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Effects of thickness and delamination on the damping in honeycomb–foam sandwich beams

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Abstract

In engineering applications where the use of lightweight structures is important, the introduction of a viscoelastic core layer, which has high inherent damping, between two face sheets, can produce a sandwich structure with high damping. Sandwich structures have the additional advantage that their strength to weight ratios are generally superior to those of solid metals. So, sandwich structures are being used increasingly in transportation vehicles. Knowledge of the passive damping of sandwich structures and attempts to improve their damping at the design stage thus are important. Some theoretical models for passive damping in composite sandwich structures are reviewed in this paper. The effects of the thickness of the core and face sheets, and delamination on damping are analyzed. Measurements on honeycomb–foam sandwich beams with different configurations and thicknesses have been performed and the results compared with the theoretical predictions.
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1. Introduction

A sandwich structure consists of three elements, the face sheets, the core and the adhesive interface layers. The greatest advantage of sandwich structures is that optimal designs can be obtained for different applications by choosing different materials and geometric configurations of the face sheets and cores. By inserting a lightweight core between the two face sheets, the bending stiffness and strength are substantially increased compared with a single-layer homogenous structure, without adding much weight. When the beam or plate undergoes flexural vibration, the damped core is constrained primarily to shear. This shearing causes energy to be dissipated and the flexural motion to be damped.

Since the late 1950s many papers have been published on the vibration of sandwich structures. The Ross–Ungar–Kerwin model is one of the first theories which was developed for the damping in sandwich structures [1–4]. In Kerwin's initial study, an analysis was presented for the bending wave propagation and damping in a simply supported three-layer beam [1]. One of the limitations of this analysis is that the bending stiffness of the top layer must be much smaller than that of the bottom layer. Ungar generalized the analysis in the earlier study and derived an expression for the total loss factor of sandwich beams in terms of the shear

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and structural parameters [3]. In extending the work of Kerwin, DiTaranto derived a sixth-order linear homogeneous differential equation for freely vibrating beams having arbitrary boundary conditions [5–7]. In this model, modes are completely uncoupled, which greatly simplifies the general forced vibration problem. Mead and Markus modified the theory and studied different boundary conditions in terms of the transverse displacement [8,9]. In another study, Yan and Dowell derived a set of five fourth-order partial differential equations using the principle of virtual work in the theory of elasticity [10].

All the models discussed so far only consider the contribution of the damping in the viscoelastic core to the total damping in the entire structure by using the complex form of the shear modulus of the core. An advantage of the use of complex shear modulus is that the differential equations only contain the even-order terms. So they are easy to solve. Mead conducted a comprehensive study on the comparison of these models and studied the effects of longitudinal inertia and shear deformation of the face sheets [11].

Models derived by Mindlin's theory and Timoshenko's theory both lead to a fourth-order differential equation. In Refs. [12,13] Nilsson states that due to the frequency dependence of sandwich structure properties, solutions of the fourth-order differential equation agree well with measurements at low frequency. However, as the frequency increases, the calculated results disagree strongly with measurements. Nilsson used Hamilton's principle to derive a sixth-order differential equation governing the bending of sandwich beams and studied boundary conditions for free, simply supported and clamped beams. The behavior of a sandwich structure in the low-frequency region is determined by pure bending of the entire structure. In the middle-frequency region, the rotation and shear deformation of the core become important. At high frequencies, the bending of the face sheets is dominant.

In the recent research, cores made of either honeycomb or solid viscoelastic material have been studied [12–16]. The core in this particular study was made of paper honeycomb filled with polyurethane (PUR) foam. The honeycomb material is expected to enhance the stiffness of the entire structure, while the foam improves the damping. Jung and Aref reported that sandwich structures with combined honeycomb–foam cores have higher damping than those with individual honeycomb or solid viscoelastic cores [17]. However, Jung and Aref used a static hysteretic damping model, so damping ratios are independent of frequency. This conclusion is obviously not valid. In this paper, the frequency dependence of damping in sandwich beams with foam-filled honeycomb cores is analyzed, and the effects of thickness of the face sheets and core, and delamination on damping are studied. Most of the earlier models ignore the bending and extensional effects in the core. However, this assumption is only valid for soft thin cores. In this paper, both the bending and shear effects are considered. And the shear stresses are continuous across the face sheet–core interfaces.

2. Effects of thickness and delamination

In the Ross–Ungar–Kerwin model [2,3,18,19], the loss factor is given by the following formula:

$$\eta = \frac{\beta YX}{1 + (2 + Y)X + (1 + Y)(1 + \beta^2)X^2}, \tag{1}$$

where

$$X = \frac{Gb}{p^2 t_c} S, \quad \frac{1}{Y} = \frac{E_t I_t + E_b I_b}{d^2} S, \quad S = \frac{1}{E_t A_t} + \frac{1}{E_b A_b}, \tag{2}$$

β is the loss factor of the viscoelastic material, and d is the distance between the neutral axes of the two face sheets, as shown in Fig. 1. E, I and A represent the Young's modulus, moment of inertia, and cross-sectional area. X and Y are the shear and structural parameters, respectively. Subscripts t and b denote the top and bottom face sheets, and c denotes the core.

