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Data reduction using a generalized regressive discrete Fourier series

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Abstract

With the development of optical measurement techniques it is possible to obtain vast amounts of data. In vibrometry applications in particular operational deflection shapes are often obtained with very high spatial resolution. It has long been known that it is possible to reduce (approximate) the measurement data by means of a Fourier decomposition.

One of the most common techniques for evaluating optical measurement data is by means of a Fourier analysis. It is well known that for periodic and band-limited sequences the Discrete Fourier Transform (DFT) returns the true Fourier coefficients when exactly 1 period (or a multiple) is processed. Leakage will occur when less than 1 period is considered. This gives rise to non-negligible errors, which can be resolved by using the Generalized Regressive Discrete Fourier Series (GRDFS), introduced in this article. The measured signal is represented by a model using sines and cosines. The coefficients of those sines and cosines are then estimated together with the phase and frequency on a global scale by means of a frequency domain system identification technique. By making use of the regressive technique proposed in this paper, it is possible to reduce the data in comparison to the classical Fourier decomposition by a sizeable factor.

In this article the method will be applied in particular to the reduction of data for laser vibrometer measurements performed on an Inorganic Phosphate Cement (IPC) beam (1D), as well as on an aluminium plate (2D). The proposed technique will also be validated on both 1D and 2D simulations of varying complexity.

1. Introduction

Modal analysis has for some time now been an important tool in structural analysis in many different fields of application. The measurements themselves have mostly been executed using contact-based measurement systems such as accelerometers. However, such systems have some disadvantages such as the cumbersome task of applying the accelerometers to the structure (sometimes several times when working in different patches) and of course there is always the mass loading aspect (important for lightweight structures). Recently, with the ever ongoing progress in optical techniques some interesting alternatives have been developed. This progress was made possible by improvements in computer hardware, which allows automatic processing of images

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within a reasonable time span. However this also led to entirely new measurement systems themselves such as the laser vibrometer, or scanning Laser Doppler Vibrometer (LDV). The LDV made it possible to increase the number of degrees of freedom (DOF) measured on the surface by two to three orders of magnitude. Where in the past one was limited to a few hundred measurement points it was now feasible to measure tens of thousands of points. Typically frequency response functions (FRF-matrices) are stored with 10,000 rows and 1000 columns, leading to very spatially dense operational deflection shapes. This high spatial resolution data calls for improvements in both data reduction and the identification techniques themselves.

There are many ways to achieve data reduction. An important and frequently used approach is to use a Fourier decomposition where the signal (e.g. the operational deflection shapes ODS) is approximated by means of a series of DFT lines [1]. This approximation however introduces a distortion effect called leakage. This is due to the assumption that the signal at hand is periodic within the measurement window. A common solution to this problem is using windows to reduce the effect, which however broadens the frequency resolution.

In this paper a data reduction method based on a regressive discrete Fourier transform technique [2,3] will be introduced. This technique was first tried by choosing a period for the desired signal, but by estimating the frequency, damping and residue by means of a transfer function model as in this article, much better results can be obtained. In the following section the method will be described and a comparison with a classical DFT reduction will be made. In Section 3 simulations and experiments will be shown on different examples in both one and two dimensions. The operational deflection shapes from an IPC beam and from a prepared flat plate will be analysed and finally some conclusions will be drawn in the last section.

2. Data reduction using a regressive Fourier series

The purpose of this paper is compressing the data in the operational deflection shapes using a generalized regressive Fourier series. (However the technique can be used on either spatial or temporal data.)

2.1. Background of the regressive Fourier technique

Regressive Fourier series were first introduced by Arruda [2]. To illustrate the technique introduced by Arruda the following example is useful. Every periodic sequence $s(x)$ can be represented by an infinite sum of sines and cosines. Now if this Fourier series is truncated after a finite number of terms M and the time signal is not periodic within the measured window leakage occurs. Imagine following ODS, where $\alpha = 1$:

$$s(x) = \sum_{m=1}^{M} A_m \sin(m2\pi\alpha f x) + B_m \sin(m2\pi\alpha f x).$$

The simplest form of this equation is a single sine wave. The spectrum of a sinusoid with spatial frequency $f = 4.2\,1/\mathrm{m}$ is depicted in Fig. 1. It is clear that the spectral energy of the single component is spread out over the neighboring frequencies. Now if one were to apply data reduction to this particular sequence using say the first 10 DFT-lines, major distortions will be present in the synthesized curve.

In [2] the period of the signal $s(x)$ was assumed to be known (e.g. α% larger than the measurement window). However, frequency and damping were fixed and in the current paper the possibility to estimate these two parameters as well is explored.

By using a well-chosen multiplier α for the basic angular frequency, it is possible to reduce errors in the calculation. For the particular example above, taking $\alpha = 1.05$ ($1.05 \times 4 = 4.2$) because 4 is a FFT-line, it is possible to eliminate errors completely. However, if one takes a different value for this parameter, the resulting synthesized curve will deviate from the original signal (Fig. 2). In this case, in Fig. 2(a) the relative error is 0% and in Fig. 2(b) the relative error is 3.5% (the relative error is defined as the sum of the squared difference between the sequence and its synthesis divided by the sum of the sequence squared).

