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Discussion

# Comments on "Vehicle–passenger–structure interaction of uniform bridges traversed by moving vehicles"

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Esmailzadeh and Jalili [1] have investigated the dynamics of vehicle–occupant–structure-induced vibration of bridges traversed by moving vehicles. The investigation is interesting, however, some statements are questionable. In this discussion, the discusser would like to make some remarks on Ref. [1].

## 1. Comments on Eq. (1), p. 613 of Ref. [1]

As shown in Fig. 1, p. 614 of Ref. [1], the vertical displacements of the unsprung mass $M_1$ and that of the sprung mass $M_2$, with reference to their respective vertical equilibrium positions, are $y_1(t)$ and $y_2(t)$, respectively. Therefore, the second expression of the vertical interaction force $F(t)$ acting on the moving vehicle in Eq. (1), p. 613 of Ref. [1] should read:

$$F(t) = M_1 \frac{\mathrm{d}^2 y_1(t)}{\mathrm{d}t^2} + C_2\left[\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_2(t)}{\mathrm{d}t}\right] + K_2[y_1(t) - y_2(t)]. \tag{1}$$

The above equation can be obtained as follows.

Firstly, let us consider the unsprung mass $M_1$ as a free body. All forces acting in the direction of the displacement degree of freedom of $M_1$ are shown in Fig. 1 in this discussion. It should be noted that the vertical displacement $y_1(t)$ of the unsprung mass $M_1$ is with reference to its vertical equilibrium position; therefore, the gravitational force of the unsprung mass $M_1$ cannot appear [2] in Fig. 1 in this discussion. According to the d'Alembert's principle, the equilibrium of these forces is given by

$$F_I(t) + F_{D1}(t) + F_{S1}(t) + F_{D2}(t) + F_{S2}(t) = 0, \tag{2}$$

in which

$$F_I(t) = M_1 \frac{\mathrm{d}^2 y_1(t)}{\mathrm{d}t^2}, \tag{3}$$

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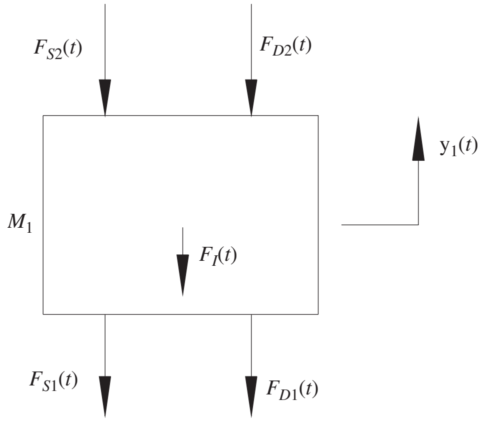


Fig. 1. Acting forces in the unsprung mass $M_1$.

$$F_{D1}(t) = C_1\left[\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - \frac{\mathrm{d}y(t)}{\mathrm{d}t}\right], \tag{4}$$

$$F_{S1}(t) = K_1[y_1(t) - y(t)], \tag{5}$$

$$F_{D2}(t) = C_2\left[\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_2(t)}{\mathrm{d}t}\right], \tag{6}$$

$$F_{S2}(t) = K_2[y_1(t) - y_2(t)]. \tag{7}$$

Secondly, substituting Eqs. (3)–(7) into Eq. (2), one obtains

$$M_1\frac{\mathrm{d}^2y_1(t)}{\mathrm{d}t^2} + C_1\left[\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - \frac{\mathrm{d}y(t)}{\mathrm{d}t}\right] + K_1[y_1(t) - y(t)] + C_2\left[\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_2(t)}{\mathrm{d}t}\right] + K_2[y_1(t) - y_2(t)] = 0. \tag{8}$$

Substituting the first expression in Eq. (1), p. 613 of Ref. [1] into Eq. (8), one finally obtains

$$F(t) = M_1\frac{\mathrm{d}^2y_1(t)}{\mathrm{d}t^2} + C_2\left[\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_2(t)}{\mathrm{d}t}\right] + K_2[y_1(t) - y_2(t)]. \tag{9}$$

Comparing Eq. (9), which is also numbered as (1), in this discussion with the second expression in Eq. (1), p. 613 of Ref. [1], one finds that the latter adds the term of the gravitational force of the unsprung mass $M_1$, which is not correct.

## 2. Comments on the second paragraph, p. 615 of Ref. [1]

It is claimed, in the second paragraph, p. 615 of Ref. [1], that when the interaction force, $F(t)$, between the moving vehicle and the bridge becomes zero, it denotes the onset of separation; and it should remain zero until the moving vehicle re-establishes contact with the bridge surface. It is claimed, in the last paragraph, p. 615 of Ref. [1], that the bridge initially is considered free of any load or deflection, and hence is horizontal at the equilibrium position under its own weight (unloaded).

Since the vertical interaction force acting on the bridge is not equal to the vertical interaction force acting on the moving vehicle, the discusser notes that the interaction force in the second paragraph, p. 615 of Ref. [1] should indicate the vertical interaction force acting on the bridge, not the vertical interaction force acting on the moving vehicle. The vertical interaction force $F_{vb}(t)$ acting on the bridge consists of the static interaction force $F_w$ and the variation of interaction force $\Delta F(t)$; their relation can be written as

$$F_{vb}(t) = F_w + \Delta F(t), \tag{10}$$

where

$$F_w = (M_1 + M_2)g, \tag{11}$$

$$\Delta F(t) = C_1\left[\frac{dy(t)}{dt} - \frac{dy_1(t)}{dt}\right] + K_1[y(t) - y_1(t)]. \tag{12}$$

The force $\Delta F(t)$ is equal to the vertical interaction force $F(t)$ acting on the moving vehicle.

## 3. Comments on Eq. (11), p. 618 and Eq. (20), p. 619 of Ref. [1]

It is claimed, in the last paragraph of p. 617 of Ref. [1], the dissipating damping forces in the bridge structure are considered as non-conservative forces in Lagrange's formulation. Consequently, the Rayleigh's dissipation function of the dissipating damping forces in the bridge structure should consider that of entire bridge structure. As a result, the first term of right-hand side in Eq. (11), p. 618 of Ref. [1], i.e., $c\dot{y}^2(x,t)$, should be replaced by $\int_0^L c[\dot{y}^2(x,t)]\,dx$.

On the other hand, the term including generalized damping for the $i$th mode of the bridge should appear in the Eq. (20), p. 619 of Ref. [1]. Therefore, the term of $C_i\dot{q}_i(t)$ should be added in the left-hand side in Eq. (20), p. 619, that is, Eq. (20), p. 619 should read as

$$N_i\ddot{q}_i(t) + C_i\dot{q}_i(t) + S_i q_i(t) + \cdots + = 0, \quad i = 1, 2, \ldots, n, \tag{13}$$

where, $C_i$ denotes generalized damping for the $i$th mode of the bridge. $C_i$ is defined by setting $i = j$ in the following equation:

$$\int_0^L c\phi_i(x)\phi_j(x)\,dx = C_i\delta_{ij}, \quad i,j = 1, 2, \ldots, n. \tag{14}$$

## References

[1] E. Esmailzadeh, N. Jalili, Vehicle–passenger–structure interaction of uniform bridges traversed by moving vehicles, *Journal of Sound and Vibration* 260 (4) (2003) 611–635.
[2] R.W. Clough, J. Penzien, *Dynamics of Structures*, second ed., McGraw-Hill, New York, 1993 p.18.