Available online at www.sciencedirect.com

SCIENCE DIRECT®

Journal of Sound and Vibration 296 (2006) 539–553

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

High frequency vibration energy transfer in a system of three plates connected at discrete points using statistical energy analysis

K. Renji*, M. Mahalakshmi

Structures Group, ISRO Satellite Centre, Bangalore, 560017 India

Received 4 January 2005; received in revised form 26 October 2005; accepted 7 January 2006
Available online 13 June 2006

Abstract

Vibration energy transfer in a system of three plates separated by a small distance and connected at a few discrete points, like solar panels in a spacecraft, is investigated. Coupling loss factors are obtained experimentally using the power injection technique. The system is then subjected to the acoustic excitation in a reverberant chamber. The measured responses of the inner plate are significant. But the measured responses of the inner plates are higher than the responses estimated based on the coupling loss factors obtained. When the system is subjected to mechanical excitation the measured responses of the inner plate closely match with the estimated responses. The difference is perhaps due to the sound radiated from the outer plates not being considered for the calculation, requiring further studies.

1. Introduction

Spacecraft experience high intensity acoustic loads during the launch. One of the elements of the spacecraft which responds largely to acoustic excitation is the solar panel as they are mounted exterior to the spacecraft and they have very low mass per unit area. Generally a few solar panels are stacked together and they are connected at a few locations through hinges and hold downs. The vibration responses of the outer most panel are expected to be higher since it is subjected to direct acoustic excitation. The inner panel is not directly coupled to the acoustic field and its responses are due to its connectivity with the outer panel. Objective of the present work is to study the vibration energy transfer in such a system. As the acoustic loads excite mainly the bending modes, energy in bending vibration only is considered.

Dynamic responses of systems at high frequencies are usually studied in the framework of statistical energy analysis (SEA) developed by Lyon [1] and others. In SEA, the entire system is considered to be an assembly of a number of structural elements, called subsystems and the power balance of these subsystems forms the basis for SEA based calculations. The system is characterized in SEA by the coupling loss factors and the dissipation loss factors of the subsystems/interconnections involved. Accuracy of the prediction of response

*Corresponding author.
E-mail address: renji@isac.gov.in (K. Renji).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.01.064

Nomenclature

E	energy of a subsystem
f	frequency, in Hz/force
f_a	actual force
f_m	measured force
f_m	applied force
j	complex operator
m	mass of a structure/equipment
M	mass of impedance head and attachment elements
$n(f)$	modal density
$\mathrm{Re}(x)$	real part of x
$\mathrm{Im}(x)$	imaginary part of x
Y	driving point admittance
Y_a	actual driving point admittance
Y_m	measured driving point admittance
Y_M	admittance due to impedance head and attachment elements
v	velocity of a structure
η_d	dissipation loss factor
η_{ij}	coupling loss factor for subsystem i to j
ϕ_{xx}	spectral density of random process x
ϕ_{xy}	cross spectral density between random processes x and y
π_{in}	input power
ω	circular frequency, in rad/s
$\langle\ \rangle_x$	average over the domain x

Note: Since the processes considered are stationary random, the dynamic variables discussed are the long time averaged quantities and in such cases the notation for the averaging is dropped. For example $\langle a^2 \rangle_t$ is written as a^2.

using SEA greatly depends on, other than the limitations of SEA, the accuracy of the available SEA parameters.

To study the high-frequency vibration energy transfer in the system under investigation, it is essential to have the information on the coupling loss factors of plates in bending vibration connected at a few discrete points. These results are not reported in the literature. In many types of structural connections the coupling loss factors cannot be estimated theoretically and they are obtained experimentally [2]. In this study these parameters of a typical system under investigation are determined experimentally. The system is then subjected to reverberant acoustic excitation and the measured responses are compared with the responses estimated using the measured coupling loss factors.

2. Details of the system

As discussed earlier, the system considered has three plates which are connected at four locations, called hold down points. They are connected by M12 allen bolts, one bolt each at each hold down point. A hole of 15 mm diameter exists at each hold down point. The adjacent plates are separated by 20 mm because of the hold down bushes. The bushes are cylindrical having outer diameter of 24.7 mm and inner diameter of 20.7 mm. The height of eight bushes, which are made of aluminium is 20 mm. The bolt passes through three plates and two bushes and tightened using nuts. All the four edges of the plates are free. The four connecting locations are away from the edges. The details are shown in Figs. 1 and 2. This configuration, that is three plates connected at discrete interior points using bolts with a small gap between them, is identical to some typical solar arrays in spacecraft. The three plates are identical and they are numbered 1, 2 and 3. Plate 2 is the middle plate. It is to be noted that plate 2 is not connected to plates 1 or 3 through the bolt but they are coupled through the bushes.

Each plate is having dimensions 1.1 m × 0.9 m. The thickness of each plate is 2 mm. The plates are made of aluminium alloy. The Young's modulus of the material is $7.2 \times 10^{10}\,\mathrm{N/m^2}$ and the Poisson's ratio is 0.3. The density of the material is $2800\,\mathrm{kg/m^3}$. The modal density of each plate is calculated [1] as 0.161/Hz and the critical frequency of acoustic radiation of the plate is calculated [1] as 6208 Hz. For estimating the critical frequency the speed of the acoustic wave in air is assumed to be 346 m/s.

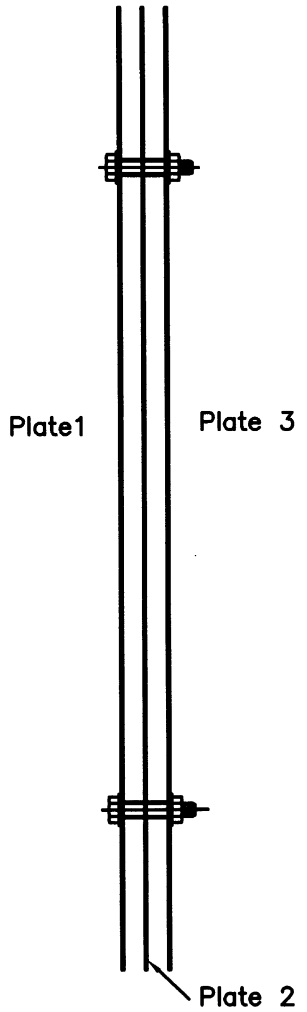

Fig. 1. A schematic view of the three-plate system.