Now in practice the factor α is usually not known beforehand so it should be estimated as well together with the residues A_m and B_m leading to a nonlinear least squares problem. This illustrates that in a general case

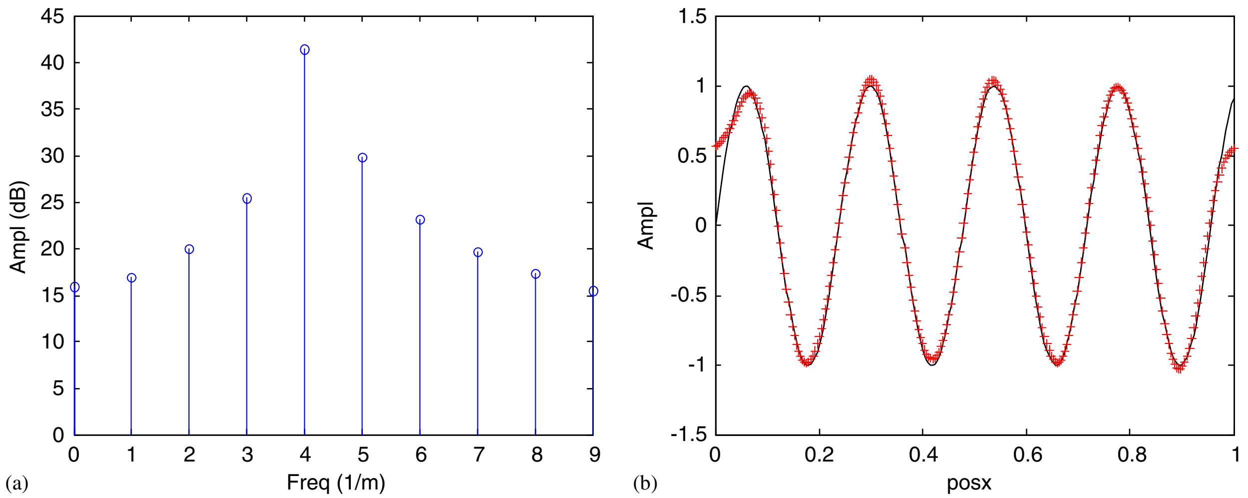

Fig. 1. (a) Spectrum of the sequence $s(x) = \sin(2\pi 4.2x)$ (first 10 DFT lines). (b) synthesis of the sequence $s(x)$ using the first 10 DFT lines (solid line indicates synthesis).

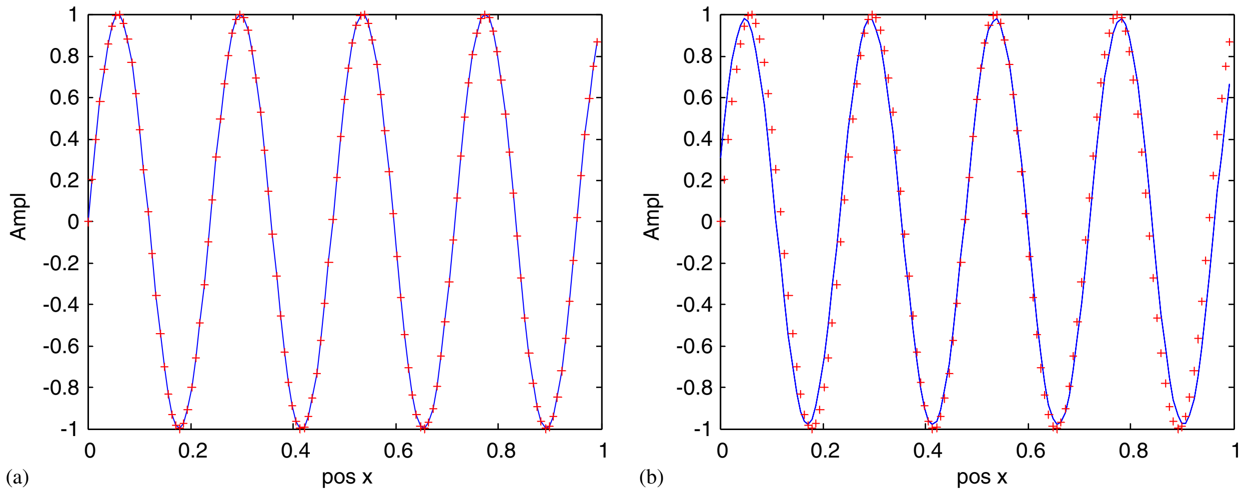

Fig. 2. (a) Synthesized curve using correct $\alpha = 1.05$; (b) synthesized curve using $\alpha = 1.025$ (solid line indicates synthesis).

where the frequencies are not known a priori, this technique does not suffice and a different approach is needed. By introducing a more general model in which the frequencies do not have to be harmonically related (unlike the previous model where there is only one α) and the harmonic functions can be damped, it will be proven possible in Section 2.3 to obtain a linear least squares problem, which resolves this issue.

2.2. Data reduction

To understand how the data reduction is achieved one can consider the following complex-valued multiharmonic function:

$$s(x) = \sum_{l=0}^{L} a_l \mathrm{e}^{-\sigma_l x + \mathrm{i}\omega_l x}. \tag{1}$$

Evaluating this function at discrete locations $x = n\,\Delta x$, Δx denoting a constant spatial resolution and $n = 0\alpha \ldots N-1$, results in the following sequence:

$$s[n] = s(n\,\Delta x) = \sum_{l=0}^{L} a_l \lambda_l^n \tag{2}$$

with $\lambda_l = \mathrm{e}^{(-\sigma_l + \mathrm{i}\omega_l)\Delta x}$ and ω_l, σ_l denoting respectively the spatial frequency and damping of component l in the operational deflection shape.

The reduction of data is possible due to the fact that the number of components L with which one models the complete sequence in (1) is much smaller than the number of measurement points N. Thus data reduction for the ODS discussed in this article means applying the proposed algorithm to the raw shape itself.

2.3. Sine estimation using a regressive Fourier technique

The technique presented in this article boils down to the following steps. By taking a discrete Fourier transform (DFT) of the acquired signal a transfer function model is derived. From this model the poles and residues (or amplitudes) are extracted. The frequency ω_l and damping σ_l can be found in the estimated poles.

The method will first be explained for the one-dimensional (1D) case [4] and then be expanded to show the 2D equations.

