



Discussion

Vibration behavior of thermally stressed composite skew plate

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1. Introduction

Vibration and stability characteristics of composite laminates under thermo-mechanical loads have attracted the attention of many researchers in recent years as they are being increasingly used in thin walled structural components of aerospace, defense, reactor vessels and other high performance application areas. These studies were reviewed and well documented by Tauchert [1] and Noor and Burton [2]. It is observed from the existing literature that, the static buckling and postbuckling behavior of thermo-mechanically loaded rectangular and circular plates have been studied by many researchers. However, limited work has been done on the dynamic characteristics of thermally stressed/buckled laminated plates. Moreover, similar studies on thermo-mechanical vibration and stability characteristics of laminated composite plates other than rectangular/circular geometry have been sparsely treated in the published literature.

Thermal load not only influences the stability characteristics of structures, but also changes its vibration frequencies. The notable recent contributions pertaining to small amplitude vibration behavior of thermally stressed/buckled rectangular composite plates may be found in Refs. [3–8]. Chang and Jen [3] studied influences of temperature change and large amplitude on the period of vibration of orthotropic rectangular plates. Noor and Burton [4] investigated the free vibration behavior of thermally stressed angle-ply composite plates using a three-dimensional thermo-elastic model. Librescu and Lin [5] employed single-term Navier-type double sine function to study the vibration behavior of thermo-mechanically loaded flat and curved composite panels taking into account interlaminar shear traction continuity requirement and considering a higher order shell theory.

For accurate solution, the assumed mode shape function in the analytical approach should have more terms, thus leading to more numerical work. Numerical methods, like the finite element method is preferable as there is no need for an a priori assumption of the mode shapes as the solution itself predicts the mode shapes. Lee and Lee [6] used the finite element method to investigate the vibration characteristics of thermally stressed composite plates in the pre- and postbuckling states. Similar studies on the vibration frequencies of thermally buckled piezolaminated composite plates, and composite plates embedded with shape memory alloy fibers were reported by Oh et al. [7] and Park et al. [8]. Moreover, the authors [6–8] employed

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Newton–Raphson method to solve the nonlinear finite element equations and reported the first three natural frequencies in the thermal postbuckling region, omitting the possibility of secondary instability. But, von-Karman plates do exhibit secondary bifurcation [9] and corresponding mode jump [10–12] in their primary postbuckling path. This secondary instability was also reported in the literature by Ganapathi and Touratier [13], Singha et al. [14,15] and Thankam et al. [16], while investigating the thermal postbuckling behavior of composite plates using the finite element method. However, studies on vibration characteristics of thermally buckled composite plates with attention to its stability considerations in the primary postbuckling path are scarce in the open literature.

Similar studies on the vibration and stability characteristics of composite skew plates have received little attention of the researchers, despite its wide application in the aerospace industry [17]. Recently Kant and Babu [18] carried out thermal buckling analysis of composite and sandwich skew plates using the finite element method, while Singha et al. [15] employed shear deformable finite element to investigate the thermo-mechanical postbuckling behavior of laminated composite skew plates. Although, linear free vibration analysis of composite skew plates attracted attention of the researchers in recent years [19,20], vibration analysis of thermally stressed/buckled composite skew plates appears to be scarce in the literature. Ganesan and Dhotarad [21] investigated the vibration behavior of thermally stressed isotropic rectangular and skew plates. However, to the best of the authors' knowledge, the work on the vibration analysis of thermally stressed/buckled composite skew plates is not yet commonly available in the published literature despite its practical significance.

In the present study, a four-noded shear flexible quadrilateral high precision plate bending element [15] is extended to analyze vibration characteristics of thermally stressed composite skew plates in the pre- and postbuckling states. As the element is free from locking phenomenon, all the energy terms are evaluated using full numerical integration scheme. The formulation includes in-plane and rotary inertia effects. The initial imperfections are not considered in the analysis. The nonlinear finite element equations are solved as a sequence of linear eigenvalue problems to trace the thermal postbuckling paths [13–15]. The small amplitude vibration characteristics of thermally stressed composite skew plate are obtained from the linearized equation of motion [6–8]. Limited parametric study is carried out to investigate the influences of the skew angle, lay-up and boundary conditions on the temperature–frequency interaction of laminated skew plates. The present study reveals the “mode shifting” or “exchange of vibration modes” in the thermal postbuckling path.