3. Measurement of driving point admittance

Driving point admittance is the most important parameter to be measured in determining coupling loss factor. Hence the accuracy of the measurement of the driving point admittance values is verified first. The modal density of a structure at frequency f, denoted by $n(f)$, is related to its driving point admittance by the relation [3]

$$n(f) = 4m\langle \mathrm{Re}(Y)\rangle_a, \tag{1}$$

where m is the mass of the structure and Y is the real part of the driving point admittance. The driving point admittance, in general, can have both real and imaginary parts, that is $Y = \mathrm{Re}(Y) + \mathrm{j}\,\mathrm{Im}(Y)$. The real part of the driving point admittance is averaged spatially. If the modal density of the structure can be theoretically determined, the experimentally obtained modal density values give the indication of the accuracy of the measurement of the driving point admittance.

3.1. Measurement technique

Driving point admittance is the ratio of the Fourier transform of the velocity of the driving point to the Fourier transform of the driving force. The admittance can be determined using the relation

$$Y = \phi_{fv}/\phi_{ff}, \tag{2}$$

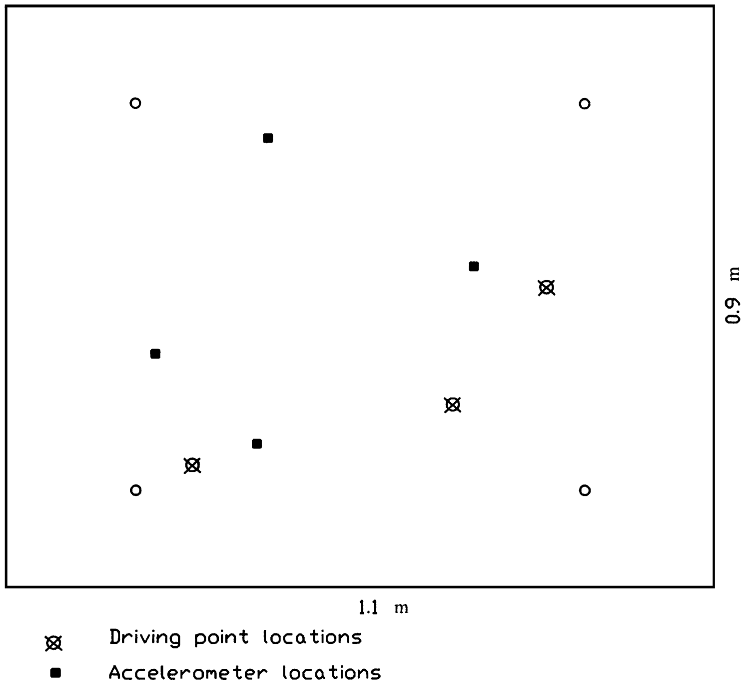

Fig. 2. Driving point and accelerometer locations.

where ϕ_{ff} is the auto-spectral density of the force and ϕ_{fv} the cross-spectral density between the force and the velocity.

The driving point admittance is normally obtained using an impedance head. Clarkson and Pope [4] had reported that the measured driving point admittance values were influenced by the mass of the impedance head and the attachment elements like the stud and cube. To take into account this effect, Brown and Norton [5] suggested the use of a correction factor as given below:

$$Y_a = Y_m/\{1 - (Y_m/Y_M)\}, \tag{3}$$

where Y_a is the actual admittance and Y_m the measured admittance. The parameter Y_M is the admittance due to the impedance head and the attachment elements. If M is the mass of the impedance head and the attachment elements, its admittance at frequency ω is given by

$$Y_M = 1/(\mathrm{j}\omega M). \tag{4}$$

The above equation is valid for a wide frequency range until the frequency of resonance of the impedance head and the attachment elements. Alternately the parameter Y_M could be determined by exciting the impedance head and the attaching stud. The measured admittance can have real and imaginary parts, that is $Y_m = \mathrm{Re}(Y_m) + \mathrm{jIm}(Y_m)$. The real and the imaginary parts of the actual admittance can be shown to be [6]

$$\mathrm{Re}(Y_a) = \mathrm{Re}(Y_m)/\{[1 + \omega M \mathrm{Im}(Y_m)]^2 + [\omega M \mathrm{Re}(Y_m)]^2\}. \tag{5}$$

$$\mathrm{Im}(Y_a) = [\omega M\{\mathrm{Re}^2(Y_m) + \mathrm{Im}^2(Y_m)\} + \mathrm{Im}(Y_m)][\mathrm{Re}(Y_a)/\mathrm{Re}(Y_m)]. \tag{6}$$

When measuring the driving point admittance of thin structural elements there could be an error due to shaker–structure interaction which could be very significant near the natural frequencies [7]. Also, this error is sensitive to external noise. It was demonstrated that these errors could be minimised by measuring the driving point admittance in the following manner [7]:

$$Y = \phi_{sv}/\phi_{sf}, \tag{7}$$

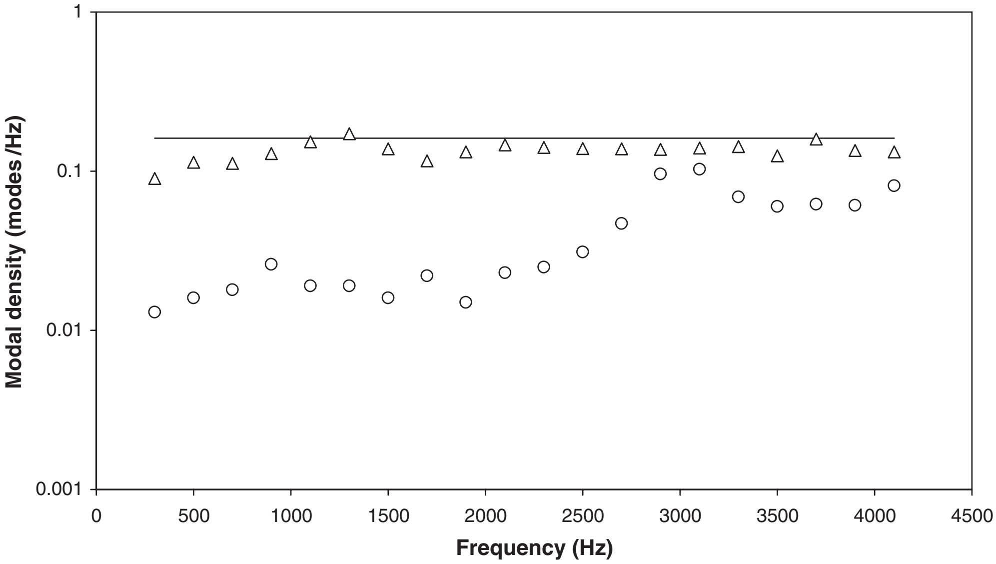

Fig. 3. Modal density of plate 1: —, theory; △, three-channel technique; ○, two-channel technique.

In the above relations s is the signal which drives the power amplifier and φ_{sv} the cross-spectral density between the drive signal and the velocity and so on. This technique is called the three-channel technique.

3.2. Experimental results

As mentioned earlier the modal density of the plate used in the present study is theoretically estimated as 0.161 modes/Hz. Modal density of plate 1 is obtained experimentally using Eq. (1) and the results are given in Fig. 3. Plate 1 is hung using thin wire ropes and the driving point admittance values are measured at three randomly selected locations. The excitation is applied to the plate using shaker systems through a stringer mounted with an impedance head. The impedance head used has a mass of 20 g. An aluminium cube having a mass of 0.6 g is bonded on the location on the plate at which it is to be excited and the impedance head is screwed on to the cube. A band limited white noise up to 5000 Hz is applied to the shaker through a power amplifier. The results are presented up to 4000 Hz with a resolution of 200 Hz. Corrections due to the mass of the impedance head and the attachment elements are applied. To arrive at the correction factors, the impedance head along with the cube is excited and it is seen that the admittance is purely imaginary as given by Eq. (4). The mass of the element to be considered in Eq. (4) is found to be 8 g. The driving point admittance is first measured using the two-channel technique and the results show that measured driving point admittance values are in large error (Fig. 3). The driving point admittance values are then measured using the three-channel technique. The results show that the measurement of the driving point admittance values is carried out with very good accuracy and the same technique is used in further experiments.

4. Coupling loss factors

For a structural system consisting of n subsystems, the SEA system matrix is $n \times n$ and there are n^2 system parameters [1]. Out of n^2 parameters, there are n dissipation loss factors, denoted by η_i, and n^2-n values of coupling loss factors, denoted by η_{ij}. The coupling loss factors of this system are determined experimentally using the power injection technique.

4.1. Experimental technique

Using the power injection method one can determine all the n^2 SEA parameters of the system experimentally. While using this technique, energies of various subsystems for a known arbitrary input power distribution are obtained. This will give n simultaneous equations in the system parameters. If the process is repeated n times for n independent power distributions, all the required n^2 parameters can be determined. The convenient way of using this experimental technique is to inject power into only one subsystem for a particular input power distribution. This is repeated for each subsystem. This ensures independent power distributions. To obtain n^2 parameters, the $n^2 \times n^2$ matrix has to be inverted. This can be carried out easily if the inverse of the system matrix is used in the power balance equations. In such a case, each experiment with a particular subsystem alone is being excited yields the elements of a column of the inverse of the system matrix. Redundancy in the estimation of the parameters can be incorporated, by using different input power distributions or using reciprocal relations of coupling loss factors or a combination of both the techniques. The number of measurements can be reduced if the dissipation loss factors of the subsystems are known and/or the reciprocal relationships between the coupling loss factors are used. The indirect coupling loss factors are also obtained while using the power injection method. This method was suggested originally by Lyon [1]. Later, Bies and Hamid [8] demonstrated this technique on two coupled plates and reported that the measured coupling loss factors were in agreement with the theoretical estimates. Clarkson and Ranky used this technique for some of the structural connections used in spacecraft [2].

Most convenient way of exciting the structure is through point excitation. One of the requirements in applying SEA is that the modal driving forces should be incoherent [9]. Fahy [10] pointed out that point excitation does not produce incoherent modal driving forces. Hence, when point excitation is used, the SEA equations are not valid. This drawback can be overcome by using many randomly located driving point positions. Bies and Hamid [8] concluded from experimental results that three randomly selected driving points would be adequate to get the average. The mean energy of the subsystem having a mass of m and spatial average of the mean square value of the velocity $\langle v^2 \rangle_x$ can be determined using the relation [1]

$$E = m\langle v^2\rangle x. \tag{8}$$

Since the measured velocity at the driving point include additional near field component, the velocity of the driving point should not be included in estimating the spatial averaged velocity as pointed out by Ranky and Clarkson [11].

The dissipation loss factors are also obtained using the power injection technique. It is important to note that the dissipation loss factors thus obtained are with the subsystems inter-connected and not with the subsystems in isolation. For conservative coupling the dissipation loss factors obtained using both the techniques match but if the coupling is non-conservative they differ. Experimentally derived system parameters may turn negative due to the presence of dissipation at the coupling points. The dissipation loss factor of a subsystem in isolation can be determined using energy method which is the most suitable method for SEA based calculations [11]. In this method the response levels at several locations are measured for a known input power. From the equality of the input power and the dissipated power, the dissipation loss factor is obtained. In other words for an input power of π_{in}, the dissipation loss factor, denoted by η_d is given by

$$\eta_d = \pi_{\text{in}}/(\omega m\langle v^2\rangle_x). \tag{9}$$

By measuring the input power and the spatial average of the mean square value of the velocity response of the structure, the dissipation loss factor of the structure can be determined using Eq. (9). Clarkson and Pope [4] demonstrated this technique for plates and cylinders. This method assumes that modal responses are independent and there is no inter-modal coupling [12]. In SEA based calculations, we require frequency averaged loss factors that can be achieved by using random excitation with suitable bandwidths.

