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Short Communication

A further study of the prediction method for aerodynamic sound produced by two in-duct elements

N. Han[a,b], X.J. Qiu[a], C.M. Mak[b,*]

[a]State Key Laboratory of Modern Acoustic and Institute of Acoustics, Nanjing University, Nanjing 210093, PR China
[b]Department of Building Services Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, PR China

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Abstract

A prediction method based on the theory of Nelson and Morfey for aerodynamic sound produced by the interaction of two in-duct elements was developed by Mak and Yang. Their predicted results were compared with the experimental results of Oldham and Ukpoho. However, there are some discrepancies between their results and those of Oldham and Ukpoho, especially, when the measured values are greater than 6 dB. In addition, some parameters such as the phase of cross-power spectral density of source volumes used in their prediction method are not obvious to be determined. In order to solve these problems, this paper is therefore an attempt to modify the prediction method of Mak and Yang by re-considering the mechanism of the sound generated by two turbulences. A modified prediction method has finally been produced and verified against the measured results of Oldham and Ukpoho.

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1. Introduction

The classical theory of aerodynamic-noise generation was first introduced by Lighthill [1], he derived that the sources of noise in a turbulent flow without boundaries are equivalent to acoustic quadrupoles and that the noise generated is related to eighth power of the flow velocity. Curle [2] extended Lighthill's theory to demonstrate that an immovable solid object in a turbulent flow with solid boundaries radiates dipole sound. Later, Gordon [3,4] discussed the sound-generating capabilities of a flow spoiler in a finite pipe and showed the three mechanisms of equivalent quadrupole, dipole and monopole radiation in the fan-jet engine. In 1970, Heller and Widnall [5] investigated the effect of enclosure in an infinite duct and analyzed a monopole and dipole near the end of a duct to develop the prediction of the sound power radiation to the free field from pipe-immersed flow spoilers.

The work of Nelson and Morfey was motivated by the additional noise when sound absorbing splitters are placed in a flow duct [6], where they devised a theoretical model for sound generation by spoilers in low speed airflow ducts by considering the fluctuation forces acting on simple strip flow spoilers. In the derivation of their prediction model, they assumed that the fluctuating forces are proportional to the steady-state

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[*]Corresponding author. Tel.: +852 2766 5856; fax: +852 2765 7198.
E-mail address: becmmak@polyu.edu.hk (C.M. Mak).

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drag forces. Oldham and Ukpoho [7] later extended their work and re-wrote the Nelson and Morfey equations by determining the open area ratio and the characteristic dimension for more complex spoilers in both circular and rectangular ducts. Oldham and Waddington further carried out on more realistic ventilation system components such as bends [8]. In later work, they found that based on aerodynamic and acoustic similarity, Nelson and Morfey's work could also be employed to predict the airflow noise generated by any common types of duct components [9]. Recently, Mak and Yang [10] have derived a prediction method for aerodynamic sound produced by two closely spaced elements in low speed airflow ducts. Mak [11] later modified their equations to determine sound power radiated by the interaction of more complicated in-duct elements. They finally presented their prediction method by introducing an interaction factor that considers the acoustic interaction of two in-duct elements in an airflow duct [15]. Recently, Mak [16] has further extended the technique to consider the acoustic interaction of multiple in-duct elements.

This paper is devoted to the further development of the Mak–Yang prediction method by considering the aerodynamic interaction of two flow spoilers in an air duct. There are two problems in their method that motivate this study. Firstly, some parameters such as the phase of cross-power spectral density of source volumes used in Mak–Yang equations are not obvious to be determined. Secondly, there are discrepancies between the measured values of Oldham and Ukpoho [12] and the predicted values of Mak–Yang equations when the measured values are greater than 6 dB [11]. There may be some other factors existing in the mechanism of the sound production and they may influence the whole sound radiation but were not included in the Mak–Yang prediction method. The purpose of the present investigation is to analyze the interaction of two turbulent areas in airflow ducts acoustically and aerodynamically, which will permit more accurate estimation of the aerodynamic sound produced by two in-duct elements.