2. Finite element formulation

Fig. 1 shows the rectangular Cartesian coordinate system along with the associated covariant base vectors ($\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$) and contravariant base vectors (${}^1\mathbf{g}, {}^2\mathbf{g}, {}^3\mathbf{g}$) for the skew plate having a and b as the length and width, and ψ as the skew angle. A four-noded shear deformable quadrilateral plate element with ten degrees of freedom ($u_1^0, u_2^0, u_3; u_{3,1}; u_{3,2}; u_{3,11}; u_{3,12}; u_{3,22}; \gamma_1$ and γ_2) per node is used in the present study. Here, u_1^0, u_2^0 , and u_3 are mid-surface displacements, γ_1 and γ_2 are rotations due to shear, and $(\cdot)_{,i}$ represents the partial differentiation of the variable preceding it with respect to ${}^i r$, the contravariant components of the position vector \mathbf{r} ($\mathbf{r} = {}^1 r \mathbf{g}_1 + {}^2 r \mathbf{g}_2 + {}^3 r \mathbf{g}_3$). The linear polynomial shape functions are employed to describe the field variables corresponding to in-plane displacements (u_1^0, u_2^0) and rotations due to shear of the middle surfaces (γ_1, γ_2), whereas, quintic polynomial function is considered for the lateral displacement (u_3) and are expressed as follows:

$$\begin{aligned} u_1^0 &= c_k ({}^1 r)^i ({}^2 r)^j; \quad i, j = 0, 1 \text{ and } k = 1, 4 \\ u_2^0 &= c_k ({}^1 r)^i ({}^2 r)^j; \quad i, j = 0, 1 \text{ and } k = 5, 8 \\ u_3 &= c_k ({}^1 r)^i ({}^2 r)^j + c_k ({}^1 r)^m ({}^2 r)^n + c_k ({}^1 r)^n ({}^2 r)^m; \\ &\quad i, j = 0, 1, 2, 3; \quad m = 0, 1; \quad n = 4, 5; \text{ and } k = 9, 32 \\ \gamma_1 &= c_k ({}^1 r)^i ({}^2 r)^j; \quad i, j = 0, 1 \text{ and } k = 33, 36 \\ \gamma_2 &= c_k ({}^1 r)^i ({}^2 r)^j; \quad i, j = 0, 1 \text{ and } k = 37, 40, \end{aligned} \quad (1)$$

where c_k are constants and are expressed in terms of nodal displacements in the finite element discretization. The full integration scheme with 6×6 Gaussian integration rule is adopted for computing the element mass

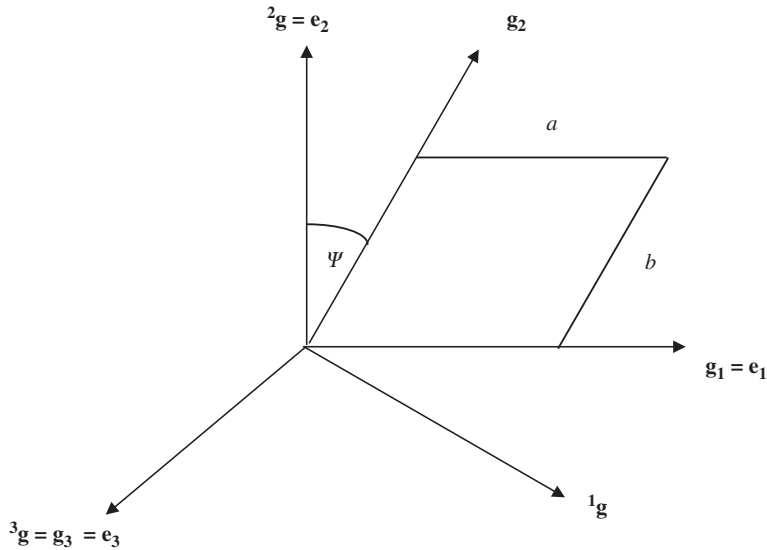


Fig. 1. The oblique coordinate system for the skew plate.

matrix whereas, 4×4 Gaussian integration rule is used to calculate the element stiffness matrix. The in-plane and rotary inertia effects are included in the present analysis. The details of element formulation are reported elsewhere [15] and are not given here for the sake of brevity. It has good convergence properties and has no spurious rigid modes.

Following standard procedure, the governing equation of motion of the plate is written as

$$[M]\{\ddot{\delta}\} + \left[[K] + \frac{1}{2}[N1] + \frac{1}{3}[N2] + \lambda[K_{gm}] \right] \{\delta\} = \lambda\{F^T\}, \tag{2}$$

where $[M]$ is mass matrix, $[K]$ the linear stiffness matrix, $[N1]$ and $[N2]$ are first- and second-order nonlinear incremental stiffness matrices, respectively; $[K_{gm}]$ and $\{F^T\}$ are geometric stiffness matrix and load vector due to unit temperature rise, λ the temperature parameter and $\{\delta\}$ the displacement vector.

3. Solution procedure

To analyze the free vibration of thermally buckled laminated plates, the solution of differential equation (2) is assumed to be sum of a time-independent and a time-dependent solutions such as $\{\delta\} = \{\delta_s\} + \{\delta_t\}$. Here, $\{\delta_s\}$ is time-independent particular solution, which means the static thermal postbuckling deflection, and $\{\delta_t\}$ is the time-dependent homogeneous solution with small magnitude. Here, the subscripts ‘s’ and ‘t’ indicate the static and dynamic displacements, respectively. Substituting the assumed solution into the equation of motion (2), two sets of equations are obtained [6–8] as

$$\left[[K] + \frac{1}{2}[N1] + \frac{1}{3}[N2] + \lambda[K_{gm}] \right] \{\delta_s\} = \lambda\{F^T\}, \tag{3}$$

$$[M]\{\ddot{\delta}\} + [[K] + [N1] + [N2] + \lambda[K_{gm}]]\{\delta_t\} = \{0\}. \tag{4}$$

Here, Eq. (3) represents the static thermal postbuckling behavior whereas Eq. (4) represents the small amplitude vibration of thermally stressed/buckled plate. It may be noted that, the sum of stiffness matrices in Eq. (4) equals the tangent stiffness matrix.

