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Rayleigh waves in a double porosity half-space

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Abstract

Based on the governing equations of double-porosity media extended from Biot's theory, the propagation characteristics of Rayleigh waves are discussed in this paper. Characteristic equations of Rayleigh waves in a double porosity half-space with the pervious boundary are derived by assuming that both the coupling mass coefficient ρ_{23} and coupling permeability term $k^{(12)}$ are equal to zeros. The dispersion and attenuation properties of Rayleigh waves in a double porosity medium are analyzed numerically. The effects of the porosity and fracture permeability on the behavior of the propagation of Rayleigh waves are investigated. It is found that Rayleigh waves in a double porosity half-space are dispersive and attenuated during propagation. The Rayleigh wave speed is always less than the phase velocity of P1 and shear waves, and always higher than the phase velocity of P3 wave.

1. Introduction

The propagation characteristics of Rayleigh waves in fluid-saturated porous media are an important problems not only from the point of view of applications such as geophysics, hydrology and acoustics, but also at the theoretical level. It is well known that Biot [1,2] first established the fundamental theory for wave propagation in porous media. Biot's theory predicts that there exists three kinds of bulk waves, i.e., fast compressional wave, slow compressional wave and one kind shear wave. Deresiewicz [3] first discussed Rayleigh waves in fluid-saturated porous media. Jones [4] studied the problem of Rayleigh waves in porous-saturated solid with a pervious surface based on Biot's theory, but his results were criticized by Tajuddin [5]. Tajuddin [5] investigated Rayleigh waves in a poroelastic half-space with the pervious and impervious boundaries and gave the curves of the Rayleigh wave speed vs. Poisson's ratio. Liu and De Boer [6] discussed the Rayleigh waves in a fluid-saturated porous half-space based on the theory of mixtures and gave the dispersion and attenuation curves. The effects of permeability parameters on the propagation of Rayleigh waves were also studied. Sharma and Gogna [7] studied Rayleigh waves in a anisotropic liquid-saturated porous solid but the dissipation is ignored. Liu and Liu [8] investigated the influence of anisotropy of the solid skeleton on the propagation characteristic of Rayleigh waves in an orthotropic fluid-saturated porous medium based on Biot's theory.

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It has been confirmed that in some materials, such as most rocks and some acoustic materials, there mainly exist two kinds of porosities: matrix porosity which is also called storage porosity, occupies a substantial fraction in the reservoir but has a very low permeability, and fracture or crack porosity which occupies very little volume but has a very high permeability. Wave propagation problems in double porosity media have received more and more attention in recent years. The double porosity media theory has been applied in various fields such as oil extraction, geological exploration and water resources exploitation. The double porosity model was first proposed by Barenblatt [10] to express fluid flow in hydrocarbon reservoirs and aquifers. Warren and Root [11] made an improvement to this model, which allows for coupling between the rock deformation and the fluid flow. Wilson and Aifantis [12,13] published a series papers on consolidation of saturated double porosity media. They studied the wave propagation problem in saturated fractured porous media and showed that there exist three kinds of longitudinal waves and one transverse wave. According to the Mixture theory, Tuncay and Corapcioglu [14,15] used a volume averaging technique to investigate the wave propagation in fractured porous media saturated by two immiscible fluids based on the double-porosity approach. Their discussions showed the existence of three compressional and one rotational waves. Beskos [16] got analogous results. Based on ideas similar to those of Biot's theory, Berryman and Wang [17] derived the phenomenological equations for double porosity media and presented the method to determine the relevant coefficients [18]. Their discussions showed that all three compressional waves are attenuative, and the second compressional wave is diffusive at low frequencies while third compressional wave is diffusive at all frequencies.

At present, the works about Rayleigh waves in double porosity media have not been available in the literatures. The purpose of this paper is to discuss the propagation of Rayleigh waves in double porosity media filled with one kind of incompressible fluid. The balance equations and constitutive relations constructed by Berryman and Wang [17] are adopted. The organization of this paper is as follows. In Section 2, the governing equations of double porosity media are reviewed. In Section 3, the characteristic equations of Rayleigh waves are derived. In Section 4, based on the characteristic equations derived in Section 3, numerical calculations are performed. The dispersion and attenuation curves are given. In Section 5 some conclusions are given.

2. Governing equations

By generalizing Biot's approach, Berryman and Wang introduce the kinetic energy function T and the dissipation function D. Then according to the Lagrange principle, the equations of motion for double porosity media can be expressed as [17]

$$\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12} & \rho_{22} & \rho_{23} \\ \rho_{13} & \rho_{23} & \rho_{33} \end{pmatrix} \begin{pmatrix} \ddot{u}_i \\ \ddot{U}_i^{(1)} \\ \ddot{U}_i^{(2)} \end{pmatrix} + \begin{pmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12}+b_{23} & -b_{23} \\ -b_{13} & -b_{23} & b_{13}+b_{23} \end{pmatrix} \begin{pmatrix} \dot{u}_i \\ \dot{U}_i^{(1)} \\ \dot{U}_i^{(2)} \end{pmatrix} = \begin{pmatrix} \sigma_{ij,j} \\ -\bar{p}_{,i}^{(1)} \\ -\bar{p}_{,i}^{(2)} \end{pmatrix}, \tag{1}$$

where u_i, $U_i^{(1)}$ and $U_i^{(2)}$ are solid displacement, matrix pore fluid displacement and fracture pore fluid displacement, respectively, $\bar{p}^{(1)}$ and $\bar{p}^{(2)}$ are the average macroscopic pressure of matrix pore fluid and fracture pore fluid across interface, related to the internal real pore pressure $p^{(1)}$ and $p^{(2)}$, respectively, by

$$\bar{p}^{(1)} = v^{(1)}\phi^{(1)}p^{(1)}, \quad \bar{p}^{(2)} = v^{(2)}p^{(2)}, \tag{2}$$

where $v^{(1)}$ and $v^{(2)}$ are volume fractions of matrix and fracture pore, respectively, with $v^{(1)} + v^{(2)} = 1$, $\phi^{(1)}$ is the volume fraction of the matrix pore in matrix and the volume fraction of the matrix pore in medium is $p^{(1)}\phi^{(1)}$. The total porosity can be expressed as

$$\phi = v^{(1)}\phi^{(1)} + v^{(2)}, \tag{3}$$

where ρ_{ij} is the mass coefficient and b_{ij} is the viscosity coupled coefficient. Detailed expressions of ρ_{ij} and b_{ij} are given in Appendix A.

