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Short Communication

Chattering alleviation in vibration control of smart beam structures using piezofilm actuators: Experimental verification

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Abstract

This paper presents a new discrete-time sliding mode controller to alleviate undesirable chattering in vibration control of a flexible beam structure. A smart beam featuring a piezoelectric film is devised and its governing equation of motion is derived. A discrete-time sliding mode controller which consists of discontinuous part and equivalent part is then designed by considering the separation principle. By doing this, undesirable chattering of the beam structure can be attenuated in the settled phase. The proposed controller is experimentally realized, and both transient and forced vibration control responses are evaluated in time domain.

1. Introduction

Recently, advanced lightweight structural systems have been achieved in various research fields with the aid of high technologies in computer and material sciences. Especially, the emergence of so-called smart materials has accelerated successful development of the advanced structural systems [1,2]. So far, potential smart materials include electro-rheological fluids, magneto-rheological fluids, shape memory alloys, piezoelectric materials, and optical fibers. Most flexible structures featuring the smart materials can be easily subjected to parameter variations and disturbances [3,4]. Hence, these structural systems require robust control algorithms such as sliding mode control (SMC). The sliding mode that is the principle mode in the variable structure control system is obtained by appropriate discontinuous control laws. In the sliding mode, the system has robust property to the parameter variations and external disturbances [5–7].

In general, the SMC has been designed based on the continuous-time system. Its implementation by a digital computer, however, requires certain sampling processes. It may be very difficult that the sampling processes are exactly realized in the continuous-time sliding mode control (CSMC) system. In other words, since the sampling processes bring not only the chattering of the control input along the pre-designed sliding surfaces but also possible instability of the control system, the practical implementation of the CSMC may not be effective. This leads to the study of the discrete-time sliding mode control (DSMC) system. The main discrepancy between the DSMC and the CSMC is the determination of the existence condition of the sliding

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mode. Sarpturk et al. [8] have presented the existence condition to determine a stable sliding mode controller for the discrete-time system. This condition requires a controller gain to have upper and lower bounds. But if the system has uncertainties, there exists a region in the neighborhood of the sliding surface where the feedback gain cannot be defined. This region is called a sliding region. Furuta and Pan [9] have suggested adjustable equivalent controller to attenuate the sensitiveness to the uncertainties. However, this solution makes the sliding region enlarge. This may cause adverse chattering in the vibration control of flexible structures.

In this study, a new discrete-time sliding mode controller is proposed in order to alleviate undesirable chattering in vibration control of a flexible smart beam structure featuring a piezoelectric film actuator (piezofilm actuator in short). After deriving the governing equation of motion for the smart beam structure, a discrete-time control model with system uncertainties is constructed in a state space. A stable sliding surface is then designed followed by the formulation of a discrete-time sliding mode controller which consists of a discontinuous part and an equivalent part. In the design of the equivalent part, so called the separation principle is introduced to achieve faster reaching to the sliding surface without extension of the sliding region in which undesirable vibration chattering may be arisen. The controller is experimentally implemented and vibration control responses of the beam structure subjected to transient and forced vibrations are evaluated in time domain.

2. Formulation of control model

Consider a smart flexible cantilevered beam as shown in Fig. 1, in which the piezofilm is perfectly bonded on the upper surface of the host structure as an actuator. Upon applying a voltage, $V(t)$ to the piezofilm actuator, a resultant strain (ε_l) is produced in the beam as follows [1]:

$$\varepsilon_l = \frac{E_2 t_2 d_{31} V(t)}{t_2(E_1 t_1 + E_2 t_2)}. \tag{1}$$

Here E_1 and E_2 are Young's modulus of the beam and the piezofilm, respectively, and d_{31} denotes the piezoelectric strain constant. From the resultant strain of Eq. (1), the bending moment (M) in the smart beam is obtained as follows [10]:

$$\begin{aligned} M &= -d_{31} \cdot \frac{t_1 + t_2}{2} \cdot \frac{E_1 E_2 t_1 b}{E_1 t_1 + E_2 t_2} \cdot V(t) \\ &= c \cdot V(t), \end{aligned} \tag{2}$$

where c is a constant dependent on mechanical properties and geometry of the host structure and the piezofilm. By employing energy equations and Hamilton's principle, the governing equation of motion and

