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A new structural damage identification method

J.K. Liu*, Q.W. Yang

Department of Mechanics, Zhongshan University, Guangzhou 510275, China

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Abstract

A new method is presented to provide an insight in to the characterizations of structural damages. The present algorithm makes use of an original finite element model and a subset of measured eigenvalues and eigenvectors. The proposed method detects damages in a decoupled fashion. First, a theory is developed to determine the number of damaged elements. With the damage number determined, the localization and quantification algorithms are then developed. A plane truss structure is analyzed as a numerical example to verify the present method. Results show that the method is accurate and robust in structural damage identification when the number of measured modes is more than the number of damaged elements with or without noise.

1. Introduction

With the increasing demands for structural quality and reliability, damage identification via techniques that examine changes in measured structural vibration response is a very important topic of research. The basic idea of these techniques is that modal parameters are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in the physical properties will cause changes in the modal properties. Any damage identification method can be classified by its capabilities to characterize the damage. There are three levels of damage characterization: detection, location, and quantification.

Most prior work in damage identification is based on the modification of a structural finite element model (FEM). The goal of these methods is to use test data from the damaged structure and the correlated FEM of the undamaged structure to determine changes to the stiffness and/or mass matrices. This class of methods can be divided into three main groups: One group is the optimal matrix update methods. The objective functions are either the minimum of a norm of the property perturbation or the minimum of the rank of the property perturbation. Rodden used ground vibration test data to determine the structural influence coefficients of a structure [1]. Baruch and Itzhack developed a closed form solution for the minimal Frobenius-norm matrix adjustment to the structural stiffness matrix incorporating measured frequencies and mode shapes [2]. Chen and Garba developed a theory for assessing the occurrence,

*Corresponding author.
E-mail address: jikeliu@hotmail.com (J.K. Liu).

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location and extent of potential damage using on-orbit response measurements [3]. Kiddy and Pines proposed a method to update both mass and stiffness matrices simultaneously [4]. Chiang and Lai presented a two-stage structural damage detection method in which the residual force method and the method of simulated evolution are employed, respectively [5]. Zimmerman and Kaouk made use of a minimum rank update theory to detect structural damage [6]. The damage sites are located by the damage vectors and the damage extents are assessed by the minimum rank update theory. Doebling improved this method by computing the minimum rank updates directly to the elemental stiffness parameters [7]. Another group is sensitivity methods. These methods start with the derivatives of the eigenvalues and/or eigenvectors to changes in material and physical parameters. Wong et al. developed a perturbation method from sensitivity of eigenparameters for damage detection of large structures [8]. Messina et al. proposed an assurance criterion for detecting single damage sites [9] and extended this method to identify the relative amount of damage at multiple sites [10,11]. Shi et al. extended the multiple damage location assurance criterions by using incomplete mode shape instead of modal frequency [12]. The third group is eigenstructure assignment methods. Andry et al. presented an excellent overview of eigenstructure assignment theory and applications [13]. Zimmerman and Kaouk utilized a symmetric eigenstructure assignment algorithm to perform the partial spectral assignment [14]. Lim and Kashangaki referred to a best achievable eigenvector as a damage indicator [15]. The damage is located by computing the Euclidean distances between the measured mode shapes and the best achievable eigenvectors. The magnitude of damage is assessed by a minimum norm update theory. Kiddy and Pines used the eigenstructure assignment technique to detect damage in rotating structures [16].

A new method similar in concept to the minimum rank update theory is presented in the present paper. The proposed algorithm makes use of an original finite element model and a subset of measured eigenvalues and eigenvectors and assumes that the number of measured modes is more than the number of damaged elements and the method detects damage in a decoupled fashion. At the outset, a theory is developed to determine the number of damaged elements. The number of damage elements is equal to the number of nonzero eigenvalues of the damage matrix defined as the product of the mode shape matrix and the residual force matrix. Then the damage elements can be localized by the damage localization matrix. Finally, the damage extents can be easily obtained. A numerical example of a plane truss structure is used to verify the present method. To illustrate the practical feasibility of the proposed method, the effect of measurement noise is taken into consideration. Results show that the proposed method can localize both single and multiple damages and can determine the magnitude of damage successfully even if the measurement noise inevitably makes the damage detection more difficult.

