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Effect of rotation on Rayleigh–Lamb waves in magneto-thermoelastic media

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Abstract

In this paper propagation of magnetic-thermoelastic plane waves in an initially unstressed, homogeneous isotropic, conducting plate, rotating about the normal to its faces with uniform angular velocity, under the action of uniform magnetic field has been investigated. The generalized theory of thermoelasticity is employed, by assuming electrical behavior as quasi-static and the mechanical behavior as dynamic, to study the problem. The secular equations for both symmetric and skew symmetric waves have been obtained. The magneto-elastic shear horizontal (SH) modes of wave propagation gets decoupled from rest of the motion here, however it may not be in general possible if the rotation took place in an arbitrary direction. These waves propagate without the influence of temperature change and thermal relaxation time. At short wavelength limits, the secular equations for symmetric and skew symmetric modes reduce to Rayleigh surface wave frequency equation. Thin plate results are also deduced. Finally, the dispersion curves are computed and represented graphically for various modes of wave propagation in different theories of thermoelasticity. The amplitudes of displacement, perturbed magnetic field and temperature change are also obtained analytically, computed numerically at the end and plotted graphically. The result in case of non-rotating media, elastokinetics, magneto-elasticity and coupled magneto-elasticity has also been deduced as special cases at appropriate stages of this work.

1. Introduction

The interaction between thermal and strain fields in a conducting thermoelastic plate gives rise to the theory of dynamic coupled magneto-thermoelasticity. Increasing attention is being devoted to this theory due to its many engineering applications in the fields of magnetic storage elements, magnetic structural elements, biotechnology and corresponding measurement techniques of magneto-elasticity. The theory of thermoelasticity and thermoelastic waves in solids is well established, see Refs. [1–4]. The governing field equations in classical dynamic coupled thermoelasticity (CT) are wave-type (hyperbolic) equations of motion and a diffusion-type (parabolic) equation of heat conduction. Therefore it is seen that part of the solution of the energy equation extends to infinity, implying that if a homogeneous isotropic elastic medium is subjected to thermal or mechanical disturbances the effect of temperature and displacement fields are felt at an infinite

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distance from the source of disturbance. This shows that part of the disturbance has an infinite velocity of propagation, which is physically impossible. With this drawback in mind, some researchers, such as Lord and Shulman [5] and Green and Lindsay [6], modified Fourier law and constitutive relations so as to get a hyperbolic equation for heat conduction. These works include the time needed for the acceleration of heat flow and take into account the coupling between temperature and strain fields for isotropic materials. Dhaliwal and Singh [7] carried out a detailed survey and described various problems in the field.

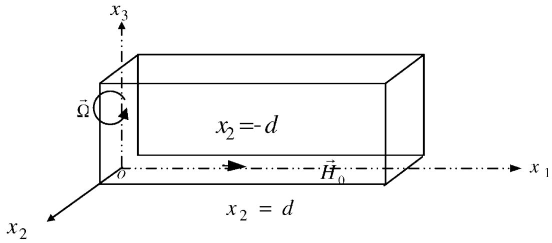
Chand et al. [8] presented an investigation on the distribution of deformation stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting, elastic half-space. Dhaliwal and Saxena [9] investigated the generalized magneto-thermoelastic waves in an infinite elastic solid with a cylindrical cavity. Ezzat [10] studied the generation of generalized magneto-thermoelastic waves by a thermal shock in a perfectly conducting half-space. Paul and Muthiyalu [11] studied magneto-thermoelastic free vibrations in an infinite plate and verified their results numerically for aluminum alloy 24S-T4. Paul and Narasimhan [12] studied the vibrations of a thermoelastic infinite plate in a large magnetic field, in the context of coupled theory of thermoelasticity. Sharma and Chand [13] studied the transient magneto-thermoelastic waves in the context of generalized theories of thermoelasticity developed by Lord and Shulman [5], and Green and Lindsay [6]. Sharma and Chand [14] investigated the distribution of deformation, temperature field, perturbed magnetic field and stresses in vacuum as well as in elastic medium under rotation due to thermal shock acting on its boundary. Wang and Dai [15] studied the magneto thermodynamic stress and perturbation of magnetic field vector in an orthotropic thermoelastic cylinder. Sharma and Pal [16] studied the Rayleigh–Lamb waves in magneto-thermoelastic homogeneous isotropic plates. Chandrasekharaiah [17] investigated the propagation of magneto-elastic transverse surface waves in an internal stratum. Dhaliwal and Sherief [18] extended the theory of generalized thermoelasticity developed by Lord and Shulman [5] to anisotropic elastic bodies. Nayfeh and Nasser [19] discussed the propagation of surface waves in homogeneous isotropic solids in the context of coupled and generalized thermoelastic bodies. Noda et al. [20] derived a formulation of generalized thermoelasticity for one-dimensional problems. Sherief and Ezzat [21] investigated the thermal shock problem in magneto-thermoelasticity with thermal relaxation.

The effect of rotation on elastic waves, both partial and surface, has been studied by many authors [22–24]. Ting [25] investigated the interfacial waves in a rotating anisotropic elastic half-space by extending the Stroh [26] formalism. He obtained explicit expressions of the polarization vector and the secular equation for monoclinic material half-space rotating about the normal to the plane of symmetry. Fang et al. [27] investigated the effect of rotation on surface acoustic waves in a piezoelectric half-space. It is shown that a piezoelectric material may not permit propagation of more than one rotation-perturbed surface wave even if both Rayleigh and Bleustein–Gulyaev waves are permissible in the absence of rotation. Fang et al. [28] investigated the effect of rotation on the characteristics of waves propagating in a piezoelectric plate. The rotation sensitivity of the wave dispersion relations for polarized ceramic plates was analyzed in details in the context of gyroscope applications. The effect of rotation on frequency shift in case of long and short waves have also been explored. Zhou and Jiang [29] studied the effects of Coriolis force and centrifugal force on acoustic waves in a piezoelectric half-space.

The present paper deals with the study of magneto-thermoelastic waves in a rotating, homogeneous isotropic, conducting plate in the context of generalized theories of thermoelasticity [5,6]. The plate is assumed to rotate about the normal to its faces with uniform angular velocity which allows the decoupling of magneto-elastic SH modes. The secular equations for symmetric and skew symmetric modes have been derived and discussed. The short wavelength and thin plate results have also been deduced and discussed. The analytical expressions for amplitudes of displacement, perturbed magnetic field and temperature change have been derived. The results obtained theoretically have been computed numerically and presented graphically for carbon steel [14] material plate.

2. Formulation of the problem

We consider an infinite homogeneous isotropic, electrically and thermally conducting, plate of thickness 2d initially at uniform temperature T_0 in contact with the vacuum. We take the origin of the coordinate system (x_1, x_2, x_3) on the middle surface of the plate with x_3 axis along the normal of the plate. The $x_1 - x_3$ plane is choosen to coincide with the middle surface and x_2-axis is taken along the thickness, as shown in Geometry below.



Geometry of the problem

The surfaces $x_2 = \pm d$ are assumed to be stress free, insulated or isothermal boundaries. In addition to this we also assume that the electromagnetic field is quasi-static. We assume that plate is rotating about the normal to its faces with uniform angular speed $\mathbf{\Omega} = (0, 0, \Omega)$ and initial magnetic field $\mathbf{H}_0$ is acting along x_1-direction which vanishes at the boundaries. When the medium undergoes dynamical deformation, the two additional terms namely,

(i) the time-dependent part of the centripetal acceleration $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ and
(ii) the Coriolis acceleration $2(\mathbf{\Omega} \times \dot{\mathbf{u}})$, where $\mathbf{u}$ is the displacement vector,

which do not appear in case of non-rotating medium will also appear in the governing equations here.

