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A prediction method of the acoustical properties of multilayered noise control materials in standing wave-duct systems

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Abstract

A new experimental approach, herein referred to as hybrid multilayer prediction, for evaluating acoustical properties of multilayered treatments of noise control materials, such as the absorption ratio and transmission loss, is presented. The two-cavity and two-load methods (TLMs) were performed in a special standing wave duct with two configurations of two- and four-microphone holders. By referring to theoretical expressions and standard approaches, such as the standing wave ratio method from the literature, The validity of these two methods for measuring the transfer matrix was investigated, and some empirical conditions of using limits for the two-cavity and TLMs, based on great amounts of experimental data, were put forth. Based on the total four-pole transfer matrices calculated by combining the two-cavity method and the TLM, some prediction examples for a set of multilayered material treatments were conducted. The prediction results suggest that the newly proposed hybrid prediction method is feasible and effective and that it can be used directly to predict the acoustical properties of an exceedingly thick sample or a multilayer treatment consisting of variable materials. In view of engineering applications, the method may be used for optimizing the in situ designs of multilayered material systems or other noise-control configurations, such as automotive mufflers.

1. Introduction

Increased public concern regarding noise pollution has yielded the introduction of multiple sound-absorbing or -isolating materials for noise and vibration control in vehicles and industrial applications. These materials, which include glass wool, polymeric fibrous materials, and various types of foams, alone or with viscoelastic materials, may be found in automotive linings, in seats, under carpets, in cavity interiors, etc. In application, sound-absorbing or -isolating materials are treated as multilayered configurations in order to optimize acoustical characteristics or to meet the demands of structural design. Engineering practice has shown that an appropriately designed multilayered treatment of materials can give good noise control results. Accordingly, there has been interest in the measurement of the acoustical properties of these multilayered systems, from the study of the noise control impact of each layer in the system to design phase predictions of

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the acoustical properties of the whole multilayered system. Absorption ratio and transmission loss are the most important acoustical properties in engineering, and they have been widely studied for the past few decades. The properties, associated with the characteristic impedance and propagation constant, represent the sound reflection and isolation capability of a single- or multilayered treatment of materials, respectively. There has been a great deal of relevant research on basic theory, standing wave duct design and experimental procedures.

Theoretically, based on some assumptions, acoustical material can be modeled in three ways: i.e., rigid, elastic and limp [1,2], depending on whether the bulk Young's modulus of the material's solid phase is greater, smaller, or of the same order as the fluid Bulk modulus. Based on these models, the characteristics of sound propagation through single or multilayered treatments of porous materials have been predicted with theoretical calculations [3–5] and numerical simulations such as FEM and BEM [6,7]. Delany and Bazley [8] stated that complex wave propagation and characteristic impedance could typically be expressed in terms of flow resistance, wavenumber, air density and sound frequency. This empirical expression was later confirmed through a large amount of experimental data by Qunli [9]. These prediction methods are somewhat difficult to use in practice, because they need the mechanical and acoustical parameters of all of the layers involved in the non-acoustical experiments, such as flow resistance and structure factors. Therefore, it is essential that a more feasible and effective method for predicting multilayer material system design be found.

In this research, we attempted to predict the acoustical properties of multilayered configurations through experimental methods that were based on standing wave-duct systems. Thus, background theories related to the experimental field are briefly described here. Scott [10] described a straightforward method for evaluating the propagation constant and characteristic impedance of a porous acoustical material. Seybert and Ross [11] investigated a two-microphone random excitation technique, wherein the sound absorption coefficient was directly calculated by the measured transfer functions. This technique has been accepted by some standards, for instance, the ASTM E1050. The four-microphone method has been preferred for measuring transmission loss, and examples of its agreement between theoretical simulations and experimental results have been widely documented [7,12]. The two- and four-microphone methods use the conventional standing wave-ratio method (SWRM) in their calculation procedures.

