Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Sound and Vibration 294 (2006) 238–248
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Frictional slip of an elastic layer on a half-space caused by an anti-plane wave of an arbitrary form

Gui-Lan Yu*, Yue-Sheng Wang, Guang-Shou Li

School of Civil Engineering and Architecture, Beijing Jiaotong University, Beijing 100044, PR China

Received 15 February 2005; received in revised form 18 October 2005; accepted 7 November 2005
Available online 4 January 2006

Abstract

The slip between an elastic layer and a half-space caused by an anti-plane wave of arbitrary form is examined. The incident wave is assumed to be sufficiently strong so that friction may be broken, and the local slip may take place at the interface. The analysis is restricted to the sub-critical angle of incidence. Fourier analysis is employed to cast the mixed boundary value problem to a recurrence relation. The calculation of the local slip velocities and the interface traction, together with the determination of the extent and location of slip zones, which are unknown before the solution of the problem, are discussed. As an example, a parabolic pulse incident wave is considered. The distribution of the slip zones, the interface shearing traction and slip velocities are calculated in detail. The results show that the problem involves a lot of features different from those for an infinite medium due to the existence of the free surface of the elastic layer. For instance, multiple slip zones may appear due to the reflection of waves at the free surface, and the slip zones become closer and closer when the thickness of the layer decreases. On the other hand, when the thickness of the layer is infinity, the distances between the slip zones become infinitely large, and the results reduce to those obtained for the infinite medium.

1. Introduction

Consider two solids bonded with friction. An elastic wave strikes the interface from one solid. If the wave is strong enough, the friction may be broken and some local slip and separation may take place at the interface. The problem is of practical importance in seismology, earthquake engineering and mechanics of laminated composites, etc. It is known, however, that the local slip and separation intervals are unknown before the solution of the problem, and therefore the mathematical procedure is very difficult. Few literatures can be found concerning this kind of problem. Miller [1] and Miller and Tran [2,3] analyzed the transmission of elastic waves through a frictional contact interface by using an approximate method—the method of equivalent linearization developed by Iwan [4] in a general sense. They restricted the analysis to the sub-critical angle of incidence and neglected the possible separation of the interface by assuming that the pressure is sufficiently large. Comninou et al. developed an exact approach based on the Fourier analysis and made an extensive investigation on this kind of problems. Their works include the interaction of a harmonic or

*Corresponding author. Tel.: +86 10 51688277; fax: +86 10 51687248.
E-mail address: glyu@center.njtu.edu.cn (G.-L. Yu).

0022-460X/$ - see front matter © 2005 Published by Elsevier Ltd.
doi:10.1016/j.jsv.2005.11.003

inharmonic wave (SH, P or SV) with a smooth or frictional contact interface [5–13] and the surface waves propagating along such interfaces [14,15]. The presence of local separations is considered and the analysis is allowed for both sub-critical and super-critical angle of incidence. The contact interface (smooth or frictional) is termed 'unilateral interface' by Comninou et al. because the boundary conditions involve inequalities. The associated problem is termed 'unilateral problem'. Relatively, the welded interface and the associated problem are called, respectively, 'bilateral interface' and 'bilateral problem'. Comninou et al. discovered many new aspects in physics of the unilateral problems. For instance, the reflected and refracted waves are distorted in relation to the incident wave and contain modes of all higher frequencies. Using the exact method, we studied the re-polarization of elastic waves at a frictional interface, which is another interesting phenomenon for unilateral problems [16–18].

It is noted, however, that all of the above mentioned works are restricted to the case of an infinite medium. When one of the half-spaces is replaced by a layer of finite thickness, the mathematical procedure will become very complex due to the involved characteristic length. Few papers can be found to attack such problems except the one by Miller [19], which analyzed the propagation of Love-type surface waves in an elastic layer bonded by Coulomb friction to an elastic half-space and obtained the asymptotic results when the attenuation in the system is very small. Recently, we studied the propagation of harmonic SH waves in a layered half-space with a frictional contact interface by employing the exact method similar to that of Comninou and Dundurs' [20,21]. It is shown that the results are drastically different from those for two semi-infinite dissimilar media because of the presence of the characteristic length. In this paper, we will extend the method to the interaction between an elastic layer and a half-space with a frictional interface under the action of an SH wave of arbitrary form for the sub-critical angle incident case.

2. Problem formulation

The problem treated here is shown in Fig. 1. An elastic layer with thickness H is forced on an elastic half-space by the applied pressure p_0 and at the same time loaded by the shearing traction q_0. Let μ and c represent the shear modulus and the transverse wave speed, respectively. The asterisk refers to the quantities of the layer. An SH wave with arbitrary form strikes the frictional contact interface from the half-space at the angle of incidence θ_0. The Coulomb friction model, with static and kinetic friction coefficients as f_s and f_k, is adopted.