The basic governing equations of generalized isotropic thermoelasticity and electromagnetic interactions, in the absence of body forces and heat sources are:

$$(\lambda + \mu)\nabla\nabla.\mathbf{u} + \mu\nabla^2\mathbf{u} + \mu_0\sigma\mathbf{e} \times \mathbf{H}_0 + \mu_0^2\sigma(\dot{\mathbf{u}}.\mathbf{H}_0)\mathbf{H}_0 - \mu_0^2\sigma\mathbf{H}_0^2\dot{\mathbf{u}} - \beta^*\nabla\left(T + \delta_{2k}t_1\dot{T}\right)$$
$$= \rho(\ddot{\mathbf{u}} + \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{u} + 2\mathbf{\Omega} \times \dot{\mathbf{u}}), \tag{1}$$

$$K\nabla^2 T - \rho C_e\left(\dot{T} + t_0\ddot{T}\right) = T_0\beta^*(\dot{e} + t_0\delta_{1k}\ddot{e}), \tag{2}$$

$$\nabla \times \mathbf{e} = 0, \quad \nabla \times \mathbf{h} = \sigma\left(\mathbf{e} + \mu_0\dot{\mathbf{u}} \times \mathbf{H}_0\right), \quad \nabla.\mathbf{h} = 0 \tag{3}$$

where $\mathbf{H}_0 = (\mathrm{H}, 0, 0)$, $\mathbf{u}(x_1, x_2, x_3, t) = (u_1, u_2, u_3)$ is the displacement vector, $T(x_1, x_2, x_3, t)$ is temperature change, $\mathbf{\Omega} = (0, 0, \Omega)$ is the angular velocity of rotation. Here $\mathbf{e}$ and $\mathbf{h}$ are the perturbations in the electric and magnetic fields, σ is the electrical conductivity, μ_0 is magnetic permeability, λ, μ are *Lame's* parameters, ρ and C_e are, respectively, the density and specific heat at constant strain; K is the thermal conductivity, e is dilatation, and $\beta^* = (3\lambda + 2\mu)\alpha_t$, α_t being linear thermal expansion. Here δ_{1k} is Kronecker delta in which $k = 1$, for Lord–Shulman (LS) theory and $k = 2$ corresponds to Green–Lindsay (GL) theory of thermoelasticity.

We define the quantities:

$$x_j = \omega^* x_j/c_1, \quad t' = \omega^* t, \quad u_j' = \rho\omega^* c_1 u_j/\beta T_0, \quad T' = T/T_0,$$

$$h_j' = h_j/H, \quad e_j' = e_j/\mu_0 c_1 H, \quad t_0' = \omega^* t_0, \quad t_1' = \omega^* t_1, \quad d' = \omega^* d/c_1,$$

$$\varepsilon_T = \beta^{*2} T_0/\rho c_e(\lambda + 2\mu), \quad \varepsilon_v = \rho c_e/\beta^*, \quad \varepsilon_H = \varpi^*/\mu_0\sigma c_1^2, \quad c_2^2 = \mu/\rho,$$

$$R_H = \mu_0 H^2/\rho c_1^2, \quad \omega^* = c_e(\lambda + 2\mu)/k, \quad c_1^2 = \lambda + 2\mu/\rho, \quad \Omega' = \frac{\Omega}{\omega}, \tag{4}$$

where c_1, c_2 are, respectively, the longitudinal and shear wave velocities in the solid plate, ω^* is characteristic frequency of the solid plate, ε_T is thermoelastic coupling constant, ε_H is thermoelastic-deformation and electromagnetic coupling constant, R_H is magnetic pressure number. The effect of magnetic field is to increase the thermoelastic coupling constant by an amount $(1 + R_H/\varepsilon_H)$. Upon introducing the quantities (4) in Eqs. (1)–(3), the non-dimensional basic governing equations of motion and heat conduction, are given by

$$u_{1,11} + (1 - \delta^2)u_{2,12} + \delta^2 u_{1,22} - \ddot{u}_1 + \Omega^2 u_1 + 2\Omega\dot{u}_2 = (T + t_1\delta_{2k}\dot{T})_{,1}, \tag{5}$$

$$(1-\delta^2)u_{1,12}+u_{2,22}+\delta^2 u_{2,11}-\ddot{u}_2-\frac{R_H}{\varepsilon_H}\dot{u}_2-2\Omega\dot{u}_1+\Omega^2 u_2=(T+t_1\delta_{2k}\dot{T})_{,2}, \tag{6}$$

$$\delta^2(u_{3,11}+u_{3,22})-\frac{R_H}{\varepsilon_H\varepsilon_T\varepsilon_V}e_2-\frac{R_H}{\varepsilon_H}\dot{u}_3-\ddot{u}_3=0, \tag{7}$$

$$T_{,11}+T_{,22}-(\dot{T}+t_0\ddot{T})=\varepsilon_T\left[\dot{u}_{1,1}+\dot{u}_{2,2}+\delta_{1k}t_0(\ddot{u}_{1,1}+\ddot{u}_{2,2})\right], \tag{8}$$

$$\begin{gathered} e_{2,1}-e_{1,2}=0, \quad e_1-\varepsilon_H h_{3,2}=0, \quad e_2+\varepsilon_T\varepsilon_V\dot{u}_3+\varepsilon_H h_{3,1}=0, \\ \varepsilon_T\varepsilon_V\dot{u}_2+\varepsilon_H(h_{2,1}-h_{1,2})=0, \quad h_{1,1}+h_{2,2}=0. \end{gathered} \tag{9}$$

Here dashes have been suppressed for convenience. The comma notation is used for spatial derivates and superposed dot represents time differentiation. The non-dimensional boundary conditions on the plate surfaces $x_2=\pm d$ are expressed as

$$\begin{gathered} u_{1,2}+u_{2,1}=0, \quad u_{2,2}+\left(1-2\delta^2\right)u_{1,1}-R^* h_1-(T+\delta_{2K}t_1 T)=0, \\ T_{,2}+H^* T=0, \quad h_3=0, \quad u_{3,2}=0, \quad h_1\pm iR_H/\bar{R}_H h_2=0, \end{gathered} \tag{10}$$

where $R^*=\left(\mu_0-\bar{\mu}_0\right)H^2/\beta^* T_0$, $\bar{R}_H=\bar{\mu}_0 H^2/\rho c_1^2$ and $\bar{\mu}_0$ is the permeability of the free space and H^* is the surface heat transfer coefficient of the medium. Here $H^*\to 0$ corresponds to thermally insulated boundaries of the plate and $H^*\to\infty$ refers to isothermal one.

3. Solution of the problem

In order to solve the problem we assume the solution of the form

$$(u_j,e_j,h_j,T)=\left(\bar{u}_j,\bar{e}_j,\bar{h}_j,T\right)\exp\left\{i\xi(x_1+mx_2-ct)\right\}, \tag{11}$$

where $c=\omega/\xi$ is the phase velocity, ω being the circular frequency and ξ is the wavenumber . Upon adopting the procedure and approach of Sharma and Pal [11], after lengthy but straight forward algebraic reductions and manipulations, we obtain

$$u_1=\sum_{q=1}^{4}\left[A_q c_q-B_q s_q\right]\exp\left\{i\xi(x_1-ct\right\}, \tag{12}$$

$$u_2=\sum_{q=1}^{4}iV_{2q}\left[A_q s_q+B_q c_q\right]\exp\left\{i\xi(x_1-ct\right\}, \tag{13}$$

$$h_1=\sum_{q=1}^{4}V_{3q}\left[A_q c_q-B_q s_q\right]\exp\left\{i\xi(x_1-ct\right\}, \tag{14}$$

$$h_2=\sum_{q=1}^{4}iV_{4q}\left[A_q s_q+B_q c_q\right]\exp\left\{i\xi(x_1-ct\right\}, \tag{15}$$

$$T=\sum_{q=1}^{4}V_{5q}\left[A_q c_q-B_q s_q\right]\exp\left\{i\xi(x_1-ct\right\}, \tag{16}$$

$$u_3=\sum_{q=5}^{6}\left[A_q c_q-B_q s_q\right]\exp\left\{i\xi(x_1-ct\right\}, \tag{17}$$

$$e_2 = \sum_{q=5}^{6} iV_{7q}\left[A_q c_q + B_q s_q\right] \exp\left\{i\xi(x_1 - ct\right\}, \tag{18}$$

$$e_1 = \sum_{q=5}^{6} V_{8q}\left[A_q s_q - B_q c_q\right] \exp\left\{i\xi(x_1 - ct\right\}, \tag{19}$$

$$h_3 = \sum_{q=5}^{6} iV_{9q}\left[A_q c_q + B_q s_q\right] \exp\left\{i\xi(x_1 - ct\right\}, \tag{20}$$

where

$$V_{2q} = \begin{cases} m_q\left(m_q^2 - \beta^2\right)/(m_q^2 - \beta_H^2), & q = 1, 2, 4, \\ -\left[\delta^2 m_q^2 - \alpha^2 + \tau_1 c V_{5q}\right]/m_q(1 - \delta^2), & q = 3, \end{cases}$$
$$V_{3q} = -m_q V_{4q}, \quad V_{34} = 0,$$

$$V_{4q} = \frac{c\in_T\in_V}{\in_H(1 + m_q^2)} V_{2q}, \quad q = 1, 2, 3, \quad V_{44} = 0, \tag{21}$$

$$\begin{gathered} V_{5q} = \begin{cases} -\left[\delta^2 m_q^2 - \alpha^2 + m_q V_{2q}(1 - \delta^2)\right]/\tau_1 c, & q = 1, 2, 4, \\ -\left[(\delta^2 m_q^2 - \alpha^2)(m_q^2 - \delta^2\beta_H^2) - m_q^2(1 - \delta^2)^2\right]/\delta^2\tau_1 c(m_q^2 - \beta_H^2), & q = 3, \end{cases} \\ V_{7q} = -\xi^2\varepsilon_H\varepsilon_T\varepsilon_V\delta^2\left(m_q^2 - \beta_H^2\right)/R_H, \quad q = 5, 6, \quad V_{8q} = V_{7q}/m_q, \quad q = 5, 6, \\ V_{9q} = V_{7q}/i\xi\varepsilon_H m_q^2, \quad q = 5, 6, \end{gathered} \tag{22}$$

$$\alpha^2 = c^2\left(1 + \Gamma^2\right) - 1, \quad \beta^2 = \frac{c^2}{\delta^2}\left(1 + \Gamma^2\right) - 1, \quad \beta_H^2 = \frac{c^2}{\delta^2}\left(1 + i\omega^{-1}\frac{R_H}{\varepsilon_H} + \Gamma^2\right) - 1,$$
$$\tau_1 = t_1\delta_{2k}i\omega^{-1}, \quad \Gamma^2 = \frac{\Omega^2}{\omega^2}, \quad s_q = \sin\left(\xi m_q x_2\right), \quad c_q = \cos\left(\xi m_q x_2\right).$$