The four-pole transfer matrix approach to acoustical property prediction has been frequently mentioned in Refs. [12–14] for the prediction of flow acoustical systems, such as noise control partitions, automotive mufflers, etc. Yaniv [15] achieved the two-cavity method (TCM) of obtaining the elements in a transfer matrix, whereby tests are performed twice on the same material sample backed by a rigid wall and by a one-quarter wavelength air cavity terminated by a rigid wall, respectively. Ulsuno et al. [16] combined the two-microphone configuration with the TCM to broaden the measuring range to include every frequency of interest. This method has been used to calculate the transfer matrix for practical applications in engineering, although it may occasionally be inaccurate when the sample material is highly dissipative. For the prediction of transmission loss in engineering applications, the two-load method (TLM) has been adopted more often than its ameliorated version, the two source-location method [17], because the latter requires that the location of the sound source be changed during measurement.

In this paper, the experimental methods that may be used in standing wave-duct systems for measuring or predicting the absorption ratio and transmission loss are classified as: the SWRM, the TCM, and the TLM. The three methods are cross-listed with their microphone requirements in Table 1. As previously mentioned, SWRM is a standard method for measuring the absorption ratio and transmission loss of a sample, regardless of whether it is a single- or multilayer material or treatment, respectively. However, due to the small specimens and limited sound power used in standing wave-duct systems, the direct measurement of the absorption ratio and especially transmission loss of some exceedingly thick samples, and most multilayered material treatments become impractical. As supplements, some multilayer prediction methods are needed. TCM and TLM approaches involving transfer matrix calculations may be candidate methods for prediction purposes. However, their application conditions, which have no clear definition in the existing literature, need to be further studied. The popularity of using multilayered material configurations in modern noise control designs is increasing. Therefore, a method for predicting these configurations and optimizing their technique parameters, such as the density, thickness of each layer, and the mode of combination, is necessary. This paper aims to meet these demands by reviewing the practicability of the aforementioned methods for predicting the

Table 1
Candidate measurement methods for standing wave duct systems

Method/ microphone	Standing wave ratio method (SWRM)	Two-cavity method (TCM)	Two-load method (TLM)
2 microphones	Absorption ratio, acoustic impedance	Absorption ratio, acoustic impedance, characteristic impedance, propagation constant, transmission loss	Absorption ratio, transmission loss
4 microphones	Transmission loss	×	Absorption ratio, transmission loss

× means no corresponding method.

acoustical properties of multilayer noise control materials and by proposing a new concept for experimental hybrid multilayer prediction (EHMP). The proposed method can be used directly in the design stage to predict or optimize noise control configurations. Some of the conclusions contained herein may benefit the development of acoustical experimental equipment.

2. Basic theory

Sound in an appropriately designed standing wave duct may be always assumed as being stationary plane waves with zero mean flow speed propagating in air. If a coordinate system is chosen so that this plane wave propagates along the x-axis, the complex acoustic pressure $p(x,t)$ and the associated particle velocity $v(x,t)$ of the medium are

$$p(x,t) = A\mathrm{e}^{j(\omega t-kx)} + B\mathrm{e}^{j(\omega t+kx)}, \tag{1}$$

$$v(x,t) = \frac{1}{Z_0}[A\mathrm{e}^{j(\omega t-kx)} - B\mathrm{e}^{j(\omega t+kx)}], \tag{2}$$

where t is the time, A and B are the amplitudes of the incident and reflected waves, ω the angular frequency, k the wavenumber ($k=\omega/c$), Z_0 the air characteristic impedance at 20 °C ($Z_0=\rho c$), and ρ and c the air density and sound speed in the air, respectively. In standing wave ducts, the transfer matrix $[T]$ can be used to relate the sound pressures and normal particle velocities on the two surfaces of a material sample, or a treatment of multilayer materials, i.e.

$$\begin{Bmatrix} p_u \\ v_u \end{Bmatrix} = [T]\begin{Bmatrix} p_d \\ v_d \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{Bmatrix} p_d \\ v_d \end{Bmatrix}, \tag{3}$$

where p_u and v_u are the sound pressure and normal particle velocity of the medium in the upstream side, p_d and v_d are the sound pressure and normal particle velocity of the medium in the downstream side, and T_{11}, T_{12}, T_{21} and T_{22} are the elements in the transfer matrix. The four-pole transfer matrix conception is the basis for the predictions in this paper.