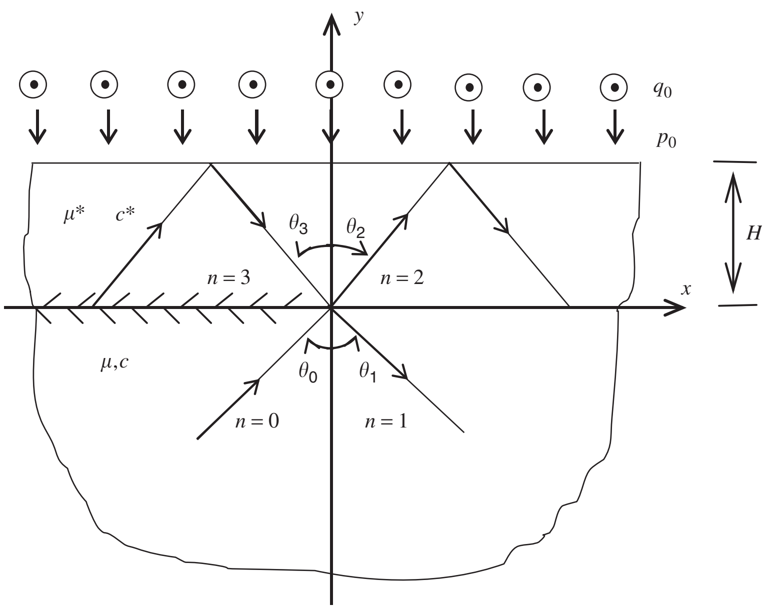

Fig. 1. An SH wave strikes a frictional contact interface at the angle of incidence θ_0 between an elastic layer and a half-space. $n = 0,1,2,3$ indicates the incident, reflected and refracted waves, respectively.

Generally $f_k \leqslant f_s$. The indices (1), (2), (3) which may appear as superscripts are used to distinguish various refracted and reflected waves. If we denote the shear traction and slip velocity at the interface as S and V, the unilateral boundary conditions can be written as

$$|S| = f_k p_0, \quad \mathrm{sign}(S) = \mathrm{sign}(V) \tag{1}$$

in the slip zones, and

$$V = 0, \quad |S| \leqslant f_s p_0 \tag{2}$$

in the stick zones. It is assumed that the wave has propagated for such a long time that the system is in steady state. The incident wave is taken as an arbitrary form

$$u_z^{(0)} = F_0(\varsigma_0), \tag{3}$$

where

$$\varsigma_0 = k(x \sin\theta_0 + y \cos\theta_0 - ct) \tag{4}$$

with k representing the wavenumber. It is seen that the disturbance caused by the incident wave propagates along the interface at the velocity $v = c/\sin\theta_0$. Therefore, we may formulate the problem in the moving coordinates (η, y), where

$$\eta = \varsigma_0|_{y=0} = k(x \sin\theta_0 - ct). \tag{5}$$

We construct the solution by adding a corrective solution to the results for the bilateral interface. Denote the bilateral solution as (u_z, τ_{yz}) and the corrective solution as $(\bar{u}_z, \bar{\tau}_{yz})$. Then the unilateral solution is $(u_z, \tau_{yz}) + (\bar{u}_z, \bar{\tau}_{yz})$. Thus. V and S may be expressed as

$$V(\eta) = \left[\dot{\bar{u}}_z^{(2)} + \dot{\bar{u}}_z^{(3)} - \dot{\bar{u}}_z^{(1)}\right]_{y=0}, \tag{6}$$

$$S(\eta) = q_0 + [\tau_{yz}]_{y=0} + [\bar{\tau}_{yz}^{(1)}]_{y=0} = q_0 + [\tau_{yz}]_{y=0} + [\bar{\tau}_{yz}^{(2)} + \bar{\tau}_{yz}^{(3)}]_{y=0}, \tag{7}$$

where $[\tau_{yz}]_{y=0}$ is the shearing traction generated by the incident wave at a bilateral interface which is given in the next section.

Comninou and Dundurs [9] demonstrated that Snell's law holds for both bilateral and unilateral interfaces. Thus, we have

$$\theta_1 = \theta_0, \quad \theta_2 = \theta_3, \quad c/\sin\theta_0 = c^*/\sin\theta_2. \tag{8}$$

When $\theta_0 > \theta_{\mathrm{cr}} = \sin^{-1}(c/c^*)$ the incident wave is totally reflected. The angle θ_{cr} is termed critical angle. In the present paper, we only consider the sub-critical angle of incidence.

3. Bilateral solution for sub-critical angle of incidence

Consider an incident wave with the form given in Eq. (3). Then the reflected and refracted waves have the forms

$$u_z^{(n)} = F_n(\varsigma_n), \quad n = 1, 2, 3, \tag{9}$$

where ς_n are given by

$$\begin{aligned} \varsigma_1 &= k(x \sin\theta_1 - y \cos\theta_1 - ct), \\ \varsigma_n &= k^*(x \sin\theta_n + (-1)^n y \cos\theta_n - c^* t), \quad n = 2, 3 \end{aligned} \tag{10}$$

and $F_n(\varsigma_n)$ are undetermined functions.