In Eqs. (12)–(22) the characteristic roots $m_q^2 \quad (q = 1, 2, 3, 4, 5, 6)$ are given by

$$m_i^2 = \left(a_i^2 c^2 - 1\right), \quad i = 1, 2, 3, \quad m_4^2 = -1, \quad m_{5,6}^2 = \frac{1}{2}\left[\frac{c^2}{\delta^2} \pm \sqrt{\frac{c^2}{\delta^2} - 4\beta_H^2}\right] - 1, \tag{23}$$

where

$$\begin{gathered} a_1^2 + a_2^2 + a_3^2 = \left(1 + \Gamma^2\right)\left(1 + \frac{1}{\delta^2}\right) + \tau_0 - i\omega\tau_1\tau_0'\varepsilon_T + \frac{1}{\delta^2}\left(i\omega^{-1}\frac{R_H}{\varepsilon_H} + \Gamma^2\right), \\ \sum a_1^2 a_2^2 = \tau_0\left(1 + \Gamma^2\right) + \frac{1 + \tau_0 + \Gamma^2}{\delta^2} - \frac{i\omega\tau_1\tau_0'}{\delta^2}\varepsilon_T + \left(i\omega^{-1}R_H/\varepsilon_H + \Gamma^2\right)\left[\frac{1 + \tau_0 + \Gamma^2}{\delta^2}\right] \\ a_1^2 a_2^2 a_3^2 = \frac{\tau_0}{\delta^2}\left[1 + \left(1 + \Gamma^2\right)\left(i\omega^{-1}R_H/\varepsilon_H + \Gamma^2\right)\right], \\ \tau_0 = t_0 + i\omega^{-1}, \quad \tau_1 = t_1\delta_{2k} + i\omega^{-1}, \quad \tau_0' = t_0\delta_{1k} + i\omega^{-1}. \end{gathered} \tag{24}$$

In the absence of rotation ($\Gamma = 0$), we have

$$
\begin{aligned}
a_1^2 + a_2^2 + a_3^2 &= 1 + \frac{1}{\delta^2} + \tau_0 - i\omega\tau_1\tau_0'\varepsilon_T + \frac{i\omega^{-1}R_H}{\delta^2 \ \varepsilon_H}, \\
\sum a_1^2 a_2^2 &= \tau_0 + \frac{1+\tau_0}{\delta^2} - \frac{i\omega\tau_1\tau_0'}{\delta^2}\varepsilon_T + \frac{i\omega^{-1}R_H}{\varepsilon_H}\left(\frac{1+\tau_0}{\delta^2}\right), \\
a_1^2 a_2^2 a_3^2 &= \frac{\tau_0}{\delta^2}\left(1 + \frac{i\omega^{-1}R_H}{\varepsilon_H}\right).
\end{aligned} \tag{25}
$$

In the absence of magnetic field ($R_H \to 0$), Eqs. (25) reduce to

$$a_1^2 + a_2^2 = 1 + \tau_0 - i\omega\tau_1\tau_0'\varepsilon_T, \quad a_1^2 a_2^2 = \tau_0, \quad a_3^2 = \frac{1}{\delta^2}. \tag{26}$$

In this case the shear vertical (SV) wave also gets decoupled from rest of the motion and the characteristic roots correspond to their counterparts in generalized thermoelasticity [30].

4. Derivation of secular equation

Invoking the boundary conditions (10) at the surfaces $x_2 = \pm d$ of the plate and using solutions (12)–(20) we obtain a system of 12 simultaneous linear equations in each case which has a non-trivial solution if the determinant of the coefficients of amplitudes $[A_1, A_2, A_3, A_4, A_5, A_6, A_1, B_2, B_3, B_4, B_5, B_6]^{\mathrm{T}}$ vanishes. This, after applying lengthy algebraic reductions and manipulations, leads to the following secular equations for a stress free, thermally insulated and isothermal plate:

$$\frac{T_5}{T_6} = \left[\frac{m_6 V_{95}}{m_5 V_{96}}\right]^{\pm 1}, \tag{27}$$

$$
\begin{vmatrix}
D_{11} & D_{12} & D_{13} & D_{14} \\
D_{21}T_1^{\pm 1} & D_{22}T_2^{\pm 1} & D_{23}T_3^{\pm 1} & D_{24}T_4^{\pm 1} \\
D_{31} & D_{32} & D_{33} & 0 \\
D_{41} & D_{42} & D_{43} & D_{44}
\end{vmatrix}
= -H^*
\begin{vmatrix}
D_{11} & D_{12} & D_{13} & D_{14} \\
D_{21}T_1^{\pm 1} & D_{22}T_2^{\pm 1} & D_{23}T_3^{\pm 1} & D_{24}T_4^{\pm 1} \\
D_{31} & D_{32} & D_{33} & 0 \\
V_{51}T_1^{\pm 1} & V_{52}T_2^{\pm 1} & V_{53}T_3^{\pm 1} & V_{54}T_4^{\pm 1}
\end{vmatrix}, \tag{28}
$$

where $D_{1q} = m_q V_{2q} + 1 - 2\delta^2 - \dfrac{R^* V_{3q}}{i\xi} + \tau_1 c V_{5\mathrm{q}}$,

$$D_{2q} = m_q + V_{2q}, \quad D_{3q} = V_{3q}T_q^{\mp 1} + i\frac{R_H}{\bar{R}_H}V_{4q}, \quad D_{4q} = m_q V_{5q} \tag{29}$$

and $T_q = \tan(\xi m_q d), \quad q = 1, 2, 3, 4.$

Here $H^* \to \infty$ refers to isothermal boundaries of the plate and $H^* \to 0$ corresponds to thermally insulated boundaries of the plate. Here the superscript $+1$ corresponds to skew- symmetric and -1 to that of symmetric modes of wave propagation in the plate. The Eqs. (27) and (28) are the secular equations for the propagation of modified guided magneto-thermoelastic. We refer such waves as magneto-thermoelastic plate waves rather than lamb waves, whose properties were firstly derived by Lamb in 1917 for isotropic solids in elastokinetics. The secular Eqs. (28) govern the symmetric and skew symmetric motion of the plate with stress free thermally insulated isothermal boundaries and Eq. (27) is the secular equation of decoupled magneto-elastic shear horizontal (SH) modes of wave propagation in the plate. These modes are not influenced and affected by thermal variations and thermal relaxation times as expected.

The secular equation (28) in case of isothermal and thermally insulated plate, respectively, become

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{D_{22}G_2}{D_{21}G_1}\left[\frac{T_2}{T_3}\right]^{\pm 1} - \frac{D_{24}G_4}{D_{21}G_1}\left[\frac{T_4}{T_3}\right]^{\pm 1} = \frac{-D_{23}G_3}{D_{21}G_1}, \tag{30}$$

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{D_{12}G_2'}{D_{11}G_1'}\left[\frac{T_2}{T_3}\right]^{\pm 1} - \frac{D_{14}G_4'}{D_{11}G_1'}\left[\frac{T_4}{T_3}\right]^{\pm 1} = -\frac{D_{13}G_3'}{D_{11}G_1'}, \tag{31}$$

where

$$\begin{aligned}
G_1 &= D_{33}T_3^{\mp 1}(D_{12}V_{54} - D_{14}V_{52}) - D_{32}T_2^{\mp 1}(D_{13}V_{54} - D_{14}V_{53}),\\
G_2 &= D_{33}T_3^{\mp 1}(D_{11}V_{54} - D_{14}V_{51}) - D_{31}T_1^{\mp 1}(D_{13}V_{54} - D_{14}V_{53}),\\
G_3 &= D_{32}T_2^{\mp 1}(D_{11}V_{54} - D_{14}V_{51}) - D_{31}T_1^{\mp 1}(D_{12}V_{54} - D_{14}V_{52}),\\
G_4 &= D_{32}T_2^{\mp 1}(D_{12}V_{53} - D_{13}V_{51}) - D_{31}T_1^{\mp 1}(D_{12}V_{53} - D_{13}V_{52}) + D_{33}T_3^{\mp 1}(D_{12}V_{51} - D_{11}V_{52}).
\end{aligned} \tag{32}$$

Here G_q' can be obtained from G_q by replacing V_{5q} with m_qV_{5q} and D_{1q} with D_{2q}, $q = 1,2,3,4$.