Substituting S in the expression for Y, we have

$$Y = \frac{3r(1 + r)}{1 + r^3}, \quad r = \frac{t_t}{t_b}. \tag{3}$$

Differentiating the loss factor with respect to the structural parameter Y gives

$$\frac{\partial \eta}{\partial Y} = \beta X \frac{1 + 2X + (1+\beta^2)X^2}{[1 + (2+Y)X + (1+Y)(1+\beta^2)X^2]^2} > 0, \tag{4}$$

which is always positive. That means the loss factor is a monotonically increasing function of the structural parameter Y.

Setting

$$\frac{\mathrm{d}Y}{\mathrm{d}r} = 3\left[\frac{1+2r}{1+r^3} - \frac{3r^2(r+r^2)}{(1+r^3)^2}\right] = 0, \tag{5}$$

we obtain $r = \pm 1$. So when $r = 1$, the loss factor has a maximum value. Then we can define $t_t = t_b = t_f$, where the subscript f stands for the face sheets. In this paper, only symmetric sandwich structures have been studied, as shown in Fig. 1.

Similarly, by taking the derivative of the loss factor η with respect to the shear parameter X, an optimal value of the shear modulus G can be calculated to obtain maximum damping. That means, in an intermediate range of core shear modulus value, the beam or plate damping has its highest value.

Fig. 2 illustrates some of the samples studied. The material for the face sheet is a carbon fiber-reinforced composite. The Young's modulus of such a material aggregated with epoxy is 60 GPa, similar to that of aluminum. So it is very stiff. Paper honeycombs are manufactured by processing paper with resin to make it water resistant. This produces a low-cost core, but one which has very good mechanical properties. PUR foams have low thermal conductivity and diffusion coefficients, giving them very good thermal insulation properties. Another advantage of PUR foams is that they can be produced in finite size blocks as well as in situ, thus providing an integrated manufacturing process in conjunction with the manufacture of the sandwich elements [20].

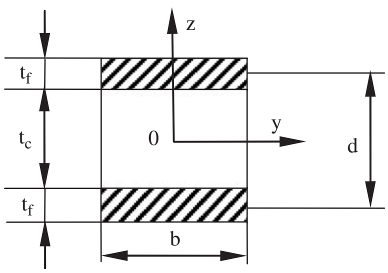


Fig. 1. Cross-section of a symmetric sandwich beam.

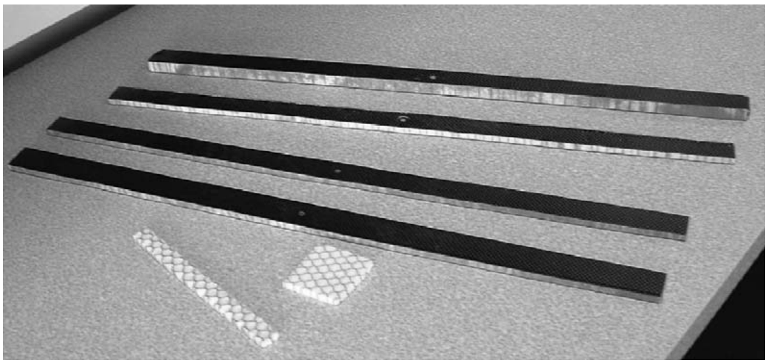


Fig. 2. Foam-filled honeycomb cores and sandwich beams.

From a static point of view, the bending stiffness of a sandwich beam can be expressed as

$$
\begin{aligned}
D &= \int E z^2 \,\mathrm{d}A = \int_{-t_f-(t_c/2)}^{-t_c/2} E_f z^2 b \,\mathrm{d}z + \int_{-t_c/2}^{t_c/2} E_c z^2 b \,\mathrm{d}z + \int_{t_c/2}^{t_f+(t_c/2)} E_f z^2 b \,\mathrm{d}z \\
&= 2\int_{t_c/2}^{t_f+(t_c/2)} E_f z^2 b \,\mathrm{d}z + 2\int_0^{t_c/2} E_c z^2 b \,\mathrm{d}z \\
&= b\left[\frac{E_f t_f^3}{6} + \frac{E_f t_f d^2}{2} + \frac{E_c t_c^3}{12}\right] = 2D_f + D_0 + D_c, \qquad (6)
\end{aligned}
$$

where b is the width of the beam, t_f and t_c are the thicknesses of the face sheet and core, E_f and E_c are the Young's moduli of the face sheet and core, and $d = t_f + t_c$. D_f is the bending stiffness of a face sheet about its own neutral axis, D_0 is the stiffness of the face sheets associated with bending about the neutral axis of the entire sandwich, and D_c is the stiffness of the core [20].

