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Experimental evaluation of a piezoelectric vibration absorber using a simplified fuzzy controller in a cantilever beam

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Abstract

This study presents a novel resonant fuzzy logic controller (FLC) to minimize structural vibration using collocated piezoelectric actuator/sensor pairs. The proposed fuzzy controller increases the damping of the structures to minimize certain resonant responses. The vibration absorber is first experimentally examined by a cantilever beam test bed for impulse and near-resonant excitation cases. Moreover, the effectiveness of the new fuzzy control design to a state-of-the-art control scheme is compared through the experimental studies. The experimental results indicate the proposed controller is highly promising for this application field. Our results further demonstrate that the fuzzy approach is much better than traditional control methods. In summary, a novel vibration absorption scheme using fuzzy logic has been demonstrated to significantly enhance the performance of a flexible structure with resonant response.

1. Introduction

Reducing the vibrations of structures by piezoelectric damping materials has been extensively studied in aeronautical, mechanical and civil engineering [1–3]. Consequently, the feasibility of adopting piezoelectric materials as actuators and sensors for controlling vibrations in flexible structures has received increasing attention [4,5]. Piezoelectric materials provide inexpensive, reliable, and non-intrusive means of actuating and sensing vibrations in flexible structures. Recent advances in piezoelectric actuators, based on the converse piezoelectric effect, make them highly promising for the active control of vibrations, especially for suppressing or isolating vibrations.

When adopted in flexible structures, piezoelectric materials are bonded to the body of the structure using strong adhesive material. Notably, piezoelectric actuators or sensors are spatially distributed over the surface that is being sensed and/or controlled, unlike discrete actuators and sensors that are often applied to control flexible structures [6]. The research reported in Ref. [7] introduced a class of resonant controllers to minimize structural vibration with collocated piezoelectric actuator–sensor pairs. All of these studies are limited to the classical vibration control of a laminated beam, and none have presented disturbances compensation techniques.

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A flexible structure is a distributed parameter system of infinite order, but must be approximated by a lower-order and controlled by a finite-order controller, because of the limitations of the onboard computer, the inaccuracy of sensors and system noise. The model reduction schemes and their methodologies are shown in Refs. [8,9]. However, the large-scale system has been limited to a reduced-order truncated system. Each step of linearization, delay approximation, decomposition and model reduction has introduced a degree of uncertainty into the system, moving the model away from the real physical situation. The above discussion also reveals that frequent, simplifying assumptions make the problem at hand too uncertain to be practical. A large-scale system should be designed and analyzed based on the best available knowledge rather than the simplest available model to treat system uncertainties. Therefore, a large-scale system is better treated through knowledge-based algorithms such as fuzzy logic and neural networks. Furthermore, the popular LQ control strategy (linear optimal control with quadratic cost function) has been demonstrated to be deficient when a system moves significantly into the nonlinear behavior range. Modified design techniques, which nonetheless keep the system model as realistic as possible need to be developed [10].

Additionally, control systems should accommodate noisy input measurements and uncertainty in system parameter values. One promising strategy, the use of fuzzy control, is inherently robust and can deal with both linear and nonlinear structural behavior.

The application of fuzzy set theory in vibration control has attracted increasing attention [11–14]. Fuzzy controllers provide a simple and robust framework for specific nonlinear control laws that accommodate uncertainty and imprecision.

Despite their contributions, the above studies fail to provide any examples of the application of the fuzzy control theory in harmonic excitations with variable frequency. Therefore, this investigation presents a class of resonant fuzzy absorbers that can be adopted to minimize structural vibration using collocated piezoelectric actuator/sensor pairs. Experimental validation on a cantilever beam is presented, showing the effectiveness of the proposed controller. Hence, this work explores a methodology for designing fuzzy controllers, and measures the system performance and expresses it by fuzzy variables.

2. Modeling the compound system

Fig. 1 shows a flexible structure with a number of piezoelectric actuator–sensor pairs attached to it. This study considers a general mechanical structure integrated with a piezoelectric actuator. Additionally, the local vibration in the structure is assumed to be detected using a piezoelectric sensor. This work uses piezoelectric patches as sensors by placing the actuators and the sensors at the same location on both sides of the beam. Assume the actuator generates forcing moments that vibrate the beam. This strain then induces an electric charge inside the piezoelectric sensor due to the piezoelectric effect. The piezoelectric actuators and sensors have length L_{px} , width L_{py} and thickness h_p. The relationship $h_p \ll h$ is assumed, since this is true for the patches applied in experiments. Hence, beam properties can be assumed to be uniform. However, approximation methods such as the finite element method can also be applied to deal with some general non-uniform structures.

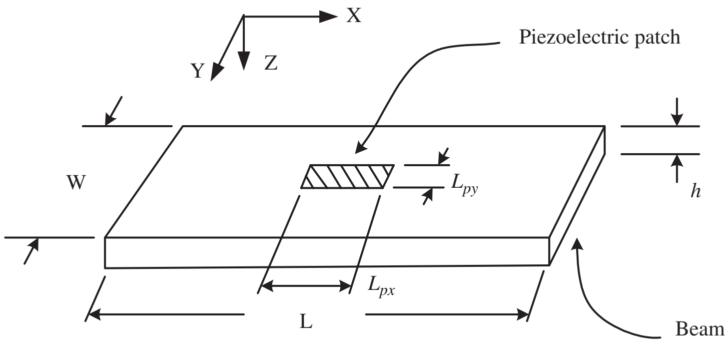

Fig. 1. A beam with a piezoelectric patch attached.

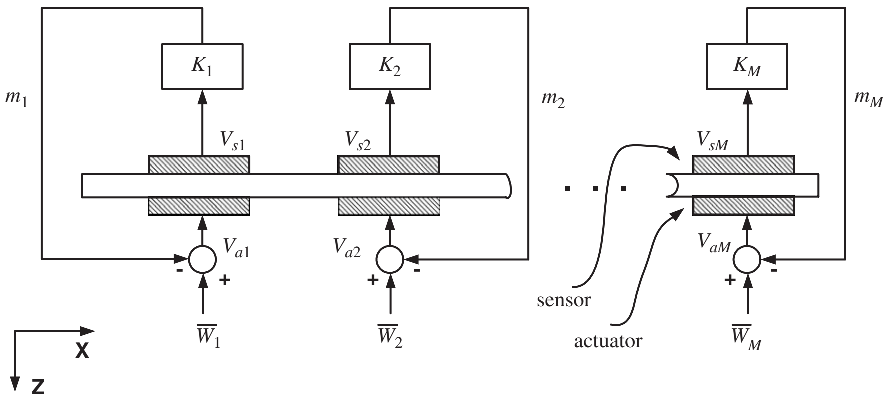

Fig. 2. A beam with a number of collocated piezoelectric patches.

Consider M collocated actuator–sensor pairs distributed over the structure, as illustrated in Fig. 2. Voltages applied to actuating patches are denoted by $V_a(t) = [\, V_{a1}(t) \quad \cdots \quad V_{aM}(t)\,]^{\mathrm{T}}$.