When considering wave propagation problem, the cross-coupling coefficients between matrix pores and fractures equal zero and can be neglected, that is, $\rho_{23}=0$ and $b_{23}=0$. Then Eq. (1) can be reduced to

$$\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12} & \rho_{22} & 0 \\ \rho_{13} & 0 & \rho_{33} \end{pmatrix}\begin{pmatrix} \ddot{u}_i \\ \ddot{U}_i^{(1)} \\ \ddot{U}_i^{(2)} \end{pmatrix} + \begin{pmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12}+b_{23} & 0 \\ -b_{13} & 0 & b_{13}+b_{23} \end{pmatrix}\begin{pmatrix} \dot{u}_i \\ \dot{U}_i^{(1)} \\ \dot{U}_i^{(2)} \end{pmatrix} = \begin{pmatrix} \sigma_{ij,j} \\ -\bar{p}_{,i}^{(1)} \\ -\bar{p}_{,i}^{(2)} \end{pmatrix}. \tag{4}$$

Similar to Biot's original equations, linear relations among the solid strain, the fluid content and the pore pressure are given by [19]

$$\begin{pmatrix} e \\ -\zeta^{(1)} \\ -\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} -p_c \\ -p^{(1)} \\ -p^{(2)} \end{pmatrix}, \tag{5}$$

where $e=\nabla\cdot\boldsymbol{u}=u_{i,i}$ is the overall volume strain, $\xi^{(1)}$ and $\xi^{(2)}$ are increments of matrix and fracture fluid content which are related to the displacements by $\zeta^{(1)}=-v^{(1)}\phi^{(1)}\nabla\bullet(\boldsymbol{U}^{(1)}-\boldsymbol{u})$ and $\zeta^{(2)}=-v^{(2)}\nabla\bullet(\boldsymbol{U}^{(2)}-\boldsymbol{u})$. p_c, $p^{(1)}$ and $p^{(2)}$ are the external confining pressure, the fluid pressure in the matrix pore and the fluid pressure in the fracture, respectively. The coefficients have the relations $a_{ij}=a_{ji}$ and are related to material properties by

$$\begin{aligned} &a_{11}=1/K, \quad a_{12}=-\alpha^{(1)}K_s^{(1)}1/K_1K_s, \\ &a_{13}=-\alpha/K-a_{12}, \quad a_{22}=v^{(1)}\alpha^{(1)}/B^{(1)}K^{(1)}, \\ &a_{23}=-v^{(1)}\alpha^{(1)}/K^{(1)}-a_{12}, \\ &a_{33}=v^{(2)}/K_f+v^{(1)}/K^{(1)}-(1-2\alpha)/K^{(1)}+2a_{12}, \end{aligned} \tag{6}$$

where K and $K^{(1)}$ are the jacketed frame bulk moduli of the whole and the matrix, respectively, K_s and $K_s^{(1)}$ are the unjacketed bulk moduli for the whole and the matrix, $\alpha=1-K/K_s$ and $\alpha^{(1)}=1-K^{(1)}/K_s^{(1)}$ are the corresponding Biot–Willis parameters. $B^{(1)}$ is Skempton's pore-pressure buildup coefficient [20]. K_f is the pore fluid bulk modulus.

Eq. (5) can be rewritten as

$$\begin{pmatrix} -p_c \\ -p^{(1)} \\ -p^{(2)} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}\begin{pmatrix} e \\ -\zeta^{(1)} \\ -\zeta^{(2)} \end{pmatrix} = A\begin{pmatrix} e \\ -\zeta^{(1)} \\ -\zeta^{(2)} \end{pmatrix}, \tag{7}$$

where A is the inverse matrix of the coefficient matrix of Eq. (5).

In addition, the solid stress can be expressed as [17]

$$\nabla\bullet\sigma=\left(K_u+\frac{1}{3}G\right)\nabla e+G\nabla\bullet\varepsilon+K_u\nabla\left[B^{(1)}\zeta^{(1)}+B^{(2)}\zeta^{(2)}\right], \tag{8}$$

where

$$K_u=\frac{K}{1-3K\left[\beta^{(1)}B^{(1)}+\beta^{(2)}B^{(2)}\right]} \tag{9}$$

is the undrained bulk modulus for double porosity media. $\beta^{(1)}$ and $\beta^{(2)}$ are poroelastic expansion coefficients.

Combining Eqs. (5), (8) and (9), we can get the following motion equations expressed by u, $U^{(1)}$ and $U^{(2)}$:

$$\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12} & \rho_{22} & 0 \\ \rho_{13} & 0 & \rho_{33} \end{pmatrix} \begin{pmatrix} \ddot{u} \\ \ddot{U}^{(1)} \\ \ddot{U}^{(2)} \end{pmatrix} + \begin{pmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12} & 0 \\ -b_{13} & 0 & b_{13} \end{pmatrix} \begin{pmatrix} \dot{u} \\ \dot{U}^{(1)} \\ \dot{U}^{(2)} \end{pmatrix}$$
$$= \begin{pmatrix} G\nabla^2\mathbf{u} + (K_u + \frac{1}{3}G)\nabla e + K_u\nabla(B^{(1)}\zeta^{(1)} + B^{(2)}\zeta^{(2)}) \\ v^{(1)}\phi^{(1)}\nabla(A_{12}e - A_{22}\zeta^{(1)} - A_{23}\zeta^{(2)}) \\ v^{(2)}\nabla(A_{13}e - A_{23}\zeta^{(1)} - A_{33}\zeta^{(2)}) \end{pmatrix}. \tag{10}$$

Considering the Helmholtz resolution of each of the three displacement vectors in the form:

$$\boldsymbol{u} = \nabla\phi_s + \nabla\boldsymbol{\psi}_s, \quad \boldsymbol{U}^{(1)} = \nabla\phi_1 + \nabla\boldsymbol{\psi}_1, \quad \boldsymbol{U}^{(2)} = \nabla\phi_2 + \nabla\boldsymbol{\psi}_2, \tag{11}$$

where φ_α and $\boldsymbol{\psi}_\alpha(\alpha = s, 1, 2)$ are potential functions of the solid skeleton, fluid in matrix pores and fluid in fractures, respectively. In a plane problem, the vector potential $\boldsymbol{\psi}_\alpha(\alpha = s, 1, 2)$ is simply degenerated to a scalar function $\psi_\alpha(\alpha = s, 1, 2)$.

Applying the divergence operation to Eq. (10), we obtain:

$$\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12} & \rho_{22} & 0 \\ \rho_{13} & 0 & \rho_{33} \end{pmatrix} \begin{pmatrix} \ddot{\varphi}_s \\ \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{pmatrix} + \begin{pmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12} & 0 \\ -b_{13} & 0 & b_{13} \end{pmatrix} \begin{pmatrix} \dot{\phi}_s \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix}$$
$$= \nabla^2 \begin{pmatrix} K_u + \frac{4}{3}G + K_u(v^{(1)}\phi^{(1)}B^{(1)} + v^{(2)}B^{(2)}) & -K_u v^{(1)}\phi^{(1)}B^{(1)} & -K_u v^{(2)}B^{(2)} \\ v^{(1)}\phi^{(1)}(\bar{A}_{12} - \bar{A}_{22}v^{(1)}\phi^{(1)} - \bar{A}_{23}v^{(2)}) & \bar{A}_{22}(v^{(1)}\phi^{(1)})^2 & \bar{A}_{23}v^{(1)}\phi^{(1)}v^{(2)} \\ v^{(2)}(\bar{A}_{13} - \bar{A}_{23}v^{(1)}\phi^{(1)} - \bar{A}_{33}v^{(2)}) & \bar{A}_{23}v^{(1)}\phi^{(1)}v^{(2)}\phi^{(2)} & \bar{A}_{33}(v^{(2)})^2 \end{pmatrix} \begin{pmatrix} \varphi_s \\ \varphi_1 \\ \varphi_2 \end{pmatrix}. \tag{12}$$

