

# Identification of modal parameters of a mdof system by modified L–P wavelet packets

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## Abstract

In this paper a methodology for identification of modal parameters of a structural system using wavelet analysis is proposed. The proposed technique differs from the other works on using wavelet for this problem in the choice of the basis function. A modified form of Littlewood–Paley (L–P) basis function is used for the identification of the parameters. This basis has the advantage being more closely representing a vibrating signal. Further it is localized in frequency and hence can be used to detect the frequency and the associated parameters better. With a modification, it is well suited for sub-band coding to detect the parameters with desired accuracy. The current work identifies modal parameters such as natural frequencies and mode shapes of a linear multi-degree of freedom (mdof) system using the wavelet transform. It utilizes wavelet transform to identify natural frequencies and the corresponding mode shapes from the transient response of the system under ambient vibration condition. The estimated natural frequencies and the mode shapes are found to be close to the theoretical values for two simulated 3 and 5 dof systems. This demonstrates the effectiveness of the proposed methodology for system identification.

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## 1. Introduction

Identification of structural systems involves an inverse procedure to identify the structural parameters from the recorded response of real-world structures. The goal is to estimate the dynamic properties such as natural frequencies and mode shapes of vibration, energy dissipation, permanent deformation and strength deterioration of damaged structures from the responses of structures under different conditions like ambient vibration, earthquakes and several other types of excitations. System identification of structures is the preceding step for design of active and passive control of structures [1] and structural health monitoring [2–4]. Identified parameters provide design criterion for structures subjected to seismic and other loadings inducing nonlinearity in structures [5,6].

Modal analysis of the response of structures is an important tool for identification of a structural system. Natural frequencies of a structure depend on its mass and stiffness distributions. The deformation patterns at

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these frequencies or the mode shapes are informative of the natural characteristics of the structures. Analysis of response signals of structures may be performed in two different paradigms: (i) time-domain analysis and (ii) frequency-domain analysis. Several approaches to time-domain system identification has been developed like state estimation using Kalman filter [7], stochastic analysis and modeling, recursive modeling and least-squares method.

The classical method of frequency-domain analysis is by means of Fourier transform [8], and its algorithmic implementation, the discrete Fourier transformation (DFT). Though DFT has been widely used for modal analysis and other system identification tasks, it has several limitations. Fourier analysis is inherently global in nature and fails to capture the time varying nature of a phenomenon. An approach for signal analysis, which circumvents the above problem, is the time–frequency analysis [9–13]. In the framework of multi-resolution time–frequency analysis, wavelet methods developed by several researchers [14–16] have been widely popular and successful for signal analysis. Wavelets produce representation of a signal using time-limited local functions having variable scales. Wavelet analysis has recently been used for a number of system identification tasks [17]. There have been several research works in literature on the use of wavelet to identify the modal parameters [18,19]. In some of these studies modal frequency, mode shapes and modal damping were identified. Random decrement technique has been used in some studies, which is known to have certain drawbacks. A shifted version of wavelet transform proposed by Staszewski [20] was used to detect frequencies. This works better for closely spaced frequencies. In Ref. [18], modified Morlet wavelet was used which works better than the traditional Morlet wavelet. Though some studies on use of wavelets have been carried out in system identification, yet it remains to be seen how different wavelets perform or which would be the suitable wavelet.

In this paper a methodology is proposed for identification of the modal frequencies and mode shapes of a structure using the real version of the Harmonic wavelet transformation for time–frequency analysis. This works better being close to vibrating signals. Further, this can be used to develop a sub-band coding leading to the wavelet packets for better accuracy as desired. The basis is also localized in frequency and hence does not suffer from the problem of band overlapping. The technique is presented for extracting the modal parameters of a linear multi-degree of freedom (mdof) system by decomposition of the original signal into frequency bands via wavelet transform, and time-dependent analysis of each band using the basic properties of eigenvalues of vibration modes. Two example cases of 3 dof and 5 dof linear viscously damped systems have been considered here. The vibration data are generated by simulating the response of the damped mdof systems under free vibration conditions. The response signals are used to identify the modal parameters.

## 2. Harmonic wavelets

### 2.1. Wavelet transform

The wavelet transform has been used by researchers for several applications such as filtering, transient analysis, time–frequency analysis, non-stationary analysis, discontinuity detection, data compression, system identification and damage detection among many others. The capability of the wavelet for carrying out time–frequency analysis has been exploited in this paper for the identification of modal parameters of a mdof dynamical system.

