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Discussion

On the bounds of Gorman's superposition method of free vibration analysis

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The recent articles [1–3] by Professor Gorman extending the building blocks based superposition method to the analysis of in-plane vibration of plates are useful and welcome additions to the literature. The method has been successfully applied to analyse out-of-plane vibrations of plates [4–6] and vibration of open cylindrical shells [7–9], including a wide range of complicated systems such as triangular and parallelogram plates, orthotropic plates, Mindlin plates, plates with elastic supports, point supported plates, laminated plates and plates under in-plane forces [4]. The superposition method is elegant in terms of mathematical basis and physical interpretation. For the systems investigated in the literature, its rate of convergence is excellent. It may be regarded as one of the most significant contributions to the field of vibration of continuous systems in the recent decades. The purpose of this communication is to consider the question of whether this method gives bounded results: a question this discusser finds interesting for the following reason.

The Rayleigh–Ritz method is widely used in calculating the natural frequencies of continuous systems. A drawback of this method is that while it gives an upper bound result for the frequencies, the error due to discretisation cannot be calculated easily. It would be good if the superposition method could be used to obtain lower bound results in which case it would nicely complement the Rayleigh–Ritz method. This appears to be the case when using building blocks that are less constrained than the system being modelled as can be explained from the following considerations.

Consider a system A, which is modelled by applying the superposition method with n building blocks each of which contributing K terms. For convenience let us label a contributing subsystem drawn from the ith building block in its mth mode as $B_{i,m}$, where $i = 1,2, \dots, n$, and $m = 1,2, \dots, K$. These subsystems are considered as systems subject to frequency-dependent boundary actions (prescribed driving forces or driving moments or prescribed driving translations or rotations) which are applied in such a way as to satisfy the boundary conditions of A approximately using a weighted average approach:

$$A \approx \sum_{i=1}^{n} \sum_{m=1}^{K} a_{i,m} B_{i,m}.$$

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For the systems investigated by Gorman [1–6] it seems possible to select $B_{i,m}$ so that as

$$K \to \infty, \sum_{i=1}^{n} \sum_{m=1}^{K} a_{i,m} B_{i,m} \to A.$$

Let the system defined by the truncated solution in the superposition method be A'_K where K is the number of terms in the series. Thus

$$A'_K = \sum_{i=1}^{n} \sum_{m=1}^{K} a_{i,m} B_{i,m}.$$

System A'_K may be regarded as an intermediate system that is governed by the same equation of motion as A and $B_{i,m}$, but which is subject to boundary conditions that are different from those of A. The eigenvalues derived from the superposition method would be exact for system A'_K if the functions used in the base problems exactly satisfy the equation of motion in the system domain. This means the superposition method gives exact solution to a problem subject to boundary conditions that are slightly different from those of A. If the differences are only in the boundary conditions, the question then arises, whether these differences result in an increase or decrease in the natural frequencies.

If none of the boundary conditions of the base problems $B_{i,m}$ is stiffer than that of the corresponding boundary in A, it is suggested that A'_K would be a Weinstein's intermediate problem [10] for A, resulting in lower bound values for the natural frequencies. Assuming that the application of forces or moments to boundaries of the building blocks $B_{i,m}$ which are less constrained than the actual system A being modelled corresponds to solving a problem where the constraints are replaced with elastic restraints of positive stiffness, this is also to be expected from the existence and convergence theorems [11] or from Rayleigh's theorem of separation [12]. On the other extreme, if all boundary conditions of the base problems $B_{i,m}$ are stiffer than those of the corresponding boundaries in A then the natural frequencies of A'_K would be upper bound to the natural frequencies of A.

The above arguments suggest that in cases where the superposition method is based on building blocks whose boundary conditions are more flexible than those of the system being modelled, the results give a lower bound estimate of the natural frequencies. This would be the case when modelling clamped plates using building blocks consisting of simply supported boundaries subject to driving forces or translations, or slip-shear boundaries subject to driving moments or rotations. In cases where the building blocks are subject to stiffer conditions at the boundaries, upper bound results may be expected. In cases where the boundary conditions are of mixed type, it is not possible to conclude from the above reasoning whether the results would be upper bound or lower bound. In making the above suggestions, it is assumed that the systems subject to driving forces or moments may be regarded as being subject to corresponding elastic restraints and may be treated as Weinstein's intermediate problems.

It would be good if these can be verified numerically but results published so far do not provide sufficient information for this. Perhaps due to the rapid convergence of the solution, the variation of frequencies with number of terms in a building block has been reported only in some of the publications. The variation of natural frequencies of a completely free square plate with number of terms in a building block (K) is shown graphically in Fig. 1.5 of Ref. [4]. The building blocks used have boundary conditions that are stiffer than the system being modelled, and the third natural frequency decreases or remains unchanged with increasing number of terms, suggesting that the results are upper bound as discussed. If results are readily available, it would be useful if Professor Gorman or other users of the method could verify the predictions made here.

References

[1] D.J. Gorman, Free in-plane vibration analysis of rectangular plates with elastic support normal to the boundaries, *Journal of Sound and Vibration* 285 (2005) 941–966.

[2] D.J. Gorman, Free in-plane vibration analysis of rectangular plates by the method of superposition, *Journal of Sound and Vibration* 272 (2004) 831–851.

[3] D.J. Gorman, Accurate analytical type solutions for the free in-plane vibration of clamped and simply supported rectangular plates, *Journal of Sound and Vibration* 276 (2004) 311–333.
[4] D.J. Gorman, *Vibration Analysis of Plates by the Superposition Method*, World Scientific, Singapore, 1999.
[5] D.J. Gorman, Free vibration analysis of the completely free rectangular plate by the method of superposition, *Journal of Sound and Vibration* 57 (1978) 432–447.
[6] D.J. Gorman, W. Ding, The superposition-Galerkin method for free vibration analysis of rectangular plates, *Journal of Sound and Vibration* 194 (1996) 187–198.
[7] S.D. Yu, W.L. Cleghorn, R.G. Fenton, On free vibration analysis of open circular cylindrical shells, *Journal of Sound and Vibration* 188 (3) (1995) 315–336.
[8] S.D. Yu, W.L. Cleghorn, R.G. Fenton, Free vibration of open cylindrical shells with various boundary conditions, Proceedings of Fifth International Conference on Recent Advances in Structural Dynamics, University of Southampton, 1994, pp. 194–204.
[9] W.L. Cleghorn, S.D. Yu, R.G. Fenton, Accurate free vibration analysis of open circular cylindrical shells, Proceedings of the Fifth International Conference on Recent Development in Structural Dynamics, Work in Progress Papers, University of Southampton, Southampton, UK, 1994, pp. 1–5.
[10] S.H. Gould, *Variational Methods for Eigenvalue Problems*, Oxford University Press, London, 1966.
[11] S. Ilanko, Existence of natural frequencies of systems with imaginary restraints and their convergence in asymptotic modelling, *Journal of Sound and Vibration* 255 (2002) 883–898.
[12] Lord J.W.S. Rayleigh, *Theory of Sound*, first American ed, Dover, New York, 1945.