3.1. Thermal postbuckling

In the present study of symmetric laminated plates under uniform temperature rise, only the membrane forces are generated and the governing equation for the thermal postbuckling analysis is written as

$$\left[[K] + \frac{1}{2}[N1] + \frac{1}{3}[N2] + \lambda[K_{gm}] \right] \{\delta_s\} = \{0\}. \quad (5)$$

Here, the force vector in the right-hand side is zero, since the temperature effects (membrane forces) are accounted in the geometric stiffness matrix. The scalar multiplier λ , associated with geometric stiffness matrix, represents the temperature load. The thermal postbuckling problem is solved as a series of linear eigenvalue problem each one of them associated with different amplitudes of deflection [13–15].

3.2. Vibration of thermally buckled plate

Noting that the variation of displacement with time is harmonic, we could express Eq. (4) as the nonlinear algebraic equation of the form

$$[[K] + [N1] + [N2] + \lambda[K_{gm}]]\{\delta\} - \omega^2[M]\{\delta_t\} = \{0\}. \quad (6)$$

Small amplitude vibration of thermally stressed composite plate is obtained by solving Eq. (6) as a generalized eigenvalue problem. In the prebuckling state, the nonlinear incremental stiffness matrices ($[N1]$ and $[N2]$) are zero. After buckling, the incremental stiffness matrices are calculated at the updated equilibrium position, obtained from static thermal postbuckling analysis.

In the present analysis, the static thermal postbuckling temperature is obtained first and then the small amplitude free vibration frequencies of composite skew plates are evaluated at the corresponding thermally buckled state. Thereafter, the next equilibrium position is determined.

4. Results and discussions

The present study is focused on the small amplitude free flexural vibration characteristics of thin laminated composite skew plates under thermal load. Based on the progressive mesh refinement, 10×10 mesh is found to be adequate to model the full skew plate. The material properties used in the present analysis are

$$E_L/E_T = 40.0, \quad G_{LT}/E_T = 0.6, \quad G_{TT}/E_T = 0.5, \quad \nu_{LT} = 0.25, \quad \alpha_L = 10^{-06}, \quad \alpha_T = 10^{-05},$$

where E , G , ν and α are Young's modulus, shear modulus, Poisson's ratio and thermal expansion coefficient, respectively. Subscripts L and T represent the longitudinal and transverse directions, respectively, with respect to the fibers. All the layers are of equal thickness. The boundary conditions considered here are:

simply supported on all sides (SS) : $u_1 = u_2 = u_3 = 0$ along the boundary nodes,

clamped on all edges (CC) : $u_1 = u_2 = u_3 = 0$, $u_{3,1} = 0$ at ${}^1r = 0, a$

$u_1 = u_2 = u_3 = 0$, $u_{3,2} = 0$ at ${}^2r = 0, b$.

Before proceeding for the detailed study of small amplitude vibration characteristics of thermally stressed/buckled composite skew plate, the formulation developed herein is validated against the linear free vibration of laminated composite skew plate. The non-dimensional natural frequencies ($\varpi = \omega a^2 / \pi^2 h \sqrt{\rho/E_T}$; a and h are length and thickness of the plate, and ρ is the mass density) obtained for simply supported (SS) and clamped (CC) angle-ply $[45^\circ/-45^\circ/45^\circ/-45^\circ/45^\circ]$ skew plates are presented in Table 1 along with the analytical solutions [19], and they match very well. The efficacy of the present element in thermal postbuckling analysis has already been reported earlier [15]. Moreover, the element is free from locking and any spurious zero-energy mode.

Table 1

Comparison of non-dimensional natural frequencies ($\varpi = \omega a^2 / \pi^2 h \sqrt{\rho / E_T}$) of five-layered angle-ply $[45^\circ / -45^\circ / 45^\circ / -45^\circ / 45^\circ]$ laminates for various skew angles ψ ($a/b = 1$; $a/h = 1000.0$)