For a bilateral interface, i.e. a welded interface, the traction and displacement are continuous, which gives

$$\mu\left(\frac{\partial u_z^{(0)}}{\partial y} + \frac{\partial u_z^{(1)}}{\partial y}\right) = \mu^*\left(\frac{\partial u_z^{(2)}}{\partial y} + \frac{\partial u_z^{(3)}}{\partial y}\right), \quad y = 0. \tag{11}$$

$$u_z^{(0)} + u_z^{(1)} = u_z^{(2)} + u_z^{(3)}, \quad y = 0. \tag{12}$$

The traction free condition on the surface gives

$$\frac{\partial u_z^{(2)}}{\partial y} + \frac{\partial u_z^{(3)}}{\partial y} = 0, \quad y = H. \tag{13}$$

Substitution of Eqs. (3) and (9) to Eqs. (11)–(13) yields

$$\Delta\left[F_0'(\eta) - F_1'(\eta)\right] = F_2'(\eta) - F_3'(\eta), \tag{14}$$

$$F_0(\eta) + F_1(\eta) = F_2(\eta) + F_3(\eta), \tag{15}$$

$$F_2'(\eta + \gamma) = F_3'(\eta - \gamma), \tag{16}$$

where

$$\gamma = k^* H \cos\theta_2, \quad \Delta = \mu c^* \cos\theta_0/(\mu^* c \cos\theta_2). \tag{17}$$

Set $f_n(\ ) = F_n'(\ )$, $(n = 0, 1, 2, 3)$. Then Eqs. (14)–(16) can be rewritten as

$$\Delta\left[f_0(\eta) - f_1(\eta)\right] = f_2(\eta) - f_3(\eta), \tag{18}$$

$$f_0(\eta) + f_1(\eta) = f_2(\eta) + f_3(\eta), \tag{19}$$

$$f_3(\eta) = f_2(\eta + 2\gamma) \tag{20}$$

from which one can derive

$$2\Delta f_0(\eta) = (1 + \Delta) f_2(\eta) - (1 - \Delta) f_2(\eta + 2\gamma). \tag{21}$$

This is a recurrence relation from which $f_2(\eta)$ and $f_3(\eta)$ can be determined as follows:

$$f_2(\eta) = \frac{2\Delta}{1 + \Delta}\sum_{i=0}^{m}\left(\frac{1 - \Delta}{1 + \Delta}\right)^i f_0(\eta + 2i\gamma), \tag{22}$$

$$f_3(\eta) = \frac{2\Delta}{1 + \Delta}\sum_{i=0}^{m}\left(\frac{1 - \Delta}{1 + \Delta}\right)^i f_0(\eta + 2(i + 1)\gamma), \tag{23}$$

where m is given by

$$\eta + 2(m + 1)\gamma \geqslant 1, \quad \eta + 2m \leqslant 1. \tag{24}$$

As an example we suppose the incident wave has a parabolic form

$$u_z^{(0)}(\varsigma_0) = F_0(\varsigma_0) = C_0(1 - \varsigma_0^2)H(1 - |\varsigma_0|), \tag{25}$$

where C_0 in the above equation is a parameter with the same dimension of the displacements. $H(\cdot)$ is the Heaviside function. Then we have

$$\tau_{yz}^{(0)}(\varsigma_0) = \mu k \cos\theta_0 f_0(\varsigma_0) = \mu k \cos\theta_0 C_0(1 - \varsigma_0^2)H(1 - |\varsigma_0|) = A_0(1 - \varsigma_0^2)H(1 - |\varsigma_0|), \tag{26}$$

where

$$A_0 = \mu k \cos\theta_0 C_0. \tag{27}$$

So the shearing traction on the bilateral interface is obtained:

$$[\tau_{yz}]_{y=0} = \mu * k * \cos\theta_2[f_2(\eta) - f_3(\eta)] = A_0 f(\eta) \tag{28}$$

with

$$f(\eta) = \frac{2}{(1 + \Delta)C_0}\left\{f_0(\eta) - \frac{2\Delta}{1 + \Delta}\sum_{i=1}^{m}\left(\frac{1 - \Delta}{1 + \Delta}\right)^{i-1} f_0(\eta + 2i\gamma)\right\}. \tag{29}$$

For $\Delta = 1$, the above equation reduces to

$$f(\eta) = \frac{1}{C_0}[f_0(\eta) - f_0(\eta + 2\gamma)], \tag{30}$$

where A_0 is a constant with the dimension of stresses and thus $f(\eta)$ is of non-dimension.

4. Unilateral solution for sub-critical angle of incidence

For sub-critical angle of incidence, the governing equations in the moving coordinates are given by

$$-(k \cos \theta_0)^2 \frac{\partial^2 \bar{u}_z^{(1)}}{\partial \eta^2} + \frac{\partial^2 \bar{u}_z^{(1)}}{\partial y^2} = 0, \tag{31}$$

$$-(k^* \cos \theta_2)^2 \frac{\partial^2 \bar{u}_z^{(n)}}{\partial \eta^2} + \frac{\partial^2 \bar{u}_z^{(n)}}{\partial y^2} = 0, \quad n = 2, 3, \tag{32}$$

where $k^* = \omega/c^*$. Using Fourier transform technique, one may obtain

$$\bar{u}_z^{(n)} = \mathrm{Re} \int_0^\infty F_n(s) \mathrm{e}^{\mathrm{i}s\varsigma_n} \, \mathrm{d}s, \quad n = 1, 2, 3, \tag{33}$$

where

$$\begin{aligned} \varsigma_1 &= k(x \sin \theta_1 - y \cos \theta_1 - ct), \\ \varsigma_n &= k^*(x \sin \theta_n + (-1)^n y \cos \theta_n - c^* t), \quad n = 2, 3. \end{aligned} \tag{34}$$

$F_n(s)$ in Eq. (33) is complex and may be written as $D_n(s) + \mathrm{i}E_n(s)$ with D_n and E_n being real. Note that the integral from $-\infty$ to 0 does not appear because the waves propagate in the positive x-direction.