We will compare two cases in order to study the effect of the thicknesses of the face sheets and core on the damping.

1. Since the core is stiff in shear but soft generally, its Young's modulus is much smaller than that of the face sheet. By assuming $E_c \ll E_f$ and $D \approx 2D_f + D_0$, the normal stresses in the face sheets and the shear stresses in the core are

$$
\sigma_{f1} = \frac{MzE_f}{2D_{f1} + D_{01}}, \quad \tau_{c1} = \frac{TE_f t_{f1} d_1}{2(2D_{f1} + D_{01})}, \qquad (7)
$$

where M and T are the bending moment and the shear force, respectively.

2. If we assume not only that $E_c \ll E_f$ but also that the face sheets are thin, $t_f \ll t_c$, then, $D \approx D_0$. The normal stresses in the face sheet and the shear stresses in the core become

$$
\sigma_{f2} = \frac{M}{t_{f2} d_2}, \quad \tau_{c2} = \frac{T}{d_2} \qquad (8)
$$

and the normal stresses in the core and the shear stresses in the face sheets are zero. So the face sheets carry bending moments as tensile and compressive stresses and the core carries transverse forces as shear stresses.

Comparing the two cases, and assuming they have the same core thickness t_c, bending moment M, and shear force T, we obtain

$$
\frac{\tau_{c1}}{\tau_{c2}} = \frac{E_f t_{f1} d_1}{2(E_f t_{f1} d_1^2/2) + (E_f t_{f1}^3/6)} \; d_2 = \frac{d_1 d_2}{d_1^2 + (t_{f1}^2/3)} < 1 \qquad (9)
$$

and

$$
\frac{\sigma_{f1}}{\sigma_{f2}} = \frac{MzE_f}{(E_f t_{f1} d_1^2/2) + (E_f t_{f1}^3/6)} \frac{t_{f2} d_2}{M} = \frac{z t_{f2} d_2}{(t_{f1} d_1^2/2) + (t_{f1}^3/6)}.
$$

Since

$$
\max\{\sigma_{f1}\} = \sigma_{f1}\left(z = \pm\frac{d_1 + t_{f1}}{2}\right),
$$

then

$$
\frac{\max\{\sigma_{f1}\}}{\sigma_{f2}} = \frac{(2t_{f1} + t_c) t_{f2} d_2}{t_{f1} d_1^2 + (t_{f1}^3/3)} = \frac{2t_{f1} t_{f2}^2 + 2t_{f1} t_{f2} t_c + t_{f2}^2 t_c + t_{f2} t_c^2}{\frac{4}{3} t_{f1}^3 + 2t_{f1}^2 t_c + t_{f1} t_c^2}.
$$

Let $a = (t_{f1}/t_{f2})$, and $b = (t_c/t_{f2})$, then

$$\frac{\max\{\sigma_{f1}\}}{\sigma_{f2}} = \frac{2a + 2ab + b + b^2}{\frac{4}{3}a^3 + 2a^2 b + ab^2} < 1 \quad \text{if} \;\; 1.247 \;\; < a < b. \tag{10}$$

It is easy to prove that Eq. (10) is a monotonically decreasing function of a and b.

Eqs. (9) and (10) show that the thinner the face sheets are, the larger is the shear in the core and the normal stress in the face sheets. This means that, if we increase the thickness of the face sheets by a factor of more than 1.247, then the shear in the core becomes more constrained. The direct stress in the face sheets also becomes smaller because the cross-sectional area is larger.

Consider the dynamic case. Vibration energy can propagate through a sandwich structure mainly in the form of bending waves and shear waves. Since bending waves create substantial transverse displacements, bending waves couple best with the surrounding fluid and are mostly responsible for the sound radiation. However, as shown before, the shear deformation in the core is significant in sandwich structures in comparison with homogeneous materials. So shear waves must also be considered. Using either Hamilton's principle or the impedance method, a sixth-order equation can be derived to solve the speed for wave propagation in sandwich beams, C_p [12,21]:

$$\begin{aligned} &m(J\omega^2 - k'GA)\omega^2 C_p^6 - m(D + 2D_f + k'GAJ)\omega^6 C_p^4 \\ &\quad + (k'GAD - 2D_f J\omega^2)\omega^4 C_p^2 + 2D_f D\omega^6 = 0. \end{aligned} \tag{11}$$

Here m is the mass per unit length (kg/m), J is the mass moment of inertia per unit length (kg m), k' is the shear coefficient, which is $\frac{5}{6}$ for a beam with a rectangular cross-section, G is the shear modulus of the core, and, A is the cross-sectional area.

