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Analysis of resonance mechanism and conditions of train–bridge system

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Abstract

In this paper, the resonance mechanism and conditions of train–bridge system are investigated through theoretical derivations, numerical simulations and experimental data analyses. The resonant responses of the bridge induced by moving trains are classified into three types according to different resonance mechanisms: the first is related to the periodical actions of moving load series of the vertical weights, lateral centrifugal and wind forces of vehicles; the second is induced by the loading rate of moving load series of vehicles; the third is owing to the periodically loading of the swing forces of the train vehicles excited by track irregularities and wheel hunting movements. The vehicle resonance is induced by the periodical action of regular arrangement of bridge spans and their deflections. The resonant conditions are proposed and the corresponding resonant train speeds are determined. The application scopes of resonance conditions are discussed. The resonance of the train–bridge system is affected by the span, total length, lateral and vertical stiffness of the bridge, the compositions of the train, and the axle arrangements and natural frequencies of the vehicles. The resonant train speeds for some bridges are estimated and are compared with the critical train speeds obtained from the dynamic simulation of train–bridge interaction model or from the field measurements.

1. Introduction

The dynamic response of railway bridges under moving train loads is one of the fundamental problems to be solved in bridge design. On the one hand, the train running with high speed induces dynamic impact on the bridge structure, influencing their working state and service life. On the other hand, the vibration of the bridge in turn affects the running stability and safety of the train vehicles, and thus becomes an important factor for evaluating the dynamic parameters of the bridge in design. Therefore, great efforts have been constantly attached to the subject of the dynamic interactions between vehicles and bridges.

The research work on this subject has a long history of more than 150 years. Especially in the last decades, with the construction of high-speed railway bridges, the raise of train speeds and the increase of train loads, increasingly sophisticated analytical models have been developed by researchers in China and abroad [1–12].

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Based on these models, the vertical and lateral dynamic interactions of train–bridge system have been studied and many useful results have applied to practical bridge engineering [13,14].

It has been noticed that when a row of train vehicles travel through a railway bridge, the loading frequencies will change corresponding to different train speeds. The resonant vibrations occur when the loading frequencies coincide with the natural frequencies of the bridges or the train vehicles. The strong vibrations induced by the resonance of train–bridge system not only directly influence the working state and serviceability of the bridge, but also result in the reduction of the stability and safety of the moving train vehicles, deteriorate the riding comfort of the passengers, and sometimes even destabilize the ballast track on the bridge. Therefore, it is necessary to analyze this problem and to develop some methods to predict the resonant speeds of the running trains and to assess the dynamic behaviors of railway bridges in resonance conditions. Matsuura [13] systematically researched the resonance of single span bridges during the design of the Shinkansen bridges. By using the half-vehicle model, he obtained the resonant curves of several bridges with different spans. Frýba [4,5] studied in theory the dynamic interaction of a beam under moving loads and proposed the corresponding resonance formula. Li and Su [15] investigated the resonant vibration for a simply supported girder under high-speed trains, using an idealized vehicle model with a rigid body and four wheel-sets. Yang and Yau [9–11] proposed a suitable numerical model to study the resonance of a beam induced by moving loads and the corresponding effect of resonance cancellation. Ju and Lin [16] established a three-dimensional finite element model to investigate the resonant characteristics of multi-span bridges with high piers and simply supported beams under high-speed trains. They also established a finite element model of high-speed train moving on the ground and found the dominated frequency $f_n = nV/L$ of the trainloads when a train moving on the ground [17]. Yang [9], Yau [18] and Kwark [19] studied the resonance of continuous bridges due to moving trains. Guo [20] in her Ph.D. dissertation presented the resonant conditions for trains composed of a row of vehicles with different axle intervals and loads.

The resonance of train–bridge system is influenced by many factors, such as the periodically loading on the bridge of the moving load series formed by the wheel–axle weights of the train vehicles; the harmonic forces on the bridge of the moving trains excited by rail irregularities, wheel flats and hunting movements; and the periodical actions on the moving vehicles of long bridges with identical spans and their deflections, and so on. When a train travels on a bridge at a certain speed, the vehicles with lateral wind pressure may form a lateral moving load series, which may be transferred through the vehicle wheels to the bridge girder. Therefore, the static wind forces due to mean wind may also induce dynamic response of the bridge (see Fig. 1). For the curved bridge, the centrifugal forces from the car bodies will form similar lateral moving load series on the bridge as well.