The boundary conditions at the surface $y = H$ and the requirement that the shearing traction is continuous across the interface ($y = 0$) lead to

$$F_1(s) = 2i\Delta^{-1} \sin(s\gamma) \mathrm{e}^{\mathrm{i}s\gamma} F_2(s), \quad F_3(s) = \mathrm{e}^{2\mathrm{i}s\gamma} F_2(s). \tag{35}$$

Substituting Eq. (33) into Eqs. (6) and (7), and taking the real parts, we have

$$V(\eta) = -kc \int_0^\infty s[M(s) \sin(s\eta) + N(s) \cos(s\eta)] \, \mathrm{d}s, \tag{36}$$

$$S(\eta) = q_0 + A_0 f(\eta) + \mu k \cos \theta_0 \int_0^\infty s[D_1(s) \sin(s\eta) + E_1(s) \cos(s\eta)] \, \mathrm{d}s, \tag{37}$$

where

$$M(s) = -\Delta \cot(s\gamma) E_1(s) + D_1(s), \quad N(s) = E_1(s) + \Delta \cot(s\gamma) D_1(s). \tag{38}$$

Set $\phi(\eta) = -(kc)^{-1} V(\eta)$. It follows from Eq. (36) that

$$\{M(s), N(s)\} = \frac{1}{\pi s} \int_{-\infty}^{+\infty} \phi(\xi) \{\sin(s\xi), \cos(s\xi)\} \, \mathrm{d}\xi. \tag{39}$$

Then Eq. (37) reduces to

$$S(\eta) = q_0 + A_0 f(\eta) + \frac{1}{2\pi} \mu k \cos \theta_0 \int_{-\infty}^{+\infty} \phi(\xi) P(\xi, \eta) \, \mathrm{d}\xi, \tag{40}$$

where

$$P(\xi, \eta) = \int_{-\infty}^{+\infty} \Lambda^{-1}(s) \{[1 - \cos(2s\gamma)] \cos[s(\eta - \xi)] + \Delta \sin(2s\gamma) \sin[s(\eta - \xi)]\} \, \mathrm{d}s \tag{41}$$

with

$$\Lambda(s) = (\Delta^2 + 1) + (\Delta^2 - 1)\cos(2s\gamma). \tag{42}$$

Expressing $\Lambda^{-1}(s)$ as a Fourier series in terms of $2s\gamma$:

$$\Lambda^{-1}(s) = \frac{1}{2\Delta} + \frac{1}{\Delta}\sum_{n=1}^{\infty}\left(\frac{1-\Delta}{1+\Delta}\right)^n \cos(2ns\gamma) \tag{43}$$

and using the relation

$$\int_{-\infty}^{+\infty} \cos(sx)\,\mathrm{d}s = 2\pi\delta(x), \tag{44}$$

we obtain

$$S(\eta) = q_0 + A_0 f(\eta) + \frac{\mu k \cos\theta_0}{1+\Delta}\left[\phi(\eta) - 2\sum_{n=1}^{\infty}\frac{\Delta(1-\Delta)^{n-1}}{(1+\Delta)^n}\phi(\eta + 2n\gamma)\right], \quad \Delta \neq 1. \tag{45}$$

For $\Delta = 1$, Eq. (40) reduces to

$$S(\eta) = q_0 + A_0 f(\eta) + \frac{\mu k \cos\theta_0}{2}[\phi(\eta) - \phi(\eta + 2\gamma)]. \tag{46}$$

The unilateral interface conditions (1) and (2) lead to

$$\phi(\eta) = 0 \tag{47}$$

in the stick zones, and

$$\frac{\mu k \cos\theta_0}{1+\Delta}\left[\phi(\eta) - 2\sum_{n=1}^{\infty}\frac{\Delta(1-\Delta)^{n-1}}{(1+\Delta)^n}\phi(\eta + 2n\gamma)\right] = -q_0 - A_0 f(\eta) \pm f_k p_0, \quad \Delta \neq 1 \tag{48}$$

in the slip zones. For $\Delta = 1$, we have

$$\frac{\mu k \cos\theta_0}{2}[\phi(\eta) - \phi(\eta + 2\gamma)] = -q_0 - A_0 f(\eta) \pm f_k p_0. \tag{49}$$

Unlike the resulting equation for an infinite medium [11], Eq. (48) or (49) is a recurrence relation which relates the slip velocity at η to that at $\eta + 2\gamma$. The parameter γ is nothing but the number of the waves with the width $(k^* \cos\theta_2)^{-1}$ included in the thickness H of the layer. It can be also interpreted as the phase difference of the wave motion between the interface and the free surface. It embodies the effects of the characteristic length H. A numerical example will be given next.

5. Numerical examples and discussion

As a simple example, we suppose $q_0 = 0$, and the incident wave is taken as a parabolic pulse given by Eq. (25).

It is shown in Ref. [11] that only a single slip zone may occur in the region (−1,1) for an infinite medium. However, additional slip zones may appear beyond the region (−1,1) in the present case. The slip zone in the region (−1,1) is called the 'main slip zone' and denoted by the interval (α, β). The leading edge β is determined by $S(\beta) = f_s p_0$ and locates between 0 and 1. Local slip cannot take place in the region $\eta > \beta$ in any case and therefore $\phi(\eta) = 0$ for $\eta > \beta$. Furthermore, we have $S(\eta) = A_0 f(\eta) + q_0$ for $\eta > \beta$ where $f(\eta)$ is given by Eq. (29). Other values of $\phi(\eta)$ for $\eta < \beta$ can be obtained by the recurrence relation (48) or (49). The condition $S(\eta) = f_s p_0$ gives the leading edges of the slip zones and the condition $\phi(\eta) = 0$ yields the trailing edges.