B.C.	Skew angle		Modes					
			1	2	3	4	5	6
SS	0	Present	2.4339	4.9859	6.1814	8.4849	10.2506	11.6433
		Ref. [19]	2.4339	4.9865	6.1818	8.4870	10.2536	11.6464
	30	Present	2.6118	5.6890	6.8308	9.4737	11.8828	13.2258
		Ref. [19]	2.6119	5.6902	6.8316	9.4773	11.8900	13.2355
	45	Present	3.3187	6.8972	9.6861	10.7119	15.5093	16.1214
		Ref. [19]	3.3182	6.9002	9.6908	10.7206	15.5318	16.1447
CC	0	Present	3.8996	7.1426	8.4532	11.2008	13.3040	14.7223
		Ref. [19]	3.9009	7.1464	8.4585	11.2112	13.3216	14.7425
	30	Present	4.5425	8.3787	9.8764	12.8428	15.6673	17.4547
		Ref. [19]	4.5431	8.3819	9.8810	12.8533	15.6906	17.4889
	45	Present	6.2903	10.7941	14.4487	15.4235	20.9774	21.9865
		Ref. [19]	6.3048	10.8193	14.4949	15.4692	21.0620	22.0759

Thereafter, a detailed numerical study is carried out for small amplitude vibration behavior of thermally stressed laminated skew plates for which results are not available in the literature. The effects of temperature on the stability and vibration characteristics of a simply supported five-layered cross-ply $[0^\circ / 90^\circ / 0^\circ / 90^\circ / 0^\circ]$ skew plate ($a/b = 1$, $a/h = 100.0$) are reported in Fig. 2(a)–(d). Fig. 2(a) represents the static thermal postbuckling path for different values of skew angle (ψ). It is observed from the figure that, initially the plate maximum displacement increases (w_{\max}/h) with the increase in temperature followed by a sudden instability (drop in the postbuckling resistance) in the postbuckling path. Thereafter, the temperature again increases marginally with the increase in maximum displacement (w_{\max}/h). This sudden instability in the postbuckling path is termed as “secondary instability” [9,13–15] and is associated with redistribution of buckling mode shape as observed in Fig. 3, where the buckling modes before and after the secondary instability are plotted. It may be noted that, initially when the plate is in the primary post buckling path, the displacement shapes are symmetric with the maximum displacement (w_{\max}) occurring at the center of the plate. However, after transition to secondary postbuckling path, the displacement shapes become unsymmetrical with respect to plate central lines and the maximum deflection shifts towards one side of the plate in the case of cross-ply plate and towards one corner in the case of angle-ply plate [14,15].

The lowest three natural frequencies of the thermally stressed laminated simply supported skew plates are presented in Figs. 2(b)–(d) in the pre- and postbuckled states for skew angles 0° , 30° and 45° , respectively. The frequency–temperature relation is plotted up to the secondary buckling temperature, as this may be assumed as the maximum load carrying capacity. From the figures, it is observed that, all the frequencies decrease with the increase in temperature, and the first (1, 1) frequency becomes zero at the first buckling point. However, in the postbuckling region, the first frequency (1, 1) increases, while the other frequencies (1, 2) and (2, 1) decrease with the increase of temperature. The corresponding vibration modes at different amplitudes ($w_{\max}/h = 0.2, 0.4$, and 0.5) of postbuckling state for the case of square cross-ply plate are presented in Fig. 4. As the temperature increases, the fundamental frequency changes to a form corresponding to next higher mode (1, 2). Here, the crossing of frequencies between mode (1, 1) and mode (1, 2) occurs at a temperature of 96.48°C ($w_{\max}/h = 0.455$). This phenomenon may be termed as “mode shifting” (exchange of vibration modes). This exchange of vibration modes occurs in between primary and secondary instability temperature for all the cases. Moreover, the fundamental frequency (1, 2) decreases as the temperature is increased and approaches zero at the secondary instability point ($w_{\max}/h = 0.55$, $T = 110.78$).

The similar studies on stability and vibration characteristics of simply supported and clamped five-layered angle-ply $[45^\circ / -45^\circ / 45^\circ / -45^\circ / 45^\circ]$ skew plates are carried out and presented in Figs. 5 and 6, respectively. The