2.1. Two-load method

This method, as its name indicates, makes use of two loads in the measuring procedure. Two loads, the anechoic and rigid terminations, have been noted as having significant benefits in Refs. [17,18], and they are therefore selected for use in this research. Fig. 1 is the apparatus collocation for the TLM approach; only the anechoic termination case (a) is shown. Alternatively, the second load, here called case (b), may be obtained by replacing the anechoic termination shown in Fig. 1 with a rigid termination similar to that shown in Fig. A.1 in Appendix A. During the measuring procedure, one can use the four-microphone test one time (the four-microphone TLM) or the two-microphone test two times (the two-microphone TLM) for each load.

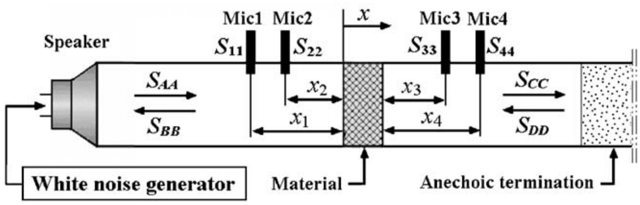

Fig. 1. Experimental setup for evaluating the acoustical properties of a material sample in standing wave ducts with the two-load method (TLM in case (a)).

Theoretically, there is no difference between the two methods when employed under a stationary sound source.

For an isentropic plane sound wave propagation across an acoustic element, the complex pressure amplitudes of the positively and negatively traveling waves, i.e., p_u^+ and p_u^- in the upstream section and p_d^+ and p_d^- in the downstream section, can be related by a pressure transfer matrix $[t]$,

$$\begin{Bmatrix} p_u^+ \\ p_u^- \end{Bmatrix} = [t]\begin{Bmatrix} p_d^+ \\ p_d^- \end{Bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}\begin{Bmatrix} p_d^+ \\ p_d^- \end{Bmatrix}. \tag{4}$$

After some derivations, the elements in the transfer matrix $[T]$, which represent a material sample in a standing wave duct, may be expressed as

$$\begin{Bmatrix} T_{11} \\ T_{12} \\ T_{21} \\ T_{22} \end{Bmatrix} = 2\begin{bmatrix} 1 & \frac{1}{Z_{dc}} & Z_{uc} & \frac{Z_{uc}}{Z_{dc}} \\ 1 & -\frac{1}{Z_{dc}} & Z_{uc} & -\frac{Z_{uc}}{Z_{dc}} \\ 1 & \frac{1}{Z_{dc}} & -Z_{uc} & -\frac{Z_{uc}}{Z_{dc}} \\ 1 & -\frac{1}{Z_{dc}} & -Z_{uc} & \frac{Z_{uc}}{Z_{dc}} \end{bmatrix}^{-1}\begin{Bmatrix} t_{11} \\ t_{12} \\ t_{21} \\ t_{22} \end{Bmatrix}, \tag{5}$$

where Z_{uc} and Z_{dc} denote the medium characteristic impedances in the upstream and downstream sections in the standing wave duct, $Z_{uc} = Z_{dc} = Z_0$.

To calculate t_{11}, t_{12}, t_{21} and t_{22}, we defined the incident transmission coefficient T_{in}, the reflected transmission coefficient T_{re} and the reflection coefficient R as

$$T_{in} = \frac{p_u^+}{p_d^+}, \quad T_{re} = \frac{p_u^-}{p_d^-}, \quad R = \frac{p_d^-}{p_d^+}. \tag{6}$$

Based on the measured data from the two cases (a) and (b), T_{in}, T_{re} and R can be carried out easily, thus, t_{11}, t_{12}, t_{21} and t_{22} are [18]

$$t_{11} = \frac{R_b T_{ina} - R_a T_{inb}}{R_b - R_a}, \quad t_{12} = \frac{T_{inb} - T_{ina}}{R_b - R_a}, \tag{7a,b}$$

$$t_{21} = R_a R_b \frac{T_{rea} - T_{reb}}{R_b - R_a}, \quad t_{22} = \frac{R_b T_{reb} - R_a T_{rea}}{R_b - R_a}, \tag{7c,d}$$

where the subscripts a and b represent the cases (a) and (b), respectively. The transfer matrix $[T]$ may be determined by substituting T_{in}, T_{re} and R into Eq. (5).