Starting from the sequence in Eq. (2′) using only one spatial component and taking the discrete Fourier transform:

$$s[n] = a\lambda^n, \tag{2′}$$

$$S[k] = \sum_{n=0}^{N-1} s[n]\mathrm{e}^{-\mathrm{i}2\pi(kn/N)} = \sum_{n=0}^{N-1} a\lambda^n z_k^{-n}, \tag{3}$$

where $z_k = \mathrm{e}^{(\mathrm{i}2\pi k/N)}$. This expression (3) can be written as follows by expanding the sum:

$$S[k] = a\frac{1-\lambda^N}{1-\lambda z_k^{-1}}. \tag{4}$$

Applying the result from Eq. (4) to the sequence in Eq. (1), which is the general form for an ODS reveals

$$S[k] = \sum_{l=1}^{L} \frac{\overline{a_l}}{1-\lambda_l z_k^{-1}} \tag{5}$$

with $\bar{a}_l = a_l(1-\lambda_l^N)$.

This result can also be applied to the particular case where the ODS is real (Eq. (6)):

$$s(x) = \sum_{l=1}^{L} a_l \mathrm{e}^{-\sigma_l x + \mathrm{i}\omega_l x} + \sum_{l=1}^{L} a_l^* \mathrm{e}^{-\sigma_l x - \mathrm{i}\omega_l x}, \tag{6}$$

Applying the result from Eq. (5) to this sequence reveals:

$$S[k] = \sum_{l=1}^{L} \frac{\bar{a}_l}{1-\lambda_l z_k^{-1}} + \sum_{l=1}^{L} \frac{\bar{a}_l^*}{1-\lambda_l^* z_k^{-1}} \tag{7}$$

where * indicates a complex conjugate.

Eqs. (5) and (7) are nothing else than pole/residue representations of the discrete Fourier transform $S[k]$. This clearly proves that it is possible to model sinusoids using a transfer function model in the frequency domain. Therefore it is possible by means of a simple least squares approach (or for more reliable results a maximum likelihood (ML) approach [5,6]) to retrieve values for the poles λ_l and the residues $\bar{a}_l$. The sinusoid frequencies can then be obtained from the poles and similarly the sinusoid amplitudes can be calculated from the residues with the following formula: $a_l = (\bar{a}_l/1-\lambda^N)$. This result also illustrates the fact that it is possible to compensate the residue estimates $\bar{a}_l$ for the leakage error. Indeed from the estimated poles λ_l and residues $\bar{a}_l$ an estimate of a_l has been derived.

To expand the technique to two dimensions one can start with Eq. (6), which can be written in a pole residue model the same way as in the 1D case:

$$S[k_x, k_y] = \sum_{l=1}^{L} \frac{\bar{a}_l}{(1-\lambda_{x,l}z_x^{-1})(1-\lambda_{y,l}z_y^{-1})}, \tag{8}$$

where $\bar{a}_l = a_l(1-\lambda_{x,l}^N)(1-\lambda_{y,l}^N)$, $z_x = e^{(i2\pi k_x/N)}$ and $z_y = e^{(i2\pi k_y/N)}$. This result is clearly very similar to the result obtained in the 1D case and can be solved in a similar fashion using a least squares (or a ML approach) to retrieve estimates for the residues $\bar{a}_l$ and for the poles $\lambda_{x,l}$ and $\lambda_{y,l}$.

3. Simulation results

3.1. 1D simulations

In a first example a single sinusoid is considered which is periodic within the measuring window (Fig. 3).

