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Short Communication

Nonlinear vibration behaviors of casing pipe in the deep water ☆

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Abstract

The vortex-induced nonlinear vibration of casing pipes in the deep water was investigated considering the loads of current and combined wave-current. The vortex-induced vibration equation of a casing pipe was set up with considering the beam mode and Morison's nonlinear fluid loads as well as the vortex-excited loads. The approach of calculating vortex-excited nonlinear vibration by Galerkin's method was proposed. The natural vibration frequencies and modes were obtained, and the response including primary resonance induced by current and the composite resonance under combined wave-current for the 170 m long casing pipe in the 160 m depth of water were investigated. The results show that the dynamics response of casing pipe obviously increases, and the complicated response behaviors of casing pipe are described under combined wave-current.

1. Introduction

The casing pipes are widely used for drilling wells in the ocean. As depth of water reaches to 100 m or more than 100 m, the relative stiffness of a casing pipe is reduced, the natural frequency of a casing pipe may be close to shedding frequency of vortex, so the resonance induced by vortex will occur. Because the diameter of casing pipe is much less than length, so a casing pipe can be simplified into the beam model. The interaction between the currents and cylinder was discussed when the fluid flows through the cylinder, and the wake oscillator-model, correlation-model, and statistical-model were presented for the prediction of dynamics response of cylinders in the uniform flow [1]. Also, the wake oscillator-model was employed to predict the vibration of cylinders in the non-uniform flow [2]. The fluid dynamics force expressed in the square of velocity in Morison formula will lead to difficulties for solving the dynamic response, so Lyons simplified the nonlinear term into linear term and the response in time domain for riser of TLP was calculated through simplifying the riser into beam model [3]. Bokaian considering the locking-in condition of vortex excited vibration, the time domain responses of vortex excited vibration for riser were calculated [4]. Yan-qiu simplified the riser of TLP into the simple support model, and the vibration of riser induced by vortex and stability of vibration were investigated considering nonlinear fluid damping force [5,6]. However, it is worth studying whether the model of the simple support beam can represent the practical restrains at the top and the bottom. Therefore, Chi and

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Yan-qiu et al. suggested the beam model with the upper moveable support and fixed support at the bottom, the dynamics responses of a riser of TLP were investigated [7]. However, the less investigations for analytical model and dynamics behavior of vibration excited by vortex under combined wave-current have been done for casing pipes in the deep water up to date. The paper developed the model and calculating method of vortex-excited vibration for casing pipe in the deep water based on the previous results of vibration excited by vortex for cylinders. The examples described the vibration response characteristics of casing pipes under combined current-wave.

2. Vibration equation and natural modes

It is assumed that wave and current flow in same direction x, and the cross-section of casing pipes is uniform. The coordinate original point is at the bottom of seas. The analytical model is illustrated in Fig. 1, the upper support represents the link between the lower deck and the casing pipe.

Where, d and l denote the height from upper deck to water surface and height of casing pipe, respectively. According to the model in Fig. 1, the vibration equation of a casing pipe can be yielded as following:

$$EI\frac{\partial^4 y}{\partial z^4} + m\frac{\partial^2 y}{\partial t^2} + c\frac{\partial y}{\partial t} = F_y(z,t), \tag{1}$$

where E and I are Young's elastic modulus and bending inertial moment of a casing pipe, respectively, in which, $I = \pi/64[D^4 - (D-2\delta)^4]$, D and δ denote the diameter and the thickness of wall for casing pipe, respectively; m is the structural mass per unit length; c denotes the damping coefficient; $F_y(z, t)$ is the total fluid force per unit length, and

$$F_y(z,t) = F_L(z,t) - F_r(z,t), \tag{2}$$

where $F_L(z, t)$ and $F_r(z, t)$ denote the vortex-excited force and the fluid damp force due to motion of a casing pipe in the direction y, respectively:

$$F_L(z,t) = \tfrac{1}{2}\rho D(V_c + u)^2 C_L \cos\omega_s t = K_L(z) C_L \cos\omega_s t \tag{3}$$

and

$$K_L(z) = \tfrac{1}{2}\rho D(u + V_c)^2, \tag{4}$$

where ρ is the density of fluid; C_L and ω_s are the lift force coefficient and the shedding frequency of vortex, respectively; u and V_c are the wave velocity in any depth of water and current velocity, respectively, i.e.