2. The residual force equation

Without loss of generality, by assuming that the mass matrix is unchanged as damage occurs, the eigenvalue equation for an n degrees of freedom FEM of a damaged structure is

$$(K_d - \lambda_{dj} M)\phi_{dj} = 0, \tag{1}$$

$$K_d = K_u - \Delta K, \tag{2}$$

where M is the mass matrix, K_u and K_d are the stiffness matrices associated with the undamaged and damaged structural models, respectively, ΔK is the corresponding changes, λ_{dj} and ϕ_{dj} are the jth eigenvalue and eigenvector of the damage structure, respectively.

Substituting Eq. (2) into (1) yields

$$(K_u - \lambda_{dj} M)\phi_{dj} = \Delta K \phi_{dj}. \tag{3}$$

Letting $b_j = (K_u - \lambda_{dj} M)\phi_{dj}$, Eq. (3) can be rewritten as

$$\Delta K \phi_{dj} = b_j, \tag{4}$$

where b_j is the jth residual force vector.

If the number of the measured modes is m, Eq. (4) can be expressed in the following form for all measured modes:

$$\Delta K \Phi = B, \tag{5}$$

where $\Phi = [\phi_{d1} \quad \phi_{d2} \cdots \phi_{dm}]$ and $B = [b_1 \quad b_2 \cdots b_m]$.

3. Determining the number of damaged elements

The perturbed global stiffness matrix ΔK can be expressed as [7]

$$\Delta K = A \Delta P A^{\mathrm{T}}, \tag{6}$$

where A is the stiffness connectivity matrix and ΔP is the elemental damage parameters matrix, they are given as

$$A = [a_1 \quad a_1 \cdots a_N], \tag{7}$$

$$\Delta P = \begin{bmatrix} \alpha_1 & & & \\ & \alpha_2 & & \\ & & \ddots & \\ & & & \alpha_N \end{bmatrix}, \quad \alpha_i \in [0, 1] \tag{8}$$

in which the $(n \times 1)$ vector a_i is the ith elemental stiffness connectivity vector, N is the total number of elements in the system, and α_i is the ith elemental damage parameter. The value of α_i is 0 if the ith element is undamaged and α_i is 1 if the ith element is completely damaged. The matrix A is unchanged as damage occurs.

Substituting Eq. (6) into (5) yields

$$A \Delta P A^{\mathrm{T}} \Phi = B. \tag{9}$$

Pre-multiplying Φ^{T} in Eq. (9) gets

$$\Phi^{\mathrm{T}} A \Delta P A^{\mathrm{T}} \Phi = \Phi^{\mathrm{T}} B. \tag{10}$$

Letting

$$C = \Phi^{\mathrm{T}} A, \quad D = \Phi^{\mathrm{T}} B, \tag{11,12}$$

substituting Eqs. (11) and (12) into Eq. (10) results in

$$C \Delta P C^{\mathrm{T}} = D, \tag{13}$$

where the $(m \times m)$ matrix D is defined as the damage matrix. The eigenvalue decomposition of D is

$$D = U \Lambda U^{\mathrm{T}}, \tag{14}$$

where

$$U = [u_1 \quad u_2 \cdots u_m], \quad \Lambda = \mathrm{diag}(\sigma_1 \quad \sigma_2 \cdots \sigma_m). \tag{15,16}$$

Substituting Eq. (14) into (13) gets

$$C \Delta P C^{\mathrm{T}} = U \Lambda U^{\mathrm{T}}. \tag{17}$$

Eq. (17) shows that the rank of ΔP should be equal to the rank of Λ, i.e., the number of nonzero diagonal entries in Λ is equal to the number of damaged elements, so we can predict the number of damaged elements by Λ (see Appendix A). When the measurement noise is considered, the number of relatively larger entries in Λ is also equal to the number of damaged elements.