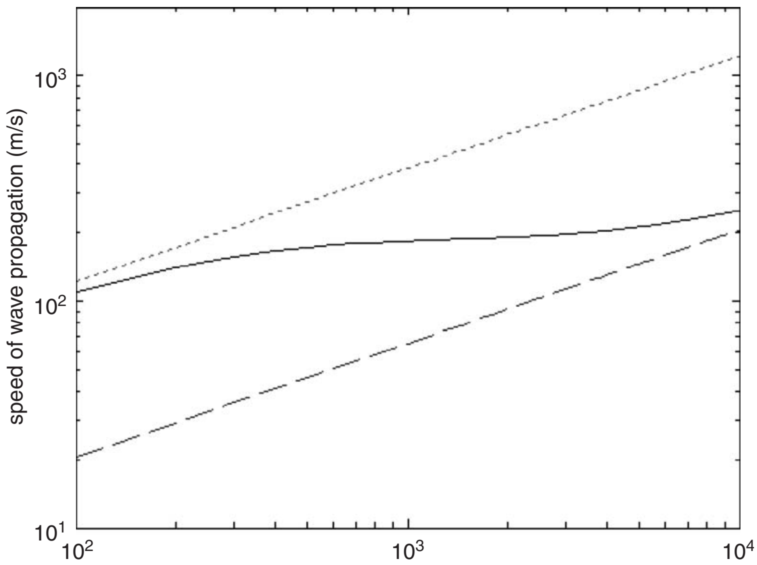
Fig. 3 compares the variation of the bending wave speed in two sandwich beams with different core materials. Case (a) corresponds to a single foam core, and case (b) a foam-filled honeycomb core. The Young's moduli of the foam core and the foam-filled honeycomb core are 10.16 and 36.4 MPa. In each plot, the solid curve represents the speed of wave propagation including the effects of shear deformation. The upper and lower straight lines depict the pure bending wave speeds of the entire sandwich structure and of the two face sheets only, respectively. Both the plots demonstrate that at low frequencies, the speed of wave in the sandwich beam is close to the speed of pure bending wave in the entire structure, while at high frequencies, it approaches the speed of the pure bending wave only propagating in the face sheets. Comparing the two plots, it can be seen that, for the sandwich beam with a single foam core, the shear deformation is only effective in the middle-frequency range. For a sandwich beam with a foam-filled honeycomb core, however, the shear deformation is still effective in the high-frequency range, because the honeycomb increases the stiffness of the core.

Therefore, in the low-frequency region the energy is dissipated by pure bending ($D \approx D_0$). With increasing frequency, more energy is dissipated due to the increased normal-to-shear coupling, in which the motion of the face sheets is mostly transformed into the shear deformation and in-plane waves in the core. Because of the viscoelastic property of the foam, the damping in the core is greater than that in the face sheets. Thus the damping has an increasing trend with frequency.

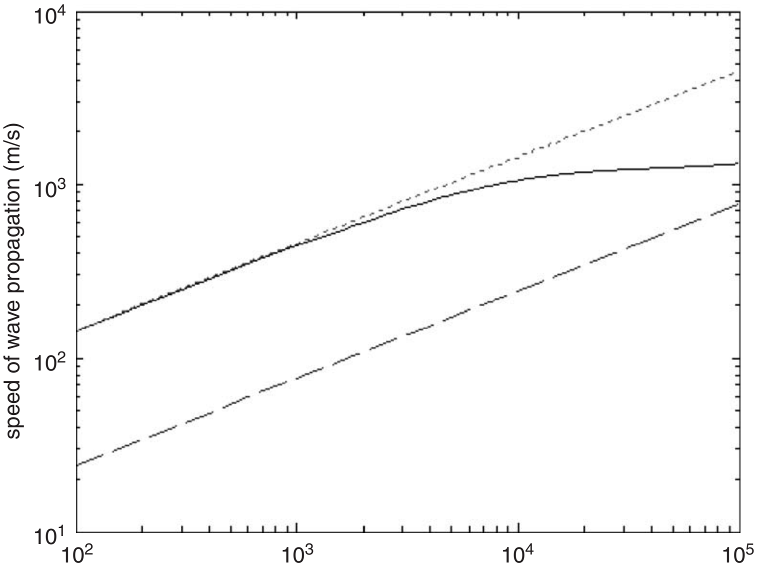
At high frequencies, if the core is very soft compared with the face sheet, the bending stiffness of the face sheets about their own neutral axes is dominant and the total damping is determined by the face sheets $\left(D \approx 2D_f\right)$. That means that the damping reaches a maximum and decreases again at high frequency [12]. However, for the material studied, the honeycomb increases the stiffness of the core compared with a core made only of foam. So the normal-to-shear coupling is still effective in the high-frequency range and thus the damping is increased substantially.