$$u = \frac{\pi H}{T_w} e^{kz'} \cos(kx + \omega_w t),$$
$$z' = z - (l - d), \tag{5}$$

$$V_c = a + bz, \tag{6}$$

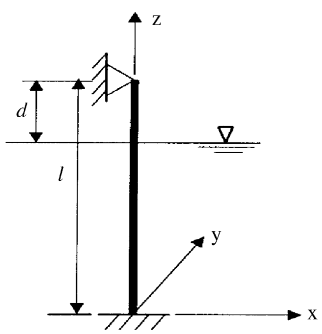

Fig. 1. Coordinate system and model.

where H and T_w are the wave height and wave period, respectively; ω_w and k denote wave frequency and wavenumber, respectively, and $k = 2\pi/L_w$, L_w is the wave length; z' represents the distance from any location of casing pipe to wave surface; a and b are positive constants and a shows the fluid velocity on the bottom of sea.

The nonlinear fluid damp force in the direction y can be expressed in the Morison's formula [2,6]:

$$F_r(z,t) = K_d C_d \,\mathrm{sgn}(\dot{y})\dot{y}^2 + m'\ddot{y}, \tag{7}$$

where $K_d = \rho D/2$; m' is the added water mass per unit length, and $m' = \rho C_a \pi D^2/4$; sgn $= +1$ or -1 depend on the negative sign or positive sign of $\dot{y}$. C_d and C_a are the coefficients of fluid damp and added water mass, respectively. Herein, $C_L = 0.6-2.4$, $C_d = 0.4-2.0$, $C_a = 1.0$ [1,6]. Dynamics deflection can be expanded as following according to Korolov function [8]:

$$y(z,t) = \sum_{n=1}^{N} y_n(t)\left(E_{\lambda_n l} C_{\lambda_n z} - D\lambda_n z\right), \tag{8}$$

where $C_{\lambda_n z}$ and $D_{\lambda_n z}$ are called as the Korolov function; $E_{\lambda_n l} = B_{\lambda_n l}/A_{\lambda_n l}$, $A_{\lambda_n l}$ and $B_{\lambda_n l}$ denote the value of Korolov function as $z = l$ [8]; $y_n(t)$ is the nth mode coordinate. The natural frequencies and vibration mode can be determined according to the boundary conditions of two ends of a casing pipe.

3. Calculation method of dynamics response

Formula (1) can be simplified into the ODE employing the Galerkin's method. Substituting formula (8) into formula (1), and multiplied by $\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)$, to integrate the equation in the region (0, l) for the two sides of equation, we have

$$\ddot{y}_n + \lambda_{Bn}^2 y_n + \frac{C_n}{\bar{m}}\dot{y}_n + \frac{1}{\bar{m} l_n} D_n = \frac{C_L}{\bar{m} l_n}\cos\omega_s t \int_0^l K_L(z)\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\mathrm{d}z \quad (n = 1, 2, \ldots, N) \tag{9}$$

and

$$\frac{C_L}{\bar{m} l_n}\cos\omega_s t \int_0^l K_L(z)\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\mathrm{d}z = A_c \cos\omega_s t + (A_w \cos 2\omega_w t + 2A_{cw}\cos\omega_w t)\cos\omega_s t, \tag{10}$$

where A_c and A_w denote the effects of fluid and wave, respectively, A_{cw} shows the coupled effects between wave and fluid, ω_w is the wave frequency; $\bar{m} = m + m'$; $C_n = 2\bar{m}\lambda_n\zeta_s$, ζ_s is the structural damp ratio; and

$$\lambda_{Bn}^2 = \frac{EI}{\bar{m}}\lambda_n^4, \quad l_n = \int_0^l \left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\mathrm{d}z, \tag{11}$$