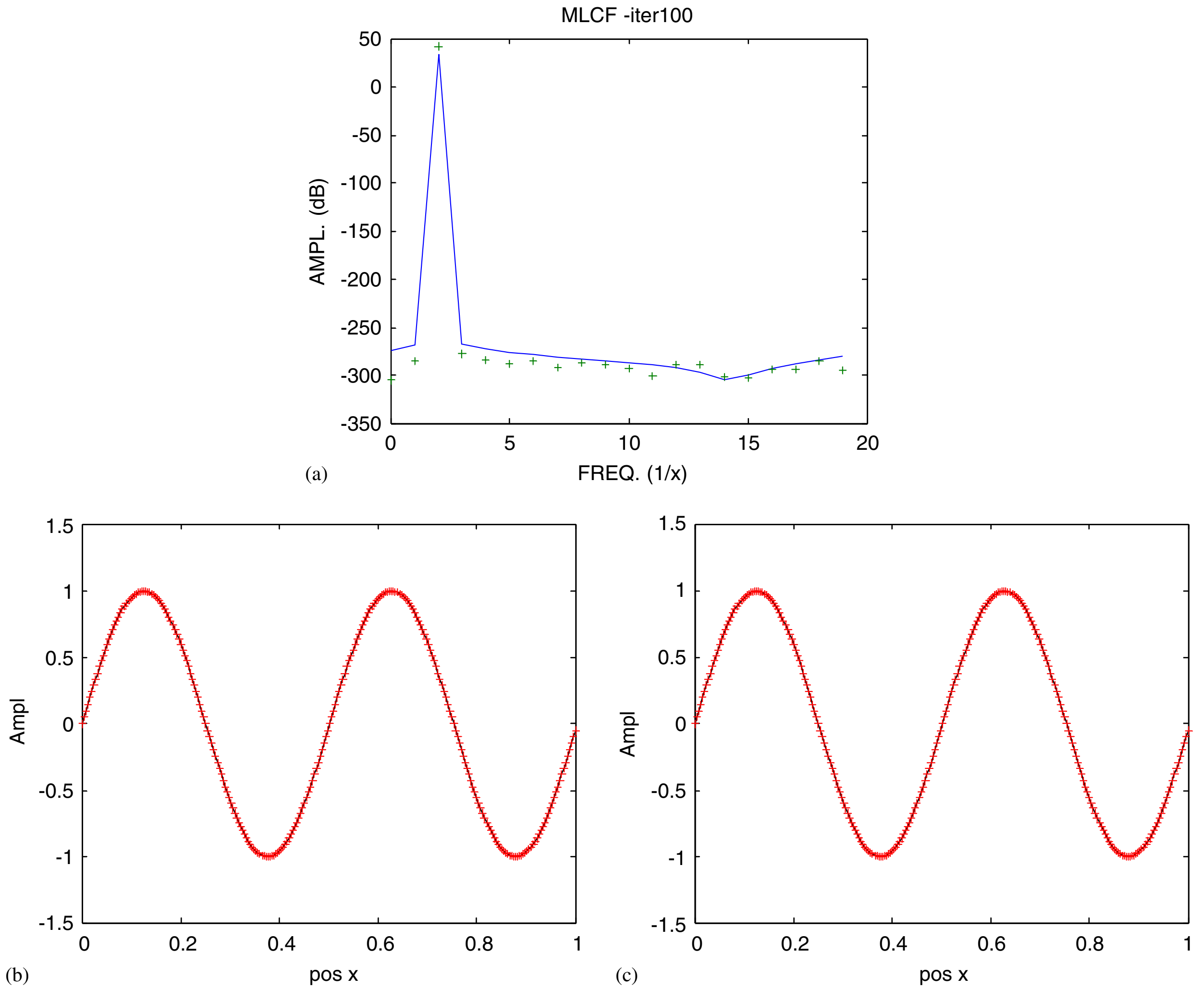

Fig. 3. Sin with freq 2 1/m; (a) frequency spectrum (solid line ML estimate); (b) classical DFT analysis 1 component; (c) GRDFS analysis 1 component (solid lines indicate synthesized signal).

It is clear that due to the choice of a periodic signal, within the measurement window no leakage will occur which means that with a DFT analysis in this case one DFT line will be sufficient to model the signal. Of course one GRDFS component will suffice to model the signal as well. Therefore no data reduction is possible.

Now consider the following example: a linear combination of 2 sinusoids with frequencies equal to 2.5 and 15.3 1/m where 20% noise was added. It is clear that this signal will not be periodic within the time window so leakage is to be expected (Fig. 4):

The frequency spectrum clearly depicts the presence of leakage. Both frequency components are spread out over neighboring frequency lines. This has a negative effect on the DFT analysis where 25 frequency lines are needed to model the signal (criterion was to take the components above the noise level). The GRDFS analysis has no such problems and can model the sequence with just 3 components (producing a similar signal error). So the obtained data compression in comparison to the DFT approach is about a factor 12. When comparing both synthesized curves to the simulated signal with no noise added to it, the DFT curve has a relative error of 4.4% and the GRDFS curve has a relative error of 0.4%. This illustrates another advantage of the regressive Fourier transform: the possibility to smooth the data and therefore be less biased to noise.

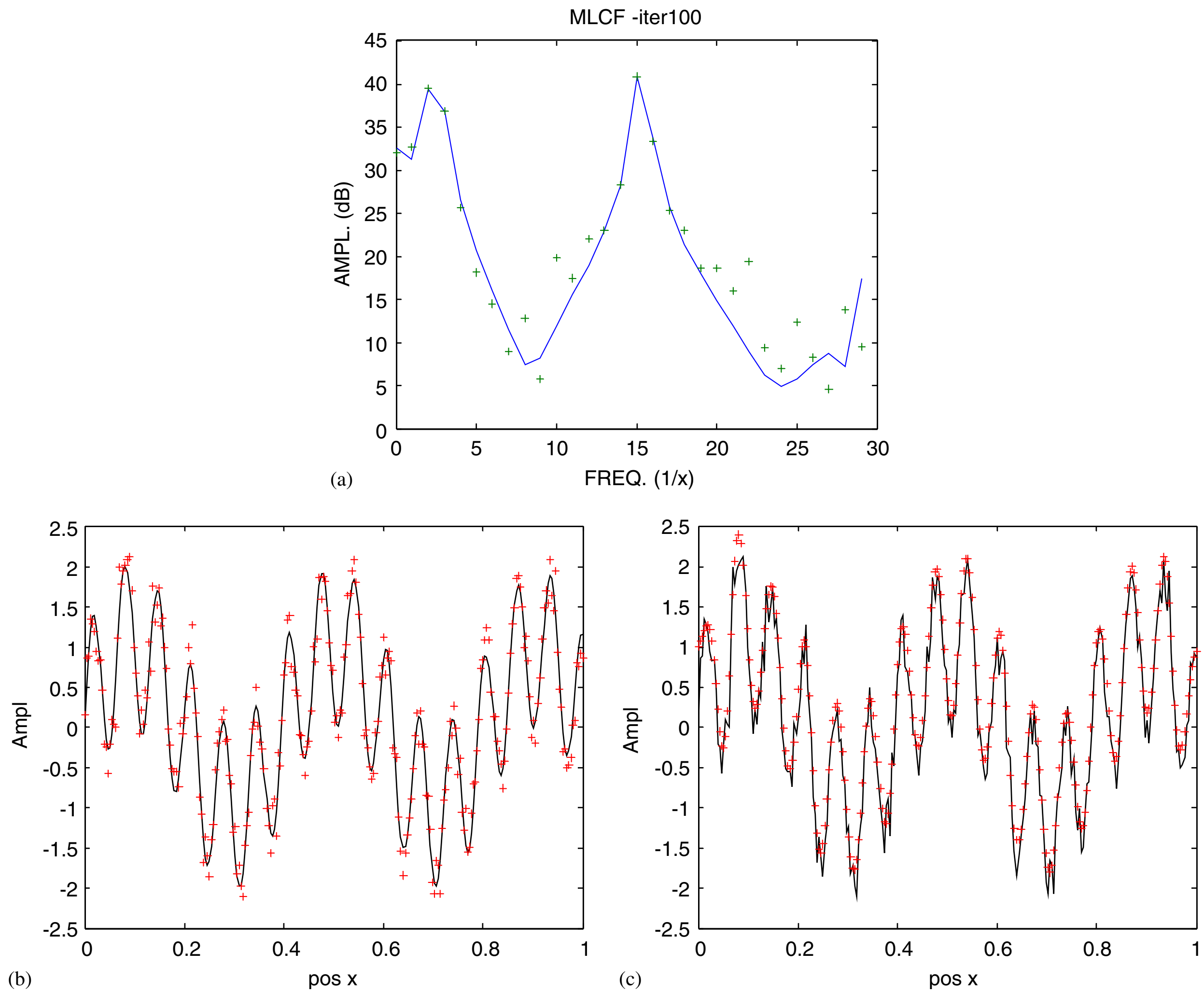

Fig. 4. Linear combination of 2 sinusoids with frequency 2.5 and 15.3 1/m and 20% noise level; (a) frequency spectrum (solid line ML estimate); (b) analysis with classical DFT approach (25 DFT lines); (c) GRDFS approach (3 components) (solid lines represent the synthesized signal).