Applying the curl operation to Eq. (10), we obtain

$$\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12} & \rho_{22} & 0 \\ \rho_{13} & 0 & \rho_{33} \end{pmatrix} \begin{pmatrix} \ddot{\boldsymbol{\psi}}_s \\ \ddot{\boldsymbol{\psi}}_1 \\ \ddot{\boldsymbol{\psi}}_2 \end{pmatrix} + \begin{pmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12} & 0 \\ -b_{13} & 0 & b_{13} \end{pmatrix} \begin{pmatrix} \dot{\boldsymbol{\psi}}_s \\ \dot{\boldsymbol{\psi}}_1 \\ \dot{\boldsymbol{\psi}}_2 \end{pmatrix} = \begin{pmatrix} G\nabla^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\psi}_s \\ \boldsymbol{\psi}_1 \\ \boldsymbol{\psi}_2 \end{pmatrix}. \tag{13}$$

Eqs. (12) and (13) describe the propagation of the compressional and shear waves in double porosity media, respectively. Since the imaginary parts of the complex wavenumbers should be positive to insure decay, it can

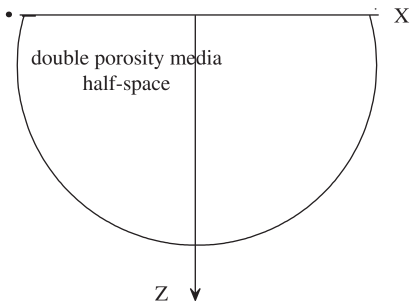

Fig. 1. A schematic drawing of the problem.

be seen from Eqs. (12) and (13) that there exists three kinds of compressional waves (P-1, P-2 and P-3 waves) and one shear wave (S wave) in double porosity medium [21].

3. Characteristic equations of Rayleigh waves

Consider a double porosity half-space with the surface $Z=0$. The geometry of the problem to be considered is illustrated in Fig. 1. The rectangular Cartesian coordinates(X, Y, Z) are selected. The positive Y-axis orients in the direction perpendicular and outward to the paper and the positive direction of Z-axis is downward.

To investigate the existence of Rayleigh waves in double porosity media, we assume the solutions of Eqs. (12) and (13) are the plane harmonic waves along the positive X-axis and can be expressed in the following forms:

$$(\varphi)=\begin{pmatrix}\varphi_s\\ \varphi_1\\ \varphi_2\end{pmatrix}=\begin{pmatrix}A_s e^{-pz}e^{i(kx-\omega t)}\\ A_{f1}e^{-pz}e^{i(kx-\omega t)}\\ A_{f2}e^{-pz}e^{i(kx-\omega t)}\end{pmatrix}, \tag{14}$$

$$(\boldsymbol{\psi})=\begin{pmatrix}\psi_s\\ \psi_1\\ \psi_2\end{pmatrix}=\begin{pmatrix}B_s e^{-sz}e^{i(kx-\omega t)}\\ B_{f1}e^{-sz}e^{i(kx-\omega t)}\\ B_{f2}e^{-sz}e^{i(kx-\omega t)}\end{pmatrix}, \tag{15}$$

where A_s, A_{f1}, A_{f2}, B_s, B_{f1}, B_{f2} are amplitudes of corresponding potential functions, k is the complex wavenumber of Rayleigh waves along X-axis, p and s are positive real numbers or complex numbers having positive real part to insure wave decay along the positive Z-axis.

The potential functions are connected with the relevant stresses σ_z, τ_{xz} and the fluid pressure terms $p^{(1)}$, $p^{(2)}$ by

$$\begin{aligned}\sigma_z=&\left(K_u+\frac{G}{3}\right)\nabla^2\varphi_s+G\left(\frac{\partial^2\varphi_s}{\partial z^2}+\frac{\partial^2\psi_s}{\partial x\partial z}\right)\\ &-K_u\left[B^{(1)}v^{(1)}\phi^{(1)}\nabla^2(\varphi_1-\varphi_s)+B^{(2)}v^{(2)}\nabla^2(\varphi_2-\varphi_s)\right],\end{aligned} \tag{16a}$$

$$\tau_{xz}=2G\frac{\partial^2\varphi_s}{\partial x\partial z}+G\left(\frac{\partial^2\psi_s}{\partial x^2}-\frac{\partial^2\psi_s}{\partial z^2}\right), \tag{16b}$$

$$-p^{(1)}=A_{21}\nabla^2\varphi_s+A_{22}v^{(1)}\phi^{(1)}\nabla^2(\varphi_1-\varphi_s)+A_{23}v^{(2)}\nabla^2(\varphi_2-\varphi_s), \tag{16c}$$

$$-p^{(2)}=A_{31}\nabla^2\varphi_s+A_{32}v^{(1)}\phi^{(1)}\nabla^2(\varphi_1-\varphi_s)+A_{33}v^{(2)}\nabla^2(\varphi_2-\varphi_s). \tag{16d}$$

Substituting Eq. (14) into Eq. (12), we get

$$\begin{pmatrix}m_{11}&m_{12}&m_{13}\\ m_{21}&m_{22}&m_{23}\\ m_{31}&m_{32}&m_{33}\end{pmatrix}\begin{pmatrix}A_s\\ A_{f1}\\ A_{f2}\end{pmatrix}=[M]\begin{pmatrix}A_s\\ A_{f1}\\ A_{f2}\end{pmatrix}=0, \tag{17}$$

where

$$
\begin{aligned}
m_{11} &= (k^2-p^2)\left\{-\left[K_u+\frac{4G}{3}+K_u(B^{(1)}v^{(1)}\phi^{(1)}+B^{(2)}v^{(2)})\right]\right\}+\rho_{11}\omega^2+\mathrm{i}\omega(b_{12}+b_{13}),\\
m_{12} &= (k^2-p^2)K_uB^{(1)}v^{(1)}\phi^{(1)}+\rho_{12}\omega^2-\mathrm{i}\omega b_{12},\\
m_{13} &= (k^2-p^2)K_uB^{(2)}v^{(2)}+\rho_{13}\omega^2-\mathrm{i}\omega b_{13},\\
m_{21} &= (k^2-p^2)\left[-v^{(1)}\phi^{(1)}\left(A_{12}-A_{22}v^{(1)}\phi^{(1)}-A_{23}v^{(2)}\right)\right]+\rho_{12}\omega^2-\mathrm{i}\omega b_{12},\\
m_{22} &= (k^2-p^2)\left[-A_{22}\left(v^{(1)}\phi^{(1)}\right)^2\right]+\rho_{22}\omega^2+\mathrm{i}\omega b_{12},\\
m_{23} &= (k^2-p^2)\left(-A_{23}v^{(1)}\phi^{(1)}v^{(2)}\right),\\
m_{31} &= (k^2-p^2)\left[-v^{(2)}\left(A_{13}-A_{23}v^{(1)}\phi^{(1)}-A_{33}v^{(2)}\right)\right]+\rho_{13}\omega^2-\mathrm{i}\omega b_{13},\\
m_{32} &= (k^2-p^2)\left(-A_{23}v^{(1)}\phi^{(1)}v^{(2)}\right),\\
m_{33} &= (k^2-p^2)\left[-A_{33}\left(v^{(2)}\right)^2\right]+\rho_{33}\omega^2+\mathrm{i}\omega b_{13}.
\end{aligned}
\tag{18}
$$