3. Damage localization and quantification

Eq. (17) can be rearranged as

$$\Lambda = U^{\mathrm{T}} C \Delta P C^{\mathrm{T}} U. \tag{18}$$

Letting

$$E = U^{\mathrm{T}} C = [e_1 \quad e_2 \cdots e_N], \tag{19}$$

where the $(m \times N)$ matrix E is defined as the damaged localization matrix whose columns corresponding to the elements and e_i is defined as the damage localization vector. When the ith element is damaged, then the vector e_i has zero elements corresponding to the zero diagonal entries in Λ (see Appendix B). If not, then the ith element is undamaged. For the vibrational data with measurement noise, the vector e_i should have smaller elements corresponding to the smaller diagonal entries in Λ if the ith element is damaged.

By using the above described localization approach, the damaged elements have been determined. Supposing that the number of the damaged elements is $q(q<m)$, and the damage localization vectors of these elements are $e_1, e_2, \cdots, e_q$, respectively, then they can be assembled as

$$S = \lfloor e_1 \quad e_2 \cdots e_q \rfloor, \tag{20}$$

where the $(m \times q)$ matrix S has $m-q$ zero arrows. Eq. (18) reduces to

$$\Lambda = S \Delta P^* S^{\mathrm{T}}, \tag{21}$$

in which

$$\Delta P^* = \begin{bmatrix} \alpha_1 & & & \\ & \alpha_2 & & \\ & & \ddots & \\ & & & \alpha_q \end{bmatrix}. \tag{22}$$

Removing the zero arrows in S, the $(m \times q)$ matrix S reduces to the $(q \times q)$ matrix S^*. Correspondingly, removing the zero diagonal entries in Λ, the $(m \times m)$ matrix Λ reduces to the $(q \times q)$ matrix Λ^*. With the above operations, Eq. (21) becomes

$$\Lambda^* = S^* \Delta P^* S^{*^{\mathrm{T}}}. \tag{23}$$

Therefore, the damage extent can be obtained as

$$\Delta P^* = S^{*^{-1}} \Lambda^* \left(S^{*^{\mathrm{T}}}\right)^{-1}. \tag{24}$$

4. Numerical example

A plane steel truss structure (shown in Fig. 1) is taken as an example to verify the proposed method. The basic parameters of the structure are as follows: $E = 200\,\mathrm{GPa}$, $\rho = 7.8 \times 10^3\,\mathrm{kg/m^3}$, $L = 1\,\mathrm{m}$, and

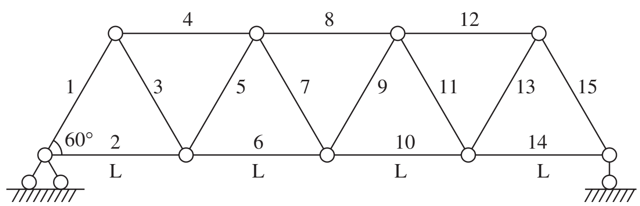

Fig. 1. A plane truss structure.

$A = 0.004\,\text{m}^2$. The first 4 modes are used in the damage identification. The mode shapes are contaminated with 5% random noise in the study of the measurement noise effect. The contaminated signal is represented as

$$\overline{\phi_{ij}} = \phi_{ij}(1 + \gamma_i^{\phi}\rho^{\phi}|\phi_{\max,j}|), \tag{25}$$

where $\overline{\phi}_{ij}$ and ϕ_{ij} are the mode shape components of the jth mode at the ith degrees of freedom with noise and without noise, respectively; γ_i^{ϕ} is the random number with a mean equal to zero and a variance equal to 1; ρ^{ϕ} is the random noise level; and $\phi_{\max,j}$ is the largest component in the jth mode shape.

4.1. Single damage

Assume that a single damage occurs in the 6th element with a stiffness loss of 20%. The eigenvalue matrix Λ of the damage matrix D without noise is

$$\Lambda = \begin{bmatrix} 177.4333 & & & \\ & 0.0000 & & \\ & & 0.0000 & \\ & & & 0.0000 \end{bmatrix}.$$

We can easily draw a conclusion that a single damage occurs because there is only one nonzero diagonal entry in Λ. Then the damage localization matrix E can be calculated by Eq. (19), which is listed in Table 1.

From Table 1, we can see that only e_6 has zero elements corresponding to the zero diagonal entries in Λ, so the 6th element is damaged and the others are undamaged. Using Eq. (24), we can obtain the damage extent of the 6th element as $\alpha_6 = 0.2000$, which is exactly the assumed value 0.2.