3.2. 1D experiments on an IPC beam

In this section an experiment carried out with a Polytec PSV 300 laser vibrometer [7] on a simple cementous composite (IPC) beam will be tackled with both techniques. The beam was excited acoustically with a swept sine and was suspended in a 'free-free' manner. For illustrative purposes we show the results of an operational deflection shape around the second mode (Fig. 5). Data reduction will again be applied directly to the deflection shape.

Starting from the estimated mode shape Fig. 5 illustrates that with a DFT analysis, using the 3 most important DFT lines of the spectrum in (a), the synthesis is at best poor (31.5% relative error). When using the GRDFS it is clear that by also estimating 3 components the synthesis is quite good (0.7% relative error). Evidently, by not only estimating the frequencies but also estimating the damping one can achieve a much better fit. Indeed, two of the three estimated components in Fig. 5(c) are exponentially heavily damped functions, one which is stable and one that is unstable. To achieve the same relative error between synthesis and measurement with the DFT analysis compared to the GRDFS with 3 components, no less than 168 DFT lines are necessary (180 lines would give an exact solution as this was the Nyquist frequency). This is a compression factor of 56, which is quite significant, especially when one considers that this is a rather straightforward mode shape.

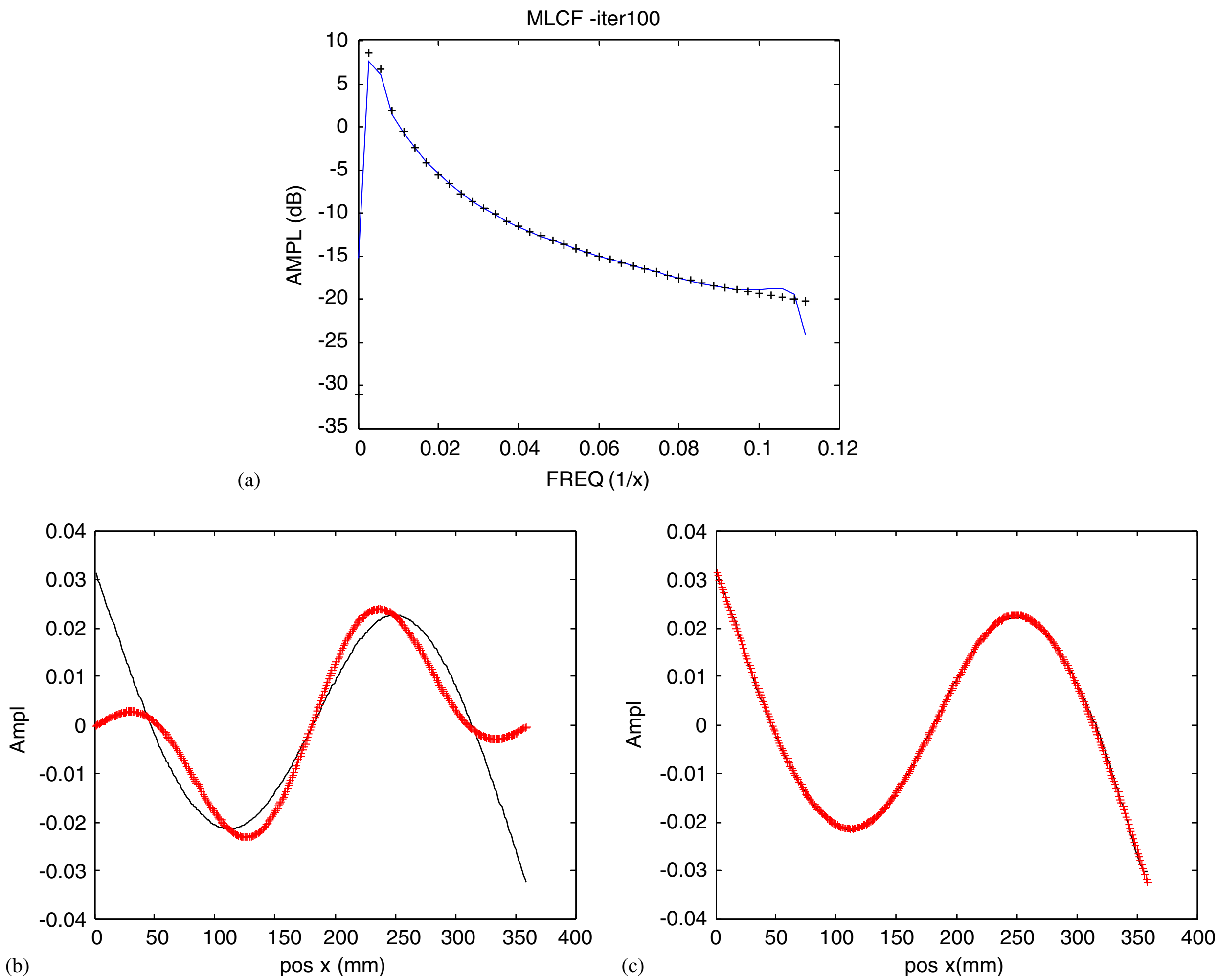

Fig. 5. Second mode shape of an IPC beam; (a) frequency spectrum of the 2nd mode shape (solid line ML estimate); (b) DFT analysis (3 most important DFT lines); (c) GRDFS analysis (3 components) [full curves indicate mode shapes and $\pm$starred curves indicate synthesis].