Substitution of Eq. (15) into Eq. (13) yields

$$
\begin{pmatrix} n_{11} & n_{12} & n_{13}\\ n_{21} & n_{22} & n_{23}\\ n_{31} & n_{32} & n_{33}\end{pmatrix}\begin{pmatrix} B_s\\ B_{f1}\\ B_{f2}\end{pmatrix}=[N]\begin{pmatrix} B_s\\ B_{f1}\\ B_{f2}\end{pmatrix}=0,
\tag{19}
$$

where

$$
\begin{aligned}
n_{11} &= -G\left(k^2-s^2\right)+\rho_{11}\omega^2+\mathrm{i}\omega(b_{12}+b_{13}),\\
n_{12} &= n_{21}=\rho_{12}\omega^2-\mathrm{i}\omega b_{12},\quad n_{13}=n_{31}=\rho_{13}\omega^2-\mathrm{i}\omega b_{13},\\
n_{22} &= \rho_{22}\omega^2+\mathrm{i}\omega b_{12},\quad n_{23}=n_{32}=0,\quad n_{33}=\rho_{33}\omega^2+\mathrm{i}\omega b_{13}.
\end{aligned}
\tag{20}
$$

The conditions for nontrivial solutions of Eqs. (17) and (19) require the determinant of matrix $[M]$ and $[N]$ must be equal to zero, respectively,

$$
\begin{aligned}
&\mathrm{Det}[M]=0 \text{ or } \alpha_1(k^2-p^2)^3+\alpha_2(k^2-p^2)^2+\alpha_3(k^2-p^2)+\alpha_4=0,\\
&\mathrm{Det}[N]=0 \text{ or } \beta_1\left(k^2-s^2\right)+\beta_2=0,
\end{aligned}
\tag{21a}
$$

where

$$
\begin{aligned}
\beta_1 &= G(\mathrm{i}\omega b_{12}-\omega^2\rho_{22})(\mathrm{i}\omega b_{13}+\omega^2\rho_{33}),\\
\beta_2 &= (\omega^2\rho_{13}-\mathrm{i}\omega b_{13})(\omega^2 b_{12}b_{13}+\mathrm{i}\omega^3 b_{12}\rho_{13}+\mathrm{i}\omega^3 b_{13}\rho_{22}-\omega^4\rho_{13}\rho_{22})\\
&\quad-(\omega^2\rho_{12}-\mathrm{i}\omega b_{12})^2(\mathrm{i}\omega b_{13}+\omega^2\rho_{33})+\mathrm{i}\omega(b_{12}+b_{13})(\omega^2\rho_{22}-\mathrm{i}\omega b_{12})(\mathrm{i}\omega b_{13}+\omega^2\rho_{33})\\
&\quad+\omega^2\rho_{11}(\omega^2\rho_{22}-\mathrm{i}\omega b_{12})(\omega^2\rho_{33}+\mathrm{i}\omega b_{13}).
\end{aligned}
\tag{21b}
$$

The explicit expressions of $\alpha_i (i=1,2,3,4)$ are not listed here for the complexity. It should be noticed that the coefficients α_i and β_i are functions of the angular frequency ω.

From Eqs. (21a, b), we can get the solution of p and s as follows:

$$
p_{(i)}^2=k^2-k_{di}^2 \quad (i=1,2,3),
\tag{22a}
$$

$$
s^2=k^2-k_s^2,
\tag{22b}
$$

where $k_{di} (i=1,2,3)$ and k_s are wavenumbers of the compressional and shear waves in double porosity media, which can be expressed as

$$
k_{d1}^2=\frac{1}{6\alpha_1}\left[-2\alpha_2+\frac{\gamma_1}{\gamma_2}+\sqrt[3]{4}\gamma_2\right],
\tag{23a}
$$

$$k_{d2}^2 = \frac{1}{12\alpha_1}\left[-4\alpha_2 - \frac{(\sqrt{3}\mathrm{i}+1)\gamma_1}{\gamma_2} + \sqrt[3]{4}(\sqrt{3}\mathrm{i}-1)\gamma_2\right], \tag{23b}$$

$$k_{d3}^2 = \frac{1}{12\alpha_1}\left[4\alpha_2 + \frac{(1-\sqrt{3}\mathrm{i})\gamma_1}{\gamma_2} + \sqrt[3]{4}(\sqrt{3}\mathrm{i}+1)\gamma_2\right], \tag{23c}$$

$$k_s^2 = -\frac{\beta_2}{\beta_1}, \tag{23d}$$

where

$$\gamma_1 = 2\sqrt[3]{2}\left(\alpha_2^2 - 3\alpha_1\alpha_2\right), \tag{24a}$$

$$\gamma_2 = \Big(-2\alpha_2^2 + 9\alpha_1\alpha_2\alpha_3 - 27\alpha_1^2\alpha_4 + \sqrt{-4\left(\alpha_2^2 - 3\alpha_1\alpha_3\right)^3 + \left(2\alpha_2^2 - 9\alpha_1\alpha_2\alpha_3 + 27\alpha_1^2\alpha_4\right)^2}\Big)^{1/3}. \tag{24b}$$

It is clear that when the dissipation is ignored, i.e., the viscosity of the fluid η equals zero, and $\rho_{12} = \rho_{13} = \rho_{23} = 0$, $\rho_{11} = \rho_{22} = \rho_{33} = \rho_s$, where ρ_s is the density of the elastic medium, Eq. (23d) is reduced to

$$c_s^2 = \frac{G}{\rho_s} \tag{25}$$

which is just the shear wave speed in elastic medium.

Assume $p_{(i)}(i = 1,2,3)$ be the three roots of Eq. (21a) with positive imaginary part. If $\Delta_r^{(n)}$ are normalized solutions of Eq. (17), the general solutions of Eq. (17) must be of the form

$$\begin{bmatrix} A_s^{(n)} & A_{f1}^{(n)} & A_{f2}^{(n)} \end{bmatrix} = \begin{bmatrix} \Delta_1^{(n)} & \Delta_2^{(n)} & \Delta_3^{(n)} \end{bmatrix} K_p^{(n)} \xi_p^{(n)}, \tag{26}$$

where $\xi_p^{(n)}(n = 1,2,3)$ are arbitrary constants,

$$K_p^{(n)} = \left[\sum_{i=1}^{3} \left|\Delta_i^{(n)}\right|^2\right]^{-1/2},$$

$$\Delta_1^{(n)} = -\begin{bmatrix} m_{13} & m_{12} \\ m_{23} & m_{22} \end{bmatrix}, \quad \Delta_2^{(n)} = -\begin{bmatrix} m_{11} & m_{13} \\ m_{21} & m_{23} \end{bmatrix}, \quad \Delta_3^{(n)} = \begin{bmatrix} m_{11} & m_{13} \\ m_{21} & m_{22} \end{bmatrix}.$$