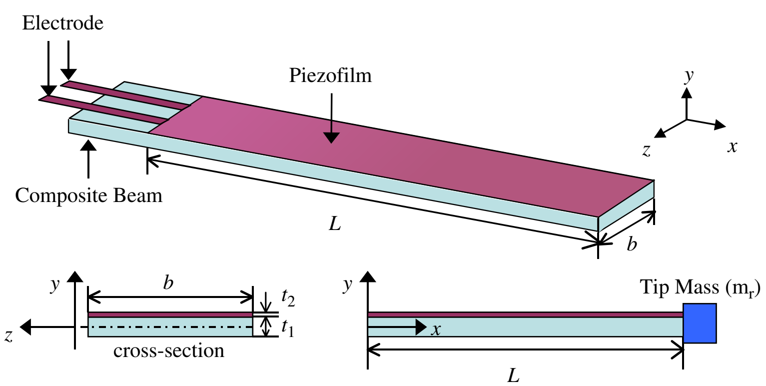

Fig. 1. Smart beam structure featuring the piezofilm actuator.

associated boundary conditions are obtained by

$$EIy^{(iv)}(x,t) + \rho A\ddot{y}(x,t) = 0, \tag{3}$$

$$y(0,t) = y'(0,t) = 0, \quad Ely''(L,t) = -cV(t), \quad Ely'''(L,t) = m_T\ddot{y}(L,t). \tag{4}$$

Here EI is the effective bending stiffness of the structure, ρA is the mass per length of the structure, and m_T denotes the tip mass. By using the assumed-mode method the displacement (or deflection) variable, $y(x,t)$ can be expressed as

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i(x) \cdot q_i(t). \tag{5}$$

The space-dependent function $\phi_i(x)$ is the eigenfunction of the ith mode, and the time-dependent function $q_i(t)$ is the generalized coordinate of the system. After applying Lagrange's equation, an infinite set of ordinary differential equations which are decoupled from each other may be obtained. Upon retaining finite number of control modes (m mode), a reduced dynamic model is obtained in the state space representation as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{f}(t), \\ y(t) &= \mathbf{E}\mathbf{x}(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} \mathbf{x} &= [q_1\ \dot{q}_1\ q_2\ \dot{q}_2\ \cdots\ q_m\ \dot{q}_m]^{\mathrm{T}} = [x_1\ x_2\ \cdots\ x_{2m}]^{\mathrm{T}}, \\ u(t) &= V(t), \quad \mathbf{f}(t) = [0\ f_1(t)\ 0\ f_2(t)\ \cdots\ 0\ f_m(t)]^{\mathrm{T}}, \\ \mathbf{A} &= \begin{bmatrix} 0 & 1 & & & \\ -\omega_1^2 & -2\zeta_1\omega_1 & & \mathbf{0} & \\ & & \ddots & & \\ & \mathbf{0} & & 0 & 1 \\ & & & -\omega_m^2 & -2\zeta_m\omega_m \end{bmatrix}, \\ \mathbf{B} &= -\frac{c}{I_t}\left[0\ \int_0^L \frac{\partial^2\phi_1}{\partial x^2}\,\mathrm{d}x\ 0\int_0^L \frac{\partial^2\phi_2}{\partial x^2}\,\mathrm{d}x\ \cdots\ 0\int_0^L \frac{\partial^2\phi_m}{\partial x^2}\,\mathrm{d}x\right]^{\mathrm{T}}, \\ \mathbf{E} &= [\phi_1(L)\ 0\ \phi_2(L)0\ \cdots\ \phi_m(L)\ 0]. \end{aligned}$$

In Eq. (6), $f_i(t)$ is unknown but bounded external force to disturb the ith mode. ω_i and ζ_i denote the natural frequency and the damping ratio of the ith mode, respectively, and I_t is the generalized mass.

From the lack of exact knowledge of model parameters, a possible variation of the parameters for the natural frequency and the damping ratio can be expressed as follows:

$$\omega_i = \omega_{i,0} + \delta\omega_i(t), \quad \zeta_i = \zeta_{i,0} + \delta\zeta_i(t). \tag{7}$$

Here $\omega_{i,0}$ and $\zeta_{i,0}$ are the nominal natural frequency and the damping ratio of the ith mode, respectively. $\delta\omega_i(t)$ and $\delta\zeta_i(t)$ are corresponding possible deviations. Now, substituting Eq. (7) into Eq. (6) yields the following uncertain dynamic model:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= (\mathbf{A}_0 + \Delta\mathbf{A}(t))\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{f}(t) \\ &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{f}(t). \end{aligned} \tag{8}$$