Therefore, it can be concluded that with an increase in the face sheet thickness, the damping in the low- and high-frequency ranges is lower, but it is still high in the middle-frequency range. On the other hand, if the thickness of the core is doubled, the damping is very much increased in the middle- and high-frequency ranges.

Damage is another mechanism which causes increased damping. Delamination introduces friction in the unbounded region of the interface. And the damping increases with the size of the delamination. Meanwhile, increased damping leads to lower natural frequencies. This effect is significant in the high-frequency range [22].



(a)



(b)

Fig. 3. Dispersion relation for sandwich beams with (a) a single foam core with Young's modulus of 10.16 MPa, and (b) a foam-filled honeycomb core with Young's modulus of 36.4 MPa. —— speed of wave in sandwich beam; ……, pure bending wave speeds of the entire sandwich beam; – – –, pure bending wave speeds of the face sheet.

In Ref. [23] a finite element program developed for a sandwich cantilever beam using NASTRAN shows that the damping increases with increasing delamination. Our experimental results presented in this paper are seen to be consistent with this prediction.

Delamination affects the stiffness of sandwich beams as well. The bending stiffness expression, Eq. (6), is derived for undamaged sandwich beams. For beams with delamination, the integral limits become smaller and the resulting bending stiffness is reduced substantially. If there is delamination on both sides of the beam, the bending stiffness is reduced more than when there is delamination only on one side. This prediction is the same as Frostig's model based on high-order elastic theory [24].

3. Experiments

We studied three intact and six delaminated beams. Their configurations are listed in Tables 1 and 2. All the other dimensions of the delaminated beams are the same: length 609.6 mm, width 25.4 mm, core thickness 6.35 mm and face sheet thickness 0.33 mm. Fig. 4 illustrates a beam with 50.8 mm delaminations on both sides.

3.1. Experimental setup

Fig. 5 shows the experimental setup for the damping measurements on sandwich composite beams. The beams were excited with white noise by a shaker mounted at the middle of the beam. The density of the sandwich material is 278 kg/m^3 and the mass of the beam A is 27.33 g. For such a light structure a general purpose accelerometer is not applicable, because the effect of mass loading is significant [25,26]. Therefore, a Polytech laser vibrometer was employed to measure the beam response. The frequency-response functions measured by a B&K accelerometer-type 4570 show that the resonance frequencies are 10% lower than those

Table 1
Configurations of intact beams

Intact beams	Length (mm)	Width (mm)	Core thickness (mm)	Face sheet thickness (mm)	Structural parameter, Y
Beam A	609.6	25.4	6.35	0.33	1229
Beam B	609.6	25.4	6.35	0.66	338
Beam C	609.6	25.4	12.7	0.33	4627

Table 2
Configurations of beams with delamination

Delaminated beams	Delamination length (mm) (percentage of length)	Delamination location
Beam D	12.7 (5)	One side
Beam E	12.7 (5)	Both sides
Beam F	25.4 (10)	One side
Beam G	25.4 (10)	Both sides
Beam H	50.8 (20)	One side
Beam I	50.8 (20)	Both sides

Fig. 4. A beam with 50.8 mm delaminations on both sides.

Fig. 5. Experimental setup for damping measurements.

measured by the laser vibrometer. The B&K PULSE system was used to analyze the signals with the Dual FFT mode and the damping ratio was determined directly.

At low frequencies, the coherence between the response and force is very poor for lightweight structures, because the surrounding airflow affects the excitation-response relationship. So it is difficult to obtain satisfactory measurements for the first mode. One solution is to excite the structures and measure the corresponding responses in extremely narrow frequency bands. In practice, both 3.125 Hz band and 1.63 Hz bands were used to excite the structures and make measurements using the zoom FFT mode. Since the beams were excited in a very narrow band, in which the excitation energy was concentrated, the airflow influence is negligible. In that way the coherence was increased up to 0.977.

3.2. Experimental results

Figs. 6 and 7 compare the receptance frequency-response functions and damping ratio of beams with single- and double-layer face sheets. Double-layer face sheets add 13% more mass to the beams. Fig. 6 shows that the vibration properties do not change very much. However, from Fig. 7 we can see that, as expected, the damping in beam B is lower than that in beam A (see Table 1) in the low- and high-frequency ranges, because the thicker face sheets constrain the deformation of the core in beam B more than in beam A. However, in the middle-frequency range, the damping ratio reaches its maximum value.