The one-dimensional Bernoulli–Euler beam describes the elastic deflection of a beam, and the partial differential equation (PDE) governs the dynamics of the homogeneous beam described as

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 z(x,t)}{\partial x^2} - C_a v_a(x,t)\right] + \rho A_b \frac{\partial^2 z(x,t)}{\partial t^2} = 0, \tag{1}$$

where E, I, A_b, and ρ denote the Young's modulus, moment of inertia, cross-sectional area, and linear mass density of the beam, respectively. The additional term is due to the moment applied to the neutral axis of the beam by the actuator piezoelectric layer, i.e., $M_a = C_a v_a(x,t)$ where C_a is a constant dependent on the actuator properties and v_a is the voltage across the actuating layer. Particularly, if the piezoelectric patches do not cover the entire beam surface, then both EI and ρA_b are functions of x. However, since the piezoelectric layers are often thin compared with the base structure, EI and ρA_b can be assumed to be uniform over the length of the beam [15].

This study assumes that each piezoelectric patch is very thin and that the beam deflects only in the y-axis. Using the modal analysis techniques, the position function $z(x,t)$ can be expanded as an infinite series of the form

$$z(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t), \tag{2}$$

where $\phi_i(x)$ denote the normalized mode shapes function and $q_i(t)$ denote the modal displacements.

A set of ordinary differential equations (ODEs) can be obtained from the PDE (1) by using the orthogonality properties and Dirac's function property. If the contribution of forcing functions generated by all M piezoelectric actuators is included, then the ODEs can be as

$$\ddot{q}_i(t) + 2\zeta_i\omega_i\dot{q}_i(t) + \omega_i^2 q_i(t) = \frac{\bar{K}}{\rho A_b}\sum_{k=1}^{M} \Psi_{ik} V_{ak}(t), \tag{3}$$

where $i = 1, 2, \ldots,$ and $q_k(t)$ represents the generalized coordinate of mode k. The subscript i denotes the ith actuator. The term $\bar{K}$ is a constant based on the properties of the beam and the piezoelectric patches and defined in Ref. [3]. The modal parameters in the above equation include damping, does not occur in the original model (Eq. (1)). Moreover, Ψ_{ki} can be written as

$$\Psi_{ik} = [\phi_i'(x_{2k}) - \phi_i'(x_{1k})], \tag{4}$$

where x_{1k} and x_{2k} denote the locations of the ends of the ith piezoelectric patch along the X-axis.

3. Fuzzy control law design

Consider the feedback control loop depicted in Fig. 2. The system is denoted as a flexible structure with M collocated piezoelectric actuator–sensor pairs attached. The ith collocated actuator–sensor pair is controlled independently by the controller K_k. The measured voltages from the piezoelectric sensors, $V_s(t)$, act as the measured inputs to the controller $K(s)$.

A flexible structure exhibits a significant response only when the excitation frequency is near a natural system frequency called a resonant frequency, since these flexible structures are often very lightly damped. Hence, minimizing the structural vibration involves minimizing the resonant responses, which occur at, or very close to, the structure's natural frequencies. An ideal controller significantly reduces the structural vibration at, and near to, the resonant frequencies of the structure.

A significant issue in designing a controller for a flexible structure is whether the developed closed-loop system has sufficient robustness to deal with uncertainties in the system model. Conventional proportional-derivative (PD) synthesis approaches are known to guarantee stability margins. Unfortunately, this controller requires a mathematical model and assumes that all system parameters are known. Effective applications in vibration control, however, require that the system dynamics can be adequately and/or accurately determined, and that the controller design can be easily implemented. The application of fuzzy logic controllers (FLCs) to achieve vibration control of smart structures has thus attracted attention, due to their ability to deal with nonlinearities, uncertainties and imprecision. Fuzzy control has become a very popular method to controller design because it enables human skills to be transferred into linguistic rules. Consequently, fuzzy control has often been applied to poorly defined systems or systems without mathematical models. Therefore, fuzzy controllers afford a simple and robust framework for a specific complex system.

Error and error change are two commonly used variables in fuzzy control. This study applies the vibration states (e) and their rate variables (Δe) as inputs, with the voltage applied to the voltage amplifier as the output. Since the ith controller handles collocated actuator–sensor pair independent of K_k, an individual fuzzy controller is easy to design for each collocated actuator–sensor pair.

In this research, a triangle-type membership function, as shown in Figs. 4 and 5, is used to convert the input and output variables into linguistic control variables. The rule base design of fuzzy subsystems is based on pre-simulation investigations. The statements employ fuzzy quantities, such as *negative big* (NB), *negative small* (NS), *zero* (ZE), *positive small* (PS) and *positive big* (PB), which require corresponding membership functions. This study applies fuzzy control rules of state evaluation, which are similar to the institutional thinking of humans:

$$R^i : \text{if } e \text{ is } A_{1i}, \quad \Delta e \text{ is } B_{1i}, \quad \text{then } u \text{ is } C_{1i}, \tag{5}$$

where e, Δe and u denote the system variables (error, error change, and output voltage), and A_{1i}, B_{1i} and C_{1i} are the linguistic values of the fuzzy variables to express the universe of discourse of the fuzzy sets. The proposed fuzzy inference method employs the max–min product composition to operate the fuzzy control rules. The fuzzy sets must be defuzzified to obtain the appropriate control output for this control system. The centroid of area method was adopted to defuzzify the output variable [11,12].

Membership function parameters are tuned off-line according to heuristic rules. The membership function breakpoints are initially chosen arbitrarily. The membership function parameters are then adjusted to produce the best performance for excitations inputs. Thus, emphasis is placed on enhancing performance in the resonant excitation operating range of the system, where the effects of the resonant degrade the system stability.

Breakpoints of membership functions of e and Δe are chosen initially to trigger excitations. The parameters of membership function u are then adjusted to attain the best performance for reference inputs in the remainder of the operating range. However, modifying the breakpoints of membership functions e, Δe, and u, changes the input rate based on the output at each time step. Hence, the breakpoints of membership functions e and Δe can be changed to affect performance over a range of reference inputs, while the breakpoints of u can be adjusted to influence performance in a certain operating region for each response. Once these parameters have been tuned, then e, Δe and u are adjusted, and the entire process is then repeated until the best overall performance is achieved. Fig. 3 illustrates a flowchart of the system parameter modification process.

