_1. Solid line, ξ_1; dashed line, ξ_2; dotted line, ξ_3.

3.2. Computational results

3.2.1. Influence of the diagonal parameters on damping ratios

Firstly, only the influence of the diagonal parameters is considered. Let $\rho_1 = \rho_2 = \rho_3 = 0$, and $\alpha^* = \beta^*$, $\gamma^* = 0$, such that matrix Λ in Eqs. (13) and (3) have the same form and α^*, β^* are equivalent tuning parameters. Fig. 4(a) shows the variation of modal damping ratios with respect to α^*. When $\alpha^* = 0$, two modes become undamped. For a structure with closely spaced natural frequencies, α is small and a little change of the parameters of the system can make α^* approach zero. Therefore, the system has undamped modes and may not be controlled.

3.2.2. Influence of the off-diagonal parameters on damping ratios

Provided $\alpha^* = \beta^* = 0.04$and $\gamma^* = 0$, the influence of off-diagonal parameters is analyzed. Let $\rho_2 = \rho_3 = 0$, and how the damping ratios vary with respect to ρ_1 is shown in Fig. 4(b). The plots show that when the equivalent parameters are large numbers, a little change of ρ_1 has slender influence on modal damping ratios and even makes low damping ratio increase. However, the low damping ratios will decrease as ρ_1 increases continually.

3.2.3. Influence of diagonal and off-diagonal parameters on control effect

In order to illustrate the influence of uncertainty parameters more clearly, three-dimensional plots are given in Fig. 5 to show the modal damping ratios varying with respect to α^* and ρ_1, where $\alpha^* = \beta^*$, $\gamma^* = 0$ and $\rho_2 = \rho_3 = 0$. When α^* is large, a little change of ρ_1 has slender influence on modal damping ratios, which is the same as case 3.2.2. But when α^* is small, the modal damping ratios vary quickly with respect to ρ_1. The reason for this situation is that when the equivalent tuning parameters are small, the mode shapes are so sensitive to the uncertainty parameters that a little change of off-diagonal parameters can make the mode shapes change greatly.

3.2.4. Relationship between the sum of damping ratios and control gain

In the discussions above, the control gains are given as $b_1h_1 = b_2h_2 = b_3h_3 = h$. From Figs. 4 and 5, one modal damping ratio increases and the other modal damping ratio decreases accordingly. It seems that the sum of all damping ratios denoted as ξ_{sum} is a constant. For a given control gain, the sum ratio ξ_{sum} varying with uncertainty parameters is shown in Fig. 6(a). The plotting is a plane, that is, uncertainty parameters can only change the values of separate damping ratios but not the sum of them. The sum of modal damping ratios only relates to the control gain and increases linearly with respect to the control gain, as shown in Fig. 6(b), where $b_1h_1 = b_2h_2 = h$ and $b_3h_3 = h_1$.

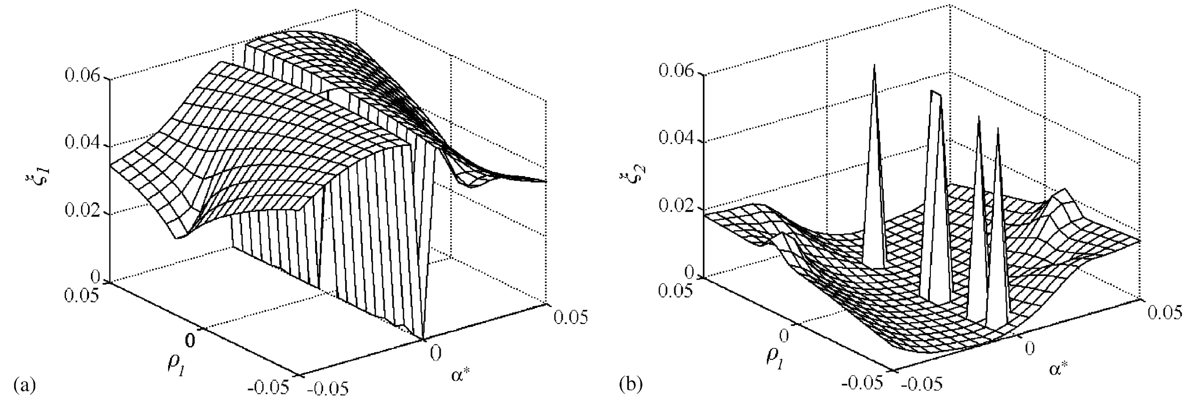

Fig. 5. The surfaces of damping ratios versus uncertainty parameters.

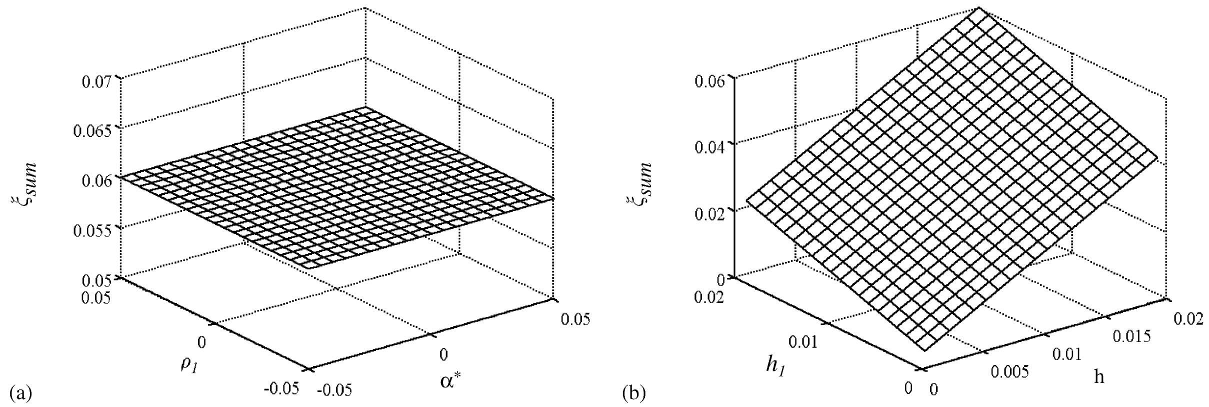

Fig. 6. (a) The sum of modal damping ratios (ξ_{sum}) versus uncertainty parameters. (b) The sum of modal damping ratios versus the control gain.

4. Conclusions

In this paper, the vibration control of a typical system with continuously closely spaced frequencies, a structure with three closely spaced frequencies, is investigated based on perturbation method. By comparing the perturbation solutions with the direct numerical ones, the efficiency of the perturbation method is proved. The behaviors of the structures with two and three closely spaced frequencies under a single actuator are compared and the influence of the spacing of the closely spaced frequencies on control effect is analyzed. It is shown that the spacing of natural frequencies is the key parameter with respect to control effect and a single input can only control one of mode of the closely spaced natural frequencies and the modal damping ratios of the rest close modes will approach zero.

The influence of uncertainty parameters on control effect is also investigated. The main conclusions are as follows: the controllability is sensitive to the diagonal parameters; when the diagonal parameters are small, the modal damping ratios vary quickly with respect to the off-diagonal parameters; the sum of all modal damping ratios only relates to the control gain and increases linearly with respect to the control gain.

Acknowledgements

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