2.2. Two-cavity method

The TCM, though originally designed for calculating the propagation constant and the characteristic impedance of a single-layer noise control material, has generally been used to determine a material's transfer matrix. The measurement apparatuses for TCM are shown in Fig. 2.

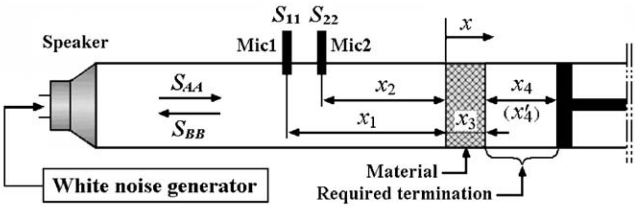

Fig. 2. Experimental setup for evaluation of the absorptive characteristics of a material sample in standing wave duct with the two-cavity method (TCM).

By measuring the sound pressures in the upstream section of the acoustic duct with two microphones, the characteristic impedance Z_c of a sample material, as a complex value, may be described as [16,19]

$$Z_c = \pm\sqrt{\frac{Z_u Z'_u(Z_d - Z'_d) - Z_d Z'_d(Z_u - Z'_u)}{(Z_d - Z'_d) - (Z_u - Z'_u)}}, \tag{8}$$

where Z_u or Z'_u is the acoustic impedance of the sample with thickness x_3 backed by an air layer with depth x_4 or x'_4, which is sandwiched between the sample and a movable piston. As viewed from the front surface of the sample, Z_d or Z'_d is the acoustic impedance of a closed tube with air depth x_4 or x'_4. Here x_4 and x'_4, the depths of the air layers, should be set to different values (nonzero) for the two iterations of the test. In terms of x_4 or x'_4, Z_u or Z'_u can be calculated by

$$Z_u = jZ_0 \frac{-H \sin kx_2 + \sin kx_1}{H \cos kx_2 + \cos kx_1}, \tag{9}$$

$$Z_d = -jZ_0 \cot(kx_4), \tag{10}$$

where H is the transfer function between microphones 1 and 2, and k, Z_0, x_1 and x_2 have the same meanings as in Eqs. (1)–(3).

The propagation constant of the sample material is also a complex value, and may be expressed as

$$\gamma = \alpha_m + j\beta_m = \frac{1}{2x_3}\ln\left(\frac{Z_u + Z_c}{Z_u - Z_c}\frac{Z_d - Z_c}{Z_d + Z_c}\right), \tag{11}$$

where γ is the propagation constant of the sample material with a real part α_m (attenuation constant) and an imaginary part β_m (phase constant). Furthermore, the transfer matrix representing this homogeneous and isotropic material sample can be computed as [1,2]:

$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos k_m x_3 & j Z_c \sin k_m x_3 \\ j \sin k_m x_3 / Z_c & \cos k_m x_3 \end{bmatrix}, \tag{12}$$

where k_m is the wavenumber in the acoustic material, $k_m = \gamma/j$. Therefore, the absorption ratio, as well as the transmission loss, may be easily calculated from this transfer matrix, and expressions will be given in the following text.

2.3. Prediction of the absorption ratio and transmission loss of multilayered treatments

For multilayered treatments, after obtaining the transfer matrix of each layer by performing TCM or TLM, a total transfer matrix describing the complete system can then be expressed by multiplying each transfer matrix of the n layers sequentially, i.e.

$$[T] = [T_1][T_2][T_3]\cdots[T_n]. \tag{13}$$

Using the four elements T_{11}, T_{12}, T_{21} and T_{22} of the total transfer matrix, one can calculate the absorption ratio and the transmission loss directly as

$$\alpha = 1 - \left|\frac{T_{11} - Z_{\mathrm{uc}} T_{21}}{T_{11} + Z_{\mathrm{uc}} T_{21}}\right|^2, \tag{14}$$

$$\mathrm{TL} = 20\log_{10}\left[\frac{1}{2}\sqrt{\frac{Z_{\mathrm{uc}}}{Z_{\mathrm{dc}}}}\left|T_{11} + \frac{T_{12}}{Z_{\mathrm{dc}}} + Z_{\mathrm{uc}} T_{21} + \frac{Z_{\mathrm{uc}}}{Z_{\mathrm{dc}}} T_{22}\right|\right], \tag{15}$$

where α is the absorption ratio of a material sample backed by a rigid wall and TL is the transmission loss.