Similarly, assume s be the root of Eq. (21b) with a positive imaginary part. If ∇_r are normalized solutions of Eq. (19), the general solutions of Eq. (19) must be of the form

$$\begin{bmatrix} B_s & B_{f1} & B_{f2} \end{bmatrix} = \begin{bmatrix} \nabla_1 & \nabla_2 & \nabla_3 \end{bmatrix} K_s \xi_s, \tag{27}$$

where ξ_s is a arbitrary constant,

$$K_s = \left[\sum_{i=1}^{3} |\nabla_i|^2\right]^{-1/2},$$

$$\nabla_1 = -\begin{bmatrix} n_{13} & n_{12} \\ n_{23} & n_{22} \end{bmatrix}, \nabla_2 = -\begin{bmatrix} n_{11} & n_{13} \\ n_{21} & n_{23} \end{bmatrix}, \nabla_3 = \begin{bmatrix} n_{11} & n_{13} \\ n_{21} & n_{22} \end{bmatrix}.$$

According Eqs. (26) and (27), the potential functions in Eqs. (14) and (15) can be expressed as

$$\begin{bmatrix} \varphi_s & \varphi_{f1} & \varphi_{f2} \end{bmatrix} = \sum_{n=1}^{3} \begin{bmatrix} \Delta_1^{(n)} & \Delta_2^{(n)} & \Delta_3^{(n)} \end{bmatrix} K_p^{(n)} \xi_p^{(n)} \mathrm{e}^{-p_{(n)}z} \mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{28a}$$

$$\begin{bmatrix} \boldsymbol{\psi}_s & \boldsymbol{\psi}_{f1} & \boldsymbol{\psi}_{f2} \end{bmatrix} = \begin{bmatrix} \nabla_1 & \nabla_2 & \nabla_3 \end{bmatrix} K_s \xi_s e^{-sz} e^{i(kx-\omega t)}, \tag{28b}$$

The free surface is supposed to be pervious, i.e.

$$\sigma_z = 0,\ \tau_{xz} = 0,\ p^{(1)} = p^{(2)} = 0 \quad \text{at } z = 0. \tag{29}$$

Substitution of Eqs. (16a–d) into Eq (29) and considering Eqs. (28a, b) leads to

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} \begin{bmatrix} \xi_p^{(1)} \\ \xi_p^{(2)} \\ \xi_p^{(3)} \\ \xi_s \end{bmatrix} = [R] \begin{bmatrix} \xi_p^{(1)} \\ \xi_p^{(2)} \\ \xi_p^{(3)} \\ \xi_s \end{bmatrix} = 0. \tag{30}$$

The explicit expressions for the elements of $[R]$ are given in Appendix B.

The condition for the nontriviality of solutions for Eq. (30) is the determinant of matrix $[R]$ must be equal to zero

$$\text{Det}[R] = \begin{vmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{vmatrix} = 0. \tag{31}$$

Separating the real and imaginary parts of Det$[R]$ and k, Eq. (31) yields

$$\text{Re}\,(\text{Det}[R]) = 0, \tag{32a}$$

$$\text{Im}\,(\text{Det}[R]) = 0. \tag{32b}$$

Considering the numerical methods used in Refs. [8,9], a root-finding technique used to solve k is given as follows. For a certain frequency, the wave numbers of four bulk waves in double porosity media, i.e., $k_{di}(i = 1, 2, 3)$ and k_s, are determinate numbers. First we set the imaginary part of the complex wavenumber of Rayleigh wave to be zero, i.e., $\text{Im}(k) = 0$, then from Eq. (32a) the real part of the complex wavenumber of Rayleigh waves, i.e., $\text{Re}(k)$, can be solved. Secondly, by substituting the value of $\text{Re}(k)$ into Eq. (32b), $\text{Im}(k)$ can also be solved. Thirdly, by substituting the value of $\text{Im}(k)$ into Eq. (32a), the new value of $\text{Re}(k)$ can be solved. Repeat the above procedure until the difference between the values of $\text{Re}(k)$(or $\text{Im}(k)$) obtained by Eqs. (32a) and (b) is within the expected error. In our calculations the error is taken to be

$$\frac{\left|\text{Re}(k_n) - \text{Re}(k_{n-1})\right|}{\left|\text{Re}(k_{n-1})\right|} < 0.01.$$

In the above iteration steps, the interval dichotomy is used to determine the new iterative values of $\text{Re}(k)$ and $\text{Im}(k)$.

It is noted that for an elastic medium, the coefficients ρ_{12}, ρ_{13}, ρ_{22}, ρ_{23}, ρ_{33}, b_{12} and b_{13}, $\phi^{(1)}$ and $v^{(2)}$ are equal to zero, and $\rho_{11} = \rho_s$, where ρ_s is the density of the elastic medium. Accordingly there exists only one appropriate root p for Eq. (21a). Then through lengthy calculation and simplification, Eq. (31) can be reduced to

$$4\left[1 - (c_R/\tilde{c}_d)^2\right]^{1/2}\left[1 - (c_R/\tilde{c}_s)^2\right]^{1/2} - \left[2 - (c_R/\tilde{c}_s)^2\right]^2 = 0. \tag{33}$$

In the above expression, $\tilde{c}_d$ and $\tilde{c}_s$ are phase velocities of the compressional and shear waves in elastic medium, respectively. Eq. (33) is just the well-known Rayleigh wave equation in elastic half-space. Therefore the classical Rayleigh waves in elastic half-space are a particular case of the Rayleigh waves in double porosity media.

Let ρ_{13}, ρ_{23}, ρ_{33}, b_{13} and $v^{(2)}$ be zero, and $\rho_{11} = \rho_s$, $\rho_{22} = \rho_f$, where ρ_s and ρ_f are the densities of the solid and fluid, respectively, we can also derive the corresponding Rayleigh wave equation in a single porosity media from Eq. (31) which is not given here for its complexity.

Table 1
Material properties [19]

Parameter	Double porosity media	Elastic media
K (GPa)	1.052	
K_s (GPa)	36.2	
$K^{(1)}$ (GPa)	8	
$K^{(2)}$	0.00775	
α	0.971	
$\alpha^{(1)}$	0.78	
$\alpha^{(2)}$	1.0	
ρ_s (kg/m^3)	3000	
ρ_f (kg/m^3)	1000	
η (Pa s)	0.001	
$\phi^{(1)}$	0.064	
$v^{(1)}$	0.9936	
$v^{(2)}$	0.0064	
$B^{(1)}$	0.847	
$B^{(2)}$	0.998	
$k^{(11)}$ (m^2)	10^{-16}	
$k^{(22)}$ (m^2)	10^{-12}	
λ (GPa)		4.5
G (GPa)		3.0
ρ (kg/m^3)		2000

It should be noticed that the imaginary part of k must be positive to insure wave decay along the X-axis direction. The phase velocity (measured in m/s) and attenuation (measured in dB/Hz s) are given, respectively, by

$$\upsilon = \frac{\omega}{|\mathrm{Re}(k)|}, \tag{34a}$$

$$\delta = 2\pi \times 8.686 \times \mathrm{Im}(k)/|\mathrm{Re}(k)|. \tag{34b}$$

4. Numerical calculations and discussion

In this section, we will use the formulas derived above to compute the phase velocity and attenuation of Rayleigh waves in double porosity half-space. The double porosity medium is chosen to be Berea sandstone. The physical parameters are taken from Ref. [19] and listed in Table 1. With the parameters listed in Table 1, the phase velocities of the four bulk waves (three kinds of compressional waves and one shear wave) in a double porosity infinite medium are calculated numerically in order to be compared with the Rayleigh wave speed in a double porosity half-space. The relations of phase velocities of the bulk waves with the wave frequency are demonstrated in Fig. 2(a)–(d).

