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Short Communication

A note on the stability and chaotic motions of a restrained pipe conveying fluid

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Abstract

In this note, the stability and chaotic motions of a standing pipe conveying fluid is studied and further compared with that of a hanging system developed by Jin [1]. The standing pipe involves elastic support and motion-limiting constraints, producing a nonlinear force on the pipe as the motion becomes large. Based on numerical calculations, bifurcation diagram, time trace and phase portrait of the oscillations are obtained. It is shown that the dynamics of the standing pipe is much richer than that of the hanging system. The effect of elastic spring stiffness on the global dynamics of the standing pipe conveying fluid is also discussed.

Keywords: Standing pipe conveying fluid; Stability; Chaotic motion

Because of its importance in engineering, the dynamics of pipes conveying fluid has been investigated widely by many investigators. Jin [1] has investigated the stability and chaotic motions of a restrained pipe conveying fluid (Fig. 1). The pipe system considered by Jin is a fluid-conveying pipe restrained by motion-limiting constraints, and a linear spring support is attached to it at the restrained point. Hence, Jin's analytical model is a modified pipe system if compared with the one analysed by Paidoussis et al. [2]. Jin studied the effect of the spring constant and some other parameters on the dynamics of the system. Attention was concentrated on the possible chaotic behaviour of the system which has been shown to occur in the case of no spring support [2].

It should be noted that the pipe is hanging vertically in Refs. [1,2]. However, the fluid-conveying pipes may be standing in engineering practice (Fig. 2). For the cantilevered pipes conveying fluid, the linear dynamics has been investigated by Paidoussis [3], both for the hanging and standing pipes (for the hanging system, the gravity parameter $\gamma > 0$; for the standing system, $\gamma < 0$). In that linear work, there existed no motion-limiting constraints, and some behaviour of the standing pipes obtained were shown to be different from that of the hanging pipes, i.e. the standing system may become buckled with a low fluid velocity. Subsequently, Li and Paidoussis [4] studied the nonlinear dynamics of pipes conveying fluid, in which the geometric nonlinearities induced by large displacements were considered. In that nonlinear study [4], it was found that the effect of the

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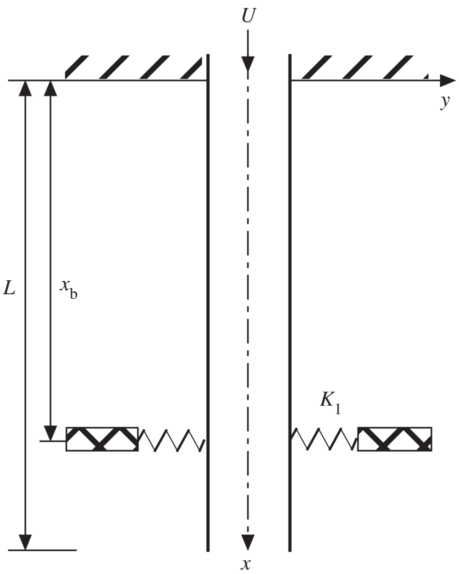

Fig. 1. Schematic of the hanging system treated in Ref. [1].

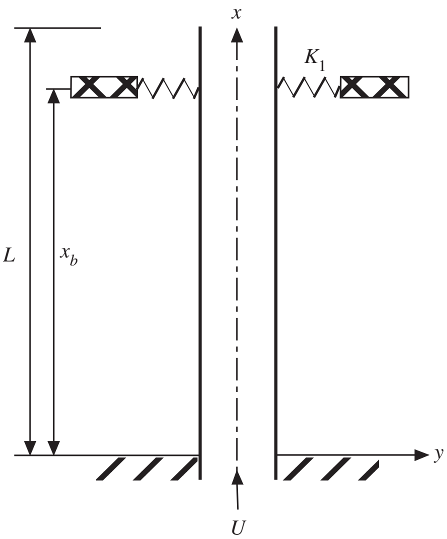

Fig. 2. Schematic of the standing system treated in this paper.

gravity parameter on the dynamics of the pipe is significant. Furthermore, a double degeneracy was detected in the parameter space of the standing system.