As we know the local slip may take place when $f_s p_0$ is lower than the absolute maximum value of the interface shearing traction. To determine the location of the slip zones for a given value of $f_s p_0$ (or $f_k p_0$), we suggest the following steps:

(i) decrease $f_s p_0$ by a small value from the maximum value of corresponding bilateral shearing traction, then make a guess of the slip zones using the bilateral solution as a guidance;
(ii) derive the interface shearing traction and slip velocities from Eqs. (45) to (49) by recurrence;
(iii) check whether the solution satisfies the boundary conditions (1) and (2); if yes, go to the next step; if no, the guessed slip zones should be corrected easily by reassuming the corresponding state conversely, then go back to step (ii);
(iv) again decrease $f_s p_0$ by a small value and make a guess of slip zones based on the bilateral solution and the solution obtained in the previous steps;
(v) repeat steps (ii)–(iv).

It is seen that the solution of the problem mainly depends on the parameters Δ, γ, q_0/A_0, $f_s p_0/A_0$ and $f_k p_0/A_0$. Next we will give some numerical results and discuss the effects of parameters Δ, γ and the kinematic locking on the solutions in detail.

5.1. The effects of parameter γ

For simplicity we neglect the kinematic locking (i.e. $f_s = f_k$) in calculation. Fig. 2 illustrates curves determining the extent and location of the slip zones for given values of $f_k p_0/A_0$ with $\gamma = 0.5, 1, 2$ and $\Delta = 2$. Figs. 3 and 4 are the same but for $\Delta = 1$ and 0.5. The symbols '+' and '−' denote the positive and negative slip direction, respectively. The shearing traction and slip velocity are shown in Fig. 5 for $\gamma = 0.5, 1$ with $f_k p_0/A_0 = 0.375$ and $\Delta = 1$. It is seen that one or more additional slip zones with opposite slip direction appears at the left of the main slip zone. As γ decreases, these slip zones become closer and closer and sharp tips occur. We believe that these behaviors are resulted by the reflection of waves at the free surface $y = H$.

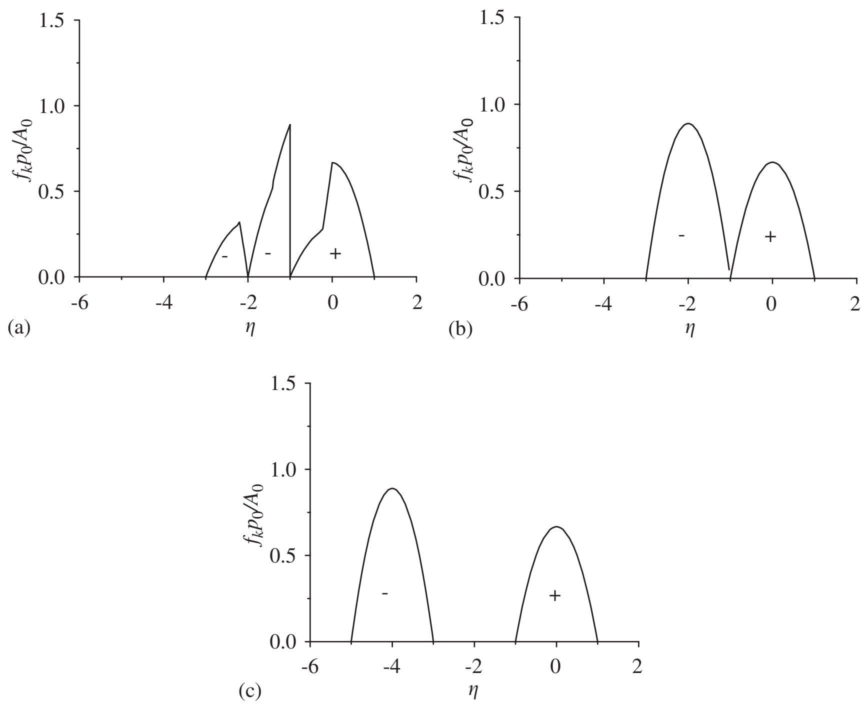

Fig. 2. Extent and location of the slip zones varying with $f_k p_0/A_0$ for $\Delta = 2$: (a) $\gamma = 0.5$; (b) $\gamma = 1$; (c) $\gamma = 2$.