The total process presented above, from the establishment of the transfer matrix to the absorption ratio and transmission loss calculation, has been partially discussed in the literature [10,20,21]. This paper, in particular, discusses the prediction of acoustical properties of multilayer materials in real applications by combining TCM and TLM using the standing wave duct.

3. Experimental procedure

Several types of experimental apparatuses were needed in this research. A typical experimental setup was used for the four-microphone measurements shown in Fig. 3; other measurements were easily made by changing the sample holder and required termination, depending on the measurement method used in the experiment (see Figs. 1 and 2). As seen in the standing wave duct system shown in Fig. 3, a loudspeaker driven by a wave file in a digital computer at the left end was used to generate broadband white noise as the sound source over the frequency range of 500–6000 Hz. The test sample was mounted in the sample holder. In contrast to previous works [1,13], the FFT analysis process together with the aforementioned procedure for calculating the acoustic properties was performed through computer programming rather than by FFT analyzers.

The materials tested in this present application were urethane foam, glass wool, polyethylene terephthalate, and rubber; the related properties of these materials are listed in Table 2. The flow resistances in MKS were measured by following the method established in the literature [19]. Using a hydraulic cutter, each cylindrical material sample was carefully cut and inserted into the sample holder to ensure a snug fit without much

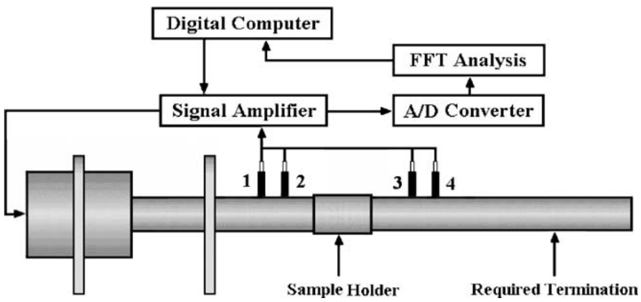

Fig. 3. Experimental apparatus (Acoustic Duct of the SCIEN Co.) for a four-microphone measurement.

Table 2
Material properties of the tested samples

Sample symbol	Material ingredient	Thickness (mm)	Density (g/m^2)	Flow resistance (ralys/m)
UF	Urethane foam	30	540	2.742×10^3
PET1	Poly ethylene terephthalate	50	1700	5.251×10^3
PET2	Poly ethylene terephthalate	50	2600	9.516×10^3
GW	Glass wool	10	1400	5.405×10^4
RUB	Rubber	1.5	3600	$+\infty$

deformation. To reduce the effect of material variability and sample mounting errors, a dozen samples of each material were tested, and their averaged results were reported. In the case of prediction, the acoustic properties of the multiple layers of a material and combined layers of different materials are also discussed below.

4. Discussion and prediction

4.1. Experimental method determination

In order to obtain exact multilayer predictions of absorption ratios and transmission losses (see Eqs. (13)–(15)), the accuracy of the transfer matrix of each layer in the multilayer system was first examined. In this paper, the measured absorption ratios and transmission losses from TCM (or TLM) were compared with those from standard measurement methods and also from the theoretical predictions from Delany's and Bazley's semi-empirical formulae [8]. Figs. 4–6 show the absorption ratios and transmission losses of the PET2 material from Table 2. As can be seen, both the absorption ratios and the transmission losses from the two-microphone TCM are very close to their theoretical results. The two- and four-microphone SWRMs, which are described in

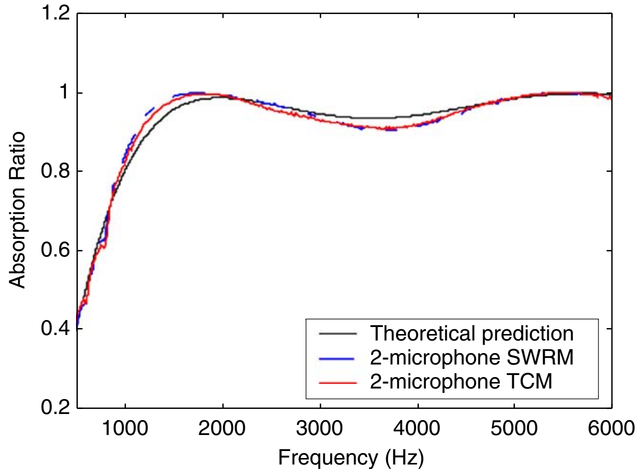

Fig. 4. Comparison of the absorption ratio of the PET2 material calculated from each discussed method.