$$A_c = \frac{C_L K_d}{\bar{m} l_n}\left[\int_0^l (a+bz)^2\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\mathrm{d}z + \frac{1}{2}\left(\frac{\pi H}{T_w}\right)^2 \int_0^l \mathrm{e}^{2kz'}\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\right]\mathrm{d}z, \tag{12}$$

$$A_w = \frac{C_L K_d}{2\bar{m} l_n}\left(\frac{\pi H}{T_w}\right)^2 \int_0^l \mathrm{e}^{2kz'}\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\mathrm{d}z, \tag{13}$$

$$A_{cw} = \frac{C_L K_d}{2\bar{m} l_n}\left(\frac{\pi H}{T_w}\right)^2 \int_0^l (a+bz)\mathrm{e}^{kz'}\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\mathrm{d}z, \tag{14}$$

$$D_n = K_d C_d \int_0^l \mathrm{sgn}(\dot{y})\dot{y}^2\left(E_{\lambda_n l} C_{\lambda_n z} - D_{\lambda_n z}\right)\mathrm{d}z. \tag{15}$$

The dynamics responses can be obtained solving formula (9) by the numerical integration. The sgn$\dot{y}(z,t)$ will change with time t and position z in integrating D_n, so sgn$[\dot{y}(z,t)]$ at $(i+1)$ step is determined by the $\dot{y}(z,t)$ at i step, and D_n is solved by the numerical integration at difference time and difference height.

4. Results of example

The parameters of the example are: $a = 0.4\,(\mathrm{m/s})$; $b = 0.001286\,(\mathrm{m/s})$; wave length $L_w = 300\,\mathrm{m}$; wave frequency $\omega_w = 0.437\,(1/\mathrm{s})$; distance from the lower deck to the bottom $l = 170\,\mathrm{m}$; diameter $D = 0.75\,\mathrm{m}$; thickness of wall $\delta = 0.02\,\mathrm{m}$; mass per unit length $m = 636\,(\mathrm{k/m})$; structural damp ratio $\zeta_s = 1.8 \times 10^{-3}$. The "wet" natural frequencies of casing pipe considering added water were calculated and shown in Table 1.

The resonance response excited by current was calculated as only the first term in the right-hand side of formula (10) is considered as $\omega_s = 0.437$, and $C_L = 2.4$, $C_D = 0.6$. The mode responses $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$ are shown in Fig. 2.

The first mode response $y_1(t)$ is larger than the response of other modes from Fig. 2 as $\omega_s = \lambda_1$. If $\omega_s = \lambda_2$, $y_2(t)$ is larger than one of other modes, but the response amplitude of second harmonic resonance is less than the amplitude of first harmonic resonance.

Fig. 3 shows the vortex-excited resonance induced by combined wave-current when $\omega_s = \omega_w = \lambda_1$.

It is known from Fig. 3 that the modes response induced by wave-current greatly increase, in order to explain the phenomena in the Fig. 3, the right-hand side of formula (10) are rewritten as following formula:

$$A_c \cos \omega_s t + (A_w \cos 2\omega_s t + 2A_{cw} \cos \omega_w t) \cos \omega_s t = A_c \cos \omega_s t + \tfrac{1}{2} A_w[(\cos(2\omega_w + \omega_s)t + \cos(2\omega_w - \omega_s)] + A_{cw}[\cos(\omega_w + \omega_s)t + \cos(\omega_w - \omega_s)t]. \quad (16)$$

It is known from formula (16), only if current is considered, the resonance frequency is ω_s, and the excited lode is $A_c \cos \omega_s t$, but as the combined wave-current is acted on the casing pipe, the resonance frequencies are $\omega_s, 2\omega_w \pm \omega_s$, which are called as the frequencies of composite resonance. It is obvious that the primary resonance and composite resonance will occur under combined wave-current, so the dynamics response of a casing pipe increase greatly. The mode responses of bending moment of casing pipe on the sea bottom are given in Fig. 4.