In wavelet analysis a signal  $x(t)$ , a function of time  $t$ , is expressed as a composition of several time localized shifted and scaled basis functions,  $\psi((t-b)/a)$  where ‘ $b$ ’ and ‘ $a$ ’ are the shifting and scaling parameters, respectively. The shift or translational parameter centers the wavelet function so that information can be obtained about the signal around the location  $t = b$ . The dilation or scale parameter, ‘ $a$ ’ can be varied to compress or extend the basis function to control the range of frequencies about which information can be obtained in the vicinity of the location  $t = b$ , by wavelet transformation. Wavelet transform converts an initial data sequence representing a chosen length of input signal  $x(t)$  into a new 2-D sequence, which consists of the coefficients  $W_\psi x(a,b)$  and is defined by

$$W_\psi x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi\left(\frac{t-b}{a}\right) dt. \quad (1)$$

The wavelet transform is an integral transform which convolutes the wavelet basis function  $\psi(t)$ , called the mother wavelet with the function  $x(t)$  being analyzed to generate the wavelet coefficients. The wavelet coefficients expressed in Eq. (1) provides temporal information of the function  $x(t)$  at a scale of ‘ $a$ ’ corresponding to the frequencies at that scale given by the Fourier transformation of  $(1/\sqrt{a})\psi((t-b)/a)$  i.e.  $\sqrt{a}\psi(a\omega)e^{-i\omega b}$ ;  $\hat{\psi}(\omega)$  being the Fourier transform of the mother basis  $\psi(t)$ . The coefficient  $W_{\psi}x(a,b)$ , at a given scale of ‘ $a$ ’, is a function of a shift parameter ‘ $b$ ’ which reflects the concentration of the frequencies corresponding to the given scale, around the time  $t = b$ . This property of wavelet coefficients will be used here to analyze vibration signals corresponding to the bands of frequencies in which the natural frequencies lie and hence are the modal bands. The analysis of the filtered signals in bands using time–frequency tools will yield the modal properties including the mode shapes of a linear dynamical system.

### 2.2. Harmonic wavelet transform

Harmonic wavelet has the mother wavelet  $\psi(t)$  whose spectrum is exactly like a box so that its Fourier transform  $\hat{\psi}(\omega)$  is defined as

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} \quad \text{if } 2\pi \leq \omega \leq 4\pi, \\ = 0 \quad \text{elsewhere.} \tag{2}$$

Then by calculating the inverse Fourier transform  $\psi(t)$ , the corresponding complex wavelet is

$$\psi(t) = \frac{(e^{i4\pi t} - e^{i2\pi t})}{i2\pi t} \tag{3}$$

with real and imaginary parts. The introduction of a complex function allows two real wavelets to be represented by a single expression. The real part of  $\psi(t)$  represents an even wavelet and the imaginary part represents an odd wavelet. The Fourier transform of the general baby wavelet, at level ‘ $j$ ’ (i.e. scaled by  $2^j$ ), and translated by ‘ $b$ ’, is defined as

$$\hat{\psi}(\omega) = \left(\frac{1}{\sqrt{2\pi}}\right)\sqrt{2^{-j}}e^{i\omega b/2^j} \quad \text{for } 2\pi \leq \frac{\omega}{2^j} \leq 4\pi, \\ = 0 \quad \text{elsewhere.} \tag{4}$$

On inverse Fourier transform, Eq. (4) gives

$$\psi\left(\frac{t-b}{2^j}\right) = \frac{(e^{i4\pi((t-b)/2^j)} - e^{i2\pi((t-b)/2^j)})}{i2\pi((t-b)/2^j)}. \tag{5}$$

The harmonic wavelets have been found particularly suitable for vibration analysis because their harmonic structure is similar to the naturally occurring vibration signals of the structures and therefore they correlate well with experimental signals [21].

### 2.3. Modified Littlewood–Paley (L–P) basis

An equivalent of the Harmonic wavelet, when the basis function is real, is L–P wavelet. This wavelet basis function is defined by

$$\psi(t) = \frac{1}{2\pi} \frac{\sin(4\pi t) - \sin(2\pi t)}{t}. \tag{6}$$

A possible variation of the wavelet is one, which retains the characteristic of the basis function (close to transient vibration signals, i.e. oscillatory and decaying) but could reduce the frequency bandwidth of the mother wavelet. Hence, the derived modified wavelet is called the modified L–P wavelet and has been proposed and used by Basu and Gupta [12,13]. The shifted and scaled version of this is called the baby

modified L–P wavelets. This wavelet basis has also been used by Basu [22,23] for damage detection in structures.

The modified L–P basis function is defined by

$$\psi(t) = \frac{1}{\pi\sqrt{\sigma-1}} \frac{\sin(\sigma\pi t) - \sin(\pi t)}{t}, \quad (7)$$

where  $\sigma$  (is a scalar)  $> 1$ . In frequency domain the wavelet basis can be represented by

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi(\sigma-1)}} \quad \text{for } \pi \leq |\omega| \leq \sigma\pi, \\ = 0 \text{ elsewhere.} \quad (8)$$

By choosing appropriate values for the bandwidth, the frequency content of the mother wavelet can be adjusted. If for numerical computation the scaling parameter is discretized as,  $a_j = \sigma^j$  (in an exponential scale), then the scaled version of the mother basis function has mutually non-overlapping frequency bands and are also orthogonal. This property can be conveniently utilized to detect natural frequencies and modal properties for the dynamical systems as can be seen in the following sections.