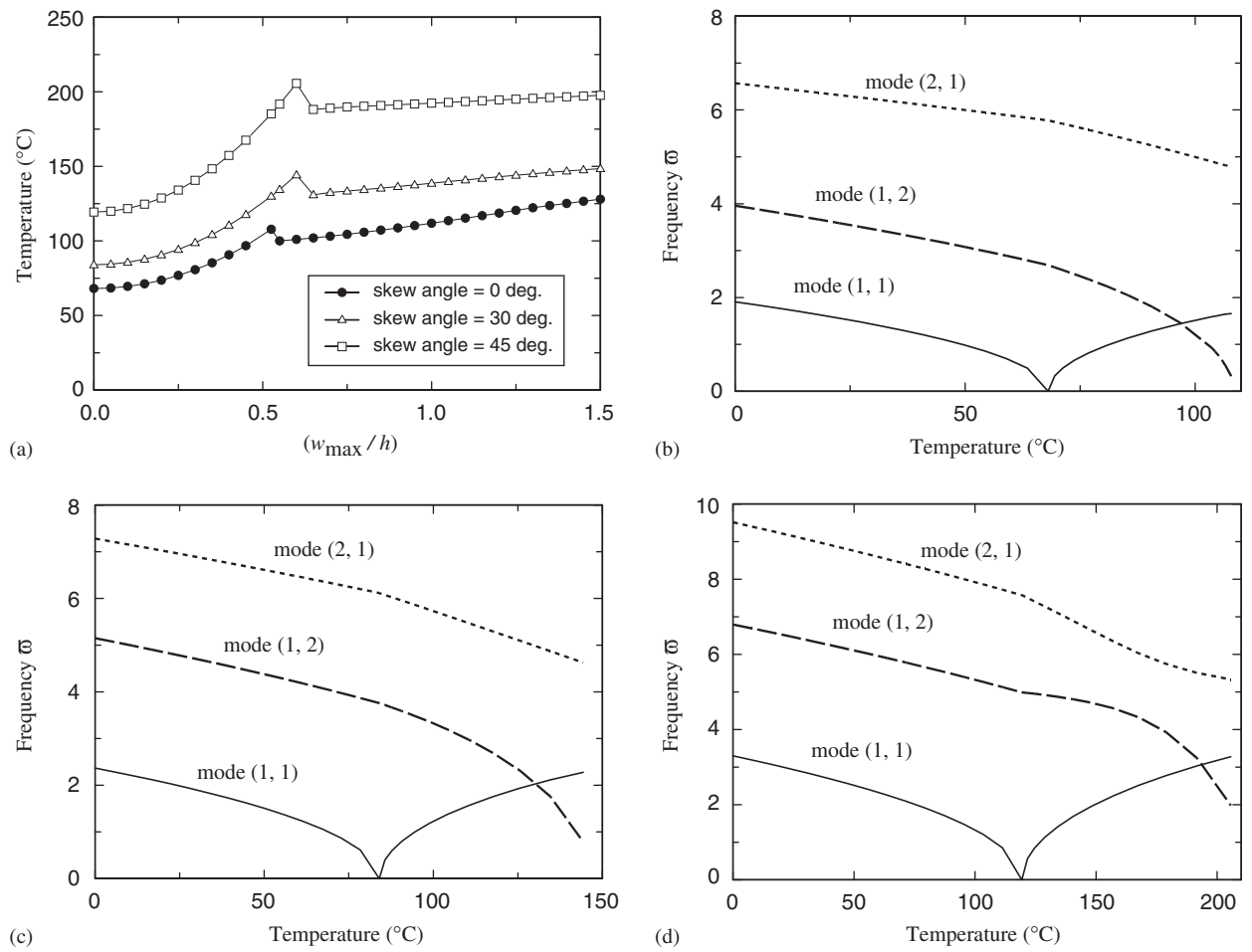


Fig. 2. Stability and vibration characteristics of thermally stressed simply supported five-layered cross-ply $[0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]$ plate for various skew angles ψ ($a/b = 1$; $a/h = 100.0$; non-dimensional frequency, $\varpi = \omega a^2 / \pi^2 h \sqrt{\rho/E_T}$). (a) Static thermal postbuckling path, (b) temperature-frequency curves ($\psi = 0^\circ$), (c) temperature-frequency curves ($\psi = 30^\circ$) and (d) temperature-frequency curves ($\psi = 45^\circ$).

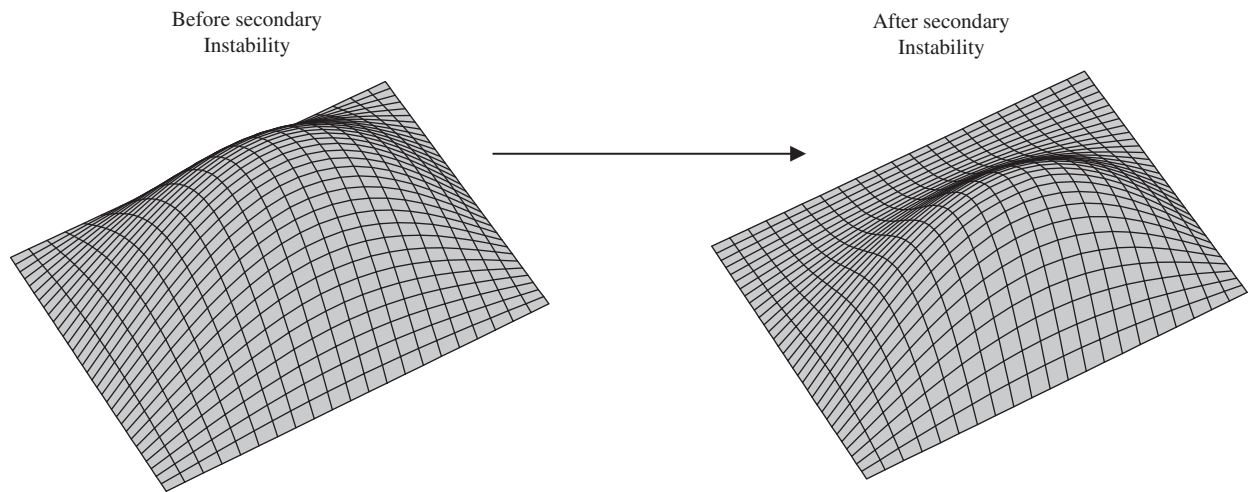


Fig. 3. The buckling mode shapes before and after the secondary instability.