It is seen that in our discussed cases the phase velocity of the fast compressional wave (P1) is about 920 m/s and slightly varies with the wave frequency. The phase velocity of the slower compressional (P2) wave is significantly dependent on frequency and its value varies approximately from 0 to 250 m/s. The phase velocity of the slowest compressional wave (P3) is very low and varies approximately from 0 to 3.5 m/s. The phase velocity of the shear wave is about 525 m/s and it almost does not change with frequency.

4.1. Effects of the permeability on the Rayleigh wave speed and attenuation

The effects of the fracture permeability on the Rayleigh wave speed and attenuation are studied first. The fracture permeability $k^{(22)}$ is taken to be 10^{-10}, 10^{-12} and 10^{-16} m^2, respectively, and other material properties remain constants as shown in Table 1. The calculation results are shown in Fig. 3(a) and (b). It is shown in

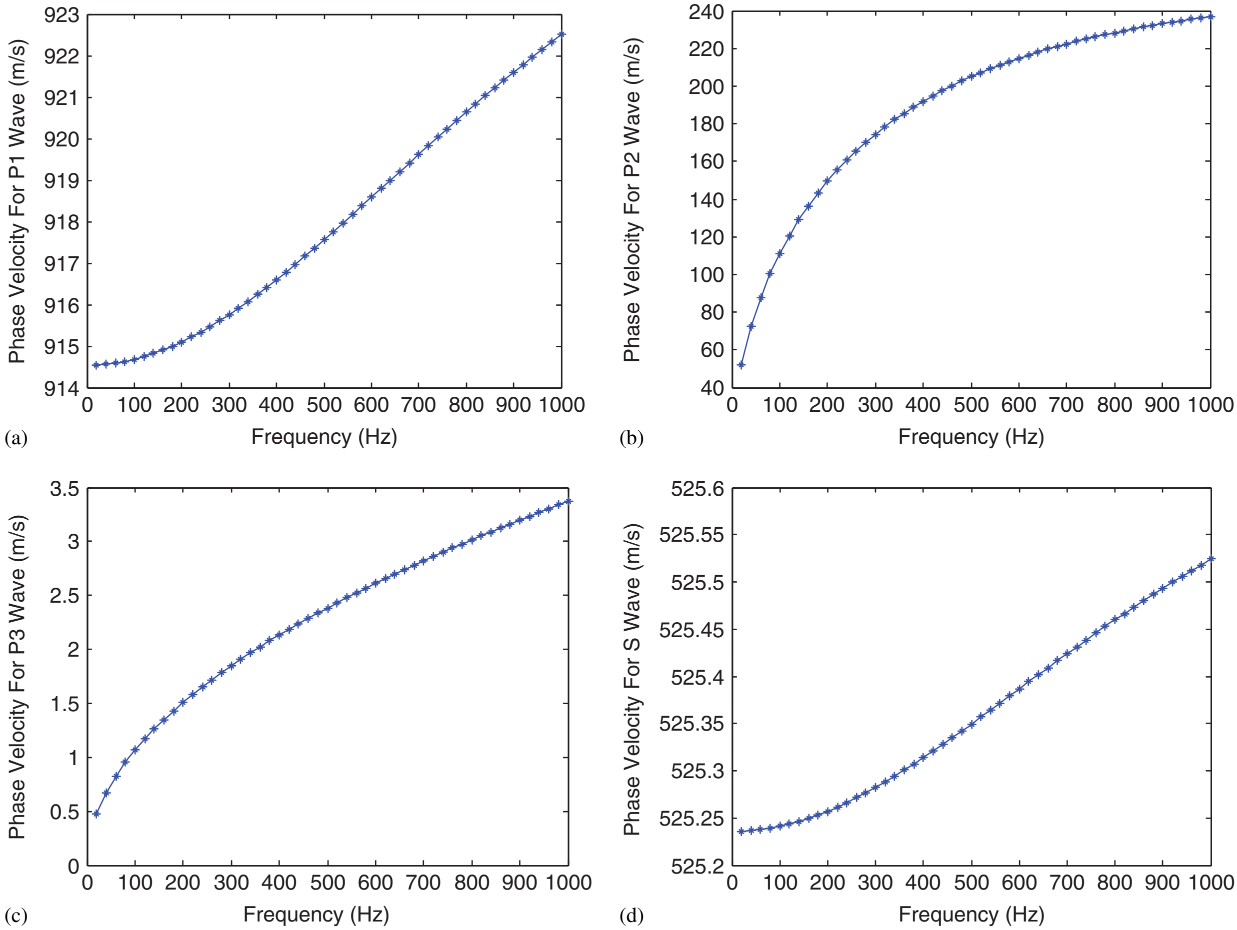

Fig. 2. (a) Variation of the phase velocity of P1 wave speed with frequency *f*. The material constants of the double porosity medium are given in Table 1. (b) Variation of the phase velocity of P2 wave speed with frequency *f*. The material constants of the double porosity medium are given in Table 1. (c) Variation of the phase velocity of P3 wave speed with frequency *f*. The material constants of the double porosity medium are given in Table 1. (d) Variation of the phase velocity of shear wave with frequency *f*. The material constants of the double porosity medium are given in Table 1.

Fig. 3(a) that unlike the classical Rayleigh wave in an elastic solid half-space, the Rayleigh wave speed in a double porosity medium is dispersive at all frequencies considered. It seems that at very high frequency (>2000 Hz, say) the Rayleigh wave speed may approach a limit value. The higher the frequency is, the higher the Rayleigh wave speed is. Comparing Fig. 3(a) with Fig. 2, it is found that the Rayleigh wave speed in double porosity media is always less than the phase velocities of the fast compressinal (P1 wave) and the shear wave. The Rayleigh wave speed is always lager than the phase velocities of the third compressional waves (P3 wave). At the low frequencies (less than several hundreds Hz), the Rayleigh wave speed may be less than the phase velocity of the slower compressional wave (P2 wave), but at the high frequencies (larger than 1000 Hz, say), the Rayleigh wave speed is larger than the phase velocity of the slower compressional wave (P2 wave).

The Rayleigh wave speed is dependent of the fracture permeability greatly. At the same frequency, the higher the fracture permeability coefficient is, the higher the Rayleigh wave speed is. And as the frequency increases, the difference is more evident.