### 3. Proposed methodology

A methodology to extract the natural frequencies, mode shapes and modal damping from the ambient free vibration response of a linear mdof system is presented here. The responses  $\mathbf{x}(t)$  of a  $n$ -dof classically damped linear system under free vibration condition is governed by the differential equation

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0} \quad (9)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the mass, damping and stiffness matrices with the over dot representing the differentiation with respect to time. To decouple Eq. (9) into  $n$ -sdof equations, the following transformation is used

$$\mathbf{x}(t) = \mathbf{\Phi}\mathbf{u}(t), \quad (10)$$

where  $\mathbf{\Phi}$  is the modal matrix or the matrix of the mode shapes and  $\mathbf{u}(t)$  is the vector of modal responses. The  $k$ th modal response can be obtained from the decoupled modal equations

$$\ddot{u}_k + 2\rho_k\omega_{n_k}\dot{u}_k + \omega_{n_k}^2 u_k = 0, \quad k = 1, 2, \dots, n. \quad (11)$$

In Eq. (11),  $\rho_k$ ,  $\omega_{n_k}$  are the  $k$ th damping ratio and natural frequency respectively. The  $m$ th dof or state of response can be represented by a linear combination of mode shapes and modal responses as

$$x_m = \sum_{k=1}^n \phi_m^k u_k, \quad m = 1, 2, \dots, n. \quad (12)$$

The solution of Eq. (11) is given by

$$u_k(t) = e^{-\rho_k\omega_{n_k}t} [C_{1_k} \cos(\omega_{d_k}t) + C_{2_k} \sin(\omega_{d_k}t)], \quad (13)$$

where  $C_{1_k}$  and  $C_{2_k}$  are arbitrary constants to be obtained from the initial conditions [i.e.  $u_k(0)$  and  $\dot{u}_k(0)$ ] and  $\omega_{d_k} = \omega_{n_k}\sqrt{1-\rho_k^2}$ . Eq. (13) can be written as

$$u_k = \alpha_k(t) \cos(\omega_{d_k}t + \theta_k), \quad (14)$$

where  $\alpha_k(t)$  is a slowly varying function of time for lightly damped system, i.e.  $\rho_k \ll 1.0$  and  $\theta_k$  is the phase angle. The response  $u_k(t)$  can be reasonably considered to be a narrow banded signal with frequencies around  $\omega_{d_k}$ . Wavelet transformation of Eq. (12) gives

$$W_\psi x_m(a, b) = \sum_{k=1}^n \phi_m^k W_\psi u_k(a, b), \quad m = 1, 2, \dots, n. \quad (15)$$

The normalized energy  $E_j(x_m)$  for the response  $x_m$ , in the frequency band corresponding to a scaling factor  $a_j = \sigma^j$  with the index 'j', can be represented by a proportional quantity as

$$E_j(x_m) \propto \frac{1}{a_j^2} \int W_{\psi}^2 x_m(a_j, b) db; \quad m = 1, 2, \dots, n. \tag{16}$$

Let the natural frequencies  $\omega_{n_1}, \omega_{n_2}, \dots, \omega_{n_n}$  be contained in the bands with scale or dilation parameter indices,  $j_1, j_2, \dots, j_n$  respectively. Since, response of the  $k$ th mode  $u_k$ , is narrow banded with frequencies around  $\omega_{d_k}$  (where  $\omega_{n_k} \approx \omega_{d_k} \in [\pi/a_{j_k}, \sigma\pi/a_{j_k}]$ , for lightly damped system) and  $\hat{\psi}(a_j\omega)$  has frequencies in the  $j$ th band within the interval  $[\pi/a_j, \sigma\pi/a_j]$

$$W_{\psi} u_k(a_j, b) = \int u_k(t) \psi\left(\frac{t-b}{a_j}\right) dt = \sqrt{a_j} \int \hat{u}_k(\omega) \hat{\psi}(a_j\omega) e^{-i\omega b} d\omega \approx 0 \quad \text{if } j \neq j_k. \tag{17}$$

On using Eqs. (15) and (17), one gets

$$\begin{aligned} W_{\psi} x_m(\sigma^j, b) &\approx 0 && \text{if } j \neq j_k, \quad k = 1, 2, \dots, n \\ &= \phi_m^k W_{\psi} u_k(a_{j_k}, b) && \text{if } j = j_k, \quad k = 1, 2, \dots, n. \end{aligned} \tag{18}$$

Thus, Eqs. (16) and (18) lead to

$$\begin{aligned} E_j(x_m) &\neq 0 && \text{if } j = j_k, \quad k = 1, 2, \dots, n, \\ &\approx 0 && \text{otherwise.} \end{aligned} \tag{19}$$

To detect the bands of frequencies in which the natural frequencies lie, the energy corresponding to each band is calculated for a particular state of response using Eq. (16). The bands, which do not contain the natural frequencies, lead to insignificant energy contribution. Hence, the first  $n$  bands with significant energy content are the bands where the natural frequencies are located. These bands are in increasing order corresponding to the first ' $n$ ' natural frequencies, i.e. the lowest frequency band has the first natural frequency and so on.