The eigenvalue matrix Λ of the damage matrix D under 5% noise is

$$\Lambda = \begin{bmatrix} 176.1587 & & & \\ & 4.8572 & & \\ & & -2.3128 & \\ & & & -0.1515 \end{bmatrix}.$$

Now we can see that the first eigenvalue is much larger than the others in Λ, so only one element is damaged. The damage localization matrix E is listed in Table 2, from which the 6th element can be easily identified to be the damaged element. Using Eq. (24), we can obtain that $\alpha_6 = 0.2018$, which has 0.9% error.

Table 1
Results for single damage (no noise)

Element	1	2	3	4	5
e_i	17.7736	−6.9276	−5.5159	−14.0446	−31.6498
	23.7118	21.8225	20.8988	−16.0819	10.6371
	−8.3739	−16.6326	−10.4654	1.1514	−7.6450
	−32.5886	19.8221	−8.2194	27.6561	29.3354
Element	6	7	8	9	10
e_i	−29.7853	−8.7337	11.2762	31.2892	6.6782
	0.0000	1.4936	−9.5602	−2.0319	−5.5459
	0.0000	0.9018	−7.6119	5.5737	1.2007
	0.0000	30.8530	−9.1716	5.7865	−21.2895
Element	11	12	13	14	15
e_i	41.0603	−27.5669	0.0411	−29.5359	30.3142
	1.5795	3.9447	9.4541	−16.2767	−11.9787
	−1.9163	−22.4757	−21.2253	15.4320	30.6215
	−16.7065	−5.4708	−14.2233	1.4674	6.3301

Table 2
Results for single damage (5% noise)

Element	1	2	3	4	5
e_i	15.5980	−5.8164	6.1574	−11.6679	−30.5403
	8.6263	−23.1739	4.7808	−9.0355	−23.1715
	17.3696	7.2732	−11.8949	−14.7210	2.2962
	−15.8045	−20.1518	16.1779	10.6755	−12.4710
Element	6	7	8	9	10
e_i	29.5434	7.2108	12.0188	31.1512	5.6173
	−1.3135	21.4160	12.4739	−5.0716	17.4581
	0.1258	0.5029	−11.3545	1.1373	−2.3730
	−0.4080	6.2692	11.6706	−0.5532	8.2616
Element	11	12	13	14	15
e_i	−39.5194	−26.8521	0.7600	29.3443	28.8274
	−8.7068	7.1708	8.0362	−7.0293	−4.0014
	−1.5862	−8.8864	−3.0341	5.0260	6.8349
	−0.8161	1.9709	−2.2174	−12.2982	3.7876

Table 3
Results for two damages (no noise)

Element	1	2	3	4	5
e_i	−26.1688	21.3461	5.1865	25.3934	59.9308
	−5.4892	20.5835	−1.9913	8.2272	7.5638
	−0.7284	2.9654	0.3988	4.0786	0.0000
	30.9060	12.8260	20.8407	−23.5955	0.0000
Element	6	7	8	9	10
e_i	23.7291	22.8481	−10.3663	−21.2076	−15.0529
	−19.1033	17.1629	1.6893	21.6297	−13.0520
	0.0000	−3.7644	11.5045	−4.0540	3.2162
	0.0000	−7.2796	−8.2188	−3.0626	0.6395
Element	11	12	13	14	15
e_i	−35.5940	24.4581	0.8792	17.4360	−29.1267
	14.6140	−12.0715	−0.6216	−26.6348	9.8905
	2.6666	17.6157	15.0738	−6.1816	−21.0704
	5.2724	1.8238	9.4846	−13.4655	−8.5057

4.2. *Two damages*

Assume that two damages occur in the 5th and 6th elements with two stiffness losses of 20%. The eigenvalue matrix Λ of the damage matrix D is

$$\Lambda = \begin{bmatrix} 830.9554 & & & \\ & 84.4293 & & \\ & & 0.0000 & \\ & & & 0.0000 \end{bmatrix}.$$

Again, the conclusion has been drawn that two damages occur because there are two nonzero diagonal entries in Λ. Then the damage localization matrix E can be calculated by Eq. (19), which is shown in Table 3.