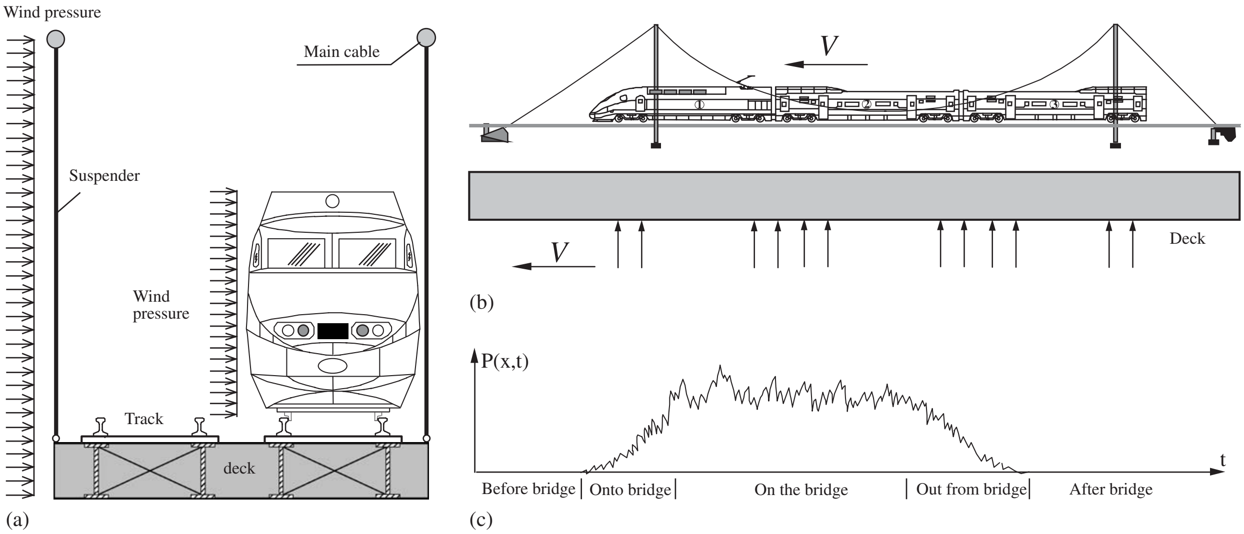

Fig. 1. Dynamic impact of train vehicles with lateral wind load moving on bridge.

The resonant vibrations of the train–bridge system are very complicated. The resonance mechanisms and conditions of the train–bridge system are analyzed in this paper, including the bridge resonance induced by moving train loads and the vehicle resonance excited by the deformation of the bridge. The predicted resonant train speeds are compared with the critical train speed from the dynamic simulation of train–bridge interaction model or the field measurements.

2. Resonance analysis of bridges

2.1. Bridge resonance induced by moving load series

2.1.1. Fundamental analysis model

The resonance of the train–bridge system is affected by the span, total length, lateral and vertical stiffness of the bridge, the compositions of the train, and the axle arrangements and natural frequencies of the vehicles. The general mechanism of bridge resonance induced by moving load series can be described as follows.

A simply supported beam without damping subjected to a series of concentrated constant loads P with identical interval d_v is analyzed (see Fig. 2b), to simulate the loading actions of a real train moving on the bridge. The train consists of several identical cars with the full length l_v of each car, the rated distance l_c between the two bogies of a car, and the fixed distance l_w between the two wheel–axles of a bogie (see Fig. 2a).

Suppose the load series travel on the beam from left to right at a uniform speed V, and the distance of the first force traveled is $x = Vt$. For the load series with identical intervals, there exists a time delay $\Delta t = d_v/Vt$ between any two successive forces. The motion equation for the beam acted on by such moving load series can be written as

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \overline{m}\frac{\partial^2 y(x,t)}{\partial t^2} = \sum_{k=0}^{N-1} \delta\left[x - V\left(t - \frac{k \cdot d_v}{V}\right)\right]P, \tag{1}$$

where L_b is the span length of the beam; E is the elastic modulus; I is the constant moment of inertia of the beam cross section; $\overline{m}$ is the constant mass per unit length of the beam; $y(x,t)$ is the displacement of the beam at position x and time t; N is the total number of moving loads; and δ is the Dirac delta function.

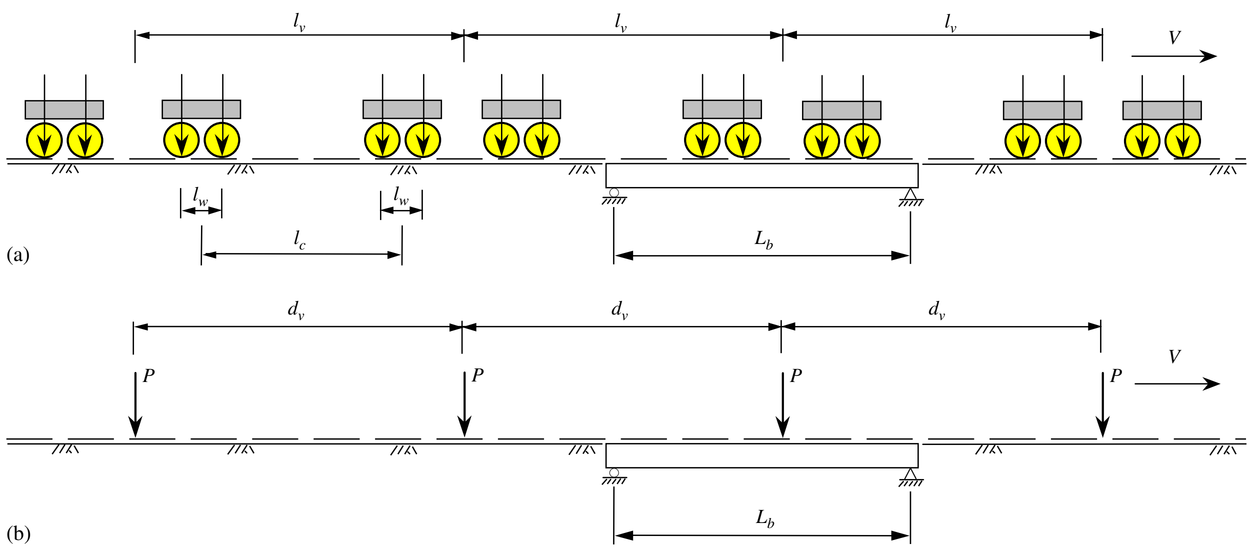

Fig. 2. Loading series of train vehicles on the bridge.