By using the zero-order-hold (ZOH) method, the continuous-time system (8) can be written by the following discrete-time version:

$$\mathbf{x}(k+1) = \mathbf{\Phi}(k)\mathbf{x}(k) + \mathbf{\Gamma}(k)u(k) + \mathbf{d}(k), \tag{9}$$

where

$$\mathbf{\Phi}(k) = \exp\left(\int_{t_k}^{t_{k+1}} \mathbf{A}(\tau)\,\mathrm{d}\tau\right) \cong \exp\left(\mathbf{A}\left(\frac{t_k + t_{k+1}}{2}\right)T\right),$$

$$\mathbf{\Gamma}(k) = \int_{t_k}^{t_{k+1}} \exp\left(\int_{\tau}^{t_{k+1}} \mathbf{A}(\alpha)\,\mathrm{d}\alpha\right)\mathbf{B}(\tau)\,\mathrm{d}\tau \cong \int_0^T \exp\left(\mathbf{A}\left(\frac{t_k + t_{k+1}}{2}\right)\tau\right)\mathrm{d}\tau\,\mathbf{B}\left(\frac{t_k + t_{k+1}}{2}\right),$$
$$\mathbf{d}(k) \cong \int_0^T \exp\left(\mathbf{A}\left(\frac{t_k + t_{k+1}}{2}\right)\tau\right)\mathbf{f}((k+1)T - \tau)\,\mathrm{d}\tau.$$

In the above, T is a sampling time and k is a sampling number. We can rewrite Eq. (9) using the nominal part and the uncertain part as follows:

$$\mathbf{x}(k+1) = (\mathbf{\Phi}_0 + \Delta\mathbf{\Phi})\mathbf{x}(k) + (\mathbf{\Gamma}_0 + \Delta\mathbf{\Gamma})u(k) + \mathbf{d}(k), \tag{10}$$

where

$$\mathbf{\Phi}_0 = \begin{bmatrix} \phi_{11,0} & \cdots & \phi_{1n,0} \\ \vdots & & \\ \phi_{n1,0} & \cdots & \phi_{nn,0} \end{bmatrix}, \quad \mathbf{\Gamma}_0 = \begin{bmatrix} \gamma_{1,0} \\ \vdots \\ \gamma_{n,0} \end{bmatrix},$$

$$\phi_{ij,0} = \frac{\phi_{ij,\min} + \phi_{ij,\max}}{2}, \quad \gamma_{i,0} = \frac{\gamma_{i,\min} + \gamma_{i,\max}}{2}, \quad i,j = 1,\ldots,n(=2m),$$

$$\Delta\mathbf{\Phi} = \begin{bmatrix} \delta\phi_{11}(k) & \cdots & \delta\phi_{1n}(k) \\ \vdots & & \\ \delta\phi_{n1}(k) & \ldots & \delta\phi_{nn}(k) \end{bmatrix}, \quad \Delta\mathbf{\Gamma} = \begin{bmatrix} \delta\gamma_1(k) \\ \vdots \\ \delta\gamma_n(k) \end{bmatrix}, \quad \mathbf{d}(k) = \begin{bmatrix} d_1(k) \\ \vdots \\ d_n(k) \end{bmatrix}.$$

And maximum parameter variations and disturbances can be defined as follows:

$$\begin{aligned} \bar{\phi}_{ij} &\equiv \max(|\delta\phi_{ij}(k)|) = \phi_{ij,\max} - \phi_{ij,0}, \\ \bar{\gamma}_i &\equiv \max(|\delta\gamma_i(k)|) = \gamma_{i,\max} - \gamma_{i,0}, \\ \bar{d}_i &\equiv \max(|d_i(k)|). \end{aligned} \tag{11}$$

3. Controller design

The control purpose is to enforce the deflection of the flexible smart beam to the zero without undesirable chattering in a settled phase. We firstly set a sliding surface in the state space as follows:

$$s(k) = \mathbf{C}\mathbf{x}(k), \tag{12}$$

where, $\mathbf{C} = [c_1 \cdots c_n]$ is surface gradient vector. Then, an equivalent controller for the nominal part $(\mathbf{\Phi}_0, \mathbf{\Gamma}_0)$ of system (10) can be established such that the state variables rest on the surface of $s(k+1) = s(k)$ for all k, given by

$$\mathbf{C}(\mathbf{\Phi}_0\,\mathbf{x}(k) + \mathbf{\Gamma}_0 u(k)) = \mathbf{C}\mathbf{x}(k).$$

Thus,

$$u_{\mathrm{eq}}(k) = \mathbf{F}_{\mathrm{eq}}\,\mathbf{x}(k) = -(\mathbf{C}\mathbf{\Gamma}_0)^{-1}(\mathbf{C}\mathbf{\Phi}_0 - \mathbf{C})\,\mathbf{x}(k). \tag{13}$$