If we write

$$c^{-1} = V^{-1} + i\omega^{-1}Q, \tag{33}$$

so that $\xi = R + iQ$, where $R = \omega/V$ and V, Q are real numbers. Also the characteristic roots m_q, $q = 1,2,3$ are, in general complex, and hence we assume that $m_q = \alpha_q + i\beta_q$, so that the exponent in the plane wave solutions (11) becomes

$$-R\left\{\frac{Q}{R}x_1 + m_q^I x_2\right\} - iR\left\{x_1 - m_q^R x_2 - Vt\right\},$$

where $m_q^R = \alpha_q - \beta_q Q/R$, $m_q^I = \beta_q + \alpha_q Q/R$. This shows that V is the propagation velocity and Q is the attenuation coefficient of the wave. Upon using representation (33) in secular equations (30) and (31), the values of propagation speed V and attenuation coefficient Q for different modes of wave propagation can be obtained. Since $c' = c/c_1$ is the non-dimensional complex phase velocity, so $V' = V/c_1$ and $Q' = c_1 Q$ are the non-dimensional phase speed and attenuation coefficient, respectively. Here dashes have been omitted for convenience.

5. Particular cases of secular equation

In this section, we deduce some particular forms of the secular equation in various theories of thermoelasticity and magneto-elasticity.

5.1. Coupled magneto-elasticity

In case of CT the thermal relaxation times vanish, i.e. $t_0 = 0 = t_1$ so that $\tau_0 = \tau'_0 = \tau_1 = i\omega^{-1}$ consequently, Eq. (24) reduce to

$$\begin{aligned}
a_1^2 + a_2^2 + a_3^2 &= 1 + \Gamma^2 + \frac{1}{\delta^2}(1+\Gamma^2) + i\omega^{-1}\left(1 + \varepsilon_T + \frac{R_H}{\delta^2\varepsilon_H}\right),\\
\sum a_1^2a_2^2 &= \frac{1+\Gamma^2}{\delta^2} + i\omega^{-1}\left(1 + \Gamma^2 + \frac{1+\Gamma^2+\varepsilon_T}{\delta^2} + \frac{R_H}{\varepsilon_H\delta^2}\right) - \frac{\omega^{-2}R_H}{\delta^2\varepsilon_H},\\
a_1^2a_2^2a_3^2 &= \frac{i\omega^{-1}}{\delta^2}\left[1 + (1+\Gamma^2)\left(i\omega^{-1}\frac{R_H}{\varepsilon_H} + \Gamma^2\right)\right].
\end{aligned} \tag{34}$$

Sub-case: When the rotation is absent ($\Gamma = 0$), we have

$$\begin{aligned}
a_1^2 + a_2^2 + a_3^2 &= 1 + \frac{1}{\delta^2} + i\omega^{-1}\left(1 + \varepsilon_T + \frac{R_H}{\delta^2\varepsilon_H}\right),\\
\sum a_1^2a_2^2 &= \frac{1}{\delta^2} + i\omega^{-1}\left(1 + \frac{1+\varepsilon_T}{\delta^2} + \frac{R_H}{\varepsilon_H\delta^2}\right) - \frac{\omega^{-2}R_H}{\delta^2\varepsilon_H},\\
a_1^2a_2^2a_3^2 &= \frac{i\omega^{-1}}{\delta^2}\left[1 + i\omega^{-1}\frac{R_H}{\varepsilon_H}\right].
\end{aligned} \tag{35}$$

The wave propagation in the plate is governed by the secular equations (27) and (28) with reduced values of the characteristic roots $m_i^2 = 1, 2, 3$ in these equations.

5.2. Magneto-elasticity

In case of uncoupled thermoelasticity (magneto-elasticity) the thermomechanical coupling constant $\in_T = 0$, which leads to $m_2^2 = \tau_0 c^2 - 1$ and m_1^2, m_3^2 are given by

$$m^4 - \left(\alpha^2 + \beta^2 + i\omega^{-1}c^2\frac{R_H}{\varepsilon_H}\right)m^2 + \alpha^2\beta_H^2 - \Gamma^2 c^4 = 0 \tag{36}$$

Sub-case: In the absence of rotation ($\Gamma = 0$), we have

$$m^4 - \left(\alpha^2 + \beta^2 + i\omega^{-1}c^2\frac{R_H}{\varepsilon_H}\right)m^2 + \alpha^2\beta_H^2 = 0. \tag{37}$$

Consequently, the secular equations are given by the equations

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{D'_{24}G_4}{D'_{21}G_1}\left[\frac{T_4}{T_3}\right]^{\pm 1} = \frac{D'_{23}G_3}{D'_{21}G_1}, \tag{38}$$

where

$$\begin{gathered} G_1 = D_{14}D_{33}T_3^{\pm 1}, \quad G_3 = D_{14}D_{31}T_1^{\pm 1}, \\ G_4 = D_{11}D_{33}T_3^{\pm 1} - D_{13}D_{31}T_1^{\pm 1}, \\ D'_{1q} = m_q V_{2q} + 1 - 2\delta^2 + iR^* V_{3q}, \quad q = 1, 2, 4, \\ D_{3q} = V_{3q}T_q^{-1} + i\frac{R_H}{\bar{R}_H}V_{4q}. \end{gathered} \tag{39}$$

5.3. Generalized thermoelasticity

In the absence of magnetic field Eqs. (30) and (31) reduce to

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_2(1 - a_1^2)}{m_1(1 - a_2^2)}\left[\frac{T_2}{T_3}\right]^{\pm 1} = \frac{-(\beta^2 - 1)^2(a_1^2 - a_2^2)}{4\beta m_1(1 - a_2^2)}, \tag{40}$$

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_1(1 - a_1^2)}{m_2(1 - a_2^2)}\left[\frac{T_2}{T_3}\right]^{\pm 1} = \frac{-4\beta m_1(a_1^2 - a_2^2)}{(\beta^2 - 1)^2(1 - a_2^2)}, \tag{41}$$

where $T_1 = \tan(m_1 d)$, $T_2 = \tan(m_2 d)$, $T_3 = \tan(m_3 d)$.

In Eq. (40) $m_3^2 = \beta^2$ and the other two roots are given by the equation

$$m^4 - \left[\alpha^2 + \gamma^2 + i\omega c^2\varepsilon_T\tau_1\tau'_0\right]m^2 + (\alpha^2\gamma^2 + i\omega c^2\varepsilon_T\tau_1\tau') = 0, \quad \gamma^2 = \tau_0 c^2 - 1. \tag{42}$$

Eqs. (40) and (41) are the same as obtained and discussed by Sharma et al. [30] and Sharma and Singh [31].

Eq. (33) in uncoupled theory ($\varepsilon_T = 0$) and Eq. (31) in the absence of magnetic field reduce to the classical case in elastokinetics as given below

$$\frac{\tan\alpha d}{\tan\beta d} = -\left[\frac{(\beta^2 - 1)^2}{4\alpha\beta}\right]^{\pm 1}. \tag{43}$$

Eq. (43) is the Rayleigh–Lamb frequency equation and has already been discussed in detail by Graff [32] and Achenbach [33].

6. Regions of the secular equations

Here depending on whether m_1, m_3, m_5 being purely imaginary or complex, the frequency Eqs. (30) and (31) are correspondingly altered as follows.

Region I: When the characteristic roots are of type $m_k^2 = -m_k'^2, k = 1, 3, 5$; which implies that $m_k = im_k'$ is purely imaginary or complex number. This ensures that the superposition of partial waves has the property of "exponential decay". In this case the secular equations are written from Eqs. (30) and (31) by replacing circular tangent functions of m_k, $k = 1,3,5$ with hyperbolic tangent functions of m'_k, $k = 1, 3, 5$.

Region II: In case two of the characteristic roots are of the type $m_k^2 = -m_k'^2, k = 1, 3$; then the frequency equation can be obtained from Eqs. (30) and (31) by replacing circular tangent functions of m_k, $k = 1,3$ with hyperbolic tangent functions of m_k', $k = 1, 3$.

Region III: In the general case the roots $m_k^2, k = 1, 3, 5$ are complex numbers, and then the frequency equation is given by Eqs. (30) and (31).