Table 1
Natural frequencies

Mode order	λ_1	λ_2	λ_3	λ_4
Frequency	0.437	1.415	2.951	5.045

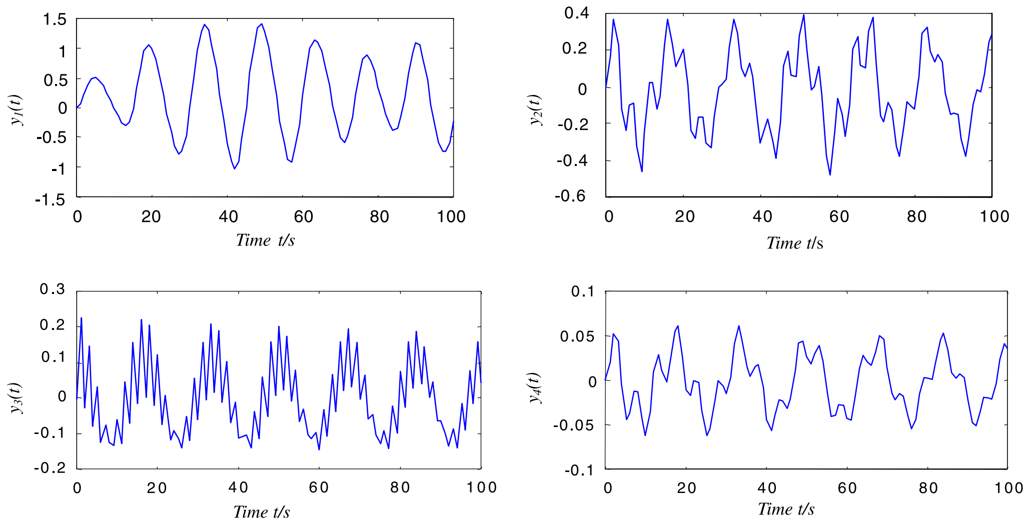

Fig. 2. First harmonic resonance induced by current.

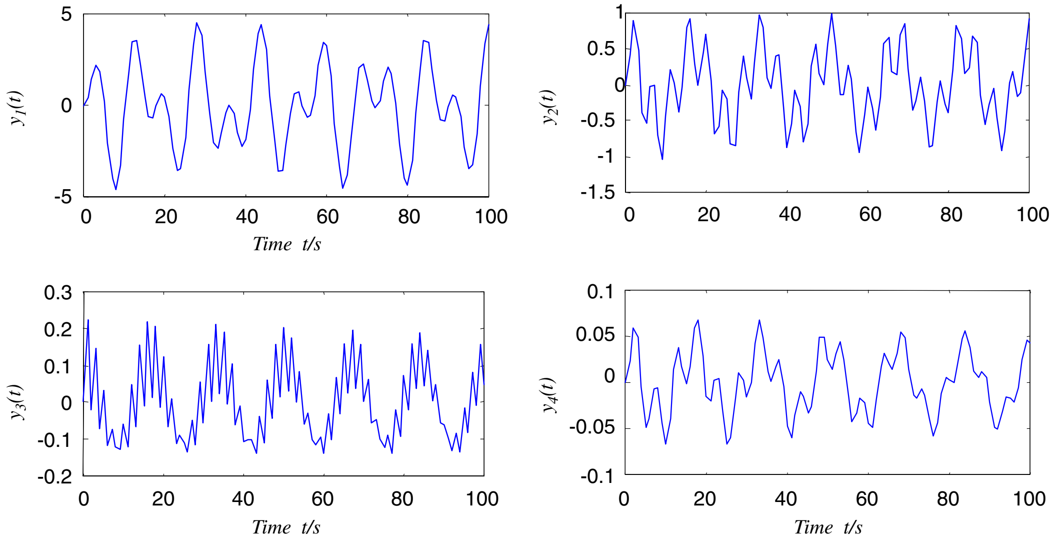

Fig. 3. Resonance induced by combined wave-current ($\omega_s = \omega_w = \lambda_1$).