However, the chosen bands may lead to bands with relatively broad interval in which the natural frequencies lie. To refine the estimates into finer intervals, so that natural frequencies could be determined to a better precision, wavelet packets are used. This is an extension of wavelet transform to provide level by level time–frequency description and is easily adaptable for the modified L–P basis. The wavelet packet enables extraction of information from signals with an arbitrary time–frequency resolution satisfying the product constraint in the time–frequency window. In this technique, to refine the estimation of the  $k$ th natural frequency,  $\omega_{n_k}$ , located in the  $j_k$ th band, i.e. with frequency band  $[\pi/a_{j_k}, \sigma\pi/a_{j_k}]$ , further re-division is carried out. If it is required to further subdivide the band in ' $M$ ' parts, then again an exponential scale is used to divide the band so that the corresponding time-domain function forms a wavelet basis function. In this approach (also sometimes, termed as sub-band coding, [24]), for the  $j_k$ th band, the mother basis for the packet,  $\psi^s(t)$  is formed with the frequency domain description

$$\begin{aligned} \hat{\psi}^s(\omega) &= \frac{1}{\sqrt{2\pi(\delta-1)}} && \text{for } \pi \leq |\omega| \leq \delta\pi \\ &= 0 && \text{elsewhere,} \end{aligned} \tag{20}$$

where  $\delta^M = \sigma$  [with  $\delta$  (a scalar)  $> 1$ ]. The corresponding time-domain description is given by

$$\psi^s(t) = \frac{1}{\pi\sqrt{(\delta-1)}} \frac{\sin(\delta\pi t) - \sin(\pi t)}{t}. \tag{21}$$

The frequency band for the  $p$ th sub-band within the original  $j_k$ th band is the interval  $[\delta^{p-1}\pi/a_{j_k}, \delta^p\pi/a_{j_k}]$ . The basis function for this is denoted by  $\psi_{a_{j_k}, b}^{\delta^p}(t)$ . The wavelet coefficient in this sub-band is denoted by  $W_{\psi_{\delta^p}, x_m}(a_{j_k}, b)$ . Using the wavelet coefficients in these sub-bands and then applying similar expression as in Eq. (19), to estimate the relative energies in the sub-bands, the natural frequencies can be obtained more precisely.

Once the natural frequencies are obtained and the corresponding bands are identified, the sub-band containing the  $k$ th natural frequency with scale parameter  $j_k$  and the sub-band parameter ‘ $p$ ’ are considered to obtain the  $k$ th mode shape. From Eq. (15), we get

$$W_{\psi_{sp}} x_m(a_j, b) = \sum_{k=1}^n \phi_m^k W_{\psi_{sp}} u_k(a_{j_k}, b); m = 1, 2, \dots, n. \tag{22}$$

Now, considering the responses of two states or dof in a mdof system, (with one arbitrarily chosen as  $m = 1$ , without loss of generality), the ratio of wavelet coefficients of the two considered degrees of freedom at an instant of time  $t = b$ , corresponding to band  $j_k$  with sub-band,  $p$ , (using Eqs. (18) and (22)) yields

$$\prod_m^{j_k} = \frac{W_{\psi_{sp}} x_m(a_{j_k}, b)}{W_{\psi_{sp}} x_1(a_{j_k}, b)} = \frac{\phi_m^k}{\phi_1^k}. \tag{23}$$

Thus it is seen that the computed ratio of the wavelet coefficients are invariant with ‘ $b$ ’. Hence, computing these ratios for different states corresponding to different values of ‘ $m$ ’ and assuming  $\phi_1^j = 1$  (without loss of generality), the mode shape for the  $k$ th mode (in  $j_k$  band with further sub-band division) can be obtained as

$$\{\phi_m^k\} = \left\{ \prod_m^{j_k} \right\}, \quad m = 1, 2, \dots, n. \tag{24}$$

Thus, mode shape for any other mode can be obtained in a similar manner for  $k = 1, 2, \dots, n$ . To estimate the modal damping of a mdof system, let us consider Eq. (14), which gives the  $k$ th modal response. This response  $u_k(t)$  is narrow banded around  $\omega_{n_k}$ , and is modulated by a slowly varying time function,  $\alpha_k(t) = A_k e^{-\rho_k \omega_{n_k} t}$  where,  $A_k$  is a constant. Thus, if this expression is used to evaluate the wavelet coefficients in Eq. (15), then the term  $\alpha_k(t)$  can be approximated by  $\alpha_k(b) = A_k e^{-\rho_k \omega_{n_k} b}$  i.e.  $\alpha_k(t)$  evaluated at  $t = b$ , and considered as a constant over the integral. This is because,  $\psi((t - b)/a)$  is more oscillatory and faster decaying as compared to  $\alpha_k(t)$  and is localized around  $t = b$ . Further, since both  $\cos(\omega_{d_k} t + \theta_k)$  and  $\psi((t - b)/a)$  are narrow banded around  $\omega_{n_k}$ , evaluation of the integral for the wavelet coefficient for the original band,  $j_k$  and the sub-band,  $p$  containing the  $k$ th natural frequency leads to