3.3. 2D simulations

To illustrate the 2D capabilities of the GRDFS technique the following simulation example was chosen: a linear combination of 6 complex sinusoids paired two by two to form a 2D image. To make matters somewhat more realistic a 10 percent noise level was added (Fig. 6).

Fig. 7 shows a comparison between the classical DFT and the GRDFS solution. Again the criterion for the DFT analysis was to take all components above the noise level, which left about 64 DFT lines (8×8) for this example. The GRDFS analysis was successful with estimating just 6 components (3 in each direction). So one can say that a compression factor of about 10 was achieved. In the reconstructed image based on the DFT analysis it is apparent that there are significant distortions near the borders due to leakage (Fig. 7(a)). Fig. 8, where a slice of both estimations is shown for $y = 20$, clearly illustrates the effect.

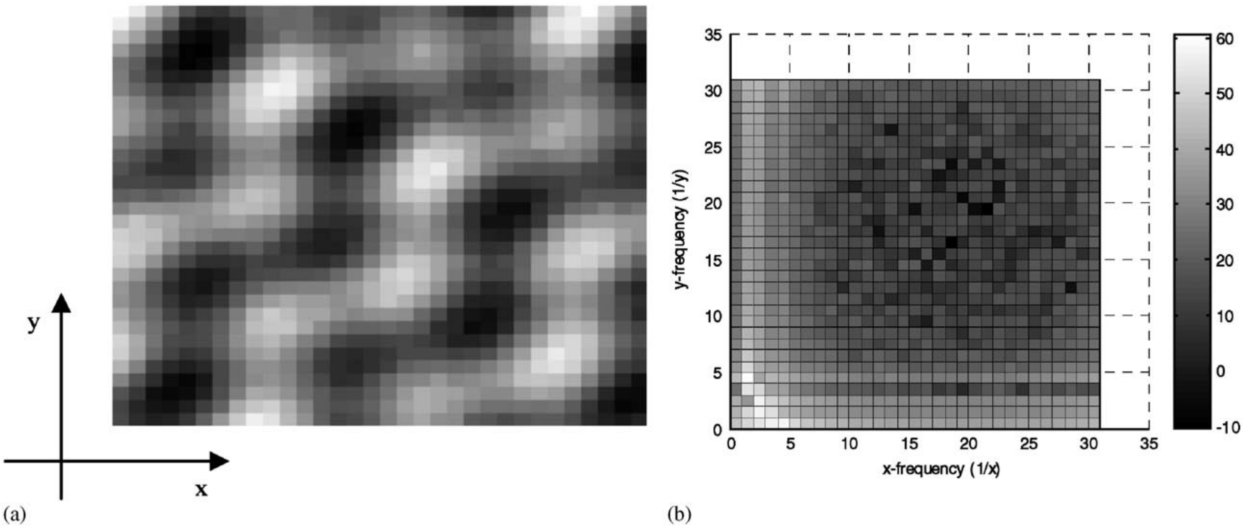

Fig. 6. (a) Image with 6 complex sinusoids with frequencies paired as followed (z_1, z_2): (0.1, 3.5)1/m; (1.5, 2.3)1/m; (3.7, 1.1)1/m with a 10% noise level; (b) frequency spectrum of image (a).

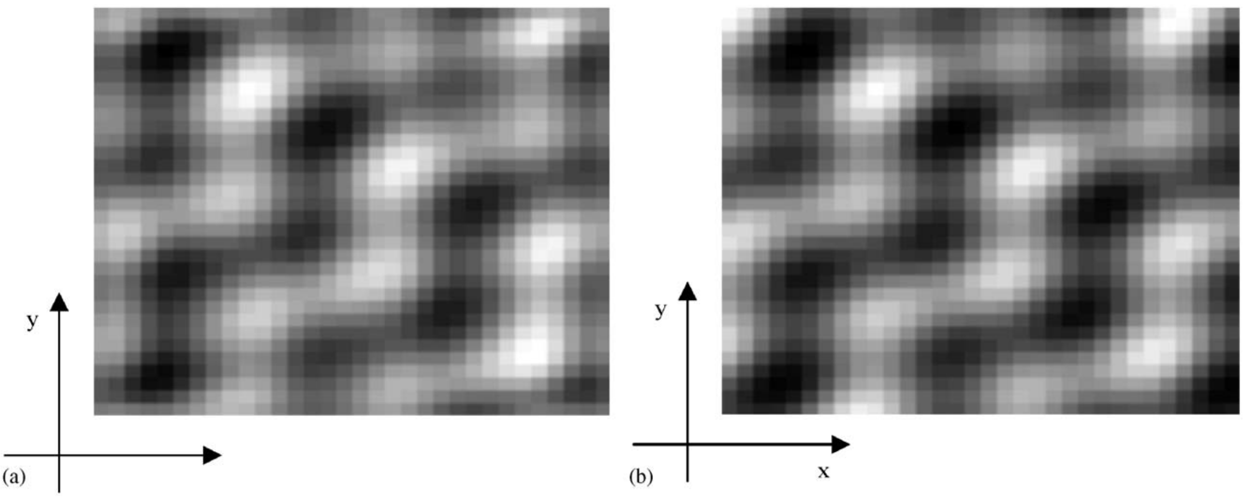

Fig. 7. (a) Synthesis of Fig. 6(a) with a classical DFT analysis (64 lines); (b) synthesis of Fig. 6(a) with a GRDFS analysis (6 components).