7. Thin plate results

When the transverse wavelength with respect to thickness is quite large $\xi d \ll 1$, regions I and II yield the results of interest in this case. In region I, the symmetric case has no roots. For skew symmetric case on retaining the first two terms in the expression of hyperbolic tangents the secular equation reduces to

$$F - \frac{\gamma'^3}{3} G = 0, \tag{44}$$

where

$$F = D_{21} G_1^* m'_1 - D_{22} G_2^* m'_2 + D_{23} G_3^* m'_3 - D_{24} G_4^* m_4,$$
$$G = D_{21} G_1^* m_1'^3 - D_{22} G_2^* m_2'^3 + D_{23} G_3^* m_3'^3 - D_{24} G_4^* m_4'^3, \quad \gamma' = \xi d.$$

Here

$$G_1^* = D_{33}^*(D_{12} V_{54} - D_{14} V_{52}) - D_{32}^*(D_{13} V_{54} - D_{14} V_{53}),$$
$$G_2^* = D_{33}^*(D_{11} V_{54} - D_{14} V_{51}) - D_{31}^*(D_{13} V_{54} - D_{14} V_{53}),$$
$$G_3^* = D_{32}^*(D_{11} V_{54} - D_{14} V_{51}) - D_{31}^*(D_{12} V_{54} - D_{14} V_{52}),$$
$$G_4^* = D_{32}^*(D_{12} V_{53} - D_{13} V_{51}) - D_{31}^*(D_{13} V_{53} - D_{13} V_{52}) + D_{33}^*(D_{12} V_{51} - D_{11} V_{52}), D_{3q}^* = V_{3q} + i\frac{R_H}{\bar{R}_H} T_q^{\mp 1}.$$

In the absence of magnetic effects the equations for thin plate results reduce to that of thermoelasticity and are given below:

Isothermal plate ($H^* \to \infty$): In this case Eq. (38) provides us

$$\alpha'^2 - \frac{\left(\beta'^2 + 1\right)^2}{4} = \frac{\gamma'^2}{3} \left[\left(m_1'^2 + m_2'^2\right) \alpha'^2 - m_1'^2 m_2'^2 - \frac{\left(\beta'^2 + 1\right)^2}{4} \beta'^2 \right],$$

which further implies that

$$c = 2\delta \sqrt{1 - \delta^2} \left[1 + \frac{1}{12} \gamma^2 F^* \right], \quad F^* = 1 - 4\delta^4 (1 - i\omega \varepsilon_T \tau_1 \tau'_0. \tag{45}$$

Thermally insulated plate ($H^* \to \infty$): In case of thermally insulated boundaries of the plate Eq. (38) leads to

$$\left(\beta'^2 - 1\right)^2 + \frac{4}{3} \gamma'^2 \beta'^4 - \frac{\alpha'^2 \gamma'^2}{3} \left(\beta'^2 + 1\right)^2 = 0,$$

where $\beta'^2 = 1 - \dfrac{c^2}{\delta^2}$.

This upon discarding the terms of order higher than c^4/δ^4, leads to

$$c = 2\delta\gamma\sqrt{\frac{(1-\delta^2)}{3}}. \tag{46}$$

This result, with linear dependence of c on γ agrees with that derived from classical plate theory in elastokinetics [32,33] and of course pertains to the flexure vibration and represents only single vibrational modes in limited frequency range in the overall frequency spectrum. No effect of thermomechanical coupling has been observed on thin plates in this case.

In region II the antisymmetric case has no roots and for symmetric case only first term is retained in the expansion of tangents and hyperbolic tangents. Therefore, in region II the thin plate results for stress free isothermal plate are given by equation

$$D_{21}G_1m_1' - D_{22}G_2m_2' + D_{23}G_3m_3 - D_{24}G_4m_4' = 0. \tag{47}$$

In the absence of magnetic effect the secular equation (47) for thin plate results reduce to that of thermoelasticity as below:

Isothermal plate ($H^* \to \infty$): In case of plate with isothermal boundaries, we have

$$\left(\beta^2 - 1\right)^2 = 4\beta^2,$$

which further implies that

$$c = 2\delta(1.707, \quad 0.2929)^{1/2}. \tag{48}$$

Thermally insulated plate ($H^* \to \infty$): For plates with thermally insulated boundaries, we have

$$\alpha'^2 - m_1'^2 - m_2'^2 = \frac{4m_1'^2 m_2'^2}{\left(\beta^2+1\right)^2},$$

which implies that

$$c = 2\delta\sqrt{1 - \frac{\delta^2}{1+\varepsilon_T}\left\{\frac{P}{2}\left(1 \pm \sqrt{1 - \frac{Q}{P^2}}\right)\right\}}, \tag{49}$$

where

$$P = 1 + \frac{1}{4i\omega^{-1}\delta^2\left(1+\varepsilon_T-\delta^2\right)}, \quad Q = \frac{1-\delta^2(1+\varepsilon_T)}{i\omega^{-1}\delta^2\left(1+\varepsilon_T-\delta^2\right)},$$

Thus the phase velocity is given by $c = 2\delta\sqrt{1-\delta^2/(1+\varepsilon_T)}$, which is the thin plate or plane stress analogue of the bar velocity of longitudinal rod theory. In general, here the wave mode depends upon the thermoelastic coupling parameter, whose phase velocity is given by Eq. (49). Thus in the case of thin plates fundamental symmetric mode S_0 becomes dispersionless, the phase velocity is equal to the group velocity $\left(\approx 2\delta\sqrt{1-\delta^2/(1+\varepsilon_T)}\right)$. The fundamental skew symmetric (A_0) mode meanwhile becomes the flexural or bending wave of the plate with its phase velocity. For a thin plate, A_0 mode is essentially a transverse mode, i.e. the z-component of the displacement dominates. While on the contrary in case of S_0 mode x-component dominates.

8. Waves at short wavelength

$$D_{11}''G_1' - D_{12}''G_2' - D_{14}''G_4' + D_{13}''G_3' = 0, \tag{50a}$$

$$D_{21}'G_1' - D_{22}'G_2'' - D_{24}'G_4'' + D_{23}''G_3'' = 0, \tag{50b}$$

where

$$\begin{aligned}
G_1' &= D_{33}'(D_{22}V_{54} - D_{24}V_{52}) - D_{32}'(D_{21}V_{54} - D_{24}V_{51}),\\
G_2' &= D_{33}'(D_{21}V_{54} - D_{24}V_{51}) - D_{31}'(D_{23}V_{54} - D_{24}V_{53}),\\
G_3' &= D_{32}'(D_{21}V_{54} - D_{24}V_{51}) - D_{31}'(D_{22}V_{54} - D_{24}V_{52}),\\
G_4' &= D_{32}'(D_{21}V_{53} - D_{23}V_{51}) - D_{31}'(D_{22}V_{53} - D_{23}V_{52}) + D_{33}'(D_{21}V_{52} - D_{22}V_{51}),\\
G_1'' &= D_{32}'\left(-D_{44}'D_{13} + D_{43}'D_{13} + D_{43}'D_{14}\right) + D_{33}'\left(D_{44}'D_{12} - D_{42}'D_{14}\right),\\
G_2'' &= D_{31}'\left(-D_{44}'D_{13} + D_{43}'D_{14}\right) + D_{33}'\left(D_{44}'D_{11} - D_{41}'D_{14}\right),\\
G_3'' &= D_{31}'\left(-D_{12}D_{44}' + D_{14}D_{42}'\right) + D_{32}'\left(D_{11}D_{44}' - D_{14}D_{41}'\right),
\end{aligned} \tag{51}$$

$$G_4'' = D_{31}'\left(-D_{12}D_{44}' + DD_{42}'\right) - D_{32}'\left(D_{11}D_{43}' - D_{13}D_{41}'\right) + D_{33}'\left(D_{11}D_{42}' - D_{12}D_{41}'\right). \tag{52}$$

D_{pq}'' can be obtained from D_{pq} by replacing α, m_j, $j = 1,2,3$ and α', m'_j, $j = 1,2,3$.

In case of magneto-elasticity Eq. (50) reduce to

$$\begin{gathered}
D_{11}''G_1^* - D_{14}''G_4^* + D_{13}''G_3^* = 0,\\
D_{21}''G_1^* - D_{24}''G_4^* + D_{23}''G_3^* = 0,\\
G_1^* = D_{24}D_{23}', \quad G_3^* = D_{24}D_{31}', \quad G_4^* = D_{21}D_{33}' - D_{23}D_{31}'
\end{gathered} \tag{53}$$

where $D_{3q}' = V_{3q} + R_H^* V_{4q}$, $D_{4q}' = m'_q V_{5q}$, $q = 1,2,3,4$.