$$W_{\psi_{sp}} x_m(a_{j_k}, b) = \tilde{K}_{p_k} e^{-\rho_k \omega_{n_k} b}, \tag{25}$$

where  $\tilde{K}_{p_k}$  is a factor depending on the  $p$ th sub-band containing the  $\omega_{n_k}$  natural frequency. On evaluating Eq. 25 at ‘ $b$ ’ and  $(b + 2\pi/\omega_{n_k})$  and taking logarithm of the ratio of these expressions, the modal damping is obtained as

$$\rho_k = \frac{1}{2\pi} \ln \frac{W_{\psi_{sp}} x_m(a_{j_k}, b)}{W_{\psi_{sp}} x_m[a_{j_k}, (b + 2\pi/\omega_{n_k})]}. \tag{26}$$

**4. Mdf model and results**

A mdof model is used to simulate the displacement response and to show the application of the proposed identification methodology. The mdof system, as shown in Fig. 1, is considered. The displacement of the  $i$ th mass relative to the support is denoted by  $x_i(t)$ . At first, simulation is carried out for a 3 dof system ( $n = 3$ ). The masses are  $m_1 = 300$  kg,  $m_2 = 200$  kg and  $m_3 = 200$  kg and the spring stiffnesses are  $k_1 = 36\,000$  N/m,  $k_2 = 24\,000$  N/m and  $k_3 = 36\,000$  N/m respectively. The damping ratio is assumed to be 5% for all modes.

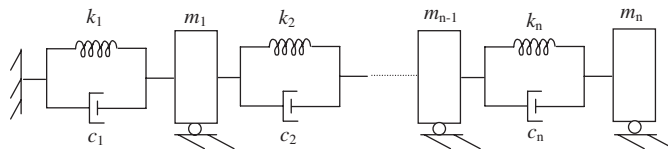


Fig. 1. mdof model.

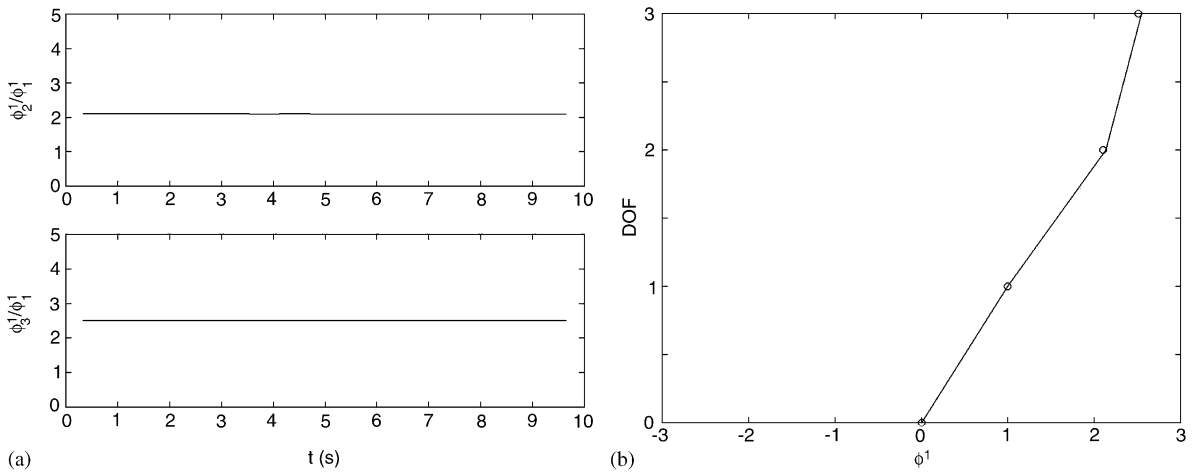


Fig. 2. (a) Ratio of modal responses at first natural frequency, (b) actual and estimated first mode shape of the 3 dof system ('\_' actual, 'o' estimated).