It is illustrated in Fig. 3(b) that Rayleigh waves in double porosity media are slightly damped. In the discussed case the attenuation of Rayleigh waves varies approximately from 0 to 0.12. Since Rayleigh wave is a superposition of the compressional and shear waves near the surface, and the attenuations of three compressional (P1–P3) waves and one shear wave increase with the frequency, the attenuation of Rayleigh waves along the direction *X*-axis (the direction of propagation) also increases with a rise in frequency. At the

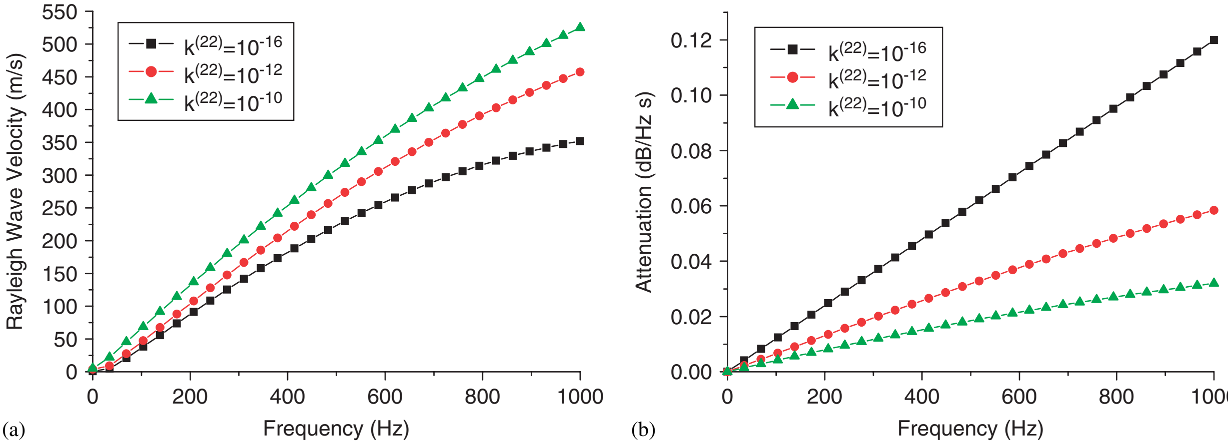

Fig. 3. (a) Variation of Rayleigh wave speed in a double porosity half-space. with frequency f at different fracture permeability ($k^{(22)}$). The material constants of the double porosity medium are given in Table 1. (b) Variation of attenuation of Rayleigh wave in a double porosity half-space with frequency f at different fracture permeability ($k^{(22)}$). The material constants of the double porosity medium are the same as in (a).

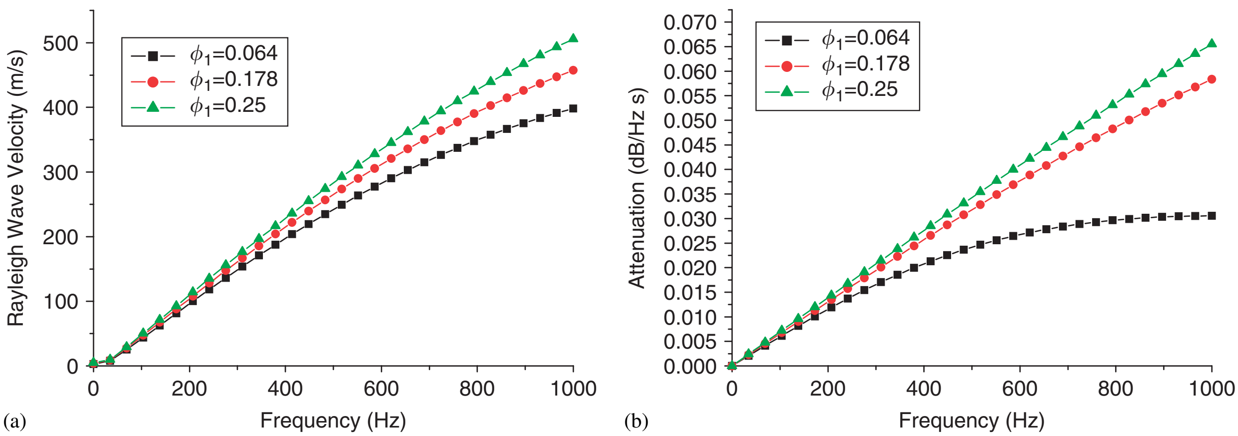

Fig. 4. (a) Variation of Rayleigh wave speed in a double porosity half-space with frequency f at different matrix porosities. The material constants of the double porosity medium are given in Table 1. (b) Variation of attenuation of Rayleigh wave in a double porosity half-space with frequency f at different porosities. The material constants of the double porosity medium are the same as in (a).

same frequency, the higher the fracture permeability is, the lower the attenuation coefficient is, since the permeability coefficient characterizes the dissipation between the solid skeleton and the liquid. As the frequency increases, the difference is also more evident.

Since the fluid in double porosity media stores mainly in the matrix pores, it is instructive to investigate the effect of the matrix permeability on the propagation of Rayleigh waves. The matrix permeability $k^{(11)}$ is taken to be 10^{-14}, 10^{-15} and $10^{-16}\,m^2$, respectively, and the other material properties remain constants as shown in Table 1. The calculation results reveal that the matrix permeability almost has no effect on the Rayleigh wave speed and attenuation. It can be explained as follows. Although the fluid in a double porosity medium stores mainly in the matrix pores, the fluid transports mainly through the fractures. Therefore its influence on the Rayleigh wave speed and attenuation is also small.

4.2. Effects of the matrix porosity on the Rayleigh wave speed and attenuation

In order to investigate the effect of $\phi^{(1)}$ on the Rayleigh wave speed and attenuation, the matrix porosity $\phi^{(1)}$ is taken to be 0.064, 0.178 and 0.25, respectively. The matrix permeability and the fracture permeability are taken to be 10^{-16} and $10^{-12}\,\mathrm{m}^2$, respectively, and the other material properties remain constant as shown in Table 1. Calculation results are shown in Figs. 4(a) and (b). It is shown in Fig. 4(a) that the Rayleigh wave speed depends on the matrix porosity. At the same frequency, the larger the matrix porosity is, the higher the Rayleigh wave speed is. The effect of porosity on the Rayleigh wave speed is more evident as the frequency increases. Fig. 4(b) indicates that the effect of the matrix porosity on the attenuation of Rayleigh waves is also evident. For the higher porosity coefficient, the attenuation coefficient is also higher, and this effect is more evident as the frequency increases. It can be explained that since more fluids flow in the matrix pores for the higher porosity $\phi^{(1)}$, although the permeability coefficient of these pores is very small, the dissipation between the solid and the liquid is also higher for the higher matrix porosity.

5. Summary and conclusions

In this paper, a theoretical analysis has been developed for the propagation of Rayleigh waves in double porosity media. The characteristic equation of Rayleigh waves with the permeable surface is derived. Through appropriate simplification, it can be degenerated to the Rayleigh wave equation in elastic solids and in single porosity media. It is found that Rayleigh waves in double porosity medium are dispersive and damped in directions of both X-axis (direction of propagation) and Z-axis. The attenuation along the direction of propagation is very small, so Rayleigh waves in double porosity medium can propagate over a long distance. By comparison, it is found that the speed of Rayleigh waves in double porosity media is always less than that of the first compressional (P1) and shear waves, whereas it is always larger than that of the third compressional (P3) wave. At low frequencies the speed of Rayleigh waves is larger than that of the second compressional (P2) wave, while at high frequencies larger than that of P2 wave. Numerical calculations reveal that the effects of the porosity and fracture permeability coefficients on the Rayleigh wave speed and attenuation coefficient are evident. For the higher fracture permeability the Rayleigh wave speed is also higher, while the attenuation coefficient is lower. And for the larger matrix porosity, the Rayleigh wave speed and the attenuation coefficient are both higher.