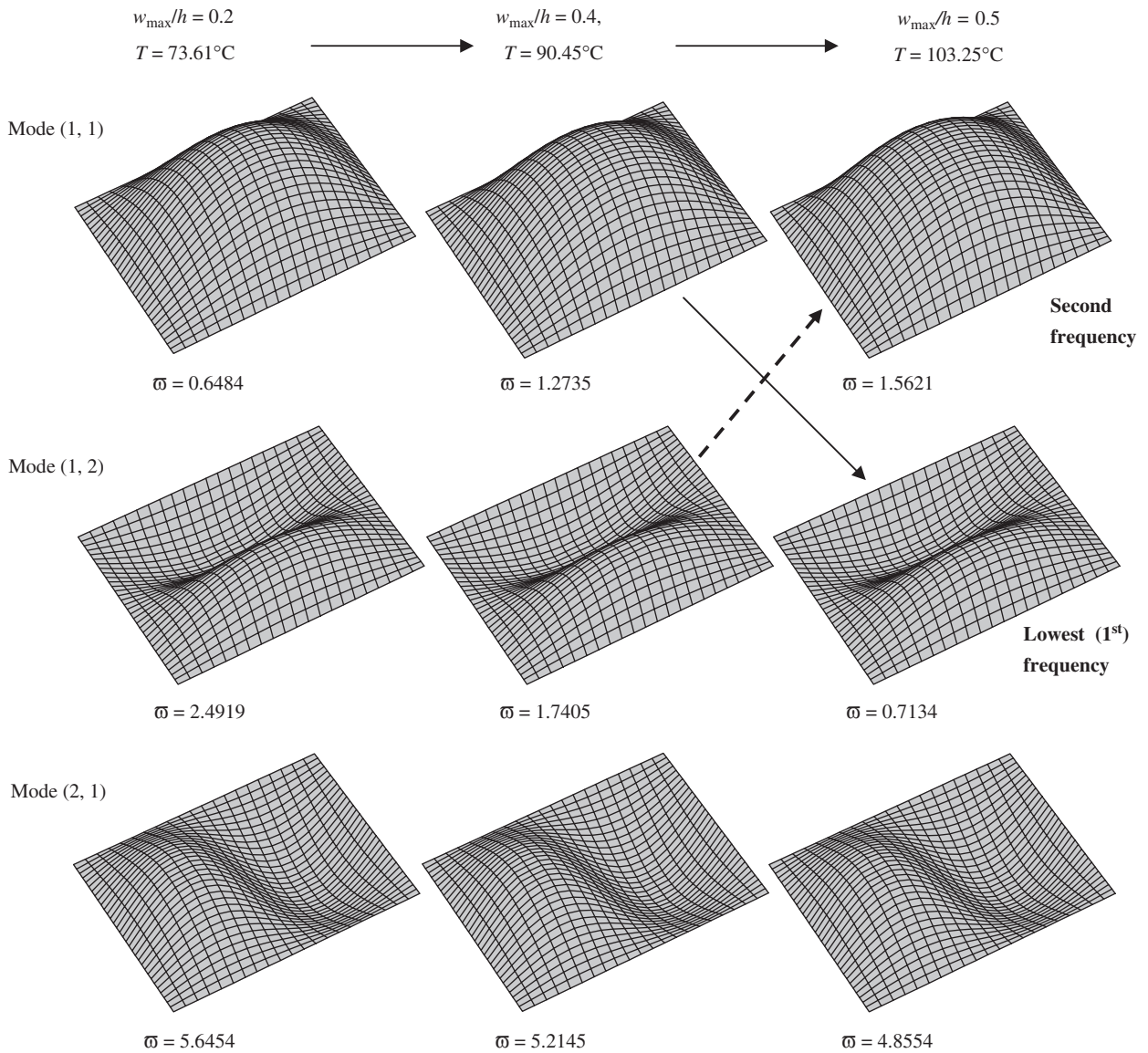


Fig. 4. The vibration mode along the postbuckling paths.

secondary instability is observed for all the cases; however, its location depends on skew angle, fiber orientation and boundary condition. The temperature–frequency interaction curve is similar for all the cases. The first frequency (1, 1) always approaches zero at the critical buckling temperature and then slowly increases with the increase of temperature. The other frequencies decrease monotonically with the increase of temperature and the second frequency (1, 2) approaches zero at the secondary buckling temperature.