Eq. (1) can be expressed in terms of the generalized coordinates as

$$\ddot{q}_n(t) + \omega^2 q_n(t) = \frac{2}{\overline{m}L} P \sum_{k=0}^{N-1} \sin \frac{n\pi V}{L_b}(t - \frac{k \cdot d_v}{V}). \tag{2}$$

The particular solution of Eq. (2) for the first vibration mode of the beam is

$$q(t) = \frac{2PL^3}{EI\pi^4} \frac{1}{1-\beta^2} \sum_{k=0}^{N-1} \left[\sin \overline{\omega} \left(t - \frac{k \cdot d_v}{V} \right) - \beta \sin \omega \left(t - \frac{k \cdot d_v}{V} \right) \right], \tag{3}$$

where $\beta = \overline{\omega}/\omega$ is the ratio of exciting frequency to the natural frequency of the beam; $1/(1-\beta^2)$ is the dynamic magnification factor; $\overline{\omega} = \pi V / L_b$ is the exciting circular frequency of the moving loads; and ω is the natural circular frequency of the beam

$$\omega = \frac{\pi^2}{L_b^2} \sqrt{\frac{EI}{\overline{m}}}. \tag{4}$$

The displacement response of the beam where only the first mode is considered can thus be expressed as

$$y(x,t) = \frac{2PL^3}{EI\pi^4} \frac{1}{1-\beta^2} \sin \frac{\pi x}{L_b} \left[\sum_{k=0}^{N-1} \sin \overline{\omega} \left(t - \frac{k \cdot d_v}{V} \right) - \beta \sum_{k=0}^{N-1} \sin \omega \left(t - \frac{k \cdot d_v}{V} \right) \right]. \tag{5}$$

The first term of the right side of Eq. (5) represents the forced response of the beam due to the moving loads while the second term the transient response due to its free vibration. According to their different mechanisms, the resonant responses of a simply supported beam subjected to moving load series can be divided into two types.

2.1.2. Bridge resonance induced by periodically loading of moving load series

First, the discussion is made for the second progression term of Eq. (5), to explain how the transient response in common sense may induce the resonance of the beam.

Before considering the second progression series, it is instructive to introduce the necessary transformation of triangular progression. For the sum of a finite triangular progression $\sin(a - ix)$, $(i = 1,2, \ldots m)$, it can be expressed as

$$\sum_{i=1}^{m} \sin(a - ix) = \sum_{i=1}^{m} [\sin a \cos ix - \cos a \sin ix]. \tag{6}$$

The two terms of Eq. (6) can be further expressed as

$$\begin{cases} \sum_{i=1}^{m} \sin ix = \sin 0.5mx \cdot \sin 0.5(m+1)x \cdot \csc 0.5x, \\ \sum_{i=1}^{m} \cos ix = \sin 0.5mx \cdot \cos 0.5(m+1)x \cdot \csc 0.5x. \end{cases} \tag{7}$$

Introducing them into Eq. (6) leads to

$$\sum_{i=1}^{m} \sin(a - ix) = \frac{\sin 0.5mx \cdot \sin[a - 0.5(m+1)x]}{\sin 0.5x}. \tag{8}$$

Now let $i = k$, $m = N - 1$, $x = \omega d_v / V$, $a = \omega t$, the progression term of the transient response in Eq. (5) becomes as the form

$$\begin{aligned} \sum_{k=0}^{N-1} \sin \omega \left(t - \frac{k \cdot d_v}{V} \right) &= \sin \omega t + \sum_{k=1}^{N-1} \sin \omega \left(t - \frac{k \cdot d_v}{V} \right) \\ &= \sin \omega t + \frac{\sin\left[(N-1) \cdot (\omega d_v / 2V)\right] \cdot \sin\left[\omega t - N \cdot (\omega d_v / 2V)\right]}{\sin(\omega d_v / 2V)}. \end{aligned} \tag{9}$$

For $\omega d_v/2V = \pm i\pi$, the second term of Eq. (9) becomes an indeterminate form 0/0, but when L'Hospital's rule is applied, the limit solution is found to be

$$\lim_{(\omega d_v/2V)\to\pm i\pi} \frac{\sin\left[(N-1)\cdot(\omega d_v/2V)\right]\cdot\sin\left[\omega t - N\cdot(\omega d_v/2V)\right]}{\sin(\omega d_v/2V)} = (N-1)\sin\omega\left[t - N\cdot(d_v/2V)\right]. \tag{10}$$

Obviously, the extreme condition with physical significance for Eq. (9) is

$$\frac{\omega d_v}{2V} = i\pi, \quad (i = 1, 2, 3, \ldots). \tag{11}$$

This is the same result as $d_v = 2i\pi V/\omega$ that was derived in a different way in Ref. [10].

Substituting this condition into Eq. (9), the limit value of the transient response term in Eq. (5) is obtained as

$$\sum_{k=0}^{N-1} \sin\omega\left(t - \frac{k\cdot d_v}{V}\right)\Bigg|_{(\omega d_v/2V)=i\pi} = N\sin\omega t. \tag{12}$$

It can be seen that each force in the moving load series may induce the transient response of the structure, and the successive forces form a series of periodical excitations. The response of the structure will be successively amplified with the increase of the number of forces traveling through the beam.