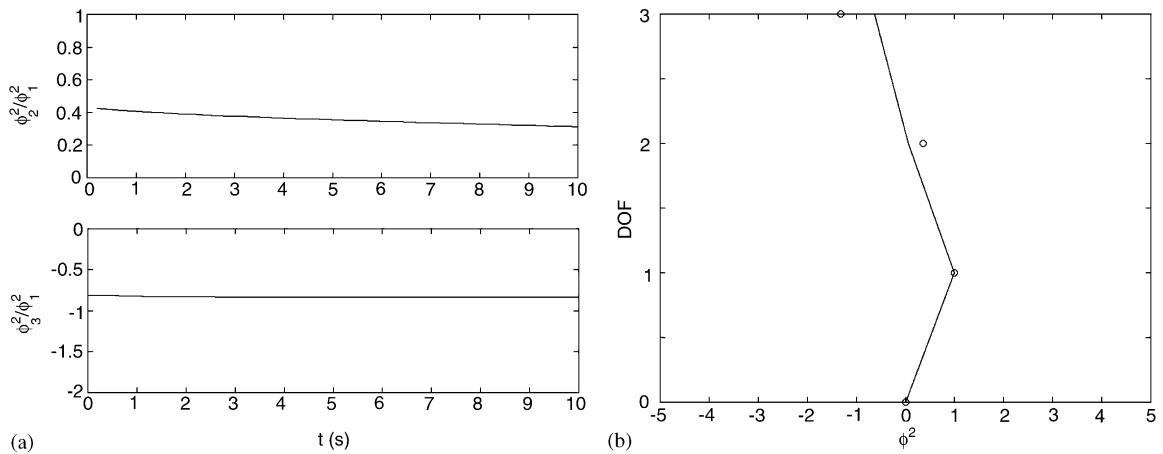


Fig. 3. (a) Ratio of modal responses at second natural frequency, (b) actual and estimated second mode shape of the 3 dof system ('\_' actual, 'o' estimated).

The system is subjected to initial displacement of  $x_1(0) = x_2(0) = x_3(0) = 1$  for the 3 dof. Using these, the ambient vibration response is simulated.

Modified L–P wavelet is used to decompose the signals into different frequency levels. Initially the response energy is calculated for each degree of freedom in frequency bands with  $\sigma = 2^{1/4}$  to broadly identify the bands that contain the natural frequencies. These bands are further divided into sub-bands using wavelet packets. Figs. 2(a), 3(a) and 4(a) represent the ratio of wavelet coefficients of displacements  $x_2(t)$  and  $x_3(t)$  with respect to the wavelet coefficients of displacement  $x_1(t)$  over time for the three frequency sub-bands containing the three natural frequencies respectively. Since the response for different dof attain same phase during modal vibration, these ratios are practically constant over time. The natural frequencies are estimated as the central frequency of the corresponding sub-bands and the corresponding mode shapes are obtained by averaging the ratios shown in Figs. 2(a), 3(a) and 4(a) using sub-band coding as discussed in Section 3. The results are summarized in Table 1. Figs. 2(b), 3(b) and 4(b) show the plot of the mode shapes estimated using the proposed method and compared with the actual for the first three modes respectively. From Figs. 2(b), 3(b) and 4(b) and Table 1, it can be noticed that the first modal frequency along with other modal parameters are

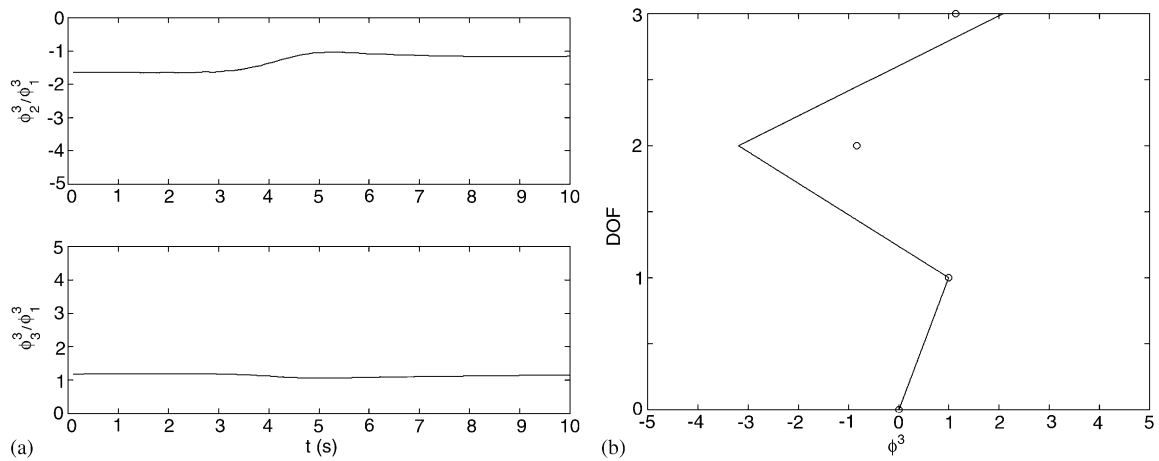


Fig. 4. (a) Ratio of modal responses at third natural frequency, (b) actual and estimated third mode shape of the 3 dof system ('\_' actual, 'o' estimated).