4.2. Measurement of input power

The most important parameter to be measured in using the power injection technique is the input power. Generally point excitation is used to excite the structure. For this kind of excitation the input power, π_{in}, is determined using the relation

$$\pi_{\text{in}} = f_p^2 \text{Re}(Y), \tag{10}$$

where ${f_p}^2$ is the mean square value of the excitation force and $\mathrm{Re}(Y)$ the real part of the driving point admittance. In the present experiment the input power is determined using Eq. (10) for which it is necessary to measure the input force and the driving point admittance.

The excitation force is usually measured using an impedance head. The mass of the impedance head and the attachment elements influence the measurement of force as in the case of driving point admittance. Following the analysis by Brown and Norton [5], the actual force, denoted by f_a, is given by

$$f_a = f_m/\{1 + (Y/Y_M)\}, \tag{11}$$

where f_m is the measured force. Considering that the driving point admittance is complex in general, the expression for the mean square value of the actual force becomes [13]

$$f_a^2 = f_m^2/\{[1 - \omega M \mathrm{Im}(Y)]^2 + [\omega M \mathrm{Re}(Y)]^2\}. \tag{12}$$

It is to be noted that the actual driving point admittance has to be used in Eqs. (12) and (10) which could be obtained from the measured driving point admittance using Eqs. (5) and (6).

4.3. Experimental results

As mentioned earlier, the coupling loss factors are determined using the power injection technique. While using this method both coupling loss factors and the dissipation loss factors are obtained. A band limited white noise up to 5000 Hz is applied and the results are presented up to 4000 Hz with a resolution of 200 Hz.

Fig. 4. Test setup for coupling loss factor measurement.

The three-plate system is hung using metal wires and the experiments are conducted (Fig. 4). Each plate is excited at three randomly selected locations and the acceleration responses are measured at four locations on each plate. The driving point locations and the acceleration response measurement locations on a typical plate are shown in Fig. 2. To excite the inner plate, a hole is made on the outer plate such a way that the stringer can pass through it and the impedance head is then attached.

As mentioned earlier the coupling loss factors are determined up to 4000 Hz. The wavelength of the bending wave in this plate is 70 mm at 4000 Hz. Wavelength at any frequency less than 4000 Hz is higher than 70 mm. The dimensions of the cube used for the excitation is 10 mm $\times$ 7 mm. It can be seen that the dimensions are far less than the wavelength and hence the excitation can be considered as point excitation.

The accelerometers used have a mass of 0.5 g each. The impedance due to the mass of the accelerometer is 12.6 N s/m at 4000 Hz and the average driving point impedance of the plate considered in the present experiment is 137.7 N s/m. Hence, the mass loading effect of the accelerometer on the measured response up to 4000 Hz is expected to be negligible. The acceleration sensitivity of these accelerometers is approximately 1.5 pC/g and the resonance frequency is 32 kHz with a useful frequency range of 5–8000 Hz (+/−5%). Hence no corrections are applied in the measured acceleration levels.

Plate 1 is excited first and the acceleration responses are measured on all the three plates. Input force and driving point admittances are measured and corrections are applied as discussed earlier. This is repeated by exciting plate 2 and then plate 3. The loss factor matrix is first obtained and then the coupling loss factors and the dissipation loss factors are deduced. The coupling loss factors η_{12}, η_{13} and η_{32} are given in Table 1.

The dissipation loss factor of plate 1, without other plates being connected, is determined using energy method and the results are given in Table 1. This is carried out as part of the experiment to determine modal density where additionally the input force and acceleration response at four locations are measured. All the precautions to be taken for the measurement of input power discussed earlier are incorporated.

Additionally the coupling loss factors in a two-plate system are also determined experimentally using the same procedure.

4.4. Discussion of results

It is to be noted that the modal densities of the plates are equal. The experimental results show that the coupling loss factors η_{31}, η_{21} and η_{23} are approximately equal to the corresponding reciprocal coupling loss

Table 1
Loss factors of the three-plate system

Frequency (Hz)	η_{13}	η_{12}	η_{32}	η_d
300	0.0192	0.0055	0.0043	0.0193
500	0.0135	0.0048	0.0034	0.0166
700	0.0083	0.0024	0.0025	0.0118
900	0.0064	0.0018	0.0014	0.0082
1100	0.0045	0.0027	0.0016	0.0094
1300	0.0041	0.0021	0.0012	0.0081
1500	0.0021	0.0014	0.0009	0.0071
1700	0.0013	0.0011	0.0007	0.0085
1900	0.0011	0.0004	0.0004	0.0044
2100	0.0012	0.0006	0.0005	0.0031
2300	0.0011	0.0006	0.0002	0.0017
2500	0.0004	0.0006	0.0002	0.0015
2700	0.0004	0.0006	0.0003	0.0030
2900	0.0003	0.0007	0.0004	0.0024
3100	0.0002	0.0009	0.0004	0.0017
3300	0.0002	0.0006	0.0003	0.0014
3500	0.0001	0.0006	0.0002	0.0014
3700	0.0001	0.0007	0.0003	0.0010
3900	0.0002	0.0007	0.0002	0.0015
4100	0.0002	0.0007	0.0002	0.0016

factors η_{13}, η_{12} and η_{32} and hence the coupling loss factors η_{13}, η_{12} and η_{32} only are presented (Table 1). The coupling loss factor η_{13} is shown in Fig. 5 the value of which at frequency f can be approximated as

$$\begin{aligned} &\text{for } f \leqslant 300\text{ Hz}, \quad \eta_{13} = 0.02, \\ &\text{for } 300\text{ Hz} < f \leqslant 1500\text{ Hz}, \quad \eta_{13} = 56f^{-1.4}, \\ &\text{for } 1500\text{ Hz} < f \leqslant 3000\text{ Hz}, \quad \eta_{13} = e^{-(0.0015f+3.9)}, \\ &\text{for } f > 3000\text{ Hz}, \quad \eta_{13} = 0.0002. \end{aligned} \tag{13}$$

The coupling loss factor η_{12} is shown in Fig. 5 the value of which at frequency f can be approximated as

$$\begin{aligned} &\text{for } f \leqslant 500\text{ Hz}, \quad \eta_{12} = 0.005, \\ &\text{for } 500\text{ Hz} < f \leqslant 2200\text{ Hz}, \quad \eta_{12} = 18f^{-1.32}, \\ &\text{for } f > 2200\text{ Hz}, \quad \eta_{12} = 0.0007. \end{aligned} \tag{14}$$

It can be seen from the results (Table 1) that the coupling loss factors η_{12} and η_{32} are almost equal, as expected for a symmetrical system.