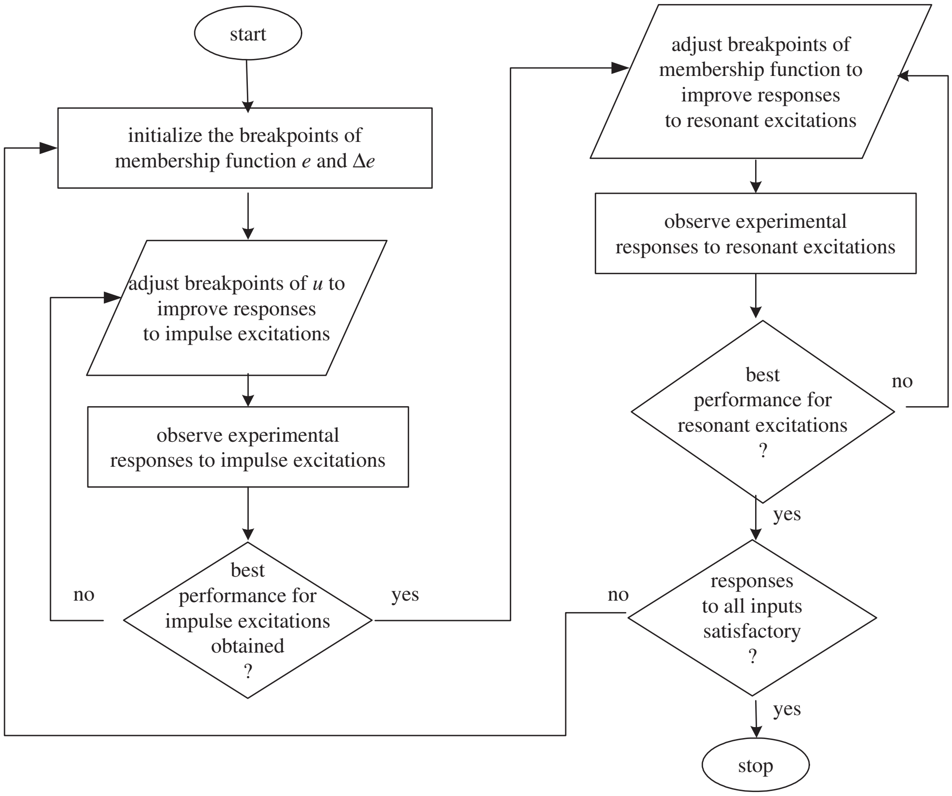

Fig. 3. Flowchart of the parameter modification process.

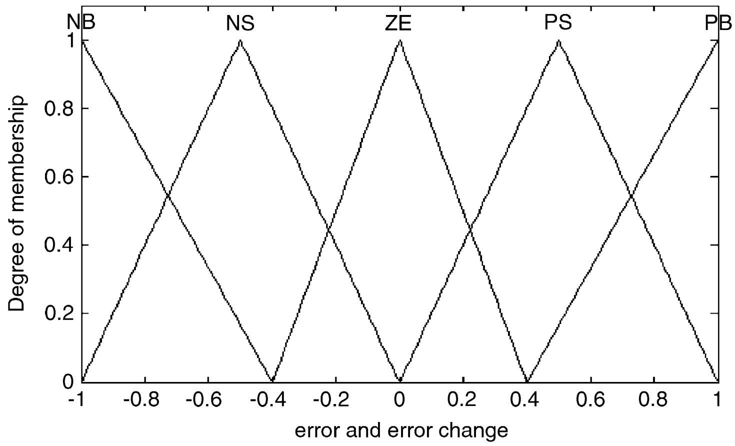

Fig. 4. Membership function for e and Δe.

Fig. 4 displays the fuzzy membership function for e and Δe, and Fig. 5 depicts the fuzzy set of the system controller input u. Table 1 illustrates the FLC design shown in Fig. 6.

4. Experimental implementation

Experiments were carried out with a piezoelectric laminate beam to assess the presented concepts. This section uses the fuzzy controller to handle the vibrations of a flexible structure using piezoelectric actuators.

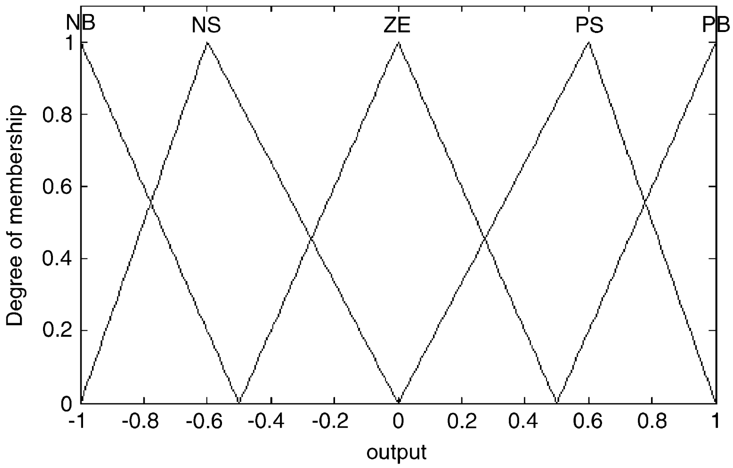

Fig. 5. Membership function for control voltage.

Table 1
Consequent table for fuzzy logic controller design

e \ Δe (u)	NB	NS	ZE	PS	PB
NB	PB	PB	PB	PS	PS
NS	PS	PS	PS	ZE	ZE
ZE	PS	PS	ZE	NE	NS
PS	ZE	ZE	NS	NS	NS
PB	NS	NS	NB	NB	NB

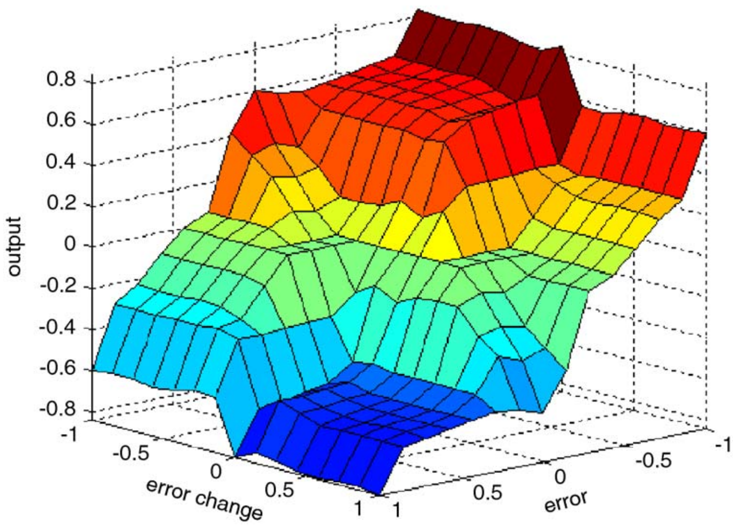

Fig. 6. Viewer surface for system.