The similar results can be obtained for higher modes of the bridge. Considering all of these modes and let $\omega_n = 2\pi f_{bn}$, the resonant condition of the bridge under moving load series can be derived from Eq. (11) as

$$V_{br} = \frac{3.6\cdot f_{bn}\cdot d_v}{i} \quad (n = 1, 2, \ldots, i = 1, 2, \ldots), \tag{13}$$

where V_{br} is the resonant train speed (km/h); f_{bn} is the nth vertical or lateral natural frequency of the bridge (Hz); d_v is the intervals of the moving loads (m), and the multiplicator $i = 1,2, \ldots$ is determined by the extreme condition Eq. (11).

Eq. (13) indicates that when a train moves on the bridge at speed V, the regularly arranged vehicle wheel–axles may produce periodical dynamic actions on the bridge with the loading period d_v/V. The bridge resonance occurs when the loading period is close to the nth natural vibration period of the bridge. A series of resonant responses related to different bridge natural frequencies may occur corresponding to different train speeds. This is defined as the first resonant condition of bridge, which is determined by the time of the load traveling through the distance d_v.

2.1.3. Bridge resonance induced by loading rate of moving load series

As for the first progression term of Eq. (5) which represents the forced response of the bridge, the only difference with the second term besides a nonzero multiplicator β is that the frequency ω is replaced by $\overline{\omega}$. An extreme condition similar to Eq. (11) can thus be directly written as

$$\frac{\overline{\omega} d_v}{2V} = i\pi \quad (i = 1, 2, 3, \ldots). \tag{14}$$

Substituting $\overline{\omega} = \pi V/L_b$ into Eq. (14), the train speed V in the numerator is counteracted with that in the denominator and thus results in the extreme condition

$$d_v = 2iL_b \quad (i = 1, 2, 3, \ldots). \tag{15}$$

The limit value of the steady-state response progression can be obtained by using this extreme condition

$$\sum_{k=0}^{N-1} \sin\overline{\omega}\left(t - \frac{k\cdot d_v}{V}\right)\Bigg|_{(\bar{\omega} d_v/2V)=i\pi} = N\sin\overline{\omega} t. \tag{16}$$

There is no train speed V expressed in Eq. (15), namely, no resonant train speed exist. Eqs. (15) and (16) show that when the interval of loads equals to $2i$ times of the bridge span, i.e. the half-wavelength formed by the beam deflection, the successive increase of the number of the passing wheel axles may gradually enlarge the bridge response. However, since the minimum axle intervals of real vehicles are much smaller than two times of the bridge span length, and the actual arrangement of wheel axles of the train vehicles is never identical, this solution is only of mathematical significance. Therefore, the resonant train speed cannot be derived in this way.

In fact, the second resonance of the simply supported beam under moving train loads can be directly determined from Eq. (5) by the dynamic magnification factor $1/(1-\beta^2)$. When the frequency ratio $\beta = 1$, i.e. $\omega_n = \overline{\omega}_n$, the dynamic magnification factor $1/(1-\beta^2)$ will become infinitive. At this time the resonant vibrations of the bridge is excited. For the simply supported beam under moving loads, the loading frequency $\overline{\omega}_n = n\pi V/L_b$, and the nth natural frequency of the beam $\omega_n = 2\pi f_{bn}$, the resonant train speed V_{br} can be described as

$$V_{br} = \frac{7.2 \cdot f_{bn} \cdot L_b}{n} \quad (n = 1, 2, \ldots), \tag{17}$$

where L_b is the length of the bridge span (m).

Eq. (17) indicates that the bridge resonance occurs when the time of the train's traveling through the bridge equals to half or n times of the natural vibration period of the bridge. This is defined as the second resonant condition of bridge, which is determined by the loading rate of the moving loads related to the bridge span.

The resonant train speed calculated from Eq. (17) is rather high. For instance, the minimum natural frequencies for the simply supported beams with moderate or small spans are between $80/L_b$ and $120/L_b$ according to the Bridge Design Code of China, and the corresponding resonant train speeds estimated by Eq. (17) are from 576 to 864 km/h, which are far higher than the current train speeds in operation. The second resonant condition, however, is of certain significance in the resonance analysis for flexible bridges such as those with high piers.

The resonance problems can be solved for other types of bridges such as continuous beams, rigid frame, and so on, in a similar way to that for the simply supported beam.

2.1.4. Examples for bridge resonance induced by moving load series

2.1.4.1. Vertical resonance of bridge. Generally, the vertical vibration resonance of the bridge is carried out for individual beams. In the dynamic analysis of the bridges on the Beijing–Shanghai High-speed Railway, the dynamic interaction model of train–bridge system was used to study the resonant responses induced by the Germany train ICE3, the France train TGV, the Japanese train E500 and the high-speed train CHT designed in China. Fig. 3 shows the simulated distribution curves of the dynamic factors versus train speed for the PC box beams with 20 and 32 m spans, where dynamic factor is defined as the ratio of the maximum dynamic to the maximum static deflection of the beam under the same load.