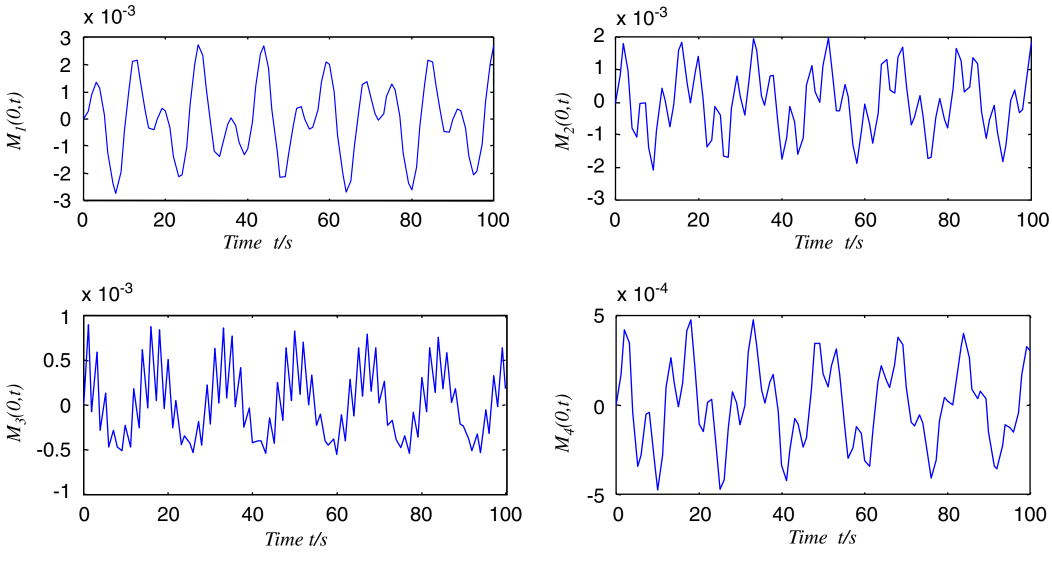

Fig. 4. Response of bending moments under combined wave-current ($\omega_s = \omega_w = \lambda_1$).

The ratios of higher-order mode (order no. >1) responses to first mode response are 20%, 4%, and 1.2%, respectively, from Fig. 3 for deflection response. But the ratios of higher modes response to first mode response are 64%, 28% and 17%, respectively, from Fig. 4 for bending moment response, which is shown that the effect of higher modes on bending moment is more important than one of higher modes on the deflection. So if the same accuracy of bending moment and deflection are excepted, the more modes should be chosen to superpose for calculation of bending moment. The history responses curves of bending moment and shear force are presented in Fig. 5.

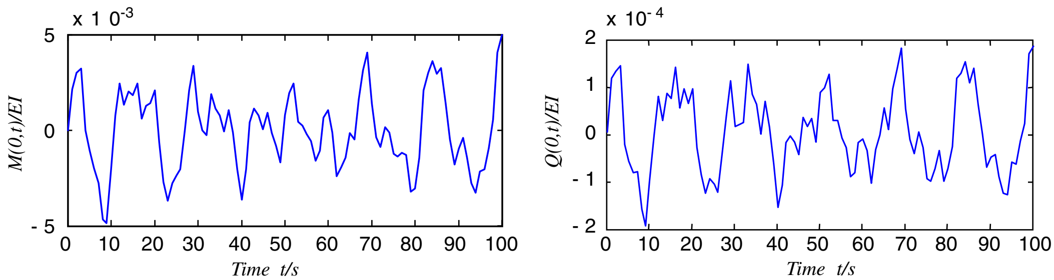

Fig. 5. Response of bending moment and shear force at the bottom.

5. Conclusions

The analytical model of nonlinear vortex-excited vibration is suggested for the casing pipes in the deep water, and the calculation approach for nonlinear dynamics response excited by vortex using Galerkin's method was put forward. The following conclusions are drawn in this paper:

(1) The composite resonance frequency occurs and the dynamic responses include the primary resonance and composite frequency resonance components, so the dynamics response of a casing pipe greatly increase, and dangerous situation of a casing pipe will take place under combined wave-current.
(2) The combined wave-current should be considered for the calculation of dynamics response as the shedding frequency of current is close to the wave frequency, and to check the dynamics strength of a casing pipe.
(3) In order to get the same accurate both the deflection and the internal forces, the chosen modes for calculation of bending moment should be more than the modes for calculation of deflection.

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