In the absence of magnetic affects the Eqs. (50a) and (50b) reduce to

$$\left(2 - \frac{c^2}{\delta^2}\right)^2 \left[m_1'^2 + m'_1 m'_2 + m_2'^2 - 1 + c^2\right] = 4\beta' m'_1 m'_2 (m'_1 + m'_2)$$

for thermally insulated plate and

$$\left(2 - \frac{c^2}{\delta^2}\right)^2 (m'_1 + m'_2) = 4\beta'(c^2 - 1 + m'_1 m'_2)$$

for isothermal surfaces of the plate.

These are merely Rayleigh surface wave equations. The Rayleigh results enter here since for such small wavelengths, the finite thickness plate appears as a half-space. Hence vibration energy is transmitted mainly along the surface of the plate [32,33].

9. Amplitudes of displacement, temperature change, electric and magnetic fields

The amplitudes of displacement, magnetic field and temperature change during symmetric mode of vibration are obtained as

$$u_1 = \left(c_1' + Lc_2' + Mc_3' + Nc_4'\right)A_1 \exp[i\xi(x_1 - ct)], \tag{54}$$

$$u_2 = \left(V_{21}s_1' + LV_{22}s_2' + MV_{23}s'_3 + Ns_4'\right)A_1 \exp[i\xi(x_1 - ct)], \tag{55}$$

$$h_1 = \left(V_{31}c_1' + LV_{32}c_2' + MV_{33}c_3' + NV_{34}c_4'\right)A_1 \exp[i\xi(x_1 - ct)], \tag{56}$$

$$h_2 = \left(V_{41}s_1' + LV_{42}s_2' + MV_{43}s_3' + NV_{44}s_4'\right)A_1 \exp[i\xi(x_1 - ct)], \tag{57}$$

$$T = \left(V_{51}c_1' + LV_{52}c_2' + MV_{53}c_3' + NV_{54}c_4'\right)A_1 \exp[i\xi(x_1 - ct)]. \tag{58}$$

Here

$$
\begin{aligned}
L &= \frac{s_1}{s_2}\left[\frac{D_{23}(D_{34}D_{41}-D_{31}D_{44})-D_{24}(D_{33}D_{41}-D_{43}D_{31})+D_{21}(D_{33}D_{44}-D_{34}D_{43})}{D_{22}(D_{33}D_{44}-D_{43}D_{34})-D_{23}(D_{32}D_{44}-D_{42}D_{34})+D_{24}(D_{32}D_{43}-D_{42}D_{33})}\right],\\
M &= \frac{s_1}{s_3}\left[\frac{D_{22}(D_{34}D_{41}-D_{31}D_{44})-D_{21}(D_{34}D_{42}-D_{44}D_{32})+D_{24}(D_{31}D_{42}-D_{32}D_{41})}{D_{22}(D_{33}D_{44}-D_{43}D_{34})-D_{23}(D_{32}D_{44}-D_{42}D_{34})+D_{24}(D_{32}D_{43}-D_{42}D_{33})}\right],\\
N &= \frac{s_1}{s_4}\left[\frac{D_{22}(D_{31}D_{43}-D_{33}D_{41})-D_{23}(D_{31}D_{42}-D_{32}D_{41})+D_{21}(D_{42}D_{33}-D_{32}D_{43})}{D_{22}(D_{33}D_{44}-D_{43}D_{34})-D_{23}(D_{32}D_{44}-D_{42}D_{34})+D_{24}(D_{32}D_{43}-D_{42}D_{33})}\right],
\end{aligned}
\tag{59}
$$

$$
\begin{gathered}
D_{3q} = V_{3q}T_q^{\mp 1} + i\frac{R_H}{\bar{R}_H}V_{4q}, \quad D_{4q} = m_q V_{5q},\\
s_q = \sin\left(\xi m_q d\right), \quad c_q = \cos\left(\xi m_q d\right), \quad s'_q = \sin\left(\xi m_q x_2\right), \quad c'_q = \cos\left(\xi m_q x_2\right).
\end{gathered}
$$

The amplitudes of displacement magnetic field and temperature change during skew symmetric mode of vibration are obtained by replacing L, M, N with L', M', N', A_q with B_q, s_q with c_q and c'_q with s'_q in Eqs. (54)–(58). These quantities in other cases of thermoelasticity can be obtained by making suitable substitutions in relavent equations.

10. Numerical results and discussion

In this section the dispersion curve obtained from secular equation (30) and amplitudes of displacement, magnetic field and temperature change given in Eqs. (54)–(58) are computed numerically for carbon steel material for which the physical data is given below [14]:

$$
\begin{gathered}
\lambda = 9.3\times10^{10}\,\mathrm{Nm^{-2}}, \quad \mu = 8.4\times10^{10}\,\mathrm{Nm^{-2}}, \quad \rho = 7.9\times10^{3}\,\mathrm{kg\,m^{-3}}, \quad T_0 = 293.1\,\mathrm{K},\\
\varepsilon_T = 0.34, \quad C_V = 6.4\times10^{2}\,\mathrm{J\,kg^{-1}\,deg^{-1}}, \quad K = 50\,\mathrm{Wm^{-1}\,K^{-1}}, \quad \alpha_t = 13.2\times10^{-6}\,\mathrm{deg^{-1}},\\
\sigma = 5.9\times10^{6}\,\Omega^{-1}\,m^{-1}, \quad H_3 = 1.0\mathrm{A\,m^{-1}}, \quad \mu_0 = 1.3\times10^{-6}\mathrm{H\,m^{-1}}.
\end{gathered}
$$

The secular equation (30) is solved numerically by iteration method to obtain the phase velocity of symmetric and antisymmetric modes of vibrations after finding the characteristic roots. The cubic equation satisfied by m_i^2, $i = 1,2,3$ is solved by using reduced Cardan's method. In general the cubic equation satisfied by m_i^2 can be written as $G(m,V) = 0$ which can be solved for 'm' for fixed values of V by Cardan's method. The secular equation (30) is transcendental equations of the form $F(V,m) = 0$. For known values of m this equation can also be solved for the phase velocity v. We have used iteration method to find the phase velocity and attenuation coefficients for different values of the wavenumber R. The adopted procedure is outlined below.

The iteration method to solve a transcendental equation $f(V) = 0$, requires to put this equation into the form $V = g(V)$, so that the sequence $\{V_n\}$ of iteration for the desired root can be easily generated as follows: if V_0 be the initial approximation to the root, then we have $V_1 = g(V_0)$, $V_2 = g(V_1)$, $V_3 = g(V_3)$, and so on. In general $V_{n+1} = g(V_n)$, $n = 0,1,2,3\ldots$. If $\left|g'(V)\right| \ll 1$, for all $V \in I$, then the sequence $\{V_n\}$ of approximations to the root will converge to the actual value $V = \zeta$ of the root, provided $V_0 \in I$, I being the interval in which root is expected. For initial value of $V = V_0 \in I$, Eq. (22) can be solved for m by Cardan's method for a particular value of the non-dimensional wavenumber Rd. The values of m are then used in the secular equation to obtain a current value of V, which is further used to generate a new approximation to V. This process is repeated time and again for a particular value of the wavenumber Rd unless the sequence of iterated approximations to the value of V converges to desired level of accuracy, i.e. $|V_{n+1} - V_n| < \varepsilon$, ε being arbitrary small number to be selected at random in order to achieve the accuracy level. This procedure is continuously repeated for different values of the non-dimensional wavenumber Rd to obtain the phase velocity. Here the sequence of the values of phase velocity has been allowed to iterate approximately for 100 iterations to make it converge in order to achieve the desired level of accuracy, viz. four decimal places here. An infinite number of roots exist for a given value of frequency, which can be obtained by giving a value of wavenumber, from the secular equation (30). Each root represents a propagating mode. Note that care must be taken in the root finding procedure, for the transcendental functions change their values rapidly. The phase velocity profiles of first four symmetric and antisymmetric modes of vibrations have been computed for various values of the non-dimensional wavenumber (Rd) from dispersion relation (30) in case of stress free, thermally insulated plate of carbon steel material. The corresponding dispersion curves and attenuation profiles for Rayleigh–Lamb type modes in case of rotating and non-rotating media are presented in Figs. 1 and 2 for symmetric and skew symmetric modes, respectively. The amplitudes of displacement, temperature change and perturbed magnetic field in case of fundamental mode for rotating and non-rotating plates have also been computed for stress free thermally insulated plate which are plotted with plate thickness in Figs. 3–6.