It is given that the natural frequencies of the 20 and 32 m PC box beams are 7.73 and 4.23 Hz, respectively. By using Eq. (13), the corresponding resonant train speeds of TGV with the average full length of each car 18.7 m can be estimated as 520 and 284.8 km/h. And the corresponding resonant train speeds are 285 and 400 km/h for ICE3, E500 and CHT whose average car lengths are all approximately 26 m. From this example,

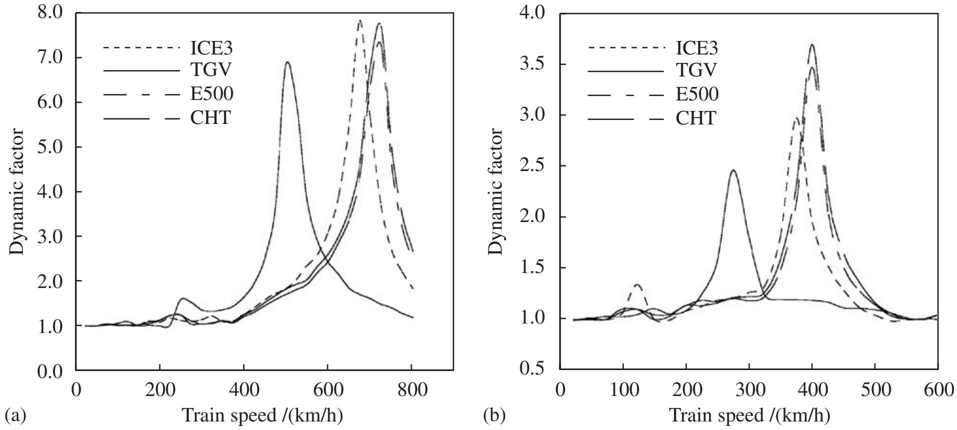

Fig. 3. Dynamic factors of simply supported beams vs train speed.

the calculation is based on $d_v = l_v$, namely the full length of vehicle is taken as the load interval, where the four axle loads of the rear bogie at the previous car and the front bogie at the following car are combined as a concentrated load. The resonant train speeds estimated by Eq. (13) are in good accordance with the critical train speeds from the simulated results, as compared in Fig. 3.

2.1.4.2. Lateral resonance of bridge. The lateral resonance analysis has special significance for bridges with high piers under moving load series induced by centrifugal forces or the lateral wind pressures. Since the lateral frequency of the bridge system is usually much lower than the vertical frequency, the critical train speed for lateral resonance is also lower.

(a) *Simply supported steel truss:* A simply supported steel truss with the span of 48 m is analyzed as an example. The moving load series are the lateral axle loads induced by wind pressures acting on vehicle bodies. The train concerned is composed of one locomotive followed by 18 passenger cars. The full length of each car is 26.57 m. The mean wind velocity is 25 m/s. The resonant train speeds are evaluated by Eqs. (13) and (17).

The lateral natural frequency of the truss is 1.86 Hz. The resonant train speed for the first resonant condition estimated by Eq. (13) is

$$V_{br1} = \frac{3.6 \times f_{bn} \times d_v}{i} = \frac{3.6 \times 1.86 \times 26.57}{1} \approx 178\,\text{km/h}.$$

The resonant train speed for the second resonant condition estimated by Eq. (17) is

$$V_{br2} = \frac{7.2 \times f_{bn} \times L_b}{n} = \frac{7.2 \times 1.86 \times 48}{1} \approx 643\,\text{km/h}.$$

To verify the evaluated resonant conditions, the dynamic responses of the truss under various train speeds are analyzed by the whole history simulations of train–bridge system, with the calculation train speeds in the range of 5–700 km/h [20]. Fig. 4 shows the distribution curve of the lateral displacements of the truss versus train speed.

The comparison between the estimated resonant train speeds and the simulated critical train speeds are in good accordance. The two types of resonant responses are obvious in the distribution curve of the lateral displacements of the truss versus train speed, where the peak values can be easily observed at the estimated resonant train speeds. The second resonant train speed, however, is of little significance because it is much higher than that in reality.

(b) *Multi-span bridge with high pier:* Since bridges often contains piers to support the bridge girders, the dynamic effects cannot be ignored in studying the bridge resonance. The lateral vibration of a bridge with piers is often analyzed as a system instead of a simply supported beam, because of the coupled movement of the piers and beams.

In the analysis, the basic global lateral deformation of the bridge deck on which the train runs can be reasonably assumed as a sinusoidal wave with a half-wavelength L_b and the frequency f_b. The moving load series are formed by the same lateral wind loads as in the first example.

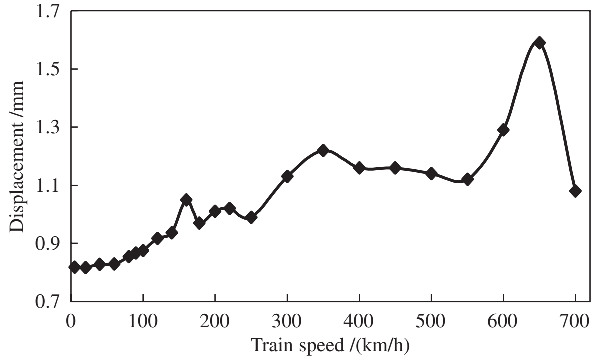

Fig. 4. Lateral displacements of steel truss vs train speed.