Acknowledgements

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Appendix A

The explicit expressions for ρ_{ij} and b_{ij} are given as follows:

$$
\begin{aligned}
\rho_{11} &= (1-\phi)\rho_s + (\tau-1)\phi\rho_f,\\
\rho_{12} &= \frac{\left[(\tau^{(2)}-1)v^{(2)} - (\tau^{(1)}-1)v^{(1)}\phi^{(1)} - (\tau-1)\phi\right]\rho_f}{2},\\
\rho_{13} &= \frac{\left[(\tau^{(1)}-1)v^{(1)}\phi^{(1)} - (\tau^{(2)}-1)v^{(2)} - (\tau-1)\phi\right]\rho_f}{2},\\
\rho_{22} &= \tau^{(1)}v^{(1)}\phi^{(1)}\rho_f,\\
\rho_{23} &= \frac{\left[(\tau-1)\phi - (\tau^{(1)}-1)v^{(1)}\phi^{(1)} - (\tau^{(2)}-1)v^{(2)}\right]\rho_f}{2},\\
\rho_{33} &= \tau^{(2)}v^{(2)}\rho_f, \quad b_{12} = \frac{\eta v^{(1)}\phi^{(1)}\left[v^{(1)}\phi^{(1)}k^{(22)} - v^{(2)}k^{(21)}\right]\rho_f}{k^{(11)}k^{(22)} - k^{(12)}k^{(21)}},
\end{aligned}
$$

$$b_{13} = \frac{\eta v^{(2)}\left[v^{(2)}k^{(11)} - v^{(1)}\phi^{(1)}k^{(21)}\right]\rho_f}{k^{(11)}k^{(22)} - k^{(12)}k^{(21)}},$$
$$b_{23} = \frac{\eta v^{(1)}v^{(2)}\phi^{(1)}k^{(12)}}{k^{(11)}k^{(22)} - k^{(12)}k^{(21)}}, \tag{A.1}$$

where $k^{(11)}$, $k^{(12)}$, $k^{(21)}$ and $k^{(22)}$ are permeability coefficients, τ, $\tau^{(1)}$ and $\tau^{(2)}$ are overall, matrix and fracture tortuosity. τ is connected with the shape factor r and porosity ϕ by the formula $\tau = 1 + r\big((1/\phi) - 1\big)$. $\tau^{(1)}$ and $\tau^{(2)}$ have the similar relations considering that ϕ is substituted by $\phi^{(1)}$ and $\phi^{(2)}$ ($\phi^{(2)} = 1$), respectively [17]. In particular, r is equal to 0.5 for spherical grains. η is shear viscosity of the fluid.

Appendix B

The explicit expressions for the elements of $[R]$ are given as follows:

$$r_{11} = \left\{\left(K_u + \frac{G}{3}\right)K_p^{(1)}\Delta_1^{(1)} - K_u K_p^{(1)}\left[B^{(1)}v^{(1)}\phi^{(1)}(\Delta_2^{(1)} - \Delta_1^{(1)}) + B^{(2)}v^{(2)}(\Delta_3^{(1)} - \Delta_1^{(1)})\right]\right\}(p_{(1)}^2 - k^2) + GK_p^{(1)}\Delta_1^{(1)}p_{(1)}^2$$
$$r_{12} = \left\{\left(K_u + \frac{G}{3}\right)K_p^{(2)}\Delta_1^{(2)} - K_u K_p^{(2)}\left[B^{(1)}v^{(1)}\phi^{(1)}(\Delta_2^{(2)} - \Delta_1^{(2)}) + B^{(2)}v^{(2)}(\Delta_3^{(2)} - \Delta_1^{(2)})\right]\right\}(p_{(2)}^2 - k^2) + GK_p^{(2)}\Delta_1^{(2)}p_{(2)}^2,$$
$$r_{13} = \left\{\left(K_u + \frac{G}{3}\right)K_p^{(3)}\Delta_1^{(3)} - K_u K_p^{(3)}\left[B^{(1)}v^{(1)}\phi^{(1)}(\Delta_2^{(3)} - \Delta_1^{(3)}) + B^{(2)}v^{(2)}(\Delta_3^{(3)} - \Delta_1^{(3)})\right]\right\}(p_{(3)}^2 - k^2) + GK_p^{(3)}\Delta_1^{(3)}p_{(3)}^2,$$

$$r_{14} = -G\nabla_1 k_s k\sqrt{k^2 - k_s^2}\,\mathrm{i}, \quad r_{21} = -2GK_p^{(1)}\Delta_1^{(1)}k\sqrt{k^2 - k_{d1}^2}\,\mathrm{i},$$
$$r_{22} = -2GK_p^{(2)}\Delta_1^{(2)}k\sqrt{k^2 - k_{d2}^2}\,\mathrm{i}, \quad r_{23} = -2GK_p^{(3)}\Delta_1^{(3)}k\sqrt{k^2 - k_{d3}^2}\,\mathrm{i}$$
$$r_{24} = -Gk_s\nabla_1\left(k^2 + k_s^2\right), \tag{B.1}$$

$$r_{31} = \left[A_{21}K_p^{(1)}\Delta_1^{(1)} + A_{22}v^{(1)}\phi^{(1)}K_p^{(1)}(\Delta_2^{(1)} - \Delta_1^{(1)}) + A_{23}v^{(2)}K_p^{(1)}(\Delta_3^{(1)} - \Delta_1^{(1)})\right](p_{(1)}^2 - k^2),$$
$$r_{32} = \left[A_{21}K_p^{(2)}\Delta_1^{(2)} + A_{22}v^{(1)}\phi^{(1)}K_p^{(2)}(\Delta_2^{(2)} - \Delta_1^{(2)}) + A_{23}v^{(2)}K_p^{(2)}(\Delta_3^{(2)} - \Delta_1^{(2)})\right](p_{(2)}^2 - k^2),$$
$$r_{33} = \left[A_{21}K_p^{(3)}\Delta_1^{(3)} + A_{22}v^{(1)}\phi^{(1)}K_p^{(3)}(\Delta_2^{(3)} - \Delta_1^{(3)}) + A_{23}v^{(2)}K_p^{(3)}(\Delta_3^{(3)} - \Delta_1^{(3)})\right](p_{(3)}^2 - k^2),$$
$$r_{34} = 0,$$
$$r_{41} = \left[A_{31}K_p^{(1)}\Delta_1^{(1)} + A_{32}v^{(1)}\phi^{(1)}K_p^{(1)}(\Delta_2^{(1)} - \Delta_1^{(1)}) + A_{33}v^{(2)}(\Delta_3^{(1)} - \Delta_1^{(1)})\right](p_{(1)}^2 - k^2),$$

$$r_{42} = \Big[A_{31}K_p^{(2)}\Delta_1^{(2)} + A_{32}v^{(1)}\phi^{(1)}K_p^{(2)}(\Delta_2^{(2)} - \Delta_1^{(2)}) + A_{33}v^{(2)}(\Delta_3^{(2)} - \Delta_1^{(2)})\Big](p_{(2)}^2 - k^2)$$

$$r_{43} = \Big[A_{31}K_p^{(3)}\Delta_1^{(3)} + A_{32}v^{(1)}\phi^{(1)}K_p^{(3)}(\Delta_2^{(3)} - \Delta_1^{(3)}) + A_{33}v^{(2)}(\Delta_3^{(3)} - \Delta_1^{(3)})\Big](p_{(3)}^2 - k^2),$$

$$r_{44} = 0.$$

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