2. The basic theory

Lighthill's acoustic analogy provides a very convenient mechanism to investigate the sound generation in turbulent area, and he arranged the Navier–Stokes equation into the form of an inhomogeneous wave equation [1]

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where $T_{ij} \simeq \rho_0 v_i v_j$ at low Mach number, and ρ_0 the density, v_i the turbulent velocity fluctuation vector in the direction of the far-field observer. The space–time correlation function of the Lighthill stress tensor components is usually employed when investigating acoustic radiation from isotropic turbulence [13]

$$R(x, d, \tau) = \overline{T_{xx}(x, t) T_{xx}(x + d, t + \tau)}. \tag{2}$$

And it usually has the form

$$R(x, d, \tau) \propto \exp\left[-\frac{\tau^2}{\tau_s^2} - \frac{(d - \tau U)^2}{l_s^2} \right]. \tag{3}$$

Here τ_s is the characteristic time, l_s the characteristic correlation length, and U the mean flow velocity. The conception of the space–time correlation function will be used to discuss the interaction of two turbulent areas, and then to determine the sound power generated by them.

After some derivations, the far-field acoustic intensity and the characteristic velocity u of the turbulent area have the relationship as follows [13]:

$$I \propto \int u^4 \, \mathrm{d}V, \tag{4}$$

where $\int \mathrm{d}V$ shows the integration over the effective volume V of the turbulence. In the work of Nelson and Morfey [6], it is assumed that the root mean square fluctuating drag force $(F_x)_{\mathrm{rms}}$ acting on the spoilers is directly proportional to the steady-state drag force $\overline{F_x}$, which can be written in terms of a drag coefficient C_D. It can be expressed as [6]

$$(F_x)_{\rm rms} = K({\rm St})\overline{F_x}, \tag{5}$$

$$\overline{F_x} = C_D(\tfrac{1}{2}\rho U_c^2)^2\sigma^2(1-\sigma)A, \tag{6}$$

where σ is the open area ratio, A the area of duct cross-section, U_c the constriction velocity, and K(St) is the ratio of the root mean square force to the steady-state force. After some derivations, the modified Nelson–Merfey equations by Oldham and Ukpoho for determining the sound power level SWL$_S$ generated by a spoiler in circular ducts are [7]:

For $f < f_0$:

$$120 + 20\log_{10}K({\rm St}) = {\rm SWL}_S - 10\log_{10}(\rho_0 A\sigma^4 C_L^2 U_c^4/16c_0). \tag{7}$$

For $f > f_0$:

$$120 + 20\log_{10}K({\rm St}) = {\rm SWL}_S - 10\log_{10}\{\rho_0\pi A^2({\rm St})^2[\sigma^2(1-\sigma)]^2 C_L^2 U_c^6/24c_0^3 r^2\} - 10\log_{10}\left[1+\frac{3c_0}{8\hat{r}f}\right], \tag{8}$$

where f_0 is the cut-off frequency of the lowest transverse duct mode, c_0 the velocity of sound outside the flow, r the characteristic dimension of the spoiler in the duct, $\hat{r}$ the radius of the duct, and $C_L = C_D(1-\sigma)$. The above equations permit the prediction of noise generated by one spoiler in air flow ducts.

3. Interaction of two turbulent areas leading to the modification of Mak–Yang equations

Assuming there are two spoilers in the duct, the effective volume V of the turbulence generated by the two spoilers can be divided into two parts V_1 and V_2, which represent the dominating effective volumes of the primary and secondary turbulences, respectively. When the primary spoiler exists alone in the duct, the sound power generated is proportional to

$$W_1 \propto u_1^4 V_1 + u_1^4 R_1^4 V_2, \tag{9}$$

where the second term denotes the sound power generated by the primary source in the dominating effective volume V_2 of the second source. If the distance d between the center of V_1 and that of V_2 is large, or the retarded time τ is long, the effect of the primary source on V_2 is decreased. It is therefore reasonable to assume the characteristic velocity correlation function and adopt the form $R_1 = \exp[-(\tau^2/\tau_S^2) - ((d-\tau U)^2/l_S^2)]$, which shows the velocity correlation between the two turbulent areas. The determination of them is difficult [13], and their exact values require further investigation. However, it can be estimated that the characteristic correlation lengths l_S may be around 3.5 m and the turbulence lifetime τ_S is $3.5/U$ for the experiment setup [12].

Similarly, if only the second spoiler exists in the duct, $R_2 = \exp[-(\tau^2/\tau_S^2) - ((-d-\tau U)^2/l_S^2)]$ and the sound power generated can be expressed as

$$W_2 \propto u_2^4 V_2 + u_2^4 R_2^4 V_1. \tag{10}$$

When the two spoilers exist in the duct at the same time, the turbulence intensity at the downstream spoiler consists of the turbulence generated by that spoiler and the turbulence generated by the upstream spoiler [12]. It is assumed here that the characteristic velocity represents the characteristic of the turbulent area. The characteristic velocity of V_1 is now determined by the velocity effect of the first turbulent area when the primary spoiler exists in the duct alone plus the velocity effect of the second turbulent area when the second spoiler exists alone. This principle is applied to V_2 in the same way. Therefore, $u_1 + u_2R_2$ and $u_2 + u_1R_1$ are employed to express the characteristic velocities of V_1 and V_2 separately, which are different from the prediction model of Mak and Yang [10]. It will be shown that this modification can improve the accuracy of the prediction method for sound generation in ducts. Assuming $V_1 = V_2 = V'$, the whole sound power has the form