Collocated piezoelectric patches are feasible in many structures subjected to pure bending loads, such as beams, plates and shells. The sensor and actuator in these structures are typically attached on opposite sides of the neutral axis of the structure. An experimental apparatus was constructed, constituting a flexible cantilever aluminum beam structure with piezoelectric patches symmetrically bonded on both sides to provide structural bending. Fig. 7 schematically illustrates the control experiment, and Fig. 8 displays the experimental apparatus. Fig. 9 shows the block diagram of the PD and fuzzy controllers. Strip-bender type BM500/120/6 piezoelectric patches were attached to the surface of the beam to serve as an actuator and sensor, respectively. The voltage from the piezoelectric sensor was used to measure the beam vibration level. Because PZT is a dual effect, bending elements can be successfully applied as vibration and force sensors as well as small electrical generators. The amplifier SVR 500 (3 channels, made by Piezomechanik), which is capable of driving highly capacitive loads, was adopted to supply necessary voltage for the actuating

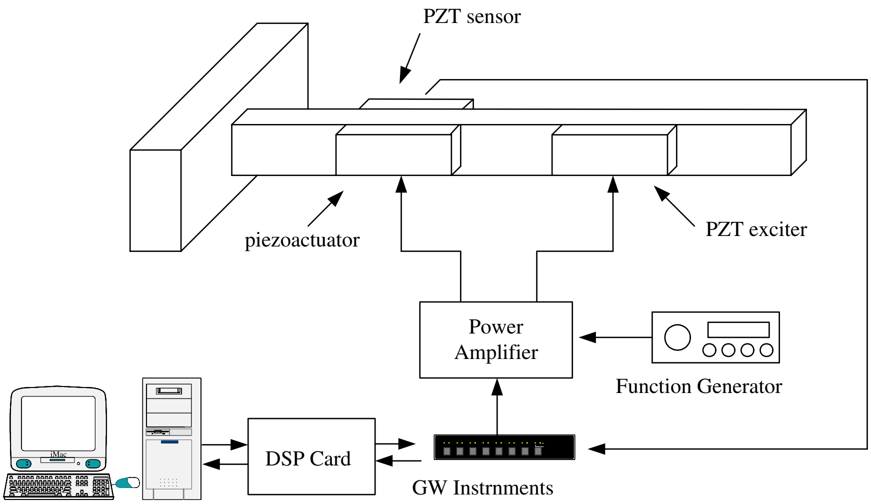

Fig. 7. Schematic diagram of the control experiment.

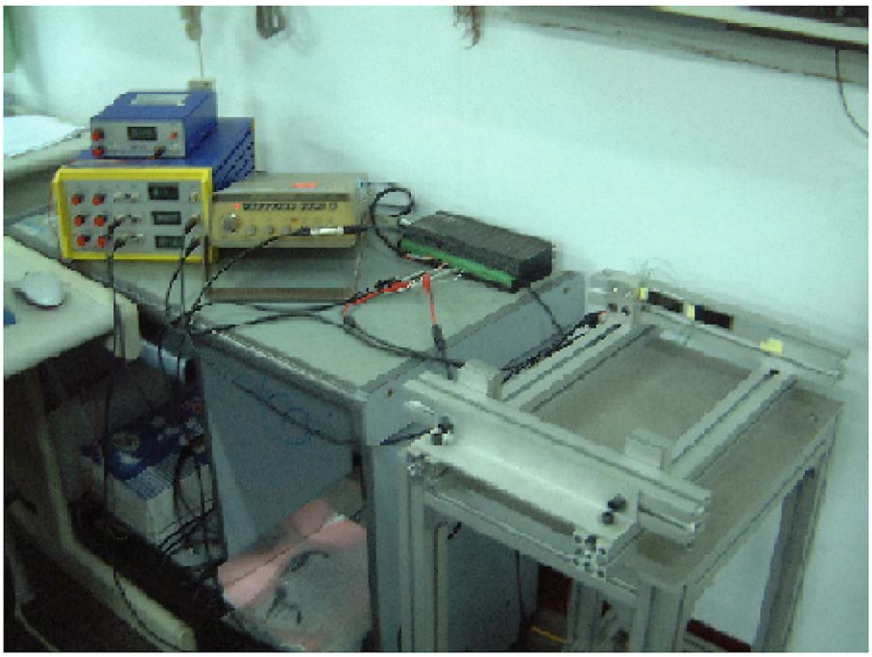

Fig. 8. Experimental apparatus.

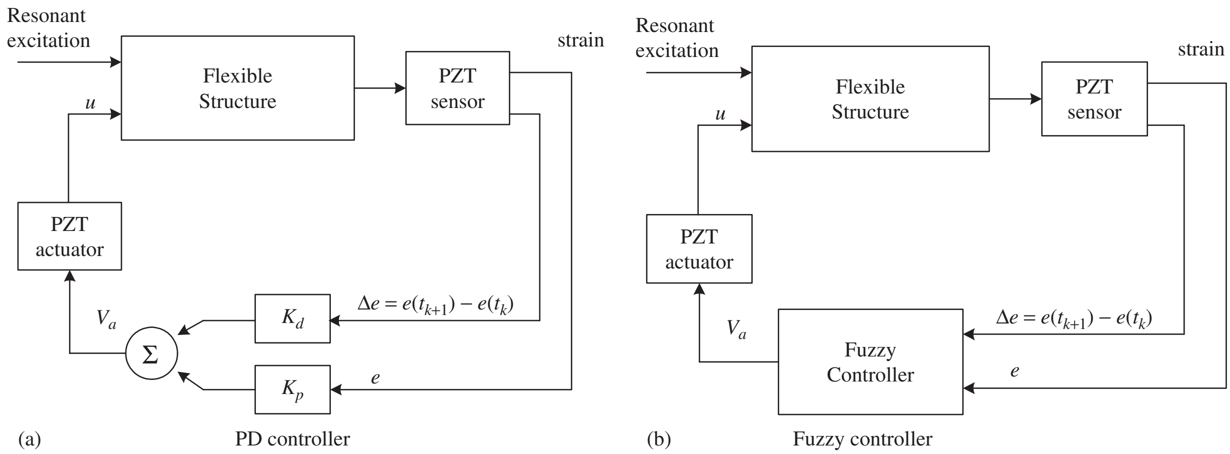

Fig. 9. Block diagram for the controllers. (a) PD controller and (b) Fuzzy controller.

piezoelectric patch. Furthermore, a testing platform fabricated from aluminum was designed as a special module that can operate with arbitrary boundary conditions (e.g. simply supported, clamped, and others). The controller was implemented using a 200PCI instruNet and Labview. The sampling time was selected as 1 ms.

In this experiment, the end of the beam was hit with a hammer to excite the beam vibration, which is called the impulse excitation in the following section. To ensure a consistent pulse amplitude, the displacement at the end of beam when the hammer strikes the beam must always be kept the same. Therefore, a limited initial vibration displacement was set at the end of the beam while in the experiment, guaranteeing a consistent pulse amplitude of the pulse. Moreover, the width of the impulse, and hence the frequency range over which the amplitudes remain constant, depends on the hardness of the hammer striker tip and the material stiffness of the system to which the hammer is applied. Therefore, a consistent pulse amplitude can be possibly generated if the same mechanical structure is used. Moreover, the beam was excited near a resonant frequency with a third piezoelectric patch located at the midpoint of the beam. Important parameters of the beam, such as resonant frequencies and damping ratio were obtained from the experimental apparatus.