Figs. 8 and 9 compare the receptance frequency-response functions and damping ratio in beams A and C. The density of the core is 156 kg/m^3. So a core which is twice as thick adds 56% more mass to the beam. Then the natural frequencies shift dramatically to lower frequencies. And the damping increases, especially in the middle- and high-frequency ranges.

Figs. 10 and 11 show the receptance frequency-response functions and damping ratio of the intact beam A and the delaminated beam D. From Fig. 11 it can be seen that the effect of delamination is more obvious in the high-frequency range. The damping increases as frequency increases.

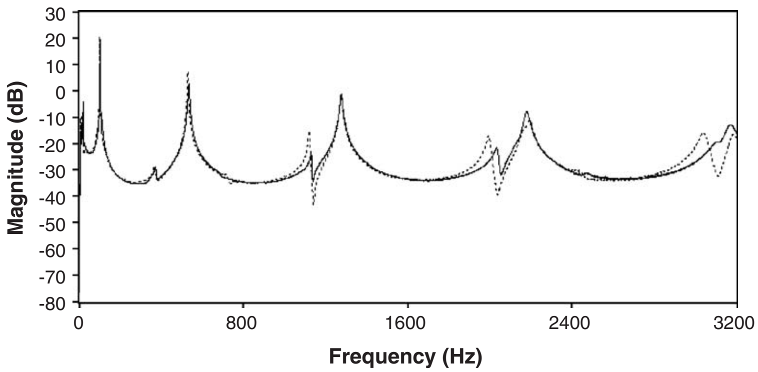


Fig. 6. Receptance FRFs of beams A and B. ———, single-layer face sheets (A); ··········, double-layer face sheets (B).

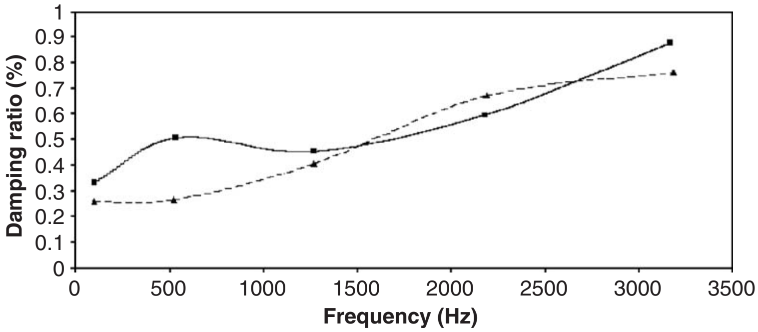


Fig. 7. Comparison of damping ratio in beams A and B. ■—■—■, single-layer face sheets (A); ▲--▲--▲, double-layer face sheets (B).

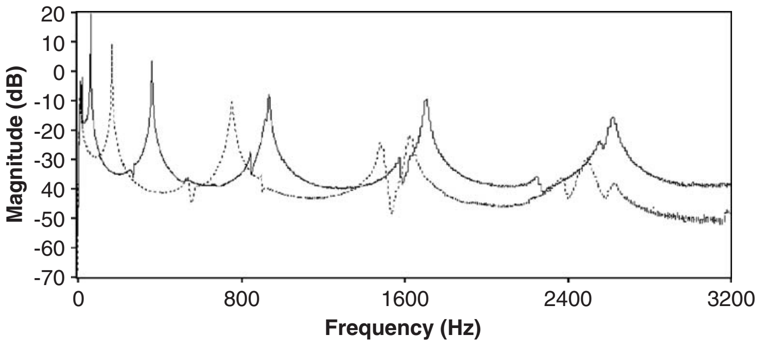


Fig. 8. Receptance FRFs of beams A and C. ———, 6.35 mm thick core (A); ··········, 12.7 mm thick core (C).

Figs. 12 and 13 show the damping ratios of the delaminated beams. With 5% delamination, the damping of each mode increases evenly. With 10% delamination, the damping ratio of the second mode is seen to be very high. With 20% delamination, both the first and the second modes have very high damping. Beams with delaminations on both sides have more damping than those with delamination only on one side.

The fundamental frequency of a cantilever beam is given by

$$f_1 = \frac{3.5160}{2\pi}\sqrt{\frac{EI}{mL^4}}, \tag{12}$$

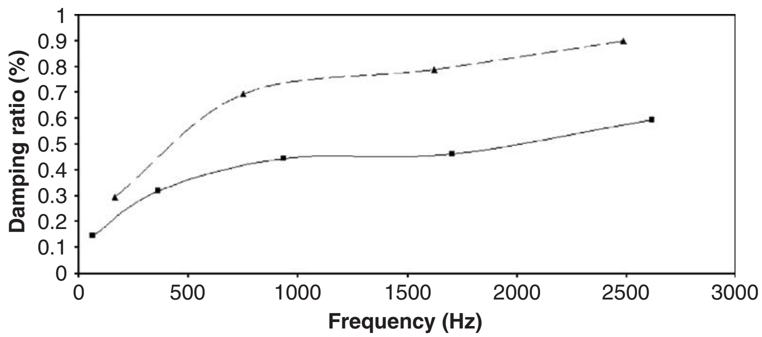


Fig. 9. Comparison of damping ratio in beam A and beam C. ■—■—■, 6.35 mm thick core (A); ▲--▲--▲, 12.7 mm thick core (C).