From Table 3, it can be seen that only e_5 and e_6 have zero elements corresponding to the zero diagonal entries in Λ, so the 5th and 6th elements are damaged and the others are undamaged. Using Eq. (24), we can obtain the damage extents as $\alpha_5 = 0.2000$ and $\alpha_6 = 0.2000$, which are exactly the assumed values (0.2, 0.2).

The eigenvalue matrix Λ of the damage matrix D under 5% noise is

$$\Lambda = \begin{bmatrix} 812.5916 & & & \\ & 83.6294 & & \\ & & -3.8671 & \\ & & & 3.6283 \end{bmatrix}.$$

We can also find that the first two eigenvalues are much larger than the others, so two elements are damaged. The damage localization matrix E is listed in Table 4, which shows that the 5th and 6th elements are the most suspected damaged elements. Using Eq. (24), we can obtain that $\alpha_5 = 0.1974$ and $\alpha_6 = 0.2117$, which have -1.3% and 5.85% errors, respectively.

4.3. Multiple damages

Assume that multiple damages occur in the 4th, 5th and 6th elements with stiffness losses all of 20%. The eigenvalue matrix Λ of the damage matrix D is

$$\Lambda = \begin{bmatrix} 1.0808 & & & \\ & 0.1717 & & \\ & & 0.0000 & \\ & & & 0.0721 \end{bmatrix} \times 10^3.$$

It is obvious that three damages occur because there are three nonzero diagonal entries in Λ. Then the damage localization matrix E can be calculated by Eq. (19), which is shown in Table 5.

From Table 5, one can see that just e_4, e_5 and e_6 have zero elements corresponding to the zero diagonal entries in Λ, so the 4th, 5th and 6th elements are damaged and the others are undamaged. The damage extents can be obtained by Eq. (24) as $\alpha_4 = 0.2000$, $\alpha_5 = 0.2000$ and $\alpha_6 = 0.2000$, which are exactly the assumed values (0.2, 0.2, 0.2).

Table 4
Results for two damages (5% noise)

Element	1	2	3	4	5
e_i	−25.4990	21.5490	−6.2136	25.1211	58.9647
	−4.5899	21.5559	0.2009	8.9515	8.1176
	16.6986	11.4034	−11.8623	−8.9445	0.0651
	34.7237	19.0742	−23.6544	−23.7739	0.9723
Element	6	7	8	9	10
e_i	−23.7531	−23.3433	−10.7548	−21.2080	−15.0058
	18.3563	−16.9302	1.8920	20.7461	−12.6742
	0.0700	7.7151	9.6766	−5.4986	3.8693
	1.8470	9.6706	−0.9331	−4.7244	2.3119
Element	11	12	13	14	15
e_i	35.7622	24.2531	1.0170	−17.2099	−28.4642
	−14.3978	−10.5229	0.7306	27.2379	6.7084
	−7.1385	20.8287	22.8002	16.1427	−28.9649
	−10.2464	14.6617	22.6595	23.2331	−25.9332

Table 5
Results of localization for multiple damages (no noise)

Element	1	2	3	4	5
e_i	−29.2076	18.8506	1.9918	37.9905	58.6388
	25.2057	7.8946	21.6326	−23.5710	9.6143
	4.7950	5.5521	4.2207	0.0000	0.0000
	−11.2694	−27.0162	−10.5273	5.5661	−9.6109
Element	6	7	8	9	10
e_i	22.8546	24.1368	−9.0118	−18.5627	−15.3200
	14.5135	−11.5592	−6.0290	−19.6612	4.8824
	0.0000	−5.4054	10.9678	−4.9484	3.5939
	15.4065	−11.0305	1.2239	−14.9160	11.0013
Element	11	12	13	14	15
e_i	−34.6150	20.2387	−3.1823	19.3240	−22.5015
	−8.9811	20.2289	15.4975	1.7364	−27.8186
	3.8390	19.4576	18.0785	−9.1460	−24.4215
	−14.4069	3.9311	−8.5761	31.4214	2.9571

Table 6
Results of localization for multiple damages (5% noise)