In this manner, the motion of the nominal system is subjected to the following sliding mode equation:

$$\begin{aligned} \mathbf{x}(k+1) &= (\mathbf{\Phi}_0 + \mathbf{\Gamma}_0 \mathbf{F}_{\text{eq}})\mathbf{x}(k) = \mathbf{\Phi}_{\text{eq}}\mathbf{x}(k), \\ s(k) &= \mathbf{C}\mathbf{x}(k) = 0. \end{aligned} \tag{14}$$

Here, one eigenvalue of $\mathbf{\Phi}_{\text{eq}}$ is 1 and the others $(n-1)$ become the predetermined desired eigenvalues. Consequently, the closed-loop nominal system with controller (13) is marginally stable in the sliding mode. On the other hand, so-called β-equivalent controller is determined from the relation of $s(k+1) = \beta s(k)$ as follows:

$$\mathbf{C}(\mathbf{\Phi}_0 \mathbf{x}(k) + \mathbf{\Gamma}_0 u(k)) = \beta \mathbf{C}\mathbf{x}(k), \quad 0 \leqslant \beta < 1.$$

This yields the following controller:

$$u_{\text{eq},\beta}(k) = \mathbf{F}_{\text{eq},\beta}\mathbf{x}(k) = -(\mathbf{C}\mathbf{\Gamma}_0)^{-1}(\mathbf{C}\mathbf{\Phi}_0 - \beta\mathbf{C})\mathbf{x}(k). \tag{15}$$

The closed-loop nominal system with the β-equivalent controller (15) is asymptotically stable inside the sliding region, since β itself is one of the closed-loop eigenvalues and it is less than 1.

As stated in Section 1, we can reduce the sliding region using the equivalent controller (13), but the system sensitivity to the uncertainties increases. On the other hand, using the β-equivalent controller (15), we can attenuate the sensitivity with a tradeoff of the large sliding region. In order to keep only advantage of each equivalent controller, in this study we propose an equivalent controller separation principle: inside the sliding region, we activate only the β-equivalent controller (15), while controller (13) is activated outside the sliding region by incorporating a discontinuous controller to be designed. Since controller (13) or (15) is designed on the basis of nominal part, the stability of the uncertain system (10) cannot be guaranteed. Thus, in order to guarantee the stability a discontinuous controller should be designed so that the following sliding mode conditions are satisfied [8]:

$$\begin{aligned} &(s(k+1) - s(k))\,\text{sgn}(s(k)) < 0, \\ &(s(k+1) + s(k))\,\text{sgn}(s(k)) > 0. \end{aligned} \tag{16}$$

Before designing the discontinuous controller, we assume that $\mathbf{C}(\mathbf{\Gamma}_0 + \Delta\mathbf{\Gamma})$ is non-singular for all k. This assumption can be then expressed by

$$\left|\sum_{i=1}^{n} c_i \gamma_{i,0}\right| > \sum_{i=1}^{n} |c_i \bar{\gamma}_i|. \tag{17}$$

Now, we claim that the state trajectory of the uncertain system (10) converges robustly into the inside of the sliding region from the outside of the sliding region, if the equivalent controller is augmented with the discontinuous controller given by

$$u_d(k) = -h(k)\,\text{sgn}(\mathbf{C}\mathbf{\Gamma}_0 s(k)) \sum_{i=1}^{n} |x_i(k)|, \tag{18}$$

where

$$\begin{cases} h_s(k) < h(k) < h_c(k): & \text{outside the sliding region}, \\ h(k) = 0: & \text{inside the sliding region}, \end{cases}$$

$$h_s(k) = \frac{\sup(H_2(k))}{\inf(H_1(k))}, \quad h_c(k) = \frac{2|s(k)| - \sup(H_2(k))}{\sup(H_1(k))}.$$

In the above equation, $\inf(H_1(k))$, $\sup(H_1(k))$ and $\sup(H_2(k))$ are defined by

$$\begin{aligned} \inf(H_1(k)) &\equiv \inf\left(\left|\mathbf{C}(\mathbf{\Gamma}_0 + \Delta\mathbf{\Gamma})\right| \sum_{i=1}^{n} \left|x_i(k)\right|\right) = \inf(|\mathbf{C}(\mathbf{\Gamma}_0 + \Delta\mathbf{\Gamma})|) \sum_{i=1}^{n} |x_i(k)| \\ &= \left(\left|\sum_{i=1}^{n} c_i \gamma_{i,0}\right| - \sum_{i=1}^{n} |c_i \bar{\gamma}_i|\right) \sum_{i=1}^{n} |x_i(k)|, \end{aligned}$$