5. Conclusions

The small amplitude vibration characteristics of thermally stressed laminated composite skew plates are studied using a shear deformable finite element. The analysis reveals the possibility of secondary instability in the primary postbuckling path. The first three natural frequencies are studied in the pre- and postbuckled states till the point of secondary bifurcation. Limited parametric study has been carried out to study the

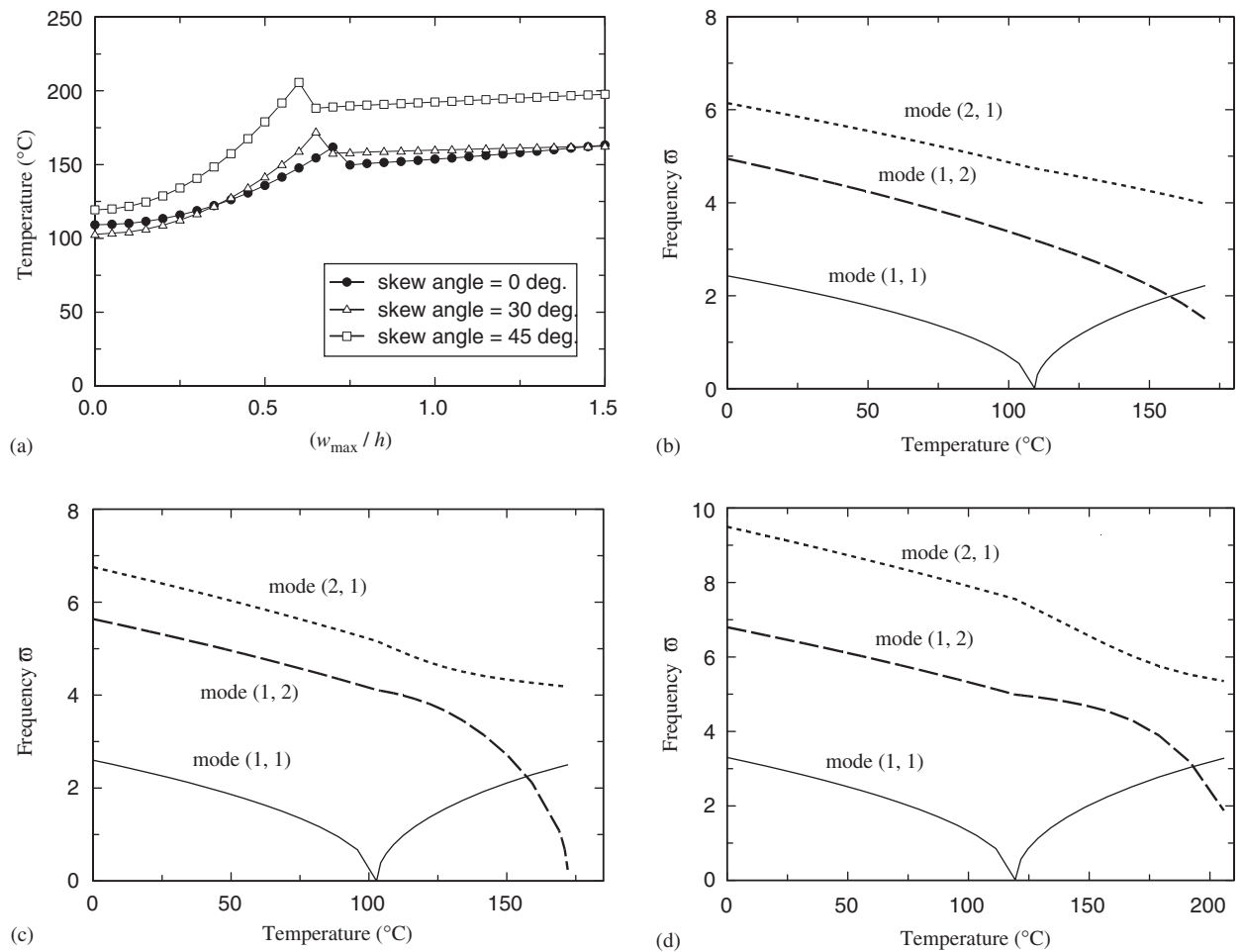


Fig. 5. Stability and vibration characteristics of thermally stressed simply supported five-layered angle-ply $[45^\circ/-45^\circ/45^\circ/-45^\circ/45^\circ]$ plate for various skew angles ψ ($a/b = 1$; $a/h = 100.0$; non-dimensional frequency, $\varpi = \omega a^2 / \pi^2 h \sqrt{\rho/E_T^0}$). (a) Static thermal postbuckling path, (b) temperature-frequency curves ($\psi = 0^\circ$), (c) temperature-frequency curves ($\psi = 30^\circ$) and (d) temperature-frequency curves ($\psi = 45^\circ$).

influences of fiber orientation, skew angle, and boundary condition on the vibration characteristics of thermally stressed composite plate. Zero natural frequencies are observed at the instability points, because of its singularity. From the above study, the following conclusions may be drawn:

- The primary postbuckling path becomes unstable at certain amplitude (w_{\max}/h) depending on skew angle, fiber orientation and boundary condition of the composite plate. This instability is termed as “secondary instability” and is associated with redistribution of buckling mode.
- The frequency corresponding to first mode (1, 1) decreases with the increase in temperature and becomes zero at the first bifurcation point, after which it increases monotonically with the increase in temperature.
- The frequencies corresponding to higher modes (1, 2) and (2, 1) decrease monotonically in the postbuckling state and frequency (1, 2) approaches zero at the secondary instability point.
- As the temperature increases, the fundamental frequency changes to a form corresponding to next higher mode (1, 2). The crossing of frequencies between mode (1, 1) and mode (1, 2) occurs in between primary and secondary instability temperature. This phenomenon may be termed as “mode shifting” (exchange of vibration modes).