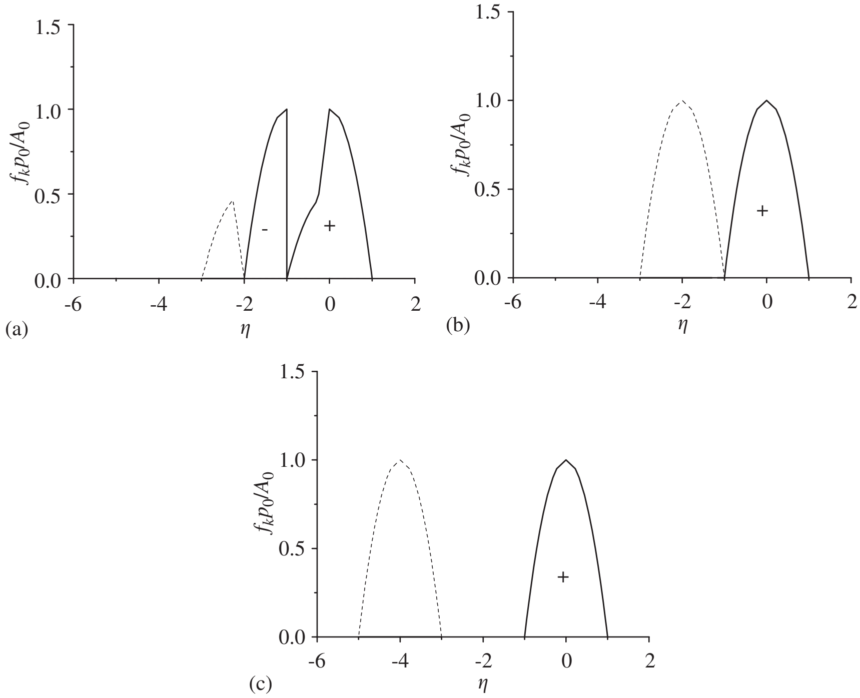

Fig. 3. Extent and location of the slip zones varying with $f_k p_0/A_0$ for $\Delta = 1$; (a) $\gamma = 0.5$; (b) $\gamma = 1$; (c) $\gamma = 2$.

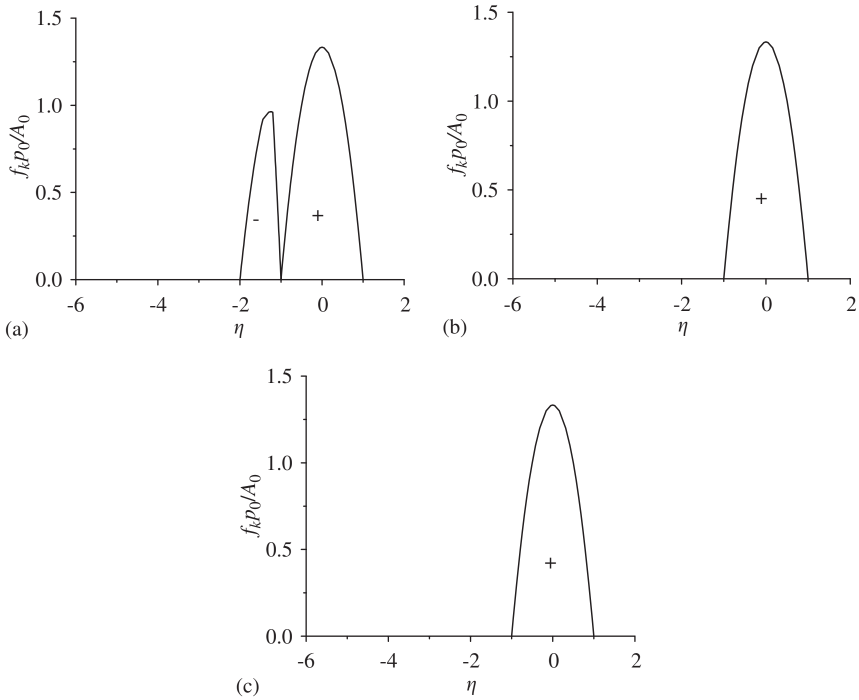

Fig. 4. Extent and location of the slip zones varying with $f_k p_0/A_0$ for $\Delta = 0.5$: (a) $\gamma = 0.5$; (b) $\gamma = 1$; (c) $\gamma = 2$.

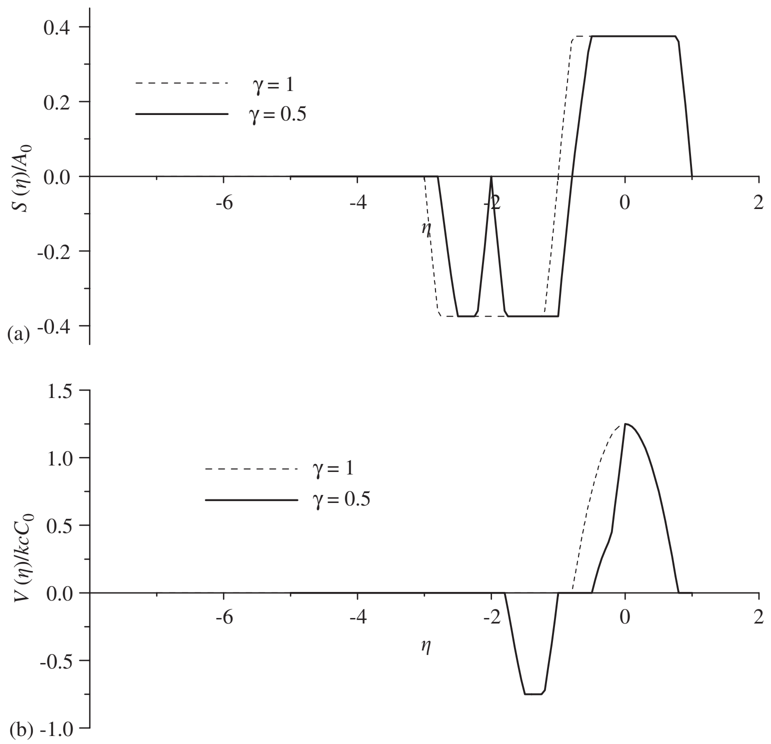

Fig. 5. Distributions of shearing traction (a) and slip velocity (b) for $f_k p_0/A_0 = 0.375$, $\Delta = 1$.