The coupling loss factor η_{13} is very much higher at lower frequencies compared to the other coupling loss factors. The reason could be that plate 3 is connected to plate 1 through the bolt as well as the bushes but plate 2 is connected to plate 1 only through the bushes. The stiffness of the bolt is significantly higher compared to that of the bushes and therefore higher vibration energy transfer is expected through the bolts compared to the bushes. At higher frequencies the structural coupling is weak. This can be concluded by comparing the coupling loss factor values with the dissipation loss factor values that are presented later. Presently no theoretical models for estimating these coupling loss factors are reported. Development of such a model is desirable especially when coupling loss factors are to be obtained for such systems but with different dynamic characteristics of the plates and connections or both.

The dissipation loss factor of plate 1 is determined using energy method and the results are given in Table 1. The experiment is conducted in air and therefore the radiation loss factor component is present in the measured loss factor. Since the critical frequency of the plate is very high, 6208 Hz, the radiation loss factor is expected to be very low in the frequency range considered, that is up to 4000 Hz. Hence the measured loss

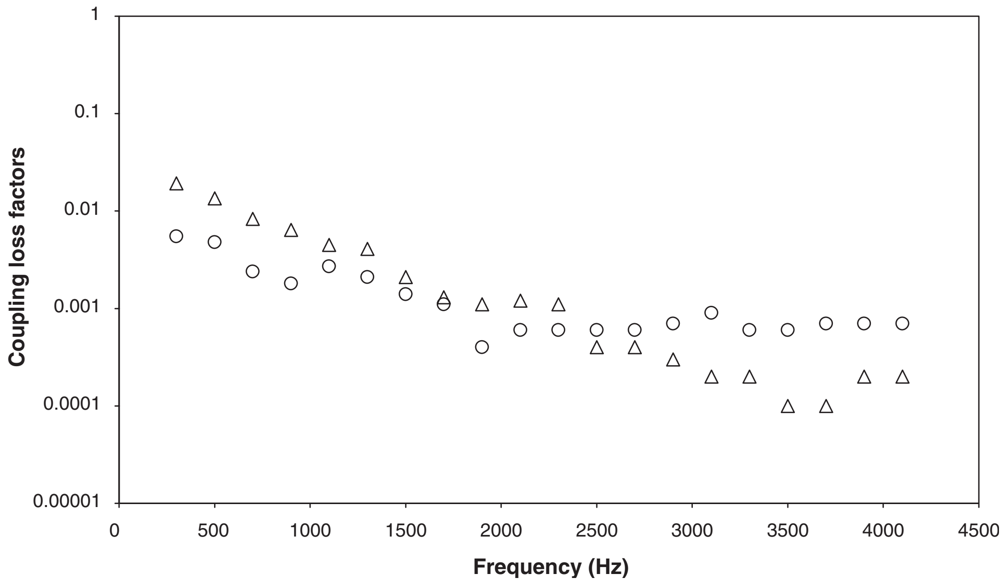

Fig. 5. Coupling loss factors of the three-plate system: △, η_{13}; ○, η_{12}

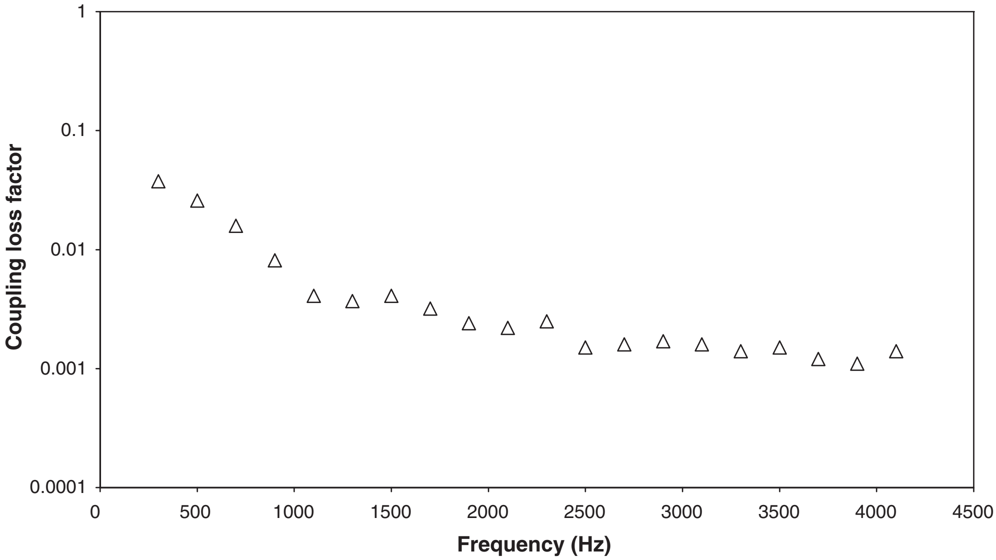

Fig. 6. Coupling loss factors of the two-plate system: △, η_{12}.

factor is considered as the dissipation loss factor. The dissipation loss factor at frequency f can be approximated as

$$\begin{aligned}&\text{for } f \leqslant 1000\ \text{Hz}, \quad \eta_d = 0.0100,\\ &\text{for } 1000\ \text{Hz} < f \leqslant 3000\ \text{Hz}, \quad \eta_d = 1200 f^{-1.7},\\ &\text{for } f > 3000\ \text{Hz}, \quad \eta_d = 0.0015.\end{aligned} \tag{15}$$

The dissipation loss factor values are approximately same as that are obtained in other studies [14]. The dissipation loss factors of the plates obtained through the power injection method (the results are not presented) are approximately same as that of the single plate. The result signifies that the coupling is conservative.