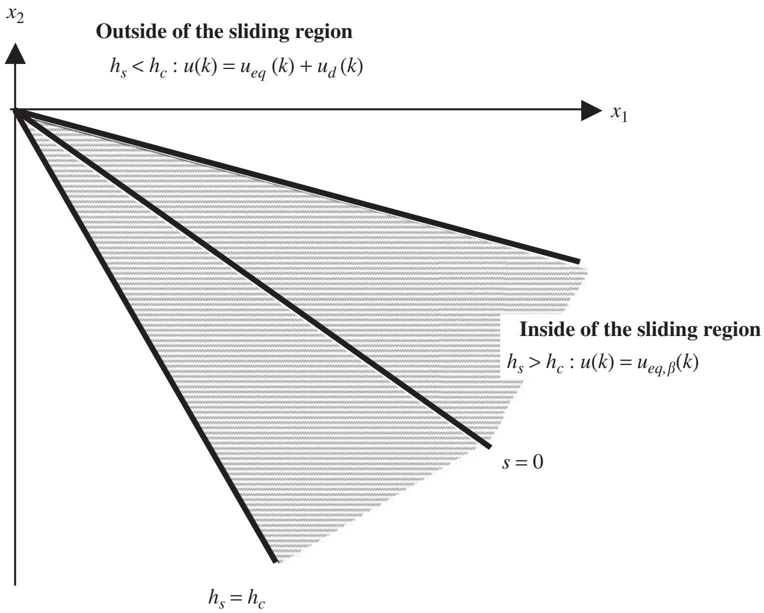

Fig. 2. Separation principle for the equivalent control action.

$$\begin{aligned}\sup(H_1(k)) &\equiv \sup\left(\left|\mathbf{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})\right| \sum_{i=1}^{n} |x_i(k)|\right) = \sup(|\mathbf{C}(\boldsymbol{\Gamma}_0 + \Delta\boldsymbol{\Gamma})|) \sum_{i=1}^{n} |x_i(k)| \\ &= \left(\left|\sum_{i=1}^{n} c_i\gamma_{i,0}\right| + \sum_{i=1}^{n} |c_i\bar{\gamma}_i|\right) \sum_{i=1}^{n} |x_i(k)|,\end{aligned}$$

$$\begin{aligned}\sup(H_2(k)) &\equiv \sup(\mathbf{C}(\Delta\boldsymbol{\Phi}\mathbf{x}(k) + \Delta\boldsymbol{\Gamma}u_{\text{eq}} + \mathbf{d}(k))) \\ &= \sup\left(\sum_{i=1}^{n}\sum_{j=1}^{n} c_i\delta\phi_{ij}(k)x_j(k) + \sum_{i=1}^{n}\sum_{j=1}^{n} c_i\delta\gamma_i(k)u_{\text{eq}} + \sum_{i=1}^{n} c_i d_i(k)\right) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} |c_i\bar{\phi}_{ij}x_j(k)| + |u_{\text{eq}}(k)| \sum_{i=1}^{n} |c_i\bar{\gamma}_i| + \sum_{i=1}^{n} |c_i\bar{d}_i|.\end{aligned}$$

By substituting the controller ($u(k) = u_{\text{eq}}(k) + u_d(k)$) into the uncertain system (10) the sliding mode conditions (16) are easily satisfied.

Fig. 2 presents the sliding region of control action. It is evident from this figure that only the β-equivalent controller (15) is activated inside the sliding region ($h_s > h_c$), while the controllers (13) and (18) are activated outside the sliding region ($h_c > h_s$). As stated earlier, this equivalent controller separation principle provides the attenuation of vibration chattering in a settled phase and also the enhancement of robustness to the system uncertainties.

4. Experimental results

In order to demonstrate the effectiveness of the proposed control methodology an experimental apparatus is established as shown in Fig. 3. The information of the tip deflection measured from a non-contacting displacement sensor (proximitor) is fed back to the micro-processor through A/D converter. Depending on the information of the tip deflection and sampling time, the tip velocity is calculated. Using these variables, control input voltage is determined in the micro-processor by means of the proposed control algorithm. The control voltage of the micro-processor is then applied to the piezofilm actuator through D/A converter and high voltage amplifier which has a gain of 1000. The dimensional and material specifications of the composite