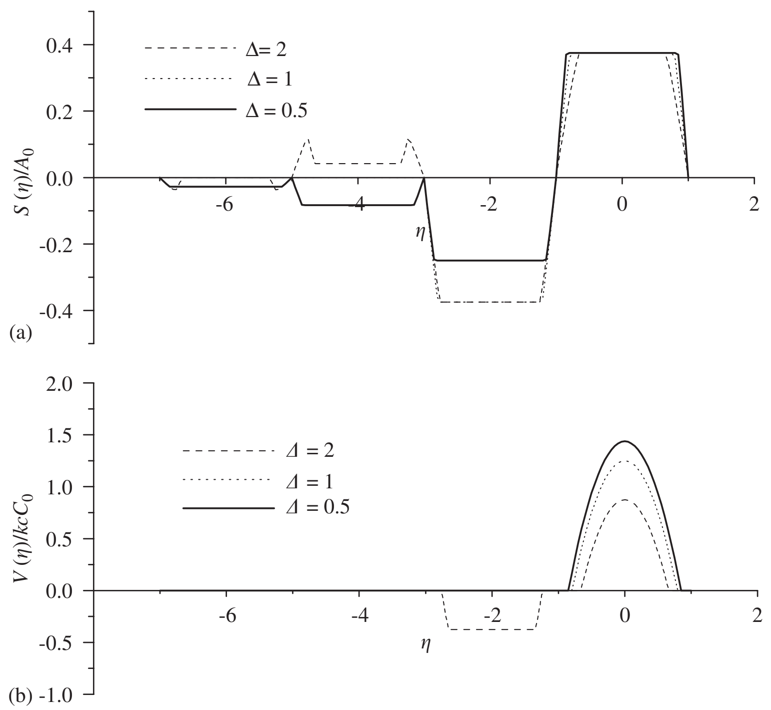

Fig. 6. Distributions of shearing traction (a) and slip velocity (b) for $f_k p_0/A_0 = 0.375$, $\gamma = 1$.

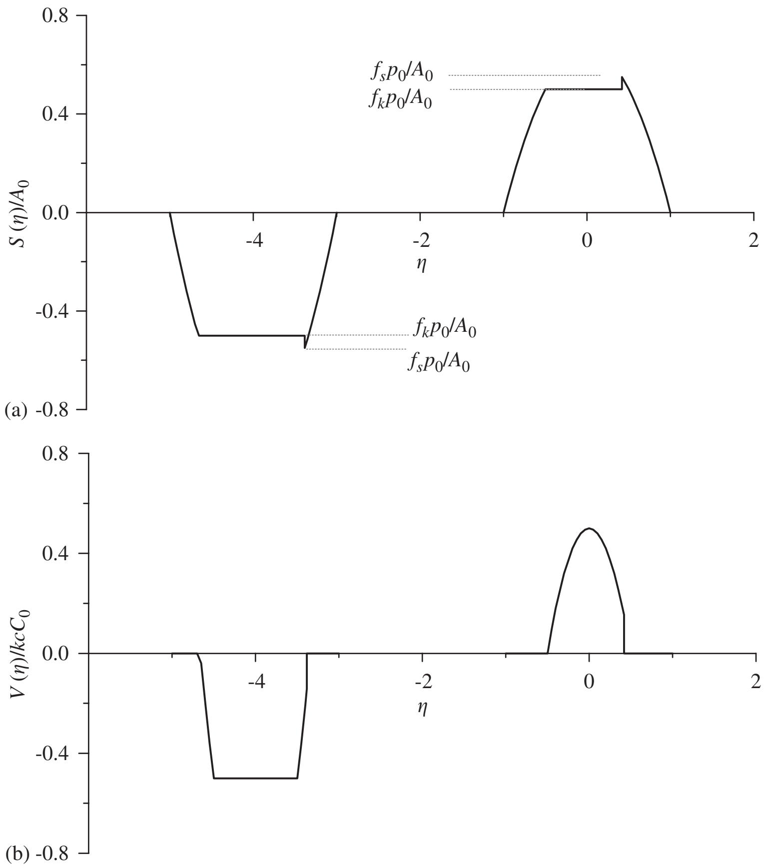

Fig. 7. Distributions of shearing traction (a) and slip velocity (b) with consideration of the effects of the kinematic locking for $\Delta = 2$, $\gamma = 2$.

This is understood if we consider the physical meaning of the parameter γ that has been mentioned before. In the limiting situation of $H \to \infty$, the distance between the slip zones will become infinitely large, and the results reduce to those obtained by Comninou and Dundurs [11] for an infinite medium.

5.2. The effects of parameter Δ

We can easily see the effects of parameter Δ on the solution by comparing the curves with the same value of γ in Figs. 2–4. It is shown that a larger Δ will lead more local slip regions except in the interval (−1,1). As the parameter Δ increases, the shearing traction and the slip velocity in the main slip zone is strongly weakened due to the appearance of the additional slip zones. For example, in Fig. 2, when the non-dimensional frictional force is greater than a certain value, say 0.65, the additional slip zone exists while the main slip zone disappears. This behavior coincides with the physical meaning of the parameter Δ that can be interpreted as the normal wave resistance ratio of the half-space to the layer. It characterizes the difficulty of the wave transmission across the interface. The smaller the value of Δ is, the more difficult it is for the waves to transmit from the half-space to the layer.