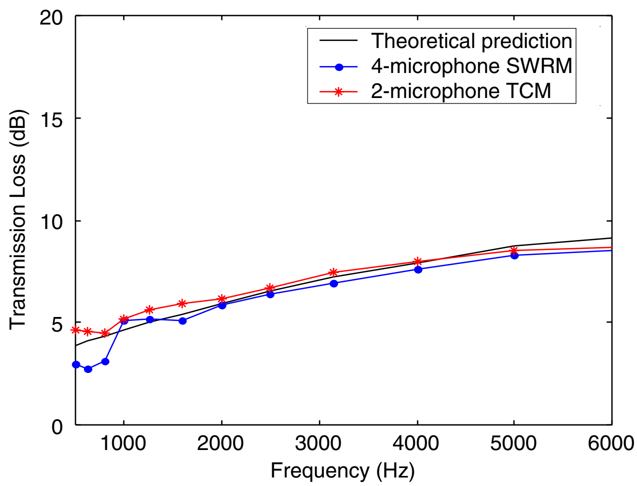

Fig. 5. Comparison of transmission losses of the PET2 material calculated from theoretical prediction, four-microphone SWRM, and two-microphone TCM.

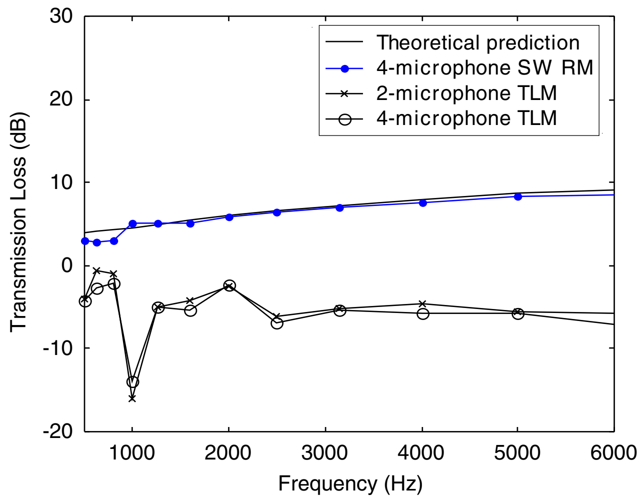

Fig. 6. Comparison of transmission losses of the PET2 material calculated from theoretical prediction, four-microphone SWRM, and two TLMs.

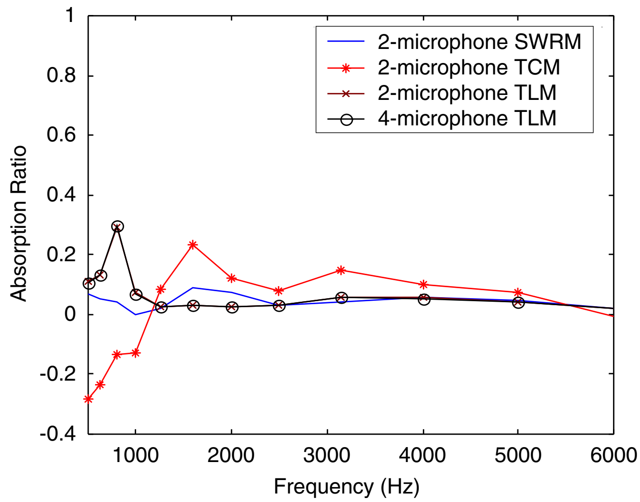

Fig. 7. Comparison of the absorption ratio of the RUB material calculated from each discussed method.