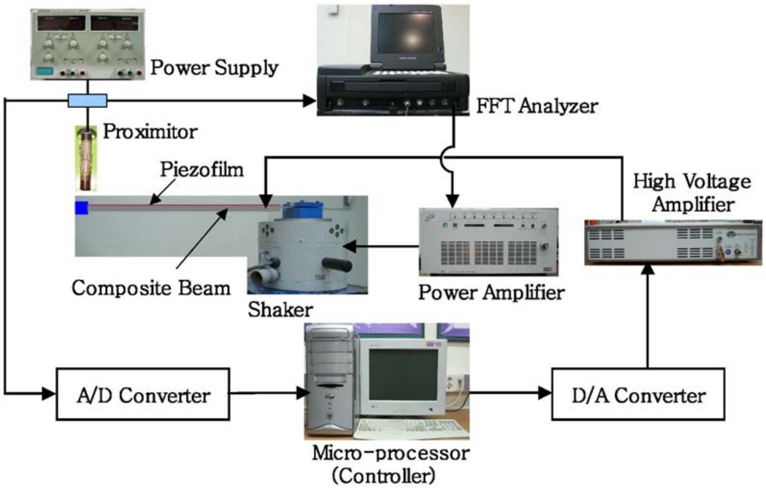

Fig. 3. Experimental setup for vibration control.

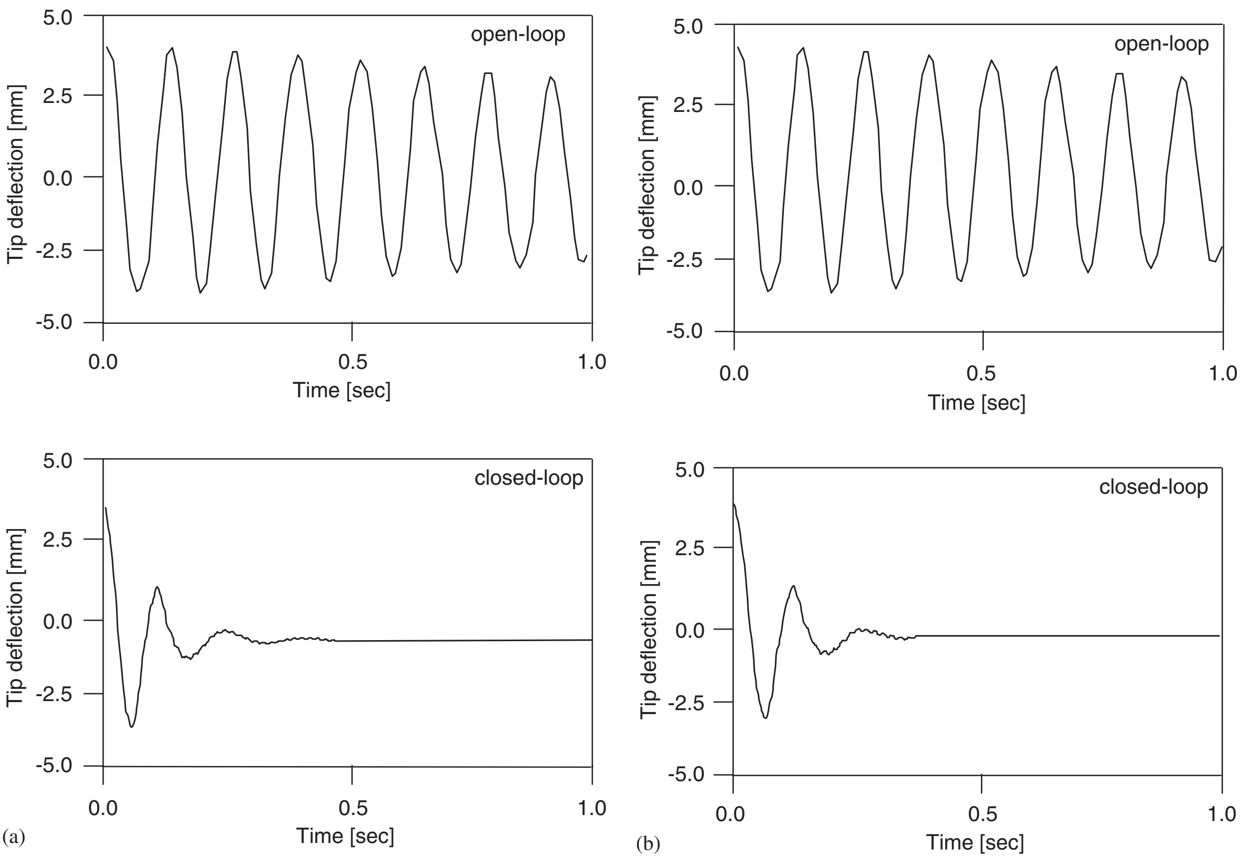

Fig. 4. Transient vibration control responses: (a) w/o the separation principle; (b) with the separation principle.