From Figs. 1a and 2a, it is observed that the phase velocity of fundamental symmetric and skew symmetric modes remains constant with variation in wavenumber in case of rotating plate indicating that this mode of wave propagation is dispersionless and hence phase velocity equals group velocity. The phase velocity of fundamental skew symmetric mode has non-zero constant value for rotating plates, where as it is noticed to be zero in case of non-rotating plate at the vanishing wavenumber and increases sharply with non-dimensional wavenumber and ultimately tends to Rayleigh wave velocity at large wavenumbers. The effect of rotation is

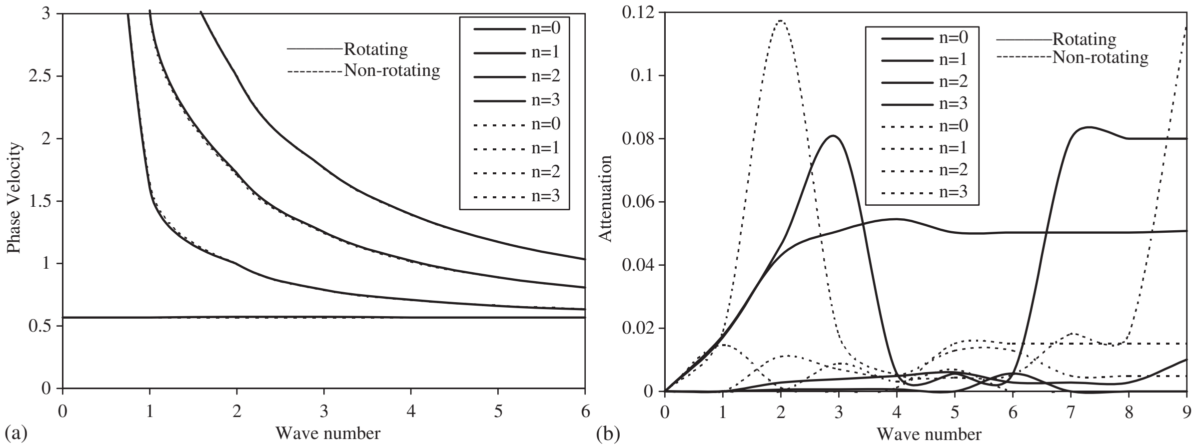


Fig. 1. (a) Dispersion curves for symmetric mode of wave propagation in a plate with stress free thermally insulated boundaries; (b) variation of attenuation coefficient for symmetric mode of wave propagation in a plate with stress free thermally insulated boundaries.

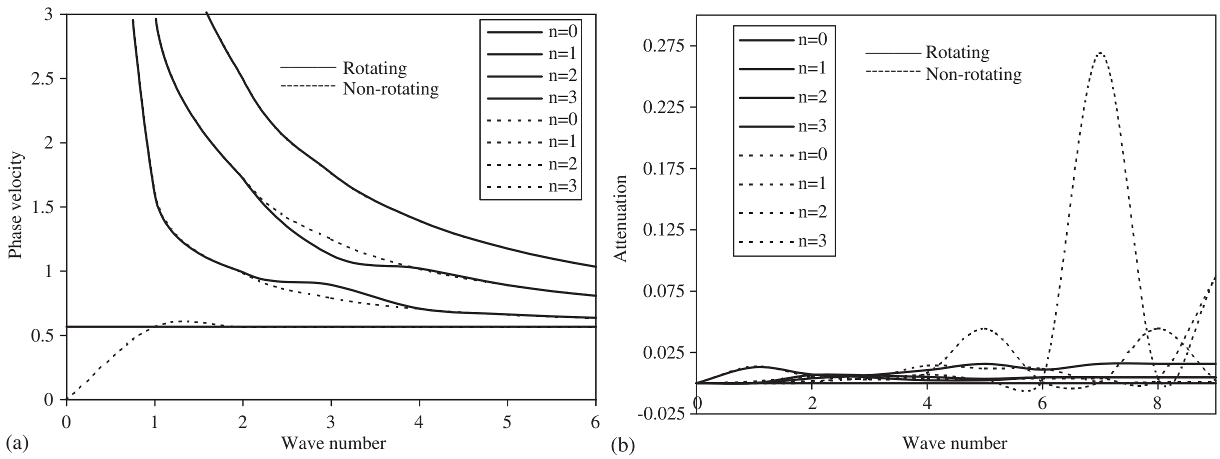


Fig. 2. (a) Dispersion curves for skew symmetric mode of wave propagation in a plate with stress free thermally insulated boundaries; (b) Variation of attenuation coefficient for skew symmetric mode of wave propagation in a plate with stress free thermally insulated boundaries.

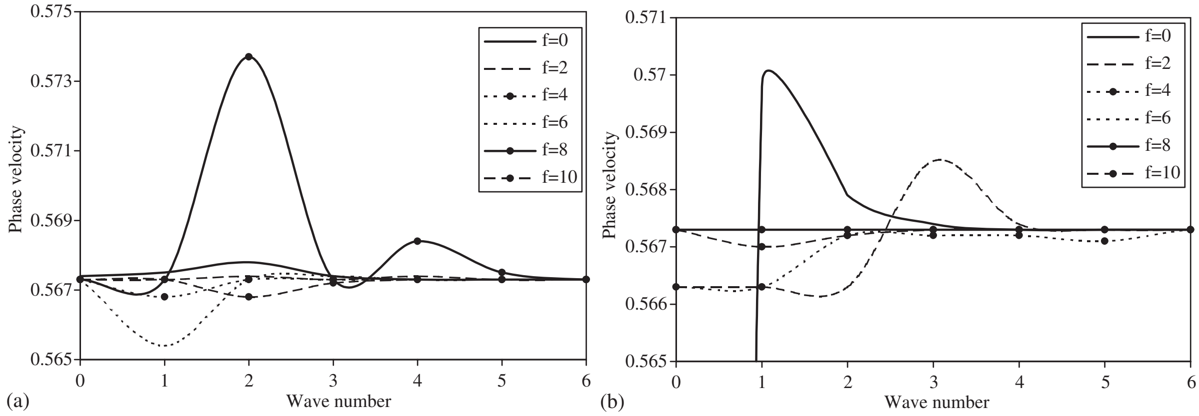


Fig. 3. variation of phase velocity with rotation for: (a) symmetric mode of wave propagation in a plate with stress free thermally insulated boundaries; (b) skew symmetric mode of wave propagation in a plate with stress free thermally insulated boundaries.

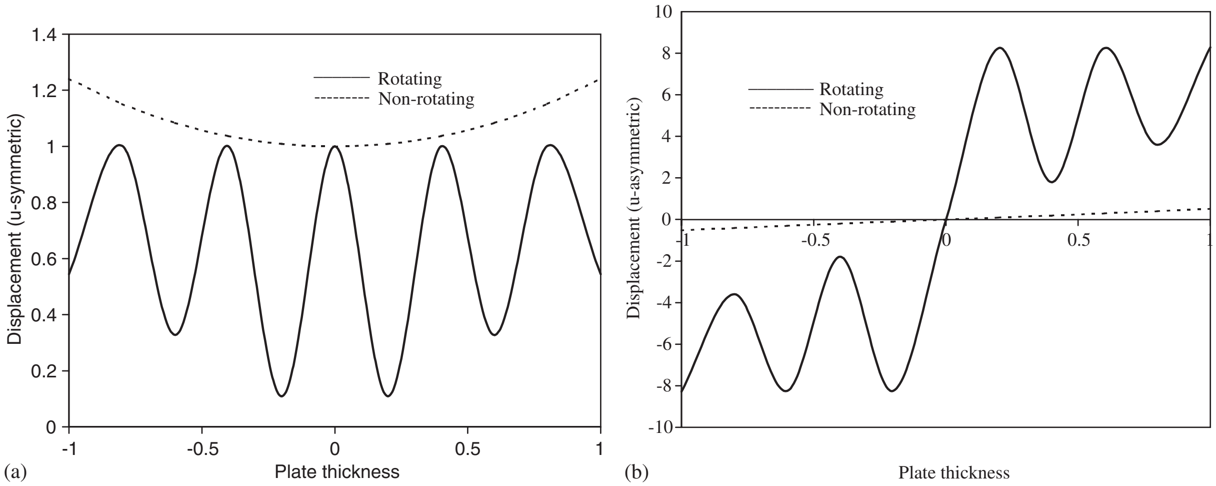


Fig. 4. Variation of x_1-component of displacement amplitude: (a) during symmetric mode with plate thickness for stress free thermally insulated boundaries; (b) during skew symmetric mode with plate thickness for stress free thermally insulated boundaries.