5. Results and discussion

Three test cases were considered—the first case was for a non-resonant excitation (impulse excitation); the second case was for an excitation near a resonant frequency of the structure, and the third case was under a resonant excitation with unexpected disturbances. The example used in this section was identical to the cantilever beam system described in Section 2. The experimental beam was a uniform aluminum beam with a rectangular cross section and experimentally fixed-free boundary conditions. The structure consisted of a 20 cm long uniform and a rectangular cross section 20×0.6 mm. The common properties of the system parameters are:

Young's modulus $= 7.1 \times 10^{10}\,\mathrm{N/m^2}$.
Density $= 2700\,\mathrm{kg/m^3}$.
Poisson's coefficient $= 0.36$.
Piezoelectric constant of PZT $= 7.664 \times 10^{8}\,\mathrm{N/C}$.
Dielectric constant of PZT $= 7.331 \times 10^{7}\,\mathrm{V\,m/C}$.
Capacitance, $C = 170{,}000\,\mathrm{pF}$.
Sampling time $= 1$ ms.

5.1. Case 1. Vibration suppression under an impulse excitation

However, the control laws presented here are obtained from physical insight of vibration absorber characteristics rather than traditional control theory. Therefore, the performance and efficiency of the fuzzy vibration absorber system was compared with a state-of-the-art active control law. Figs. 10 and 11 illustrate the experimental frequency and time response results of employing a conventional PD controller and the proposed fuzzy controller under an initial impulse excitation by the piezoelectric actuator for comparison. Fig. 10 illustrates the fast Fourier transform (FFT) response of the uncontrolled, PD-controlled and fuzzy vibration absorbers in the experimental results at the first natural frequency of the beam. The introduction of the controller significantly reduced the peak magnitude of the first vibration mode. Significantly, the performance of the proposed fuzzy controller is better than that of the traditional PD controller. Moreover, Table 2 lists the normalized root-mean-square (rms) sensor output voltage under the impulse excitation of a beam under uncontrolled, PD-controlled and fuzzy vibration absorbers. Table 2 also reveals that the proposed fuzzy absorber reduces the displacement owing to uncontrolled vibration by approximately 44%.

Fig. 11 plots the time response for sensor output voltage under an initial impulse excitation. These results indicate the effectiveness of the fuzzy controller in minimizing the structural vibration in the time domain. The fuzzy control action considerably reduces the settling time of the position response, demonstrating that the convergence rate is faster than the PD controller. Additionally, Fig. 11 shows the effectiveness of the controller effectiveness in minimizing the beam vibration in the time domain.

5.2. Case 2. Vibration suppression under resonant excitation

This section considers the case when the nominal excitation frequency is near a structural resonant frequency. The controller has a resonant nature because the beam is highly resonant, i.e., the controller attempts to use a high gain at each resonant frequency to minimize the resonant response. Figs. 12 and 14

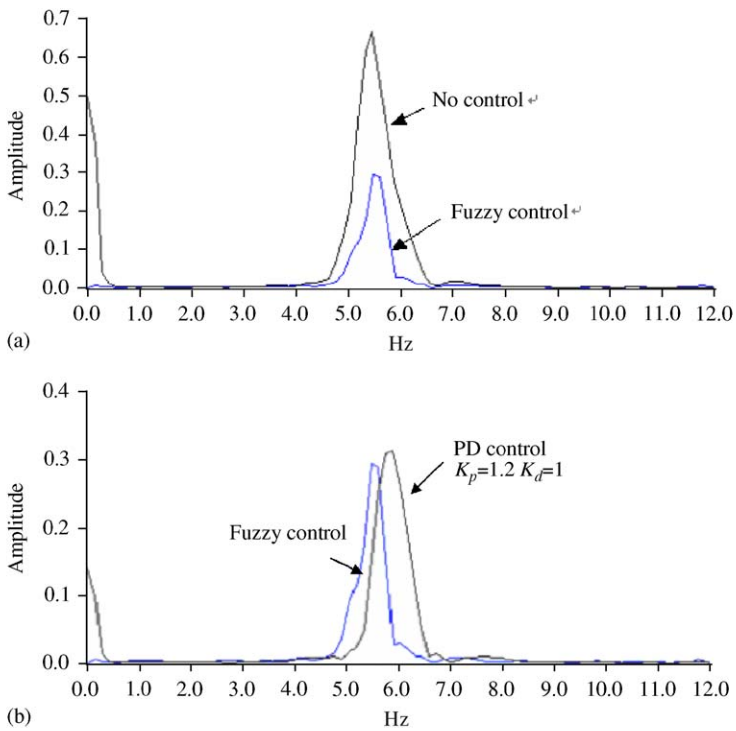

Fig. 10. (a,b) Frequency response by using FFT under an initial impulse excitation.

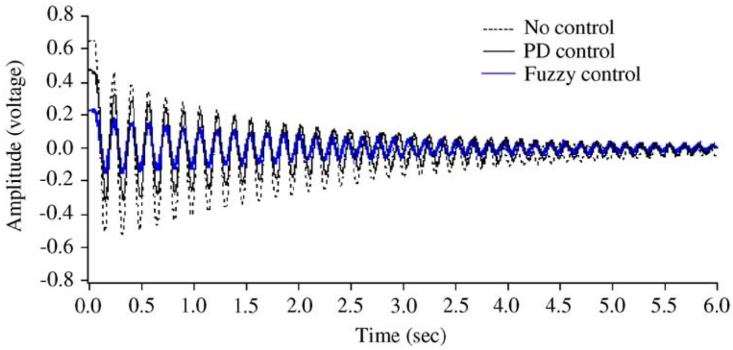

Fig. 11. Time response for sensor output voltage under an initial impulse excitation.

Table 2
Normalized rms sensor output voltages under impulse excitation

Uncontrolled (A)	PD control (B)		Fuzzy (C)	Reduction (A−C)/A (%)	Reduction (B−C)/B (%)
0.150	$K_p = 0.6$, $K_d = 0$	0.126	0.084	44	33.3
	$K_p = 1.2$, $K_d = 0$	0.105		44	20
	$K_p = 1.2$, $K_d = 1$	0.102		44	17.6