A bridge system consisting of two 32 m simply surpported beams and a 56 m high pier is analyzed as an example. The modal analysis shows that the first mode of the bridge is dominated by the lateral vibrations of the pier. With respect to the bridge lateral frequencies 0.95, 2.52 and 5.02 Hz, the resonant train speeds estimated by the first resonant condition Eq. (13) include

$$V_{br1} = \frac{3.6 \times f_{b1} \times d_v}{i} = \frac{3.6 \times 0.95 \times 26.57}{1} \approx 90.9\,\mathrm{km/h},$$

$$V_{br2} = \frac{3.6 \times f_{b1} \times d_v}{i} = \frac{3.6 \times 0.95 \times 26.57}{2} \approx 45.4\,\mathrm{km/h},$$

$$V_{br3} = \frac{3.6 \times f_{b1} \times d_v}{i} = \frac{3.6 \times 0.95 \times 26.57}{3} \approx 30.3\,\mathrm{km/h}.$$

The possible resonant train speeds estimated from the second resonant condition Eq. (17) include

$$V_{br2} = \frac{7.2 \times f_{b1} \times L_b}{i} = \frac{7.2 \times 0.95 \times 64}{1} \approx 438\,\mathrm{km/h},$$

$$V_{br2} = \frac{7.2 \times f_{b2} \times L_b}{i} = \frac{7.2 \times 2.52 \times 64}{1} \approx 581\,\mathrm{km/h},$$

$$V_{br2} = \frac{7.2 \times f_{b3} \times L_b}{i} = \frac{7.2 \times 5.02 \times 64}{1} \approx 771\,\mathrm{km/h}.$$

The dynamic responses of the bridge under various train speeds are analyzed by the whole history simulations of train–bridge system, with the calculation train speeds in the range of 5–900 km/h. Fig. 5 shows the distribution curves of the lateral displacements of the pier top versus train speed.

The curves show that the lateral resonance of the pier is obvious: the lateral displacements appear peak values at the train speeds slightly lower than the estimated ones by the first resonant condition; one can also find the peak displacements at 380 and 740 km/h, which are close to the corresponding resonant train speeds estimated from the second resonant condition. There is no obvious peak displacement at 581 km/h since the mode shape of the second mode of the bridge is very small at the pier top. Considering that the natural frequency of the bridge will decrease when loaded by the train, the estimated results are in accordance with those from the whole history simulations of train–bridge system.

Furthermore, one can estimate the responses of the bridge under the vehicle centrifugal forces. As moving load series, the vehicle centrifugal forces play the same mechanism to induce the lateral vibrations of the bridge as the mean wind pressures acting on the vehicle bodies. Thus the calculated curves in Fig. 5 can also be used for estimating centrifugal forces. According to the Fundamental Code for Design on Railway Bridge & Culvert in China, the design centrifugal force can be 15% of the static load of vehicles, which is about 2.5 times of the design wind load of vehicles. Therefore, when considering the vehicle centrifugal forces, much bigger pier-top displacements will be excited than those shown in Fig. 5.

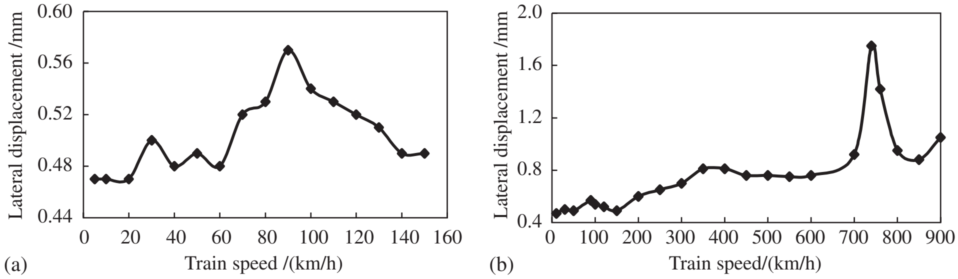

Fig. 5. Lateral displacement of high piers vs train speed.

2.2. Bridge resonance owing to the sway forces of train vehicles

The third bridge resonance is induced by the periodical actions on the bridge of the lateral moving load series owing to the sway forces of the train vehicles. The sway forces of vehicles may be excited by the track irregularities and wheel hunting movements. The resonant train speed in this case can be determined by

$$V_{br} = \frac{3.6 \cdot f_{bn} \cdot L_s}{i} \quad (n = 1, 2, \ldots,\ i = 1, 2, \ldots). \tag{18}$$

This equation is basically the same as Eq. (13) for the first resonance condition, except that d_v is replaced by L_s which represents the dominant wavelength of the track irregularities or wheel hunting movements. The multiplicators $n = 1,2,\ldots, i = 1,2, \ldots$ show that when the dominate frequency of the track irregularities or wheel hunting movements equals to the nth natural frequency or their higher harmonic frequencies, the resonance of the bridge occurs. This is called the third resonant condition of bridge.

In spite that both the track irregularities and the wheel hunting movements are of random properties, Eq. (18) can still be used to estimate the lateral resonance of the bridge induced by their dominant wavelengths. A good example is presented in Fig. 6, the distributions of the lateral displacements of the two high piers versus train speed. The data in the figure were measured in the field experiments at two real bridges on the Chengdu-Kunming Railway in China [21]. One can find that the peak values appear at certain train speeds, which are in good accordance with the estimated resonant train speeds of 33 and 51.1 km/h, respectively. The estimated resonant train speeds are calculated by Eq. (18), using the hunting wavelength $L_s = 8.5$ m of the wheels with worn tyres, the given pier heights $H = 55$ and 32 m, and the corresponding frequencies $f = 1.08$ and 1.67 Hz, respectively.