It might be stressed that, for the standing and hanging pipes, the dynamics (both linear and nonlinear dynamics) show some differences between these two models while they have the same formulations of equations of motion. In this note, a standing pipe conveying fluid subjected to elastic spring and motion-limiting constraints is considered. As will be shown below, the dynamics of a restrained standing pipe displays more interesting behaviour than that of a restrained hanging pipe.

As mentioned in the foregoing, the equation of motion of a standing system is the same as that of a hanging system. However, for a hanging system, $\gamma > 0$; for a standing system, $\gamma < 0$. Moreover, it has been shown that, for quantitatively more accurate results, the geometric nonlinearities in the pipe dynamics and at least a four-degree-of-freedom analysis are needed [5]. Since the main purpose of this note is to investigate part of the qualitative behaviour of the restrained standing system, the only nonlinearity is associated with the restraints and the two-mode expansion ($N = 2$) is adopted in the analytical model for simplicity [1]. Thus, in this note we use the following four-dimensional first-order ordinary differential equation of motion of the pipe which was obtained by Jin [1] for the system of two-degree-of-freedom:

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{F}(\mathbf{X}), \tag{1}$$

where the dot denotes differentiation with respect to τ, $\tau = [EI/(M+m)]^{1/2}t\big/L^2$; EI is the flexural rigidity of the pipe; M and m are the per-unit-length mass of the fluid and pipe, respectively; L is the length of the pipe, t the time variable; and

$$\mathbf{X} = (x_1, x_2, x_3, x_4)^{\mathrm{T}}, \quad x_1 = q_1(\tau), \quad x_2 = q_2(\tau), \quad x_3 = \dot{q}_1(\tau), \quad x_4 = \dot{q}_2(\tau),$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{bmatrix}, \quad \mathbf{F}(\mathbf{X}) = (0, 0, F_3, F_4)^{\mathrm{T}}$$

$$a_1 = -(\lambda_1^4 + u^2 c_{11} + \gamma e_{11} + k_1 g_{11}), \quad a_2 = -(u^2 c_{12} + \gamma e_{12} + k_1 g_{12}),$$

$$a_3 = -\left(\alpha\lambda_1^4 + 2\sqrt{\beta}ub_{11}\right), \quad a_4 = -2\sqrt{\beta}ub_{12}, \quad b_1 = -(u^2 c_{21} + \gamma e_{21} + k_1 g_{21}),$$

$$b_2 = -(\lambda_2^4 + u^2 c_{22} + \gamma e_{22} + k_1 g_{22}), \quad b_3 = -2\sqrt{\beta}ub_{21}, \quad b_4 = -\left(\alpha\lambda_2^4 + 2\sqrt{\beta}ub_{22}\right),$$

$$F_3 = -k_2\left[\sum_r^N \varphi_r(\xi_b)x_r\right]^3 \varphi_1(\xi_b), \quad F_4 = \varphi_2(\xi_b)F_3/\varphi_1(\xi_b),$$

in which the coefficients c_{ij}, b_{ij}, e_{ij} and g_{ij} are defined in Ref. [1]; x is the longitudinal co-ordinate, $\xi_b = x_b/L$; $u = (M/EI)^{1/2}UL$, U is the flow velocity of the fluid in the pipe; $\alpha = [EI/(M+m)]^{1/2}a/L^2$, $\beta = M/(M+m)$, $\gamma = (M+m)gL^3/EI$, g is the acceleration due to gravity; $k_1 = KL^3/EI$, $k_2 = KL^5/EI$, where K_1 and K_2 are the stiffnesses of the elastic spring and cubic spring, respectively; $\varphi_r(\xi) = \cosh\lambda_r\xi - \cos\lambda_r\xi - \sigma_r(\sinh\lambda_r\xi - \sin\lambda_r\xi)$ is the eigenfunction of the cantilever beam and λ_r represent the rth eigenvalues of the cantilever beam,

$$\sigma_r = [\sinh\lambda_r - \sin\lambda_r]/[\cosh\lambda_r + \cos\lambda_r].$$