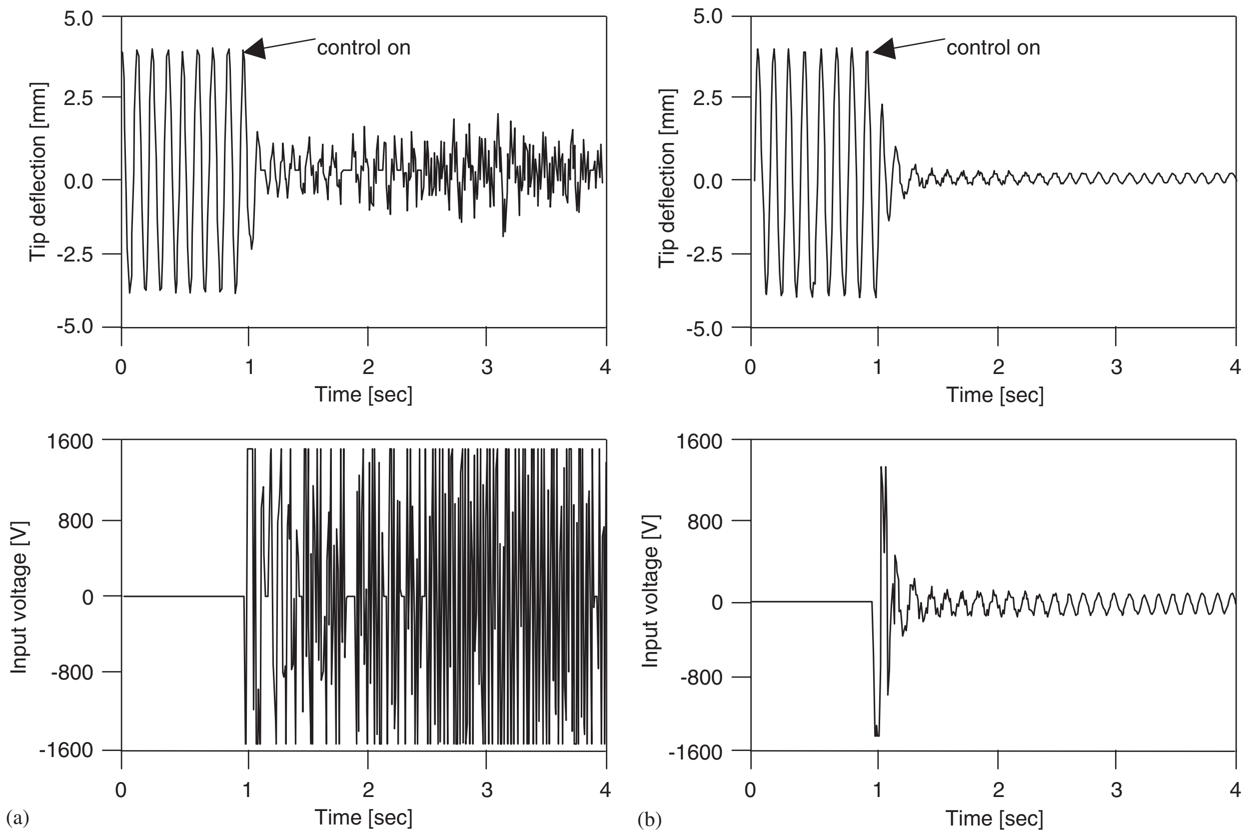

Fig. 5. Forced vibration control responses: (a) w/o the separation principle; (b) with the separation principle.

beam and the piezofilm used in this study are given as follows:

$$E_1 = 6.4\,\text{GPa},\quad t_1 = 0.65\,\text{mm},\quad b = 26.6\,\text{mm},\quad \rho_1 = 1865\,\text{kg/m}^3,$$
$$L = 170\,\text{mm},\quad E_2 = 2\,\text{GPa},\quad t_2 = 0.11\,\text{mm},\quad \rho_2 = 1780\,\text{kg/m}^3$$

and $d_{31} = 23 \times 10^{-13}(m/m)(V/m)$. Using these values, the first mode natural frequency of the structure is 11.8 Hz without the tip mass and 7.85 Hz with 2.0 g of the tip mass which is about 30% of the beam mass. Hence, the frequency change due to the tip mass can be considered as the parameter variation. In addition, we assume the damping ratio has parameter variation with 20% of the nominal value of 0.007. The desired eigenvalue is chosen as 0.5 for the sliding surface design and the system is sampled with the sampling time of 0.01 s. It is noted that in this experiment the first vibration mode is considered as dominant mode to be controlled.