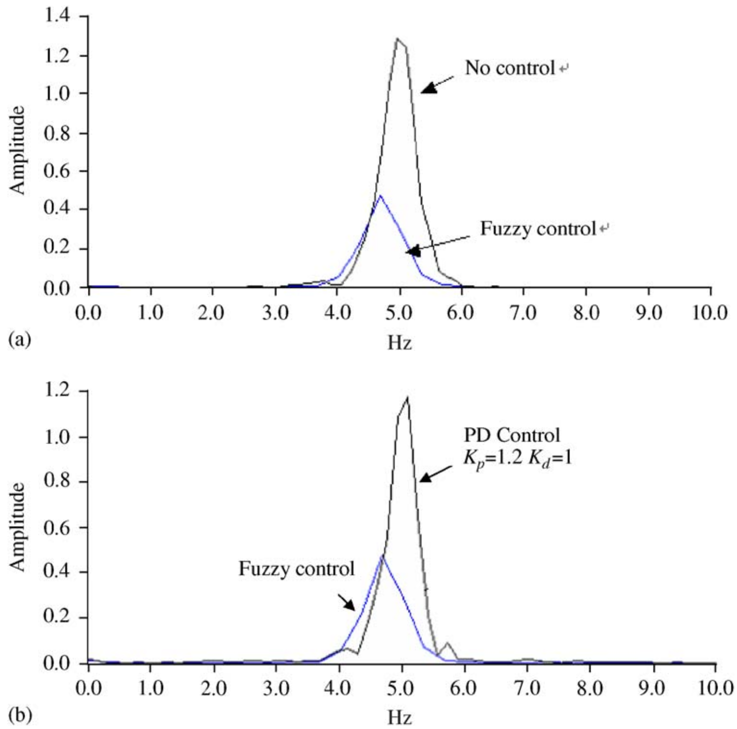

Fig. 12. (a,b) Frequency response by using FFT under 1st resonant excitation.

show the frequency response by employing the FFT of the controller for the 1st and 2nd resonant frequency. Furthermore, Figs. 12b and 14b compare the FFT of the PD and fuzzy control for experimental results at the resonant frequency of the beam. The fuzzy controller considerably reduced the resonant response of the

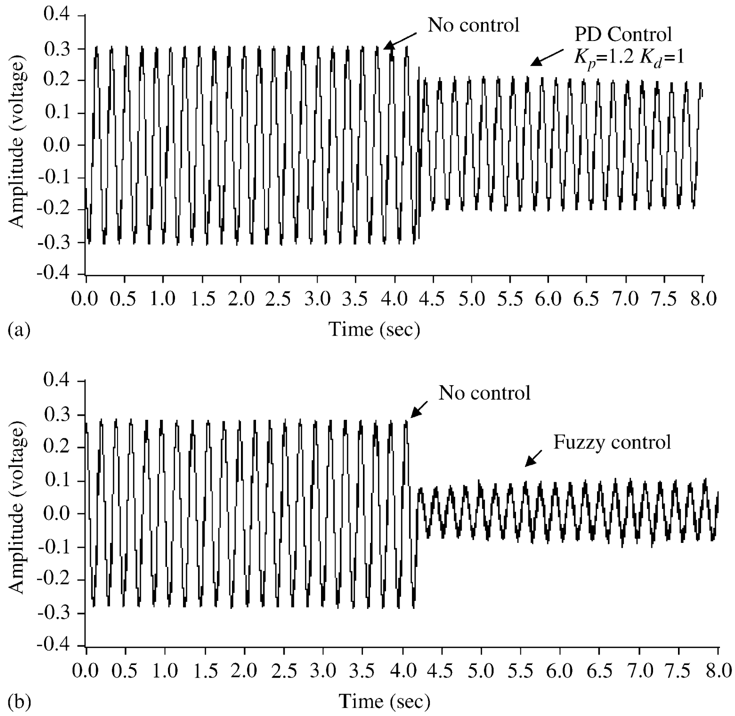

Fig. 13. (a,b) Time response for sensor output voltage under 1st resonant excitation.

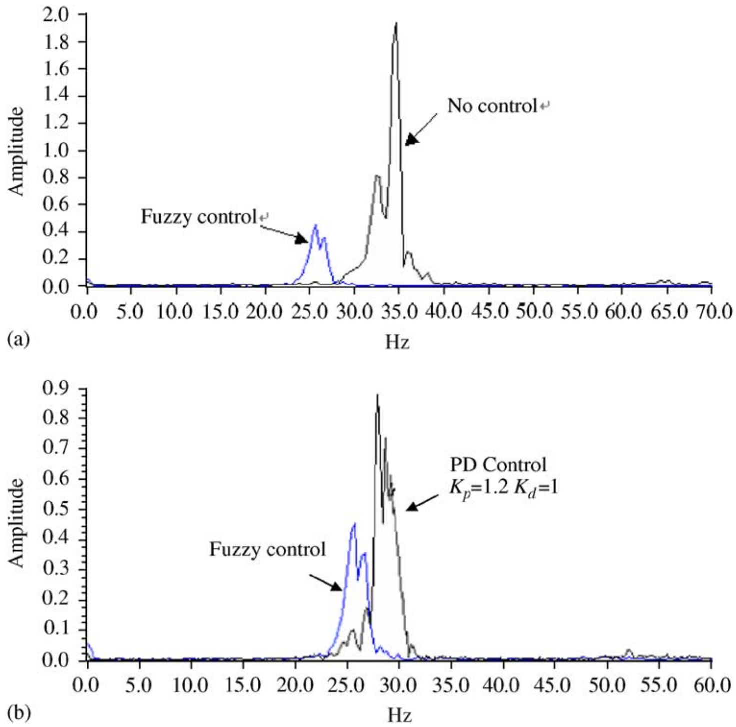

Fig. 14. (a,b) Frequency response by using FFT under 2nd resonant excitation.

vibration modes. Significantly, the amount of vibration reduction is greater for low-frequency modes than for high-frequency modes. This is beneficial since low-frequency modes often contribute significantly to the vibrations of flexible structures (see Figs. 12–14).