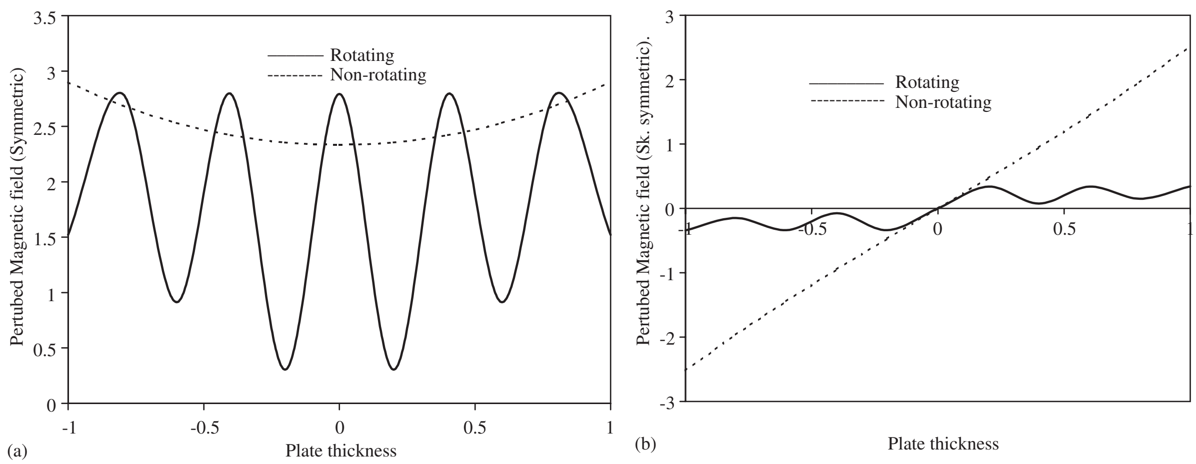


Fig. 5. Variation of amplitude of x_1-component of perturbed magnetic field: (a) during symmetric mode with plate thickness for stress free thermally insulated boundaries; (b) during skew symmetric mode with plate thickness for stress free thermally insulated boundaries.

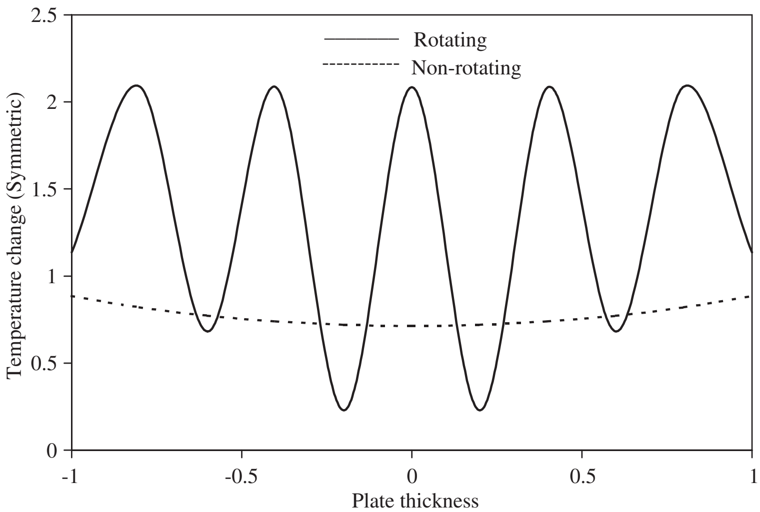


Fig. 6. Variation of temperature change during symmetric mode with plate thickness for stress free thermally insulated boundaries.

clearly observed in the instant case because of which the skew symmetric acoustic mode becomes dispersionless here. The velocity of higher symmetric and skew symmetric modes of wave propagation decreases exponentially for small values of the wavenumber and asymptotically approaches Rayleigh wave speed at higher wavenumbers in the context of various theories (CT, GL, and LS) of thermoelasticity. The effect of thermal relaxation is observed to be negligibly small on dispersion curves. The asymptotic closeness of various modes of wave propagation to Rayleigh wave velocity at higher wavenumbers establishes the fact that at small wavelength the finite thickness plate appears as a half-space and that the vibration energy is transmitted mainly along the surface of the plate. The free surfaces admit a Rayleigh-type surface wave with complex wavenumber and hence phase velocity. Consequently, the surface wave propagates with attenuation due to radiation of energy into the medium. This radiated energy will be reflected back to the center of the plate from lower and upper surfaces. Consequently, the attenuated surface wave on the free surface is enhanced by this reflected energy to form a propagation wave. In fact, the multiple reflections from upper and lower surfaces of the plate form caustics at one of the free surface and a strong stress concentration arises due to which wave field becomes unbounded in the limit $d \to \infty$. The unbounded displacement field is characterized by singularities of the circular tangent functions.

The attenuation of various modes of propagation has also been computed and represented graphically in Figs. 1b and 2b for symmetric and skew symmetric cases, respectively. For rotating plate attenuation is very small and almost constant in case of skew symmetric mode. It means signal or wave can go for large distances without attenuation. This explains why the skew symmetric mode for thin rotating plate is the choice for biosensing applications in nuclear magnetic resonance (NMR), magnetic resonance imaging (MRI) and echo planer imaging (EPI). MRI has become very important in diagnosis treatment and follow-up of various diseases like cancer, brain tumor and a valuable tool for radiotherapy. It is observed from Figs. 3a and b that the phase velocity increases initially to attain its maximum value and then again starts decreasing with increase in rotation of the plate. This can be explained by the fact that as the rotation of the plate increases, the coupling effect of various interacting fields also increases resulting in lower phase velocity. It is noticed that as the thickness of the plate increases, the phase velocity decreases. It can also be observed that the Rayleigh wave velocity is reached at lower wavenumber as the thickness increases, because the transportation of energy mainly takes place in the neighborhood of the free surfaces of the plate in this case.

The magnitude of non-dimensional amplitudes of displacement, perturbed magnetic field and temperature change in a stress free and thermally insulated plate have also been computed and are shown graphically in Figs. 4–6 for different values of the plate thickness. It is clear from Fig. 4a that the behavior of x_1-component of symmetric displacement amplitude ($u_{1\text{sym}}$) becomes oscillatory because of rotation effect in contrary to the non rotating plate. This quantity has maximum amplitude at the center of the plate which decreases with increase in plate thickness and the oscillatory nature almost vanishes at $d \to \pm\infty$ showing damping effect. Damping effect becomes more and more prominent with increase in plate thickness. The maximum displacement amplitude at center of the plate is very important from practical point of view for maintaining resonance conditions. From Fig. 4b the amplitude of skew symmetric displacement ($u_{1\mathrm{asy}}$) is observed to be quite large at plate surfaces and zero at the center of the plate. The comparison of Figs. 4a and b reveals that symmetric displacement is dominant as compared to skew symmetric one in case of non-rotating plate while this trend get reversed in case of rotating plate. The behavior of z-component of symmetric displacement ($u_{2\mathrm{sym}}$) is observed to be similar to that of skew symmetric x-component of displacement ($u_{1\mathrm{asy}}$) and the skew symmetric displacement ($u_{2\mathrm{asy}}$) follows the trend of symmetric displacement ($u_{2\mathrm{sym}}$) in rotating as well as non-rotating plates. The behavior of the amplitudes of perturbed magnetic field and temperature change are almost similar to that of displacement except that they differ in magnitude in case of non-rotating plate. However, the behavior of these quantities is oscillatory and same for rotating plates with the exception of varying magnitude. The trends of x_2-components of symmetric and skew symmetric perturbed magnetic field are found to be similar to that of skew symmetric and symmetric x_1-component in case of rotating and non-rotating plates. The magnitude of perturbed magnetic field gets suppressed in the presence of rotation for symmetric and skew symmetric vibrations. The displacement amplitudes remain almost close in all the theories of thermoelasticity, however a significant departure is observed in case of perturbed magnetic field and temperature change, although not plotted here. The temperature change is observed to be large in CT as compared to that in GL and LS theories of thermoelasticity. The perturbed magnetic field is observed to be amplified in case of GL and LS theories as compared to that in CT. Hence effect of relaxation time is quite significant on magnetic and thermal fields whereas it is negligible in case of displacement amplitudes.

11. Conclusions

The propagation of electromagnetic-thermoelastic plane waves in an initially unstressed, homogeneous isotropic, conducting plate under uniform magnetic field has been investigated in the context of generalized theory of thermoelasticity. The plate is rotating with uniform angular velocity normal to its faces. The magneto-elastic SH mode of wave propagation decouples from rest of the motion and is not influenced by thermal variations and thermal relaxation times. At short wavelength limits the secular equations for symmetric and skew symmetric modes reduce to Rayleigh surface wave frequency equation, because a finite thickness plate in such a situation behaves like a semi-infinite medium. The dispersion curves, attenuation coefficients, amplitudes of displacement, perturbed magnetic field and temperature change are computed and shown graphically for various modes of wave propagation in different theories of thermoelasticity in case of carbon–steel material plate. The numerically computed results are found to be significantly in agreement with the corresponding analytic results. Although the effect of relaxation time is observed to be quit small in phase velocity, but significant effect is observed in amplitudes of perturbed magnetic field and temperature change. The secular equation has been discussed under different situations in case of various classical and non-classical theories of thermoelasticity. The analysis carried out will be useful in the design and construction of rotating sensors and other surface acoustic wave (SAW) devices in addition to possible biosensing applications.

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