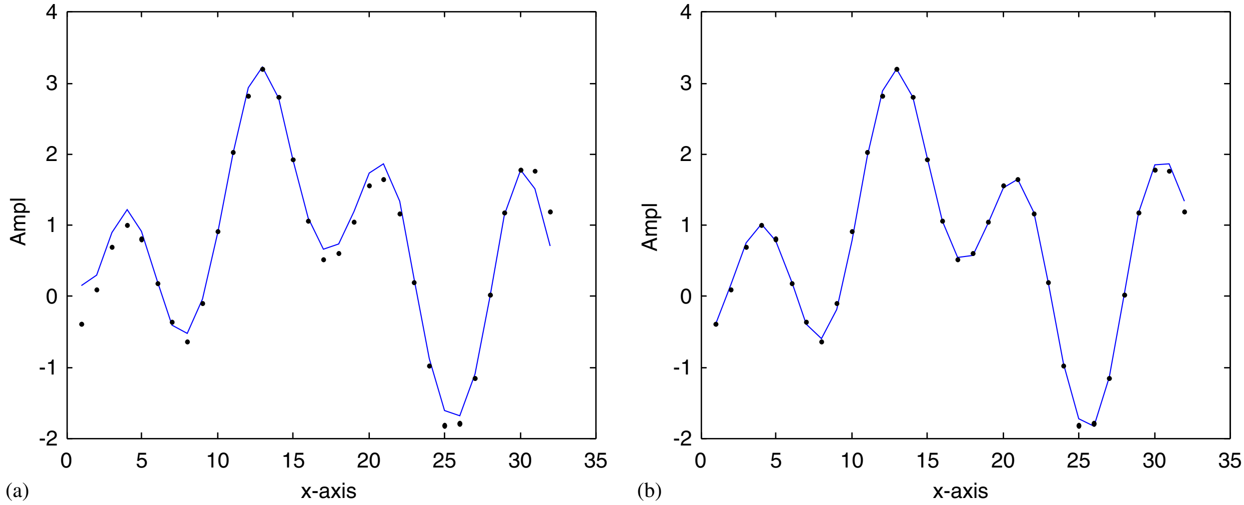

Fig. 8. (a) Slice of the image in Fig. 7(a) at $y = 20$; (b) slice of the image in Fig. 7(b) at $y = 20$ (dotted lines indicate the simulated values).

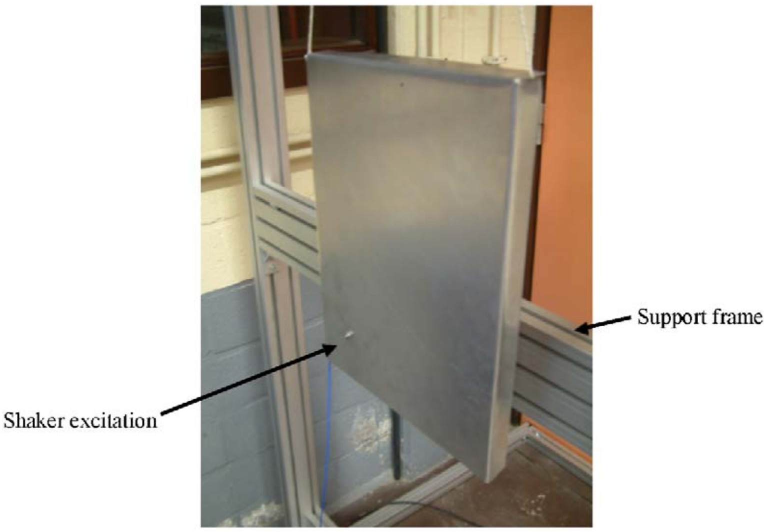

Fig. 9. Measurement setup of a flat plate with boundary conditions on 3 edges.

3.4. 2D experiments on a flat plate

In this section some 2D measurements obtained with a Polytec PSV 300 laser vibrometer [7] will be analysed. Measurements were done on a flat aluminium plate (40 cm × 30 cm), via a shaker with a swept sine excitation. Some boundary conditions were introduced on 3 of the 4 sides where the plate was stiffened by bending the edges around. The fourth side was left free (Fig. 9).

This experiment is interesting in a sense that the mode shapes will certainly not be symmetrical, so leakage is definitely to be expected. Fig. 10 shows the mode shape under inspection and its frequency spectrum (only the real parts of the images are displayed; the imaginary parts give very similar results). Again data reduction will be applied directly to the estimated mode shape.

It is obvious that the DFT analysis does not perform at all well. The slice of the signal in Fig. 12 shows that the amplitude is not quite as smooth as it should be. The contour and slice plot depict a lot of distortions

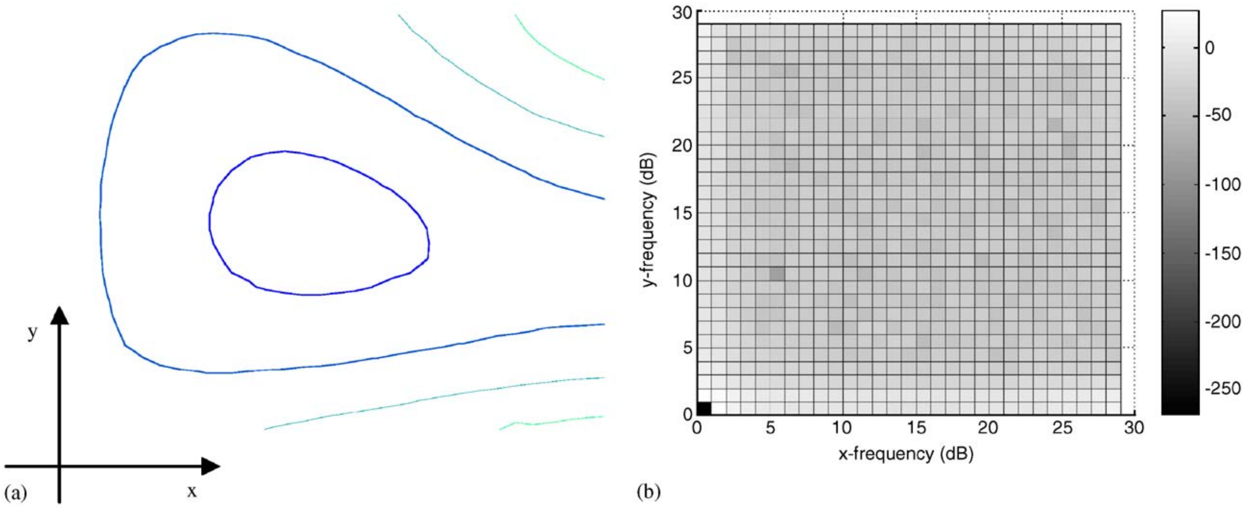

Fig. 10. (a) Contour of the first mode shape of an aluminium plate stiffened on 3 sides; (b) frequency spectrum of image (a).