Table 1  
Actual and estimated modal parameters of 3 dof system using modified L–P wavelet

Mode	Natural frequency (rad/s)		Normalized mode shape						Damping ratio (%)	
			$X_1$		$X_2$		$X_3$			
	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated
1	5.39	5.40	1.00	2.13	2.10	2.54	2.51	0.05	0.04	
2	13.99	13.40	1.00	0.06	0.36	-0.63	-0.83	0.05	0.02	
3	21.34	20.60	1.00	-3.19	-1.32	2.07	1.14	0.05	0.01	

Table 2(a)  
Actual and estimated natural frequencies and damping ratios of 5 dof system using modified L–P wavelet with 5% damping ratios in all modes

Mode	Natural Frequency (rad/s)		Damping ratio (%)	
	Actual	Estimated	Actual	Estimated
1	2.84	2.88	0.05	0.04
2	7.69	7.69	0.05	0.03
3	12.35	12.59	0.05	0.02

estimated satisfactorily, which proves the effectiveness of the proposed method. It can also be observed that although the frequency ratios of wavelet coefficients for higher modes are constant over time, the accuracy in estimation reduces for the higher modes. This is due to the fact that the energy content in bands containing the higher modal frequencies reduces with increase in mode number.

To investigate the accuracy of the estimation for higher modes in further detail, a 5 dof model with two additional masses and springs ( $m_4 = 250$  kg and  $m_5 = 350$  kg;  $k_4 = 20\,000$  N/mm and  $k_5 = 15\,000$  N/mm) are considered while the modal damping ratio is kept as 5% for all modes. The displacement response relative to the base is simulated using initial conditions  $x_4(0) = x_5(0) = 1$  along with those used in the 3 dof system. The first three modes are estimated in a similar way as in case of the 3 dof system and the results are summarized in Tables 2(a,b). Figs. 5(a,b)–7(a,b) show the corresponding modal wavelet coefficient ratios and estimated mode shapes for the first three modes. From these results, it can be observed that the modal frequencies and mode



Table 2(b)  
Actual and estimated mode shapes of 5 dof system using modified L–P wavelet with 5% damping ratios in all modes

Mode	Normalized mode shape									
	$X_1$		$X_2$		$X_3$		$X_4$		$X_5$	
	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated
1	1.00	2.40	2.37	3.22	3.18	4.45	4.39	5.48	5.39	
2	1.00	1.76	1.73	1.69	1.66	0.56	0.69	-1.49	-1.58	
3	1.00	0.59	0.89	-0.18	-0.59	-1.30	-1.02	0.51	0.63	

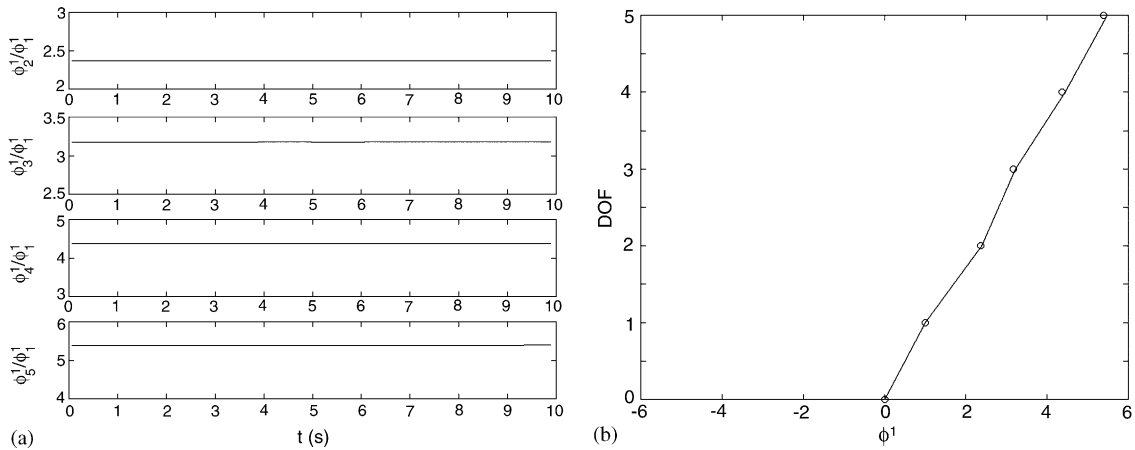


Fig. 5. (a) Ratio of modal responses at first natural frequency, (b) actual and estimated first mode shape of the 5 dof system (‘\_’ actual, ‘o’ estimated).