Element	1	2	3	4	5
e_i	−28.8329	18.4241	−1.3469	38.3043	59.4776
	24.2913	6.2325	−20.9792	−23.3287	9.4392
	4.6559	6.3066	−4.6595	0.1631	0.4648
	12.2815	26.9067	−10.7744	−5.7298	8.9867
Element	6	7	8	9	10
e_i	−22.8560	−23.5810	−8.7523	−19.1544	−15.2456
	−15.6004	12.3985	−5.6402	−20.6028	5.9543
	0.3824	4.8198	10.6549	−4.3506	3.1792
	15.7470	−10.5799	−1.7332	15.1618	−11.0828
Element	11	12	13	14	15
e_i	34.5409	20.2790	−3.0223	−19.6775	−23.1599
	9.9998	21.5101	15.8558	−3.4136	−28.9609
	−3.7182	19.5369	18.3875	9.8682	−24.6788
	−14.1227	−3.6226	8.8654	31.5709	−3.4873

The eigenvalue matrix Λ of the damage matrix D under 5% noise is

$$\Lambda = \begin{bmatrix} 1.1045 & & & \\ & 0.1720 & & \\ & & 0.0060 & \\ & & & 0.0750 \end{bmatrix} \times 10^3$$

The damage localization matrix E is listed in Table 6. The 4th, 5th and 6th elements are the most suspected damaged elements from E and Λ. Using Eq. (24), we can obtain that $\alpha_4 = 0.2030$, $\alpha_5 = 0.2089$ and $\alpha_6 = 0.1930$, which have 1.5%, 4.45% and −3.5% errors, respectively.

5. Conclusions

A decoupled damage identification algorithm is presented in this paper. The method approaches the damage identification problem in three steps: determining the number of damaged elements, localizing the damaged elements and quantifying the damage extents. A plane truss structure is used as a numerical example to illustrate the proposed method. Results demonstrate that the proposed procedure can localize and quantify the damage accurately if the number of measured modes is more than the number of damaged elements with or without noise.

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Appendix A

The statement after Eq. (17) "Eq. (17) shows the rank of ΔP should be equal to the rank of Λ" can be illuminated as follows:

From Eq. (14) one can obtain

$$\mathrm{rank}(D) = \mathrm{rank}(\Lambda). \tag{A1}$$

According to Eq. (6) we have [7]

$$\mathrm{rank}(\Delta K) = \mathrm{rank}(\Delta P). \tag{A2}$$

Substituting Eqs. (6) and (12) into Eq. (10) yields

$$\Phi^{\mathrm{T}} \Delta K \Phi = D. \tag{A3}$$

From Eq. (A3) one can obtain [6]

$$\mathrm{rank}(\Delta K) = \mathrm{rank}(D). \tag{A4}$$

Considering Eqs. (A1), (A2) and (A4) one can easily obtain

$$\mathrm{rank}(\Delta P) = \mathrm{rank}(\Lambda). \tag{A5}$$

Appendix B

The statement after Eq. (19) "When the ith element is damaged, then the vector e_i has zero elements corresponding to the zero diagonal entries in Λ" can be proven as follows:

Eq. (18) can be rewritten as

$$\begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_m \end{bmatrix} = \sum_{i=1}^{N} \alpha_i \begin{Bmatrix} e_i^1 \\ e_i^2 \\ \vdots \\ e_i^m \end{Bmatrix} (e_i^1 \quad e_i^2 \cdots e_i^m), \tag{B1}$$

where $e_i^1, e_i^2, \cdots, e_i^m$ are the elements of e_i. From Eq. (B1) one can obtain

$$\sigma_m = \sum_{i=1}^{N} \alpha_i (e_i^m)^2. \tag{B2}$$

If $\sigma_m = 0$, Eq. (B2) becomes

$$\sum_{i=1}^{N} \alpha_i (e_i^m)^2 = 0. \tag{B3}$$

Because $\alpha_i \in [0,1]$, one can obtain

$$\sum_{i=1}^{N} \alpha_i (e_i^m)^2 \geqslant 0. \tag{B4}$$

From Eq. (B4), the equal sign is valid only if

$$\alpha_i (e_i^m)^2 = 0, \quad i = 1 \sim N. \tag{B5}$$

When the ith element is damaged ($\alpha_i > 0$), Eq. (B5) is valid only if

$$e_i^m = 0. \tag{B6}$$

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