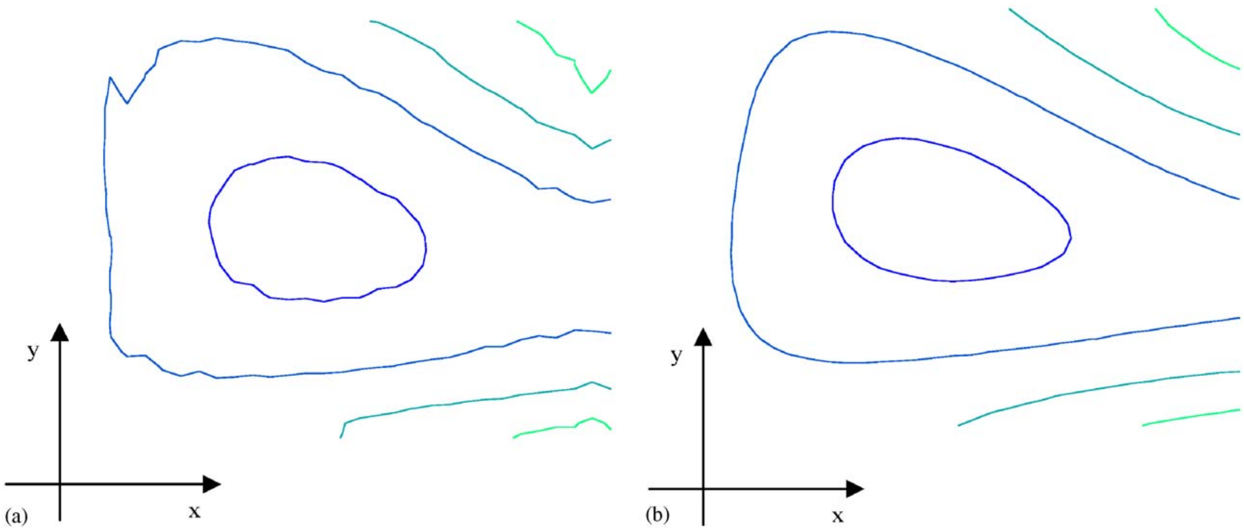

Fig. 11. (a) Contour synthesis of Fig. 10(a) with a DFT analysis (64 lines); (b) contour synthesis of Fig. 10(a) with a GRDFS analysis (6 components).

around the edges (Figs. 11 and 12). Even with this poor analysis the DFT technique still needs 10 times more components than the GRDFS method.

4. Conclusions

In this paper it has been shown that data reduction using a classical DFT decomposition can present certain distortions in the synthesis due to leakage. A regressive Fourier approach was introduced (based on a transfer function model of the measurement spectra) that did not exhibit these faults. On top of that it was shown that another major strength of this regressive technique was its ability to compress the data significantly in comparison to a traditional DFT line decomposition. Moreover it has been proven that by using the regressive Fourier technique it was possible to smooth the data and hence diminish the effect of noise. This was shown

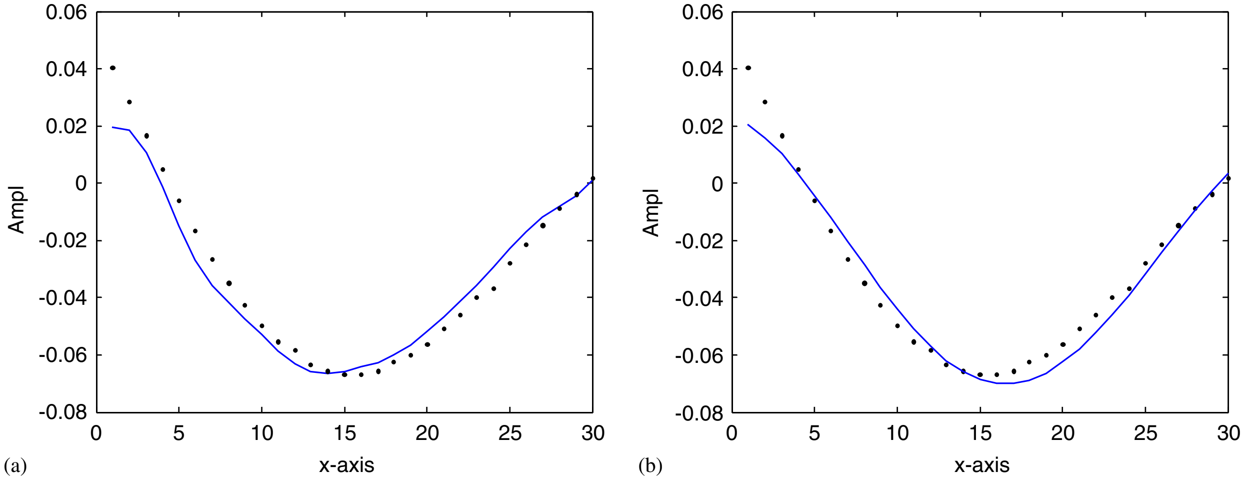

Fig. 12. (a) Slice of image in Fig. 11(a) at $y = 10$; (b) slice of image in Fig. 11(b) at $y = 10$ (dotted lines indicate the exact image).

on both simulations in one and two dimensions and experiments on an IPC beam (1D) and a prepared aluminium plate (2D).

Acknowledgments

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