The coupling loss factor η_{12} of the two-plate system is given in Fig. 6. The coupling loss factor at frequency f can be approximated as

$$\begin{aligned}&\text{for } f \leqslant 500\ \text{Hz}, \quad \eta_{12} = 0.0300,\\ &\text{for } 500\ \text{Hz} < f \leqslant 2000\ \text{Hz}, \quad \eta_{12} = 5470 f^{-1.95},\\ &\text{for } f > 2000\ \text{Hz}, \quad \eta_{12} = 0.00020.\end{aligned} \tag{16}$$

The coupling loss factors are higher compared to those in a three-plate system.

5. Response to acoustic excitation

As discussed earlier, the objective of the present study is to determine the dynamic responses of the inner panel in a stack of three panels, as in the case of spacecraft, when subjected to acoustic excitation. The coupling loss factors of this system are obtained experimentally. The responses of the inner plate, when the system is subjected to acoustic loads, are now estimated using these coupling loss factors. They are verified with the measured acceleration values.

5.1. Experimental results

The three-plate system is hung in a reverberation chamber and subjected to acoustic excitation.

The reverberation chamber is having dimensions of $10.33 \times 8.2 \times 13.0$ m. The frequency of the first acoustic mode in the chamber is 13.3 Hz. The sound pressure level (SPL) in the chamber is almost uniform in the third octave bands centered at 63 Hz and above. There are 19 modes present in the chamber in the third octave band centered at 63 Hz and there is still higher number of modes present in the higher frequency bands. The SPL values at three locations, which are away from the test specimen, are measured and the spatial average is given in Table 2.

The responses are measured at four randomly selected locations on each plate. The accelerometers used are the same as that are used in the coupling loss factor measurement experiment and the effect of the mass loading on the measured response is expected to be negligible. The spatial average values of the acceleration responses of the three plates are given in Table 2.

5.2. Discussion of results

The measured acceleration responses show that the responses of both the outer plates are the same except at very low frequencies. This is expected for a symmetrical system.

The acceleration responses of the inner plate (plate 2) are now estimated. In this calculation, the responses of the outer plates are set for the measured values and the responses of the inner plate are estimated. The measured coupling loss factor and dissipation loss factor values are used for the calculation. The estimated spatial average values of the acceleration responses are given in Table 2. It can be seen that the responses of the middle plate are also significant but lower than the outer plates. But at low frequencies the responses of the inner plate is almost equal to that of the outer plates. This is because the coupling loss factor values are very much higher at low frequencies.

The theoretically estimated responses of the inner plate are compared with the measured values in Table 2 and Fig. 7. It can be seen that except at very low frequencies the measured responses are very much higher than the estimated responses. This means that the energy of the inner plate is more than the energy transferred through the mechanical coupling. Therefore, it is expected that apart from the mechanical coupling, there could be vibration energy transfer through other coupling paths. The possibility of the inner plate having the direct acoustic excitation is very unlikely since the gap between the two plates is very small, 20 mm. The probable additional coupling could be through the sound radiated by the outer plates. The bending modes excited by the acoustic loads on the outer plates cause radiation of sound. Since the inner plate is in proximity to the outer plates, that is they are kept with a small gap, the radiated sound from the outer plates sets the inner plate in to bending modes. This requires further investigation.

Table 2
Responses of the three-plate system to acoustic excitation

1/3 octave band center frequency (Hz)	SPL (dB)	Acceleration response (g)			
		Plate 1	Plate 2		Plate 3
			Experiment	Theory	
200	125.9	1.27	0.77	0.79	0.94
250	126.1	4.82	2.50	2.79	2.79
315	128.5	9.23	4.49	5.32	5.27
400	127.3	6.86	5.43	4.66	6.31
500	128.3	8.19	8.25	6.39	9.81
630	119.3	2.86	2.98	1.86	2.89
800	116.9	2.24	2.09	1.30	2.22
1000	115.9	1.67	1.75	0.88	1.62
1250	116.0	1.68	1.65	0.92	1.59
1600	116.1	1.80	1.71	1.07	1.88
2000	116.7	1.97	1.86	1.11	1.73
2500	116.3	2.22	2.21	1.41	2.19
3150	115.5	2.38	2.65	1.63	2.30
4000	114.5	2.77	3.21	1.87	2.61

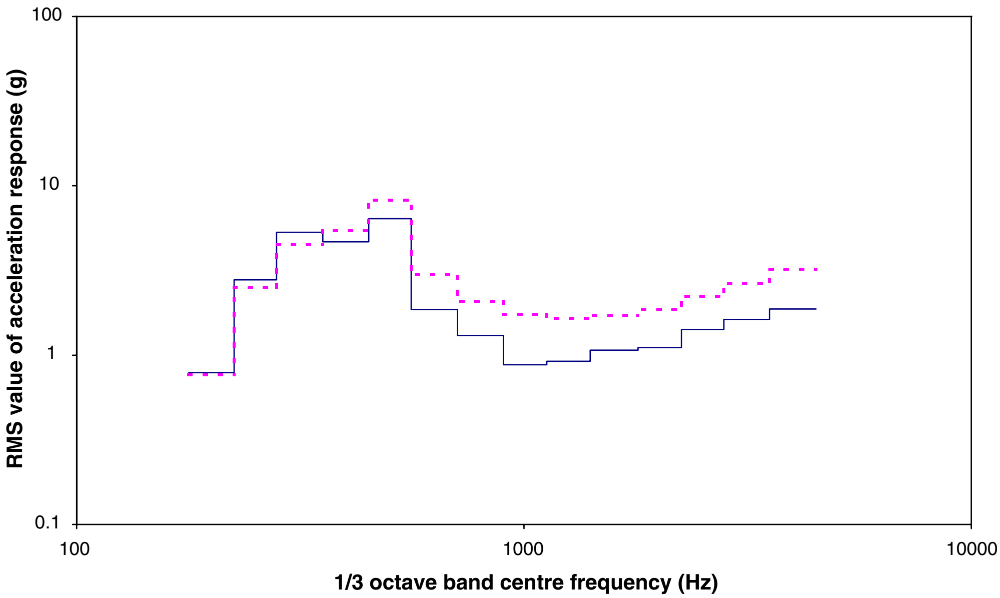

Fig. 7. Response of the inner plate during acoustic excitation: —, theory; - - - -, experiment.