$$W \propto \left[(u_1+u_2R_2)^4+(u_2+u_1R_1)^4+2(u_1+u_2R)^2(u_2+u_1R)^2\sum_{m,n}^{N}\cos(k_{mn}d)\cos(\delta-kM\bar{d})\right]V'. \tag{11}$$

The third term in the bracket is the acoustic interaction after considering the mutual impedance of the two sound sources. In the effect of the mean airflow velocity U [14], the expression of the mutual impedance is $\cos(kd)\cos(\delta(\omega)-kM\bar{d})$ in the duct, where $\delta=\theta_1(\omega)-\theta_2(\omega)$ is the difference between the phases of two sources, and $M=U/c_0$, $\bar{d}=(d/1-M^2)$.

The steady-state drag forces $\overline{F_{x1}}$ and $\overline{F_{x2}}$ acting on the two spoilers are different in most cases. When $\overline{F_{x2}}=\zeta\overline{F_{x1}}$ [11], the relationship of the characteristic velocities of the two turbulences is $u_2=\sqrt{\zeta}u_1$, and then Eq. (11) has the form

$$W \propto u_1^4\left(1+\sqrt{\zeta}R_2\right)^4+u_1^4\left(\sqrt{\zeta}+R_1\right)^4+2u_1^4\left(1+\sqrt{\zeta}R_2\right)^2\left(\sqrt{\zeta}+R_1\right)^2\sum_{m,n}^{N}\cos(k_{mn}d)\cos\left(\delta-kM\bar{d}\right). \tag{12}$$

In Eqs. (9) and (10), if the distance of the two spoilers is relatively large , $R_1^4 \ll 1$ and $R_2^4 \ll 1$. In that case, the approximate expressions of Eqs. (9) and (10) can be given as

$$W_1 \propto u_1^4 V_1, \tag{13}$$

$$W_2 \propto u_2^4 V_2. \tag{14}$$

Comparing Eq. (12) with Eq. (13), the whole sound power generated by two spoilers in the duct can be expressed in terms of W_1, thus $W \approx W_1[B_1^2+B_2^2+2B_1B_2\sum_{m,n}^{N}\cos(k_{mn}d)\cos(\delta-kM\bar{d})]$ is obtained, where $B_1=\left(1+\sqrt{\zeta}R_2\right)^2$ and $B_2=\left(\sqrt{\zeta}+R_1\right)^2$. Following Nelson and Morfey's theory [6] and the prediction model of Mak and Yang [15], the sound power $W_{\Delta F}$ radiated down in ducts due to two spoilers can be deduced in a given bandwidth for frequencies below and above the cut-off frequency of the lowest transverse duct mode:

For $f<f_0$ (plane wave propagation):

$$W_{\Delta F} \approx (\rho_0/16c_0)AK^2(\mathrm{St})[\sigma^2(1-\sigma)]^2C_D^2U_c^4[B_1^2+B_2^2+2B_1B_2\cos(kd)\cos(\delta-kM\bar{d})]. \tag{15}$$

For $f>f_0$ (multimodal wave propagation):

$$W_{\Delta F} \approx \left(\frac{\rho_0\pi}{24c_0^3}\right)\left[1+\frac{3\pi c_0(a+b)}{4\omega A}\right]\left(\frac{A}{r}\right)^2(\mathrm{St})^2K^2(\mathrm{St})[\sigma^2(1-\sigma)]^2C_D^2U_c^6[B_1^2+B_2^2+2B_1B_2Q\cos(\delta-kM\bar{d})], \tag{16}$$

where

$$Q=\frac{3k^2ab/(4\pi)(\sin e/e+2\cos e/e^2-2\sin e/e^3)+3k(a+b)/8(J_0(e)-J_1(e)/e)}{[k^2ab/4\pi+3k(a+b)/16]}.$$

The infinite-duct values of the radiated sound power level SWL_D in 1/3 octave band can be normalized. For a circular duct [7], the equations for determining the sound power level SWL_D generated by two in-duct elements are:

For $f<f_0$:

$$120+20\log_{10}K(\mathrm{St})=\mathrm{SWL}_D-10\log_{10}\left(\frac{\rho_0A\sigma^4C_L^2U_c^4}{16c_0}\right)-10\log_{10}[B_1^2+B_2^2+2B_1B_2\cos(kd)\cos(\delta-kM\bar{d})]. \tag{17}$$

For $f>f_0$:

$$\begin{aligned}120+20\log_{10}K(\mathrm{St})=\ &\mathrm{SWL}_D-10\log_{10}[\rho_0\pi A^2(\mathrm{St})^2\sigma^4C_L^2U_c^6/24c_0^3r^2]\\&-10\log_{10}\left[1+\frac{3c_0}{8\widehat{rf}}\right]-10\log_{10}[B_1^2+B_2^2+2B_1B_2Q\cos(\delta-kM\bar{d})],\end{aligned} \tag{18}$$

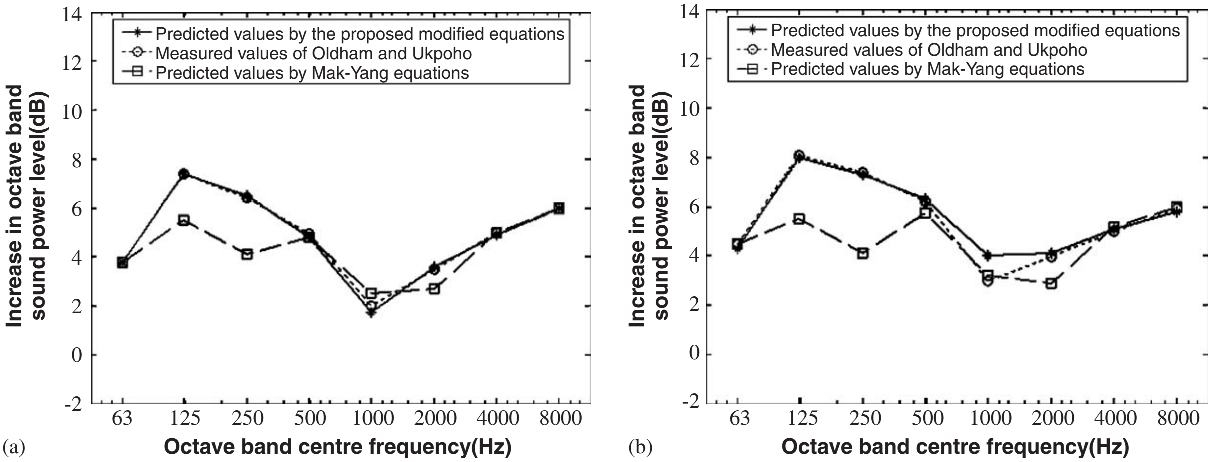

Fig. 1. Comparison of the values predicted by the proposed modified equations (assuming $\xi = 1$), the measured values of Oldham and Ukpoho and the values predicted by the Mak–Yang equations. A total of 3 m separation between two in-duct elements: (a) $U = 18.2\,\mathrm{m/s}$; (b) $U = 20\,\mathrm{m/s}$.

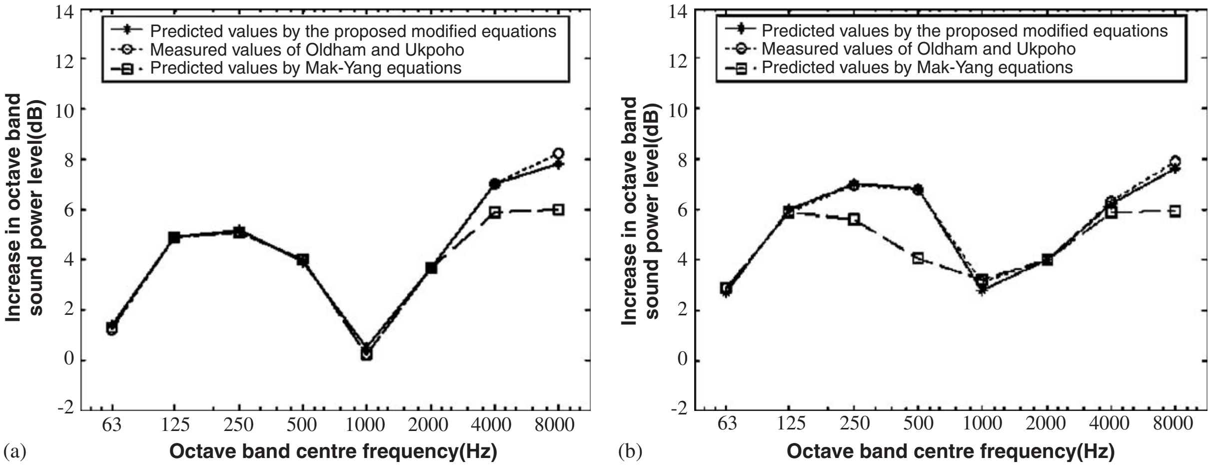

Fig. 2. Comparison of the values predicted by the proposed modified equations (assuming $\xi = 1$), the measured values of Oldham and Ukpoho and the values predicted by the Mak–Yang equations. A total of 1.5 m separation between two in-duct elements: (a) $U = 18.3\,\mathrm{m/s}$; (b) $U = 19.9\,\mathrm{m/s}$.