It should be mentioned that K_1 and K_2 represent the effect of elastic support and motion-limiting constraints, respectively. Hence, based on Eq. (1), the dynamics of the standing pipe can be obtained by using a fourth-order Runge–Kutta integration algorithm, with a step size of 0.005; the initial conditions employed were $z_1(0) = z_2(0) = 0.001, z_3(0) = z_4(0) = 0$.

As the main aim of the present work is to show the difference between the hanging and standing pipe models, the dimensionless parameter, γ, will be chosen to have a negative value (in this study $\gamma = -10$). Hence, let some other key parameters as

$$\beta = 0.2,\ \alpha = 0.005,\ \xi_b = 0.82,\ k_2 = 100,\ k_1 = 20 \tag{2}$$

This set of system parameters defined in Eq. (2) was also utilized in Ref. [1]. In what follows, it is of interest to investigate, in detail, what behaviour would occur when the dimensionless fluid velocity is varied for the standing pipe. For this purpose, calculations have produced the bifurcation diagram of Fig. 3 for the system parameters defined in Eq. (2). In this figure the displacement plotted in the ordinate is the amplitude of the two-mode approximation of the free-end displacement of the pipe (tip displacement),

$$\eta(1,\tau) \cong \varphi_1(1)x_1(\tau) + \varphi_2(1)x_2(\tau). \tag{3}$$

It should be noted that the transient solutions were discarded in the calculations. As shown in Fig. 3, the global dynamics of the standing pipe is similar to that of the restrained hanging pipe. When the fluid velocity is low, the pipe is stable with zero displacement and no buckling is found. Similar to the conclusion obtained in

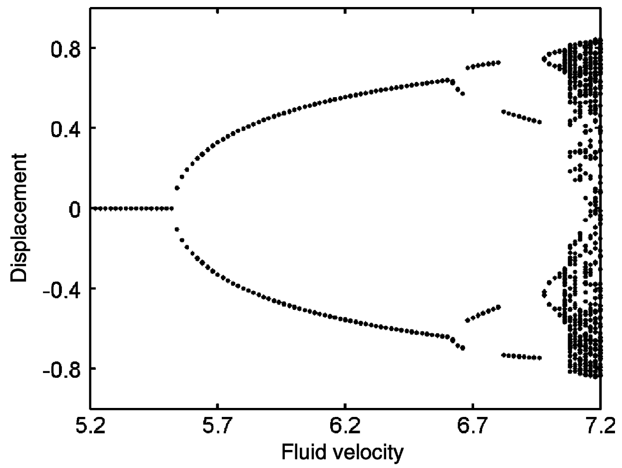

Fig. 3. Bifurcation diagram for the tip displacement of a standing system defined by $\gamma = -10$, $\beta = 0.2, \alpha = 0.005, \xi_b = 0.82$, $k_2 = 100, k_1 = 20$.

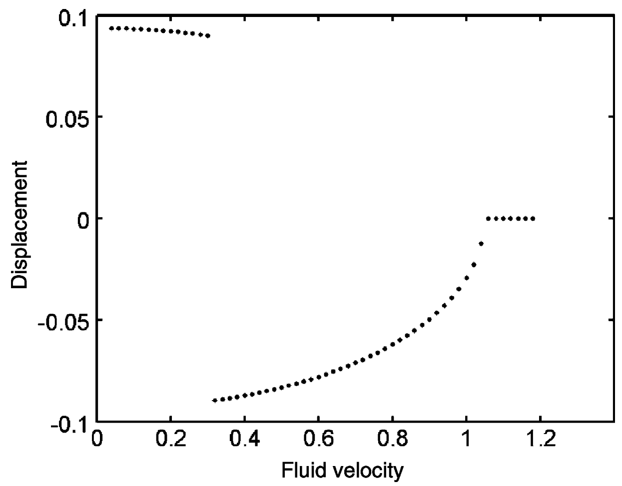

Fig. 4. Bifurcation diagram for the tip displacement of a standing system defined by $\gamma = -10$, $\beta = 0.2, \alpha = 0.005, \xi_b = 0.82, k_2 = 100$, $k_1 = 1$.