2.3. Application scopes of resonance conditions

Based on the analysis above, the resonant vibrations of bridges induced by moving trains have been classified into three mechanisms. The first is related to the intervals of the moving load series, which form the periodically loading on the bridge. The second is induced by the loading rate, i.e. the relative moving speed of the train vehicles to the bridge. The third is owing to the swing forces of the train vehicles excited by the track irregularities and wheel hunting movements.

In the above resonant conditions, the axle loads of the train vehicles are assumed to be in equidistance. While in reality, there exist several axle intervals in a real train: the full length l_v of a car, the rated center-to-center distance l_c between two bogies of a car, the fixed axle distance l_w between two wheel-sets of a bogie, and the different compositions of these distances. According to the relative lengths between the beam span or the bridge length and the above loading intervals, when Eq. (13) is used to analyze the first resonance induced by moving trains, the application scopes can be further discussed as follows (ref Figs. 2 and 7):

(1) $L_b < l_w$ When the bridge length L_b is shorter than the fixed axle distance l_w of a bogie, there can only be one wheelset at any moment on the bridge, with the shortest excitation period l_w/V and some other longer periods as $(l_v - l_c)/V$, l_v/V … However, it is only an extremal situation in theory, for there does not exist such short bridge in reality.

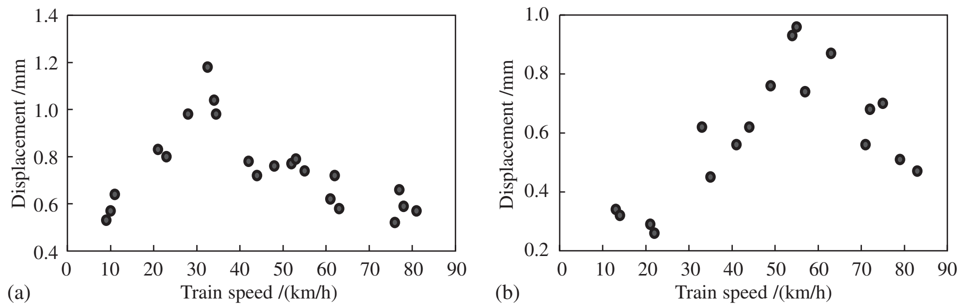

Fig. 6. Lateral displacements of piers vs train speed.

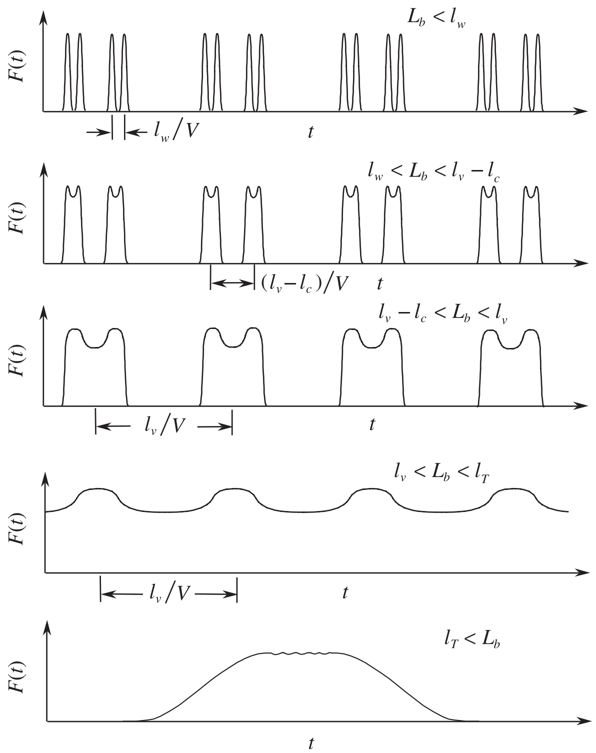

Fig. 7. Time histories of load series moving on bridge.

(2) $l_w < L_b < l_v - l_c$: When the bridge length L_b is longer than the fixed axle distance l_w of a bogie but shorter than the distance $l_v - l_c$ between the rear bogie of the previous car and the front bogie of the following car, there can still be only one wheelset at any moment on the bridge, with the main excitation period $(l_v - l_c)/V$ and some longer periods as l_v/V …, while the shorter period l_w/V is not obvious. This situation may occur for the bridges with very short spans.

(3) $l_v - l_c < L_b < l_v$: When the bridge length L_b is longer than the distance $l_v - l_c$ between the rear bogie of the previous car and the front bogie of the following car, but shorter than the full length l_v of the car, there can be two wheel-sets simultaneously on the bridge, with the main excitation period l_v/V, while the shorter periods as l_w/V and $(l_v - l_c)/V$ are not obvious. Since the full lengths are about 25 m for passengers car and 15 m for freight cars, this situation may occur for the commonly used bridges with small spans.

(4) $l_v < L_b < l_T$: When the bridge length L_b is longer than the full length l_v of a car but shorter than the total length l_T of the whole train, there can be more than one cars and two wheelsets simultaneously on the bridge, neither of the above excitation periods as l_w/V, $(l_v - l_c)/V$ nor l_v/V is obvious. This situation may occur for the common bridges with moderate spans, or for the lateral resonance analysis of the bridge as a whole.