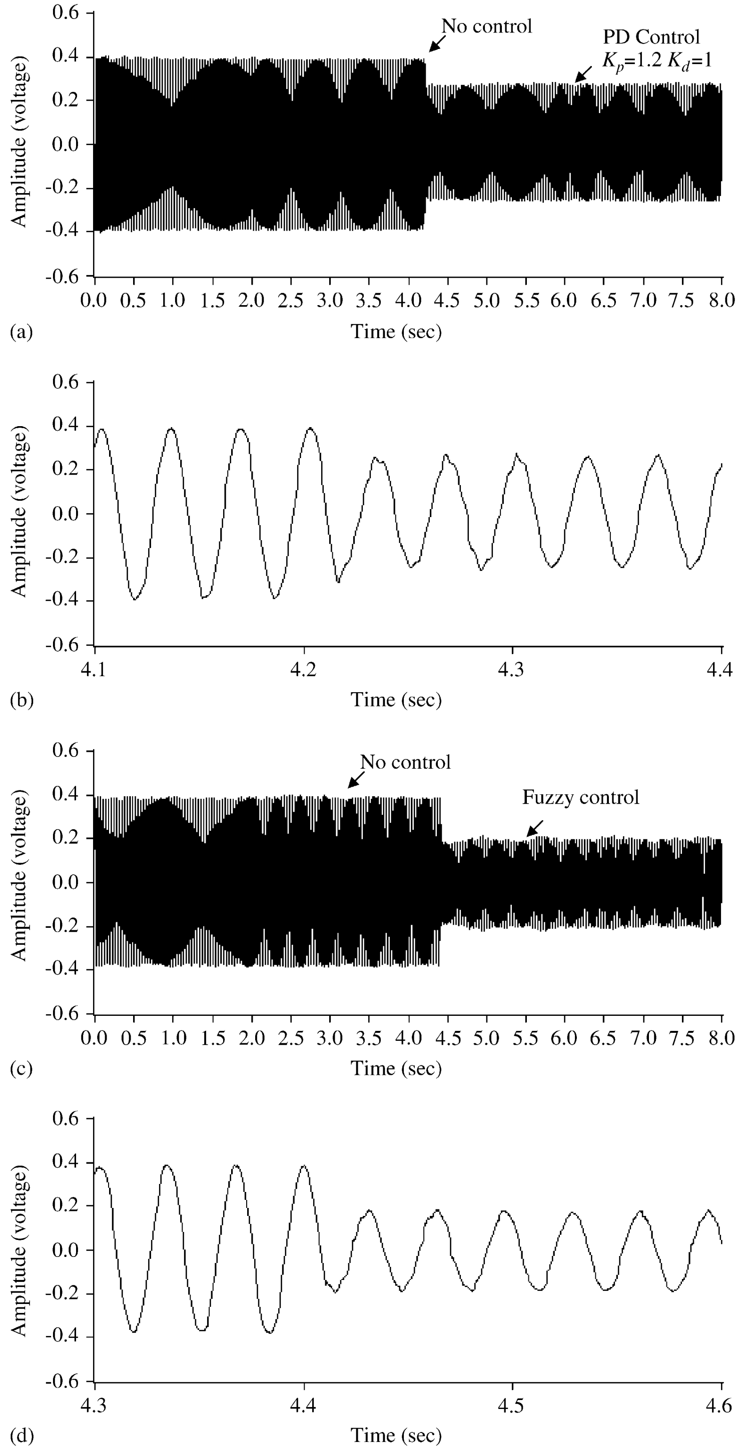

Fig. 15. (a,c) Time response for sensor output voltage under 2nd resonant excitation. (b) Time history at the moment of turning on the PD controller (2nd). (d) Time history at the moment of turning on the fuzzy controller (2nd).

Table 3
Normalized rms sensor output voltages under resonant excitation

Uncontrolled (A)	PD Control (B)		Fuzzy (C)	Reduction (A−C)/A (%)	Reduction (B−C)/B (%)
(*a*) *1st resonant frequency*					
0.250	$K_p = 1, K_d = 0$	0.141	0.050	80	64.5
	$K_p = 2, K_d = 0$	0.124		80	59.7
	$K_p = 1.2, K_d = 1$	0.108		80	53.7
(*b*) *2nd resonant frequency*					
0.269	$K_p = 1, K_d = 0$	0.223	0.077	71.4	65.5
	$K_p = 2, K_d = 0$	0.186		71.4	58.6
	$K_p = 1.2, K_d = 1$	0.186		71.4	58.6

Figs. 13 and 15 plot the time response for sensor output voltage of a beam under resonant excitation. These plots demonstrate the effectiveness of the fuzzy controller in minimizing structural vibration in the time domain, and that the convergence rate is faster than for PD control when using the fuzzy control techniques. Figs. 15b and d also illustrate the time history at the moment of switching on the PD and fuzzy controller, respectively.

Table 3 presents the normalized rms sensor output voltage with resonant excitation of a beam under uncontrolled, PD control and fuzzy vibration absorbers, and shows that the proposed fuzzy absorber reduces the displacement due to vibration of an uncontrolled absorber by approximately 71–80%. The PD control methodology only controls the vibration up to the limit given in Table 3. A fuzzy controller appears to yield a more significant improvement in displacement reduction over that obtained by PD control techniques. Hence, the controller reduced the resonant responses of the structure by increasing the system damping at those resonant frequencies.

5.3. Case 3. Vibration suppression under resonant excitation with unexpected disturbances

Figs. 16 and 17 plot the time response for sensor output voltage under resonant excitation with unexpected disturbances. Figs. 16b and d depict the time history at the moment of turning on the controller with unexpected disturbance for 1st resonant excitation, and Figs. 17b and d show the control transition when disturbance acts for 2nd resonant excitation. As before, the performance of the intelligent active fuzzy vibration absorber is not significantly affected by the change of system parameters, demonstrating the effectiveness of the fuzzy controller in minimizing structural vibration in the time domain even if it is caused by unexpected disturbances. Such a controller results in suppression of the transverse deflection of the entire structure. The active piezoelectric fuzzy vibration absorber presented here may have the performance and robustness required for such a case. Moreover, fuzzy control is a powerful and efficient means to cope with such a flexible structure system.

6. Conclusions

This work presents a novel vibration absorber method using fuzzy logic, which can significantly enhance the performance of a flexible structure with a resonant response. Among the control techniques mentioned, the control is synthesized by directly feeding back the measured strain rate signal induced from the piezo-structure. The effectiveness of the design is verified through the hardware implementation by a digital signal processing (DSP). The simplified fuzzy controller was experimentally verified in an aluminum cantilever beam under resonant excitations with unexpected disturbances. Tests were performed in the system to evaluate the performance of the proposed fuzzy controller and the traditional PD controller. The experimental results demonstrate that the proposed design can outperform PD control methods while requiring less control effort. Fuzzy control provides a simple framework to capture the effects of unexpected disturbances in a real problem