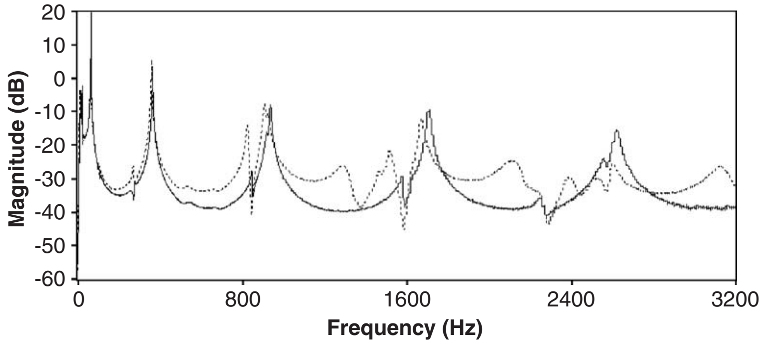


Fig. 10. Receptance FRFs of intact beam A and delaminated beam D. ———, intact (A); ·········, 5% delaminated (D).

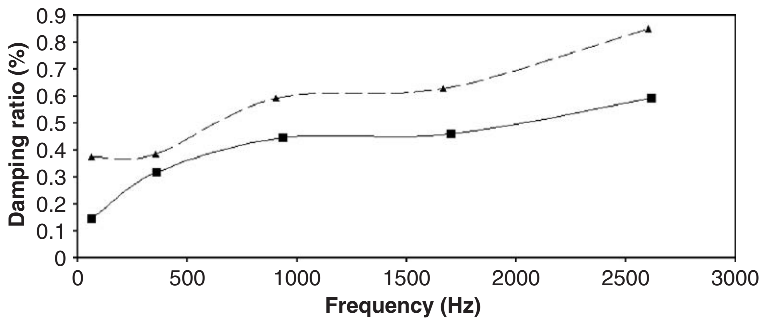


Fig. 11. Comparison of damping ratio of intact beam A and delaminated beam D. ■—■—■, intact (A); ▲--▲--▲, 5% delaminated (D).

where m is the mass per unit length and L is the length of the beam. Then the equivalent Young's modulus can be obtained by measuring the fundamental frequencies of the intact and delaminated sandwich beams. Fig. 14 shows the effect of delamination on the equivalent Young's modulus.

3.3. Discussion

First of all, it is worth noticing that high damping is not the only criterion for noise and vibration control. The overall effects of many factors such as mass, stiffness, damage tolerance and so on have to be considered

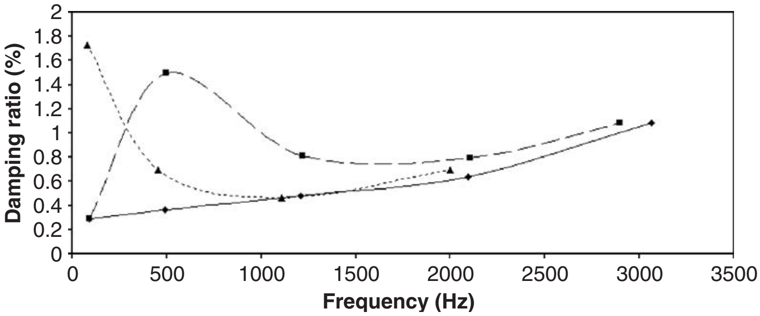


Fig. 12. Damping ratios of beams with delamination only on one side. ◆—◆—◆, 5% delamination (D); ■ – ■ – ■, 10% delamination (F); ▲······▲······▲, 20% delamination (H).

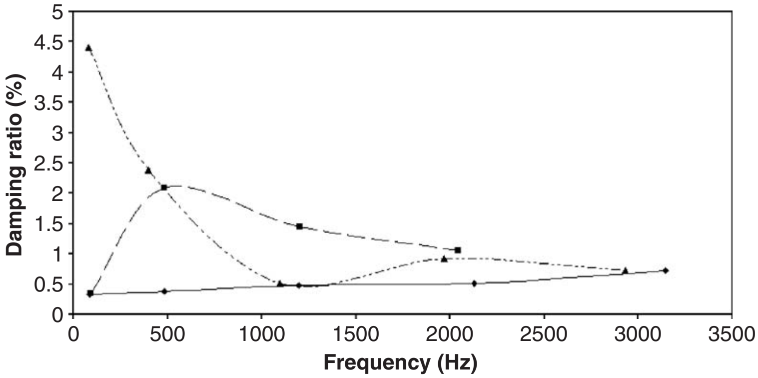


Fig. 13. Damping ratios of beams with delaminations on both sides. ◆—◆—◆, 5% delamination (E); ■ – ■ – ■, 10% delamination (G); ▲······▲······▲, 20% delamination (I).