The shearing traction and slip velocity are shown in Fig. 6 for $\Delta = 2, 1, 0.5$ when $f_k p_0/A_0 = 0.375$. In the region (−1,1), the slip velocity is smaller for a larger value of Δ. It is worth noting that the slip velocity in the main part of the additional slip zones remains a constant due to the reflection of waves at the free surface. It is an interesting case for $\Delta = 1$ shown in Fig. 5 that the shearing traction in an interval at the left of the main slip zone is equal to the slip frictional force, but the slip velocity is zero. We call this case ‘critical slip state’ and denote the interval by dashed lines in Fig. 3. It can be derived from Eqs. (46) and (49) that the critical slip zones will exist only if the parameter Δ is 1.

5.3. The effects of kinematic locking

Fig. 7 shows the shearing traction and slip velocity for $\Delta = 2$ and $\gamma = 2$ with consideration of the kinematic locking ($f_s > f_k$). As in Ref. [11] for an infinite medium, the kinematic locking results in the discontinuities in the shearing traction and slip velocity at the leading edges of the slip zones. It is easily understood that less slip zones may occur in some cases due to the kinematic locking.

Acknowledgments

The work is supported by China National Science Foundation under Grant Nos. 10025211 and 10372001.

References

[1] R.K. Miller, An approximate method of analysis of the transmission of elastic waves through a friction boundary, *Journal of Applied Mechanics* 44 (1977) 652–656.
[2] R.K. Miller, H.T. Tran, Reflection, refraction and absorption of elastic waves at a frictional interface: SH motion, *Journal of Applied Mechanics* 46 (1979) 625–630.
[3] R.K. Miller, H.T. Tran, Reflection, refraction and absorption of elastic waves at a frictional interface: P and SV motion, *Journal of Applied Mechanics* 48 (1981) 155–160.
[4] W.D. Iwan, A generalization of the concept of equivalent linearization, *International Journal of Non-Linear Mechanics* 8 (1973) 279–287.
[5] E.L. Chez, J. Dundurs, M. Comninou, Reflection and refraction of SH waves in presence of slip and friction, *Bulletin of the Seismology Society of America* 68 (1978) 999–1011.
[6] M. Comninou, J.R. Barber, J. Dundurs, Disturbance at a frictional interface caused by a plane elastic pulse, *Journal of Applied Mechanics* 49 (1982) 361–365.
[7] M. Comninou, J. Dundurs, Reflection and refraction of elastic waves in presence of separation, *Proceedings of the Royal Society of London, Series A* 356 (1977) 509–528.
[8] M. Comninou, J. Dundurs, Singular reflection and refraction of elastic waves due to separation, *Journal of Applied Mechanics* 45 (1978) 548–552.
[9] M. Comninou, J. Dundurs, Interaction of elastic waves with a unilateral interface, *Proceedings of the Royal Society of London, Series A* 368 (1979) 141–154.
[10] M. Comninou, J. Dundurs, Interface separation in the transonic range caused by a plane stress pulse, *Journal of Sound and Vibration* 62 (1979) 317–325.
[11] M. Comninou, J. Dundurs, Interface slip caused by an SH pulse, *International Journal of Solids and Structures* 16 (1980) 283–289.
[12] M. Comninou, J. Dundurs, E.L. Chez, Total reflection of SH waves in presence of slip and friction, *Journal of the Acoustic Society of America* 66 (1979) 789–793.
[13] J. Dundurs, M. Comninou, Interface separation caused by an plane elastic wave of arbitrary form, *Wave Motion* 1 (1979) 17–23.
[14] M. Comninou, J. Dundurs, Elastic interface waves involving separation, *Journal of Applied Mechanics* 44 (1977) 222–226.
[15] M. Comninou, J. Dundurs, *Elastic interface waves and sliding between two solids* 45 (1978) 325–330.
[16] Y.-S. Wang, G.-L. Yu, B.-Z. Gai, Re-polarization of elastic waves at a frictional contact interface. Part I. Incidence of an SH wave, *International Journal of Solids and Structures* 35 (1998) 2002–2021.
[17] Y.-S. Wang, G.-L. Yu, Re-polarization of elastic waves at a frictional contact interface. Part II. Incidence of an P or SV wave, *International Journal of Solids and Structures* 36 (1999) 4563–4586.
[18] G.-L. Yu, Y.-S. Wang, B.-Z. Gai, In-plane motion induced by an SH pulse at a frictional contact interface, *Mechanics Research Communications* 25 (1998) 203–210.
[19] R.K. Miller, An estimate of the properties of Love-type surface waves in a frictionally bonded layer, *Bulletin of the Seismology Society of America* 69 (1979) 305–317.
[20] G.-L. Yu, Y.-S. Wang, B.-Z. Gai, Slip with friction between an elastic layer and a substrate caused by SH waves, *Journal of Applied Mechanics* 64 (1997) 1019–1021.
[21] Y.-S. Wang, G.-L. Yu, B.-Z. Gai, Propagation of SH waves in a layered half-space with a frictional contact interface, *Bulletin of the Seismology Society of America* 88 (1998) 1300–1310.