where

$$Q = \frac{3k^2\hat{r}^2/4(\sin e/e + 2\cos e/e^2 - 2\sin e/e^3) + 3k\pi\hat{r}/8(J_0(e) - J_1(e)/e)}{[k^2\hat{r}^2/4 + 3k\pi\hat{r}/16]}$$

and other symbols were explained in Refs. [11,15]. The interaction factor β defined in Refs. [11,15] can also be rewritten as

$$\beta = \begin{cases} B_1^2 + B_2^2 + 2B_1B_2\cos(kd)\cos(\delta - kM\overline{d}), & f < f_0, \\ B_1^2 + B_2^2 + 2B_1B_2Q\cos(\delta - kM\overline{d}), & f > f_0. \end{cases} \tag{19a,b}$$

The relationship between the sound power level SWL_D due to two elements and the sound power level SWL_S due to a simple element is $\mathrm{SWL}_D = \mathrm{SWL}_S + 10\log_{10}\beta$, where SWL_S can be obtained by Eqs. (7) and (8), and the second term $10\log_{10}\beta$ on the right represents the increase in generated sound power level compared to the sound power level due to a single element. It can be found that there are several differences between the Mak–Yang equations [15] and the proposed modified equations. Firstly, some parameters such as the phase of cross-power spectral density of source volumes used in Mak–Yang equations are removed and substituted by the difference δ between the phases of two sources which has more clear physical meaning. Secondly, the aerodynamic interaction of the two turbulences is considered in the modified equations and the characteristic velocity in each turbulent area changes. For brevity, only four of the figures presented in Mak's paper [11] are given below for comparing the values predicted by the proposed modified equations, the measured values by Oldham and Ukpoho and that predicted by the Mak–Yang equations.

When two independent sound sources are put together, the increase in sound power level is 3 dB if the two sources have the same sound power. The deviation from 3 dB in Figs. 1 and 2 show that the interaction of the two turbulences plays an important part in determining the whole sound power generated by the two spoilers. It can be seen in the figures that at most frequencies, the values are greater than 3 dB. That means the respective sound power of every turbulence increases because of the positive interaction of the two turbulences. However, measured points at around 1 kHz, i.e. the cut-off frequency of the lowest transverse duct mode, the values are less than 3 dB. It indicates that the acoustic interaction of two noise sources plays a negative part in the sound power radiation. Furthermore, it can be seen from Eqs. (19a,b) that it is possible to have values more than 6 dB increase if the interaction value β is larger than 4. Figs. 1 and 2 show that at the points where the measured value is greater than 6 dB, there are the large discrepancies of the values predicted by Mak–Yang equations and the measured results of Oldham and Ukpoho (this was stated in Mak's paper [11]). However, the predicted values of the proposed modified equations agree much better with the measured values for 3 and 1.5 m separation of two duct elements. This shows that the proposed form of the characteristic velocity correlation function is reasonable and the modified equations based on the addition of the characteristic velocities of different turbulent areas are valid. However, there are a number of different types of in-duct elements in air ducts, and the accuracy of these modified predictive equations requires further investigation.

4. Conclusions

In this paper, the mechanism of the sound generated by two turbulences is investigated. The interaction of two turbulent areas is analyzed by considering the characteristic velocity correlation function. The Mak–Yang equations have only included the acoustic interaction of two flow-noise sources. By taking into account of aerodynamic interaction of two turbulences, the modified predicted equations have been developed which permit more accurate determination of sound power radiated by two closely spaced spoilers. It has been shown that the predicted values of the proposed modified equations are more close to the measured values of Oldham and Ukpoho, especially at the points where the measured increase in sound pressure level is greater than 6 dB. Further experimental/computational works are required to determine the measurable quantities such as phase difference of two noise sources. In order to arrive at a generalized prediction method, the ultimate objective is to extend this modified Mak–Yang equations to predict the aerodynamic sound power radiated by the interaction of a combination of more than two in-duct elements.

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