(5) $l_T < L_b$: When the bridge length L_b is longer than the total length l_T of the train, there can be several cars with many wheelsets simultaneously on the bridge, thus the load series can not form periodically loading to the bridge system. This situation may occur for long span bridges, or for the lateral resonance analysis of the bridge as a whole. However, the resonant conditions proposed in this paper cannot be directly used to analyze the resonant conditions for long span bridges, because the whole course of the train traveling over the bridge longer than the total length of the train is equivalent as a half-loading period, and thus no harmonic load forms. Therefore, Eqs. (13) and (17) cannot be directly used to estimate the resonant train speeds. As for the third resonant condition where the bridge resonace is excited by rail irregularites or wheel hunting, no obvious resonance can be observed for long span bridges because of the counteractions between the forces from the wheelsets moving with different phases.

Thus it can be seen that when using the above equations to analyze the train induced resonance of the bridge, the loading intervals can be the full length l_v of a car, the rated center-to-center distance l_c of a car, the fixed axle distance l_w of a bogie, and the different compositions of these distances. While for a row of train vehicles, the arrangement of the axle loads is not in equidistance, and neither equal are the values of all axle forces which are affected by the bridge damping, track irregularities and other complicated factors. Therefore, a series of resonant vibrations may be excited with different response levels when the train moving at various speeds on the bridge, and a series of corresponding resonant train speeds could be found. Therefore, the precise resonance analysis usually depends on the simulation calculations of the train–bridge dynamic interaction system according to the real conditions of the train composition, the wheel arrangement and vehicle loads.

3. Resonance analysis of train vehicles

As a row of train vehicles traveling over a bridge at speed V, the periodical actions on the vehicles can be excited by the deflections of the bridge that consists of a long series of identical spans. The loading frequency can be estimated as

$$f = V/L_b. \tag{19}$$

The vehicle resonance occurs when this loading frequency coincides with the natural frequency of the train vehicles (ref. Fig. 8), when the dynamic responses of the vehicle will be greatly amplified. The critical train speed can be written as

$$V_{vr} = 3.6 \cdot f_v \cdot L_b, \tag{20}$$

where V_{vr} is the critical train speed (km/h); f_v is the natural vertical frequency of the vehicle (Hz); L_b is the span length of the bridge (m).

The excitation of bridge deflections on the vehicles is equivalent with the mass-spring system on the ground in harmonic vibrations. The transmissibility between the amplitudes of the mass and the deflection of the beam can be estimated as [22]

$$\mathrm{TR} = \sqrt{\frac{1 + (2\xi\beta)^2}{(1 - \beta^2)^2 + (2\xi\beta)^2}}. \tag{21}$$

For a half-vehicle model with the sprung mass $M = 24t$, the equivalent spring stiffness $k = 800\,\mathrm{kN/m}$ and the damping ratio $\xi = 0.2$, the natural frequency is calculated as 0.92 Hz. At the critical train speed, i.e. $\beta = 1$, the transmissibility can be calculated as $\mathrm{TR} = 2.69$. It means that when the deflection of the beam is 2 mm, the amplitude of the vehicle will be as large as 5.38 mm. Moreover, the resonance of vehicles will in turn enlarge the dynamic impact on the bridge.

The fundamental vertical natural frequencies of the train vehicles are usually between 0.8 and 1.5 Hz. For the railway bridges with 20–40 m spans, the corresponding critical train speeds could thus be estimated as $V_{vr} = 57$–216 km/h. Therefore, it is better not to arrange long series of identical spans in the design of railway bridges, to prevent the vehicle resonance due to the bridge deflections. The other effective schemes to minimize the influence of bridge vibration on the responses of vehicles include: to control the stiffness of the bridges to

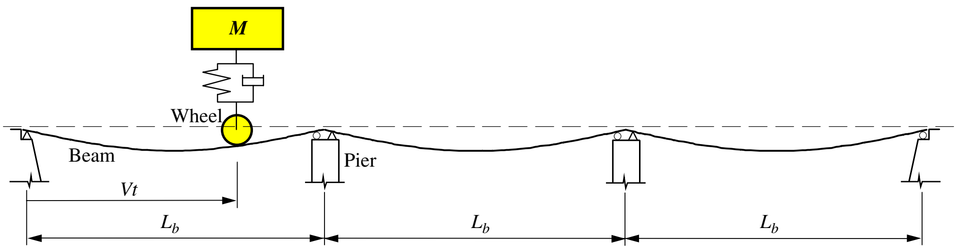

Fig. 8. Vehicle vibration induced by bridge deflection curves.

reduce the span deflections, and to apply the cancellation technique through adjusting the span length and/or the stiffness of the bridge to eliminate the resonance of the beams [11].

4. Conclusions

The resonant vibrations of train–bridge system can be divided into following types according to their generation mechanisms:

(1) Bridge resonance excited by periodically loading of moving load series of moving vehicles, due to the wheel intervals of the vehicles.
(2) Bridge resonance excited by the loading rate of moving load series of vehicles.
(3) Bridge resonance excited by the periodical actions of the sway forces of running vehicle induced by rail irregularities and hunting movements.
(4) Vehicle resonance excited by the periodical actions of regularly arranged bridge spans and their deflections.

For bridge resonance analysis, the load series consist of not only the vertical forces of axle weights of the train vehicles, but also the lateral forces transmitted from the wheels due to the centrifugal forces or wind pressures acting on the vehicles, which should be noticed.

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