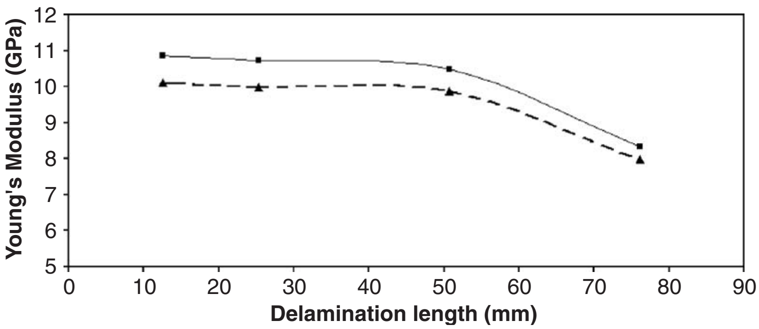


Fig. 14. Young's modulus of sandwich beams as a function of delamination length. ■—■—■, delamination on one side; ▲– –▲– –▲, delamination on both sides.

as well. High damping is usually associated with relatively low stiffness. So the trade-off between the requirement for low vibration levels and strength and stiffness must be analyzed during the design stage.

As discussed before, the system loss factor η reaches a maximum value when the shear modulus of the core has an optimal value in the intermediate range. In the Ross–Ungar–Kerwin model, the shear parameter X is inversely proportional to the core thickness t_c. This means that only when the core thickness is also in an optimal range, can the damping reach a maximum value. He and Rao reported the same prediction using a numerical simulation [27]. However, the loss factor also depends on the total bending stiffness which is also affected by the core thickness. Mead proved that the loss factor η is much less sensitive to the change of the shear parameter X when the structural parameter Y is large [28]. The shear parameter is then in a much wider range of the optimum value for maximum η. In He and Rao's study, the core is thinner than the face sheets and $Y = 27$. However, for the beams studied in this paper, the face sheets are much thinner than the cores. So the structural parameters are large as listed in Table 1.

In addition, Mead presented the relationship between the maximum loss factor $\eta_{\max}$ and the structural parameter Y:

$$\eta_{\max} = \frac{\beta Y}{(2+Y) + 2\sqrt{(1+Y)(1+\beta^2)}}. \tag{13}$$

Taking the derivative of $\eta_{\max}$ yields:

$$\frac{\mathrm{d}\eta_{\max}}{\mathrm{d}Y} = \frac{\beta\left[2+Y+2\sqrt{(1+Y)(1+\beta^2)}\right] - \beta Y\left[1+(1+\beta^2)\Big/\left(\sqrt{(1+Y)(1+\beta^2)}\right)\right]}{\left[2+Y+2\sqrt{(1+Y)(1+\beta^2)}\right]^2} = \frac{A}{B}. \tag{14}$$

The denominator B is always positive. The numerator A:

$$\begin{aligned} A &> \beta\left[2+2\sqrt{(1+Y)(1+\beta^2)} - Y\sqrt{\frac{1+\beta^2}{1+Y}} - \sqrt{1+\beta^2}\right] \\ &= \beta\left[2+2\sqrt{(1+Y)(1+\beta^2)} - \sqrt{(1+Y)(1+\beta^2)}\right] \\ &= \beta\left[2+\sqrt{(1+Y)(1+\beta^2)}\right] > 0. \end{aligned} \tag{15}$$

So the derivative (14) is always positive. This means that the loss factor increases monotonically with increasing value of Y, if other parameters are fixed. The theoretical analysis given in Section 2 and the experimental results presented in this paper agree with Mead's prediction.

4. Conclusions

Several theoretical models for the damping of sandwich structures have been reviewed. The frequency dependence of damping was analyzed for foam-filled honeycomb sandwich beams, in which the face sheets are much thinner than the core, or the structural parameters are large. The effects of thickness and delamination were studied. If the face sheet thickness increases, the damping in the low- and high-frequency ranges is lower, but it remains high in the middle-frequency range. If the thickness of the core increases, the damping is increased in the middle- and high-frequency ranges. Delamination introduces more friction in the composite beam structure and thus makes the damping increase. However, delamination also reduces the stiffness as well as the natural frequencies of sandwich structures. Experiments on beams with different configurations and with delamination were carried out. The experimental results are consistent with the analytical predictions.

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