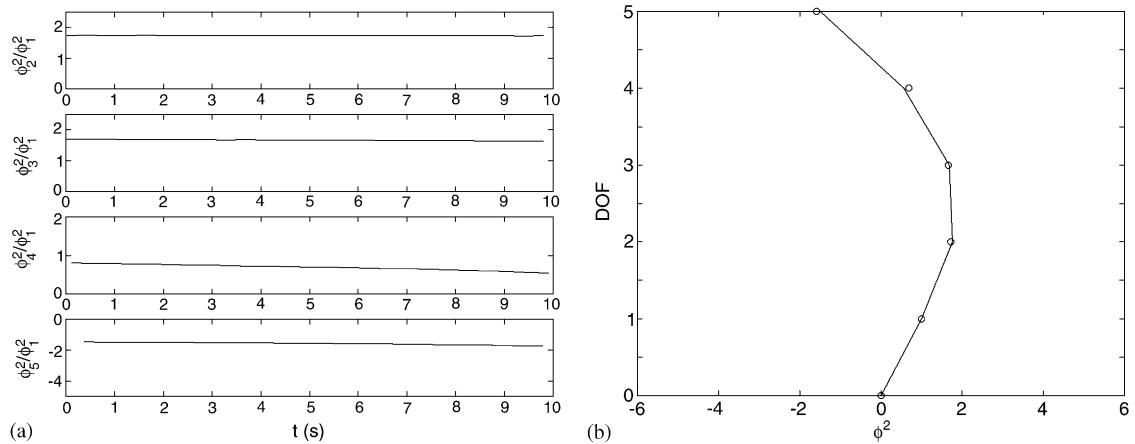


Fig. 6. (a) Ratio of modal responses at second natural frequency, (b) actual and estimated second mode shape of the 5 dof system (‘\_’ actual, ‘o’ estimated).

shapes can be estimated with high level of accuracy for the first two modes as opposed to just only the first modal parameters being estimated with high level of accuracy for the 3 dof system.

For the 5 dof system the estimation accuracies start deteriorating from the third mode onward and are poorer for the last two modes. The results are consistent with the results from the 3 dof system. This indicates that more number of modes and their modal properties can be identified with greater accuracy for systems

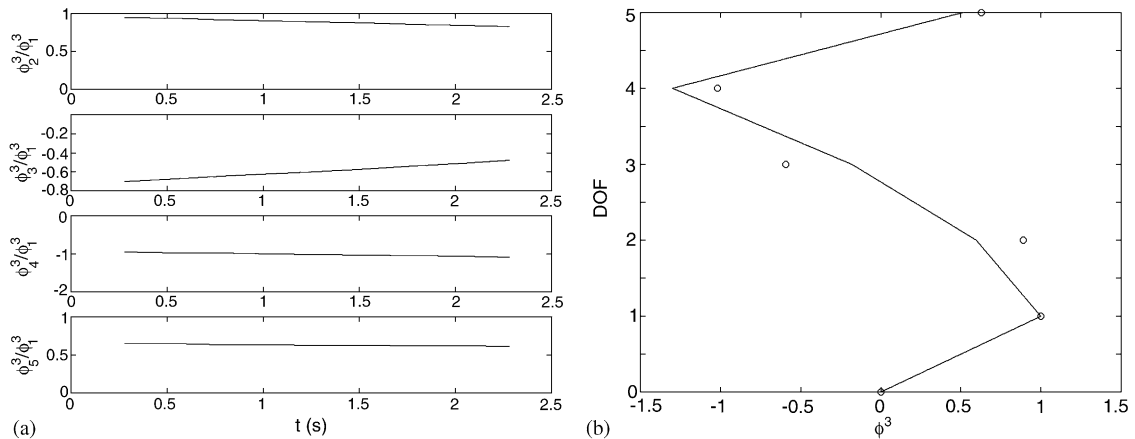


Fig. 7. (a) Ratio of modal responses at third natural frequency, (b) actual and estimated third mode shape of the 5 dof system ('\_' actual, 'o' estimated).

Table 2(c)

Actual and estimated mode shapes of 5 dof system using modified L–P wavelet with 2% damping ratios in all modes

Mode	Normalized Mode Shape									
	$X_1$		$X_2$		$X_3$		$X_4$		$X_5$	
	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated
1	1.00	2.40	2.39	3.22	3.20	4.45	4.41	5.48	5.46	
2	1.00	1.76	1.76	1.69	1.67	0.56	0.64	–1.49	–1.47	
3	1.00	0.59	0.79	–0.18	–0.38	–1.30	–1.14	0.51	0.59	

with relatively greater number of degrees of freedom. Also, modal damping ratios can be estimated with reasonable accuracy with the level of accuracy deteriorating with higher modes. The higher modal damping ratios tend to be underestimated.

To investigate the efficiency of wavelet-based system identification technique proposed, the 5 dof system is further considered with 2% damping ratios in all modes. Rest other parameters of the system and the initial conditions are kept unchanged. The first three mode shapes are estimated in a similar way as done earlier and the results are presented in Table 2c. It can be seen that the mode shapes are estimated accurately for this system even with lower level of damping as compared to the earlier one. However, it has been found that the accuracy of prediction of modal damping ratios falls for system with low level of damping. This is possibly because of the fact that the response and hence the wavelet coefficients decay at a very slow rate, posing estimation problem.

## 5. Conclusion

In this paper a wavelet-based methodology for identification of the modal parameters of a linear mdof system from ambient vibration records has been proposed. The key feature of the work is to present a technique for identifying the natural frequencies, mode shapes and associated modal damping ratios of a mdof system on the basis of wavelet packets, where the wavelet basis used is a modified form of L–P function. An efficient wavelet-based algorithm using modified L–P basis has been used in the identification methodology, which is computationally simple as well. The example cases shown that the methodology can estimate the parameters accurately. The proposed method can be extended to identify system parameters under various kinds of excitations, particularly when the input is unknown.

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