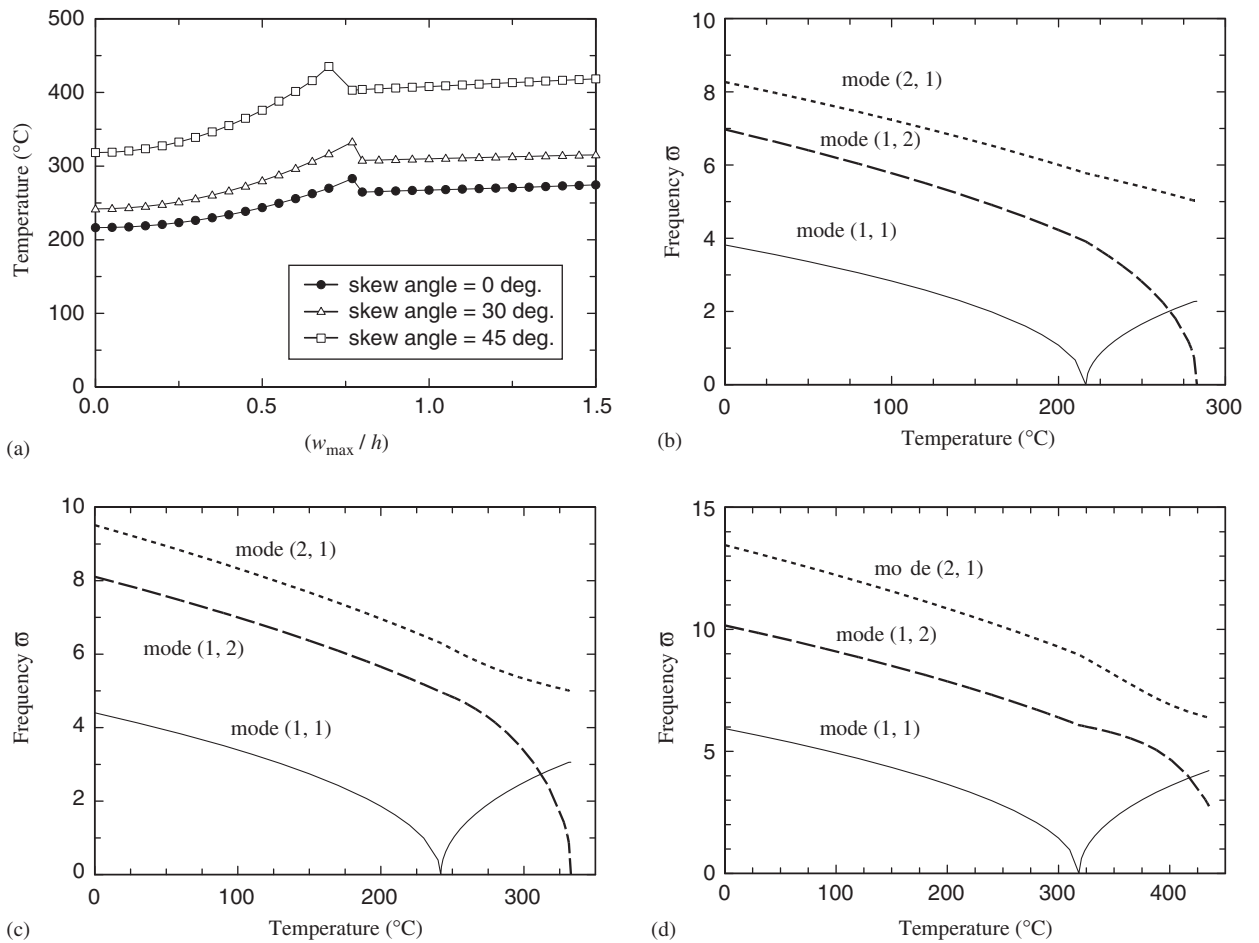


Fig. 6. Stability and vibration characteristics of thermally stressed clamped five-layered angle-ply $[45^\circ/-45^\circ/45^\circ/-45^\circ/45^\circ]$ plate for various skew angles ψ ($a/b = 1$; $a/h = 100.0$; non-dimensional frequency, $\varpi = \omega a^2 / \pi^2 h \sqrt{\rho/E_T}$). (a) Static thermal postbuckling path, (b) temperature-frequency curves ($\psi = 0^\circ$), (c) temperature-frequency curves ($\psi = 30^\circ$) and (d) temperature-frequency curves ($\psi = 45^\circ$).

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