Fig. 4 presents the measured transient vibration control responses. It is clearly observed that the imposed transient vibration is perfectly controlled in both the proposed and the conventional cases. In the transient vibration control, the effectiveness of the proposed equivalent controller separation principle is not prominent. This implies that the conventional method is enough to effectively control the transient vibration in the absence of the external disturbance. Fig. 5 presents the measured forced vibration control responses with the applied control voltages. We can clearly observe that the controller associated with the proposed equivalent controller separation principle considerably improves the system responses by attenuating the chattering of the control input and the tip deflection as well. The large chattering, in the conventional case arises from the excessive supply of the control input voltage in the settled phase. This is due to that the sliding mode motion occurs on the sliding region, not inside the sliding region. The effectiveness of the proposed control

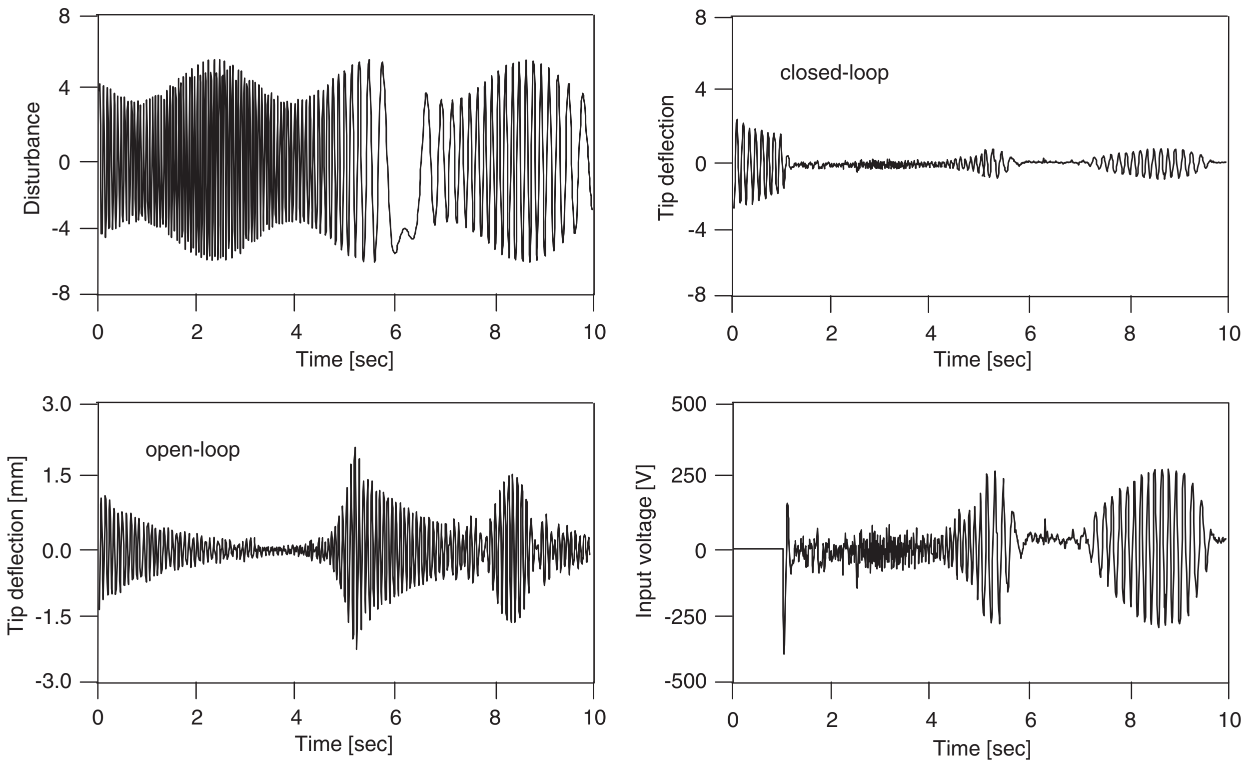

Fig. 6. Random vibration control responses with the separation principle.

methodology can be also verified in the random vibration as shown in Fig. 6. The imposed random vibration is well suppressed without exhibiting large chattering.

5. Concluding remarks

In this study, a new discrete-time sliding mode controller was formulated to attenuate the chattering of the vibration and also to achieve the robustness to the system uncertainties. In the design of the controller, an equivalent controller separation principle was employed for faster reaching to the sliding without extension of the sliding region. It has been demonstrated through experiment that the proposed controller furnishes favorable control responses such as significant reduction of the chattering, while maintaining the robustness to the parameter uncertainties. Especially, in the forced vibration control case, the equivalent controller separation principle was very effective. It is finally remarked that for the application of the proposed control system to more realistic systems, the control system may be integrated with electronic-data-processing function on a single integrated circuit chip.

Acknowledgements

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