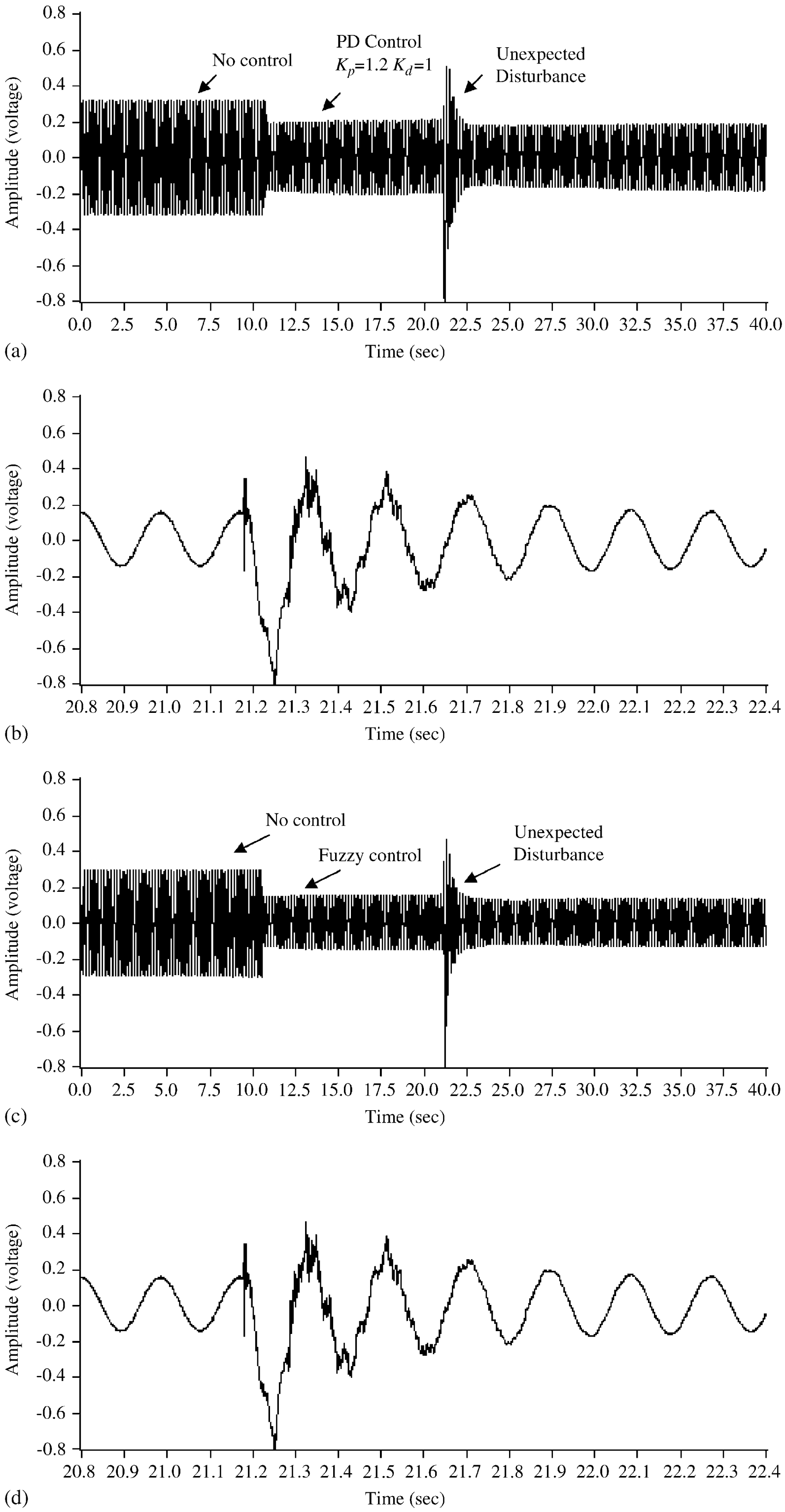

Fig. 16. (a,c) Time response for sensor output voltage under 1st resonant excitation with unexpected disturbances. (b) Time history at the moment of turning on the PD controller with unexpected disturbance (1st). (d) Time history at the moment of turning on the fuzzy controller with unexpected disturbance (1st).

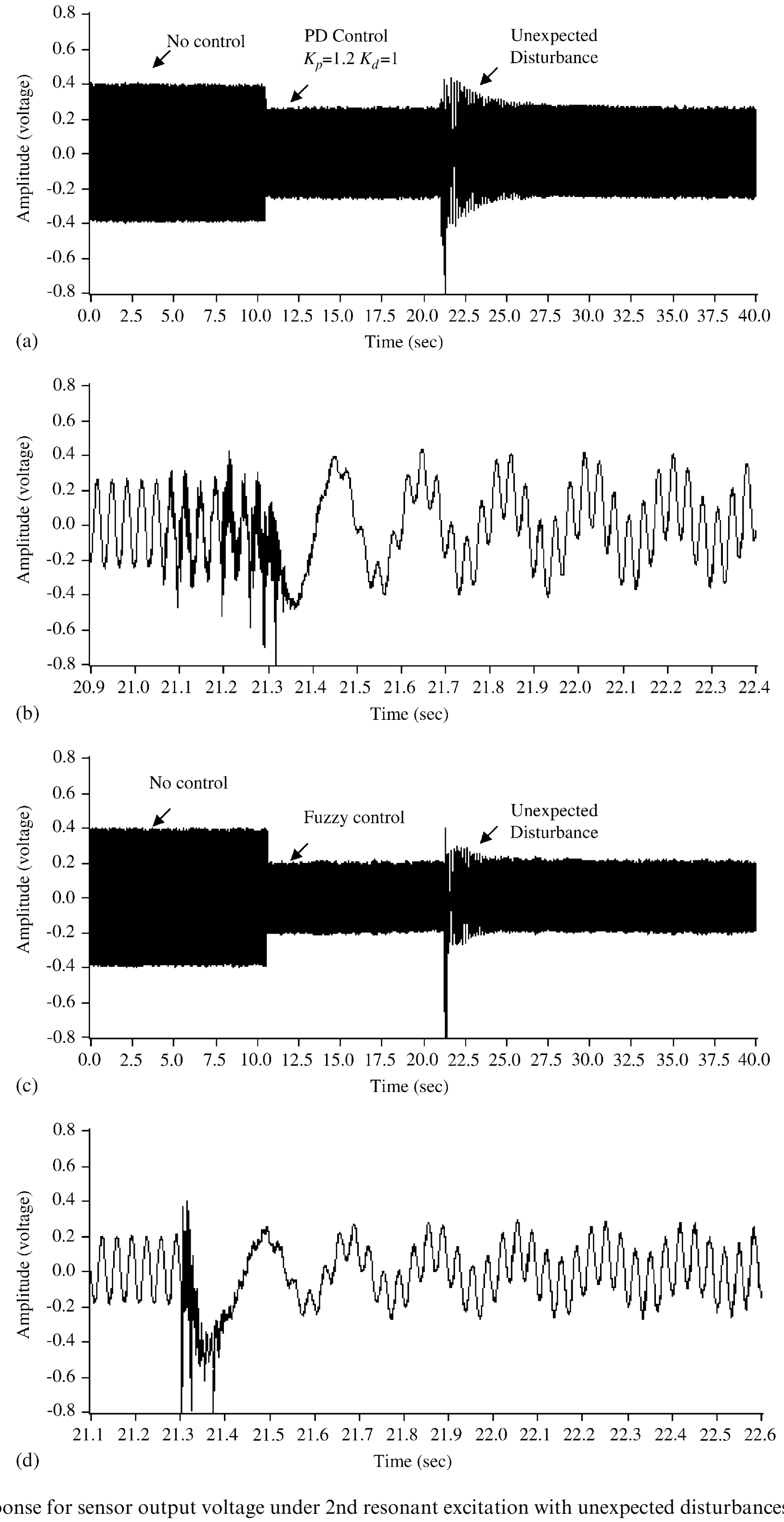

Fig. 17. (a,c) Time response for sensor output voltage under 2nd resonant excitation with unexpected disturbances. (b) Time history at the moment of turning on the PD controller with unexpected disturbance (2nd). (d) Time history at the moment of turning on the fuzzy controller with unexpected disturbance (2nd).

without an explicit model of the plant or controller, and could provide insight and design guidelines for developing an absorber system.

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