Table 3
Responses of the three-plate system to mechanical excitation

Frequency (Hz)	Acceleration response of plates (g)					
	Plate 1		Plate 2		Plate 3	
	Theory	Experiment	Theory	Experiment	Theory	Experiment
300	0.29	0.27	0.18	0.18	0.23	0.21
500	0.29	0.28	0.18	0.19	0.20	0.21
700	0.24	0.26	0.13	0.16	0.14	0.18
900	0.22	0.27	0.10	0.15	0.12	0.19
1100	0.22	0.26	0.10	0.15	0.11	0.17
1300	0.26	0.26	0.12	0.15	0.14	0.18
1500	0.33	0.32	0.15	0.16	0.18	0.19
1700	0.43	0.40	0.20	0.19	0.23	0.21
1900	0.53	0.52	0.25	0.22	0.27	0.26
2100	0.51	0.51	0.24	0.23	0.25	0.28
2300	0.34	0.34	0.16	0.19	0.16	0.21
2500	0.29	0.31	0.15	0.16	0.13	0.16
2700	0.36	0.39	0.18	0.19	0.16	0.18
2900	0.30	0.33	0.16	0.17	0.13	0.14
3100	0.22	0.21	0.12	0.12	0.09	0.09
3300	0.17	0.17	0.09	0.08	0.07	0.07
3500	0.15	0.15	0.08	0.08	0.06	0.05
3700	0.15	0.14	0.08	0.08	0.06	0.06
3900	0.13	0.12	0.07	0.07	0.05	0.05
4100	0.13	0.13	0.07	0.07	0.05	0.05

To substantiate the above conclusion the three-plate system is mechanically excited. Plate 1 is mechanically excited and the responses of all the three plates are measured. The spatial average values of the acceleration responses of the three plates are then obtained. The results are given in Table 3 and Figs. 8–10. The results show that the measured responses of the plates match very well with the responses estimated using the coupling loss factors and the dissipation loss factors obtained in this study, except in a small frequency band around 1100 Hz. But, as discussed earlier, when the plates are acoustically excited the measured responses of the inner plate are much higher than the estimated responses. This confirms that responses of the inner plate are caused due to other coupling paths in addition to the mechanical coupling with the outer plates.

6. Conclusions

The coupling loss factors of a system of three plates connected at discrete points are obtained experimentally using the power injection method. When such a system is subjected to acoustic excitation the vibration

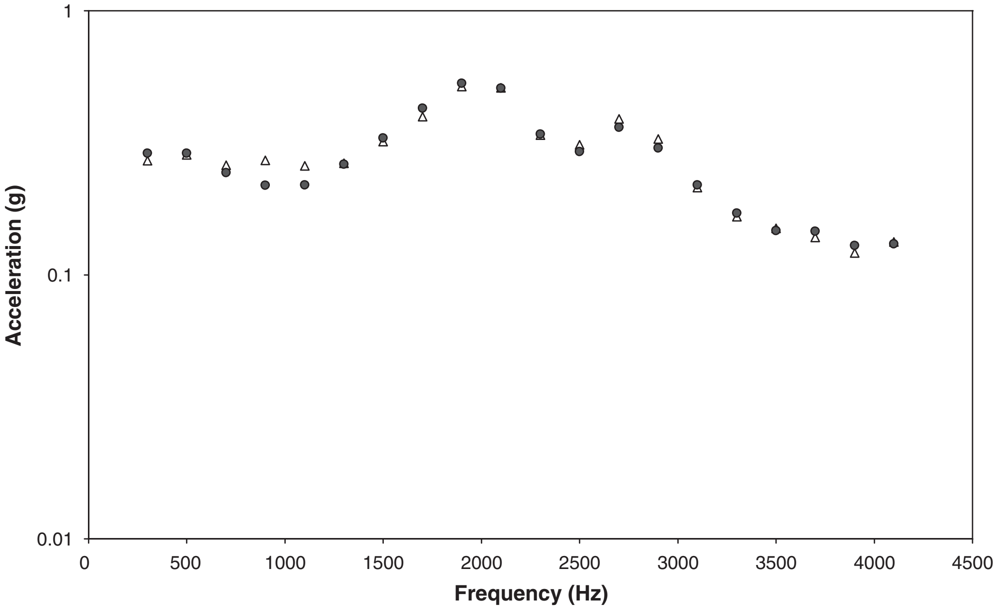

Fig. 8. Response of plate 1 during the mechanical excitation: ●, theory; △, experiment.