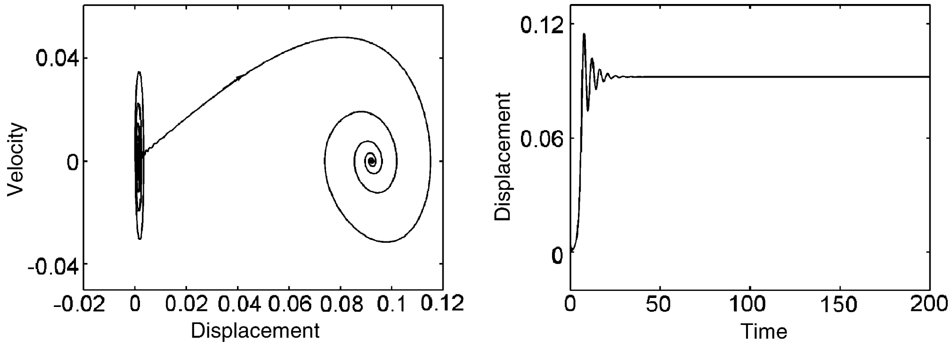

Fig. 5. Phase portrait and time response for a buckling pipe, for the system of Fig. 4, and $u = 0.2$.

Ref. [4], a lower fluid velocity (u_H) corresponding to a Hopf bifurcation can be detected via numerical calculations. The Hopf bifurcation occurs at $u = u_H \approx 5.5$ for a restrained standing pipe, which is much lower than that of a hanging pipe (in the hanging system $u_H \approx 6.8$). Moreover, as can be seen in Fig. 3, the fluid velocities corresponding to the pitchfork and the period-doubling bifurcations are also lower than those of the restrained hanging pipe. Thus, a much lower fluid velocity can induce chaotic motions for the restrained standing pipe.

Nevertheless, if the elastic spring constant is small (e.g. $k_1 = 1$) or even zero, the dynamics of the standing pipe is much richer, as can be seen in Fig. 4. In this case, the standing system is buckled when u is low. This may be caused by the weight of the fluid-conveying pipe itself. The corresponding phase portrait and time response are shown in Fig. 5. This buckling phenomenon has also been found in the dynamics for the standing pipes without restraints [3,4]. Then, increasing u causes the system to regain stability, subsequently to undergo a limit cycle motion and finally to lead to chaos through period-doubling bifurcations. Moreover, it can be seen in Fig. 4 that, a discontinuity (jump) arises in the parameter space of u, the nature of which is not understood. This discontinuity indicates that the static deflection of the standing pipe may suddenly change from a positive displacement to a negative one at a certain fluid velocity.

Note that the mass ratio β in this work is chosen to be $\beta = 0.2$, this is just for a convenient comparison between the hanging and standing systems. The standing pipe may convey air in engineering practice, thus the corresponding mass ratio is very small. However, more extensive calculations show that a restrained standing pipe conveying air behaves like the restrained pipe conveying fluid with a large mass ratio (e.g. $\beta = 0.2$).

Thus, for a standing fluid-conveying pipe restrained by elastic support and motion-limiting constraints, its dynamics is much richer than that of a restrained hanging pipe. For a large value of elastic spring stiffness (in this study $k_1 = 20$), the global dynamics of the standing system is similar to that of a hanging system; but the standing system is much easier to lose its stability in the parameter space of u. If, however, the stiffness of the elastic spring is very small, the standing pipe may be subject to buckling when the fluid velocity is low, and a discontinuity can be detected in the bifurcation diagram when the fluid velocity is chosen to be the variable parameter.

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