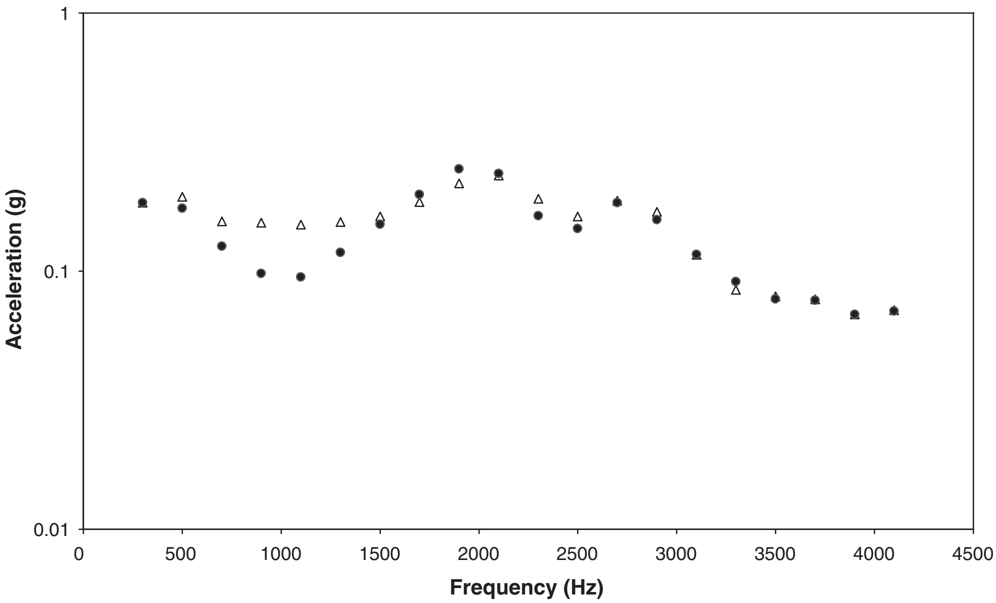

Fig. 9. Response of plate 2 during the mechanical excitation: ●, theory; △, experiment.

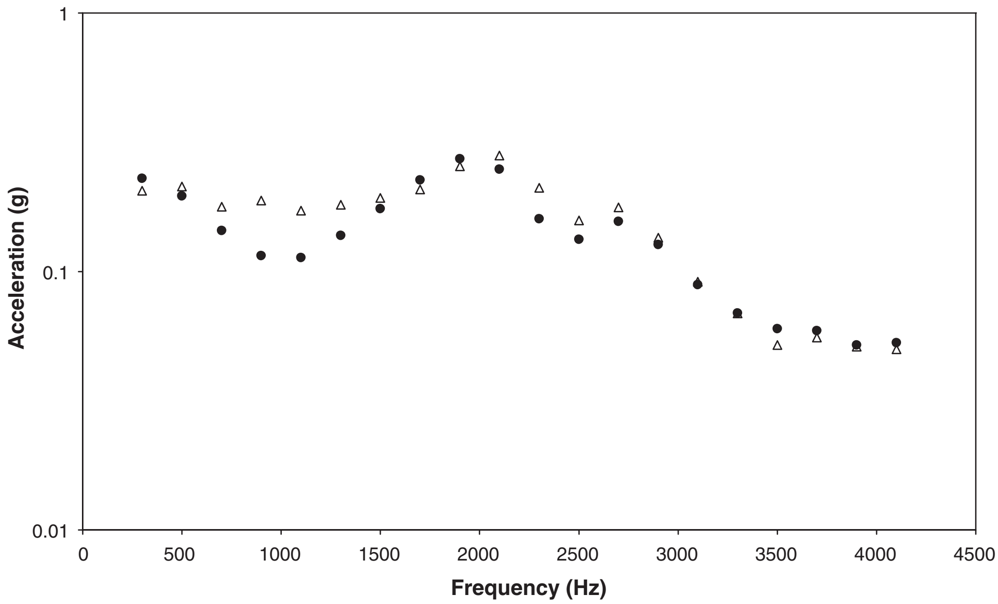

Fig. 10. Response of plate 3 during the mechanical excitation: ●, theory; △, experiment.

responses of the inner plate are found to be significant. Also the measured responses of the inner plate are very much higher than the responses estimated using the coupling loss factors and dissipation loss factors, signifying additional coupling paths other than the mechanical connection.

Acknowledgements

Authors wish to thank Dr. P.S. Nair, Deputy Director, Mechanical Systems Area, ISRO Satellite Centre, Bangalore, India for his valuable suggestions during the course of this work. The help rendered by the staff of Acoustic Test Facility, National Aerospace Laboratories, Bangalore, India in conducting the acoustic test is greatly acknowledged.

References

[1] R.H. Lyon, *Statistical Energy Analysis of Dynamical Systems: Theory and Applications*, MIT Press, Cambridge, MA, 1975.
[2] B.L. Clarkson, M.F. Ranky, On the measurement of the coupling loss factors of structural connections, *Journal of Sound and Vibration* 94 (1984) 249–261.
[3] B.L. Clarkson, The derivation of modal densities from point impedances, *Journal of Sound and Vibration* 77 (1981) 583–584.
[4] B.L. Clarkson, R.J. Pope, Experimental determination of modal densities and loss factors of flat plates and cylinders, *Journal of Sound and Vibration* 77 (1981) 535–549.
[5] K.T. Brown, M.P. Norton, Some comments on the experimental determination of modal densities and loss factors for SEA applications, *Journal of Sound and Vibration* 102 (1985) 588–594.
[6] K. Renji, Experimental modal densities of honeycomb sandwich panels at high frequencies, *Journal of Sound and Vibration* 237 (2000) 67–79.
[7] K.T. Brown, Measurement of modal density: an improved technique for use on lightly damped structures, *Journal of Sound and Vibration* 96 (1984) 127–132.
[8] D.A. Bies, S. Hamid, In situ determination of loss and coupling loss factors by the power injection method, *Journal of Sound and Vibration* 70 (1980) 187–204.
[9] A.J. Keane, W.G. Price, Statistical energy analysis of strongly coupled systems, *Journal of Sound and Vibration* 117 (1987) 363–386.
[10] F.J. Fahy, Energy flow between oscillators: a special case of point excitation, *Journal of Sound and Vibration* 11 (1970) 481–483.

[11] M.F. Ranky, B.L. Clarkson, Frequency average loss factors of plates and shells, *Journal of Sound and Vibration* 89 (1983) 309–323.
[12] M.P. Norton, R. Greenhalgh, On the estimation of loss factors in lightly damped pipeline systems: some measurement techniques and limitations, *Journal of Sound and Vibration* 105 (1986) 397–423.
[13] K. Renji, S. Shankarnarayan, Loss factors of composite honeycomb sandwich panels, *Journal of Sound and Vibration* 250 (2002) 745–761.
[14] D.C.G. Eaton, *Structural acoustic design manual*, ESA PSS-03-1201 Issue 1, 1987.