Appendix A, obtained an approximate absorption ratio and an acceptable transmission loss, respectively, whereas the absorption ratios and the transmission losses from the two TLMs were significantly different from the theoretical predictions. Note that the two TLMs results are nearly equal and that their absorption ratios, which seem divergent, cannot be shown on the same plot in Fig. 4. Generally speaking, for the PET2 material, the TCM can be regarded as the best method. The two- and four-microphone SWRM approaches, in the sample standing wave-duct system, have enough accuracy and dependability to be used as standard methods for providing reference values for the absorption ratio and the transmission loss in the preceding sections.

Figs. 7 and 8 show the tested results of each method using the RUB sample from Table 2. In contrast to the results shown in Figs. 4–6, the TCM results shown in Figs. 7 and 8 deviate significantly from one another. However, the TLMs, especially the four-microphone TLM, offer results that are much closer to those measured by the two- and four-microphone SWRMs, except for some differences in the low frequency range. In Fig. 7 both the two- and four-microphone TLMs gave more accurate absorption ratios, which are in contrast with the conclusion from Fig. 4. Comparing the transmission loss curves in Figs. 5 and 6 with those in Fig. 8, a conclusion can be obtained that the two-microphone TCM is suitable for the PET2 material but the

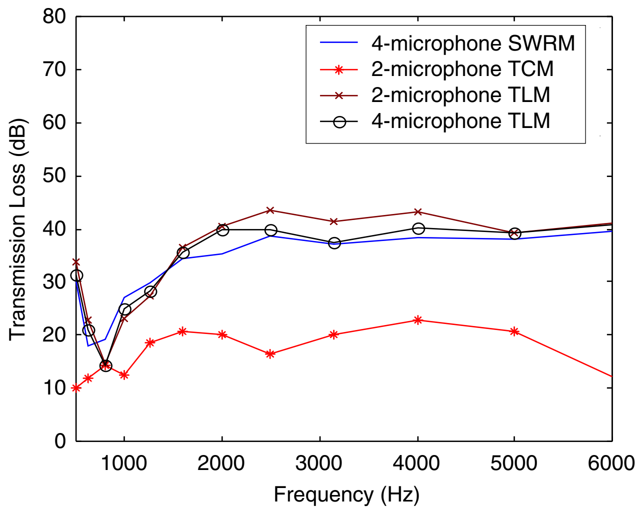

Fig. 8. Comparison of the transmission loss of the RUB material calculated from each discussed method.

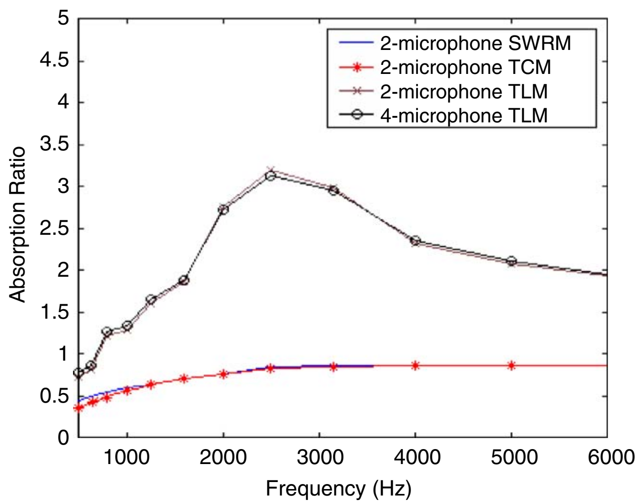

Fig. 9. Comparison of the absorption ratio of a three-layered treatment of GW calculated from each discussed method.

four-microphone TLM for transmission loss measurement of the RUB. The measured absorption ratio and transmission loss of the treatment with three-layered GW samples from Table 2, which may also be regarded as a single GW sample with a thickness of 30 mm, are shown in Figs. 9 and 10. The absorption ratio and transmission loss were obtained by applying the TCM or TLM to each layer and substituting the calculated combined transfer matrix in Eq. (13) into Eqs. (14) and (15). Aside from some deviations in the frequencies <1300 Hz, TCM and TLM are adequate methods for evaluating the absorption ratio and transmission loss, respectively. Here, a trend may be assumed that one can select a reasonable study method to measure the absorption and transmission loss properties, according to the flow resistance of a material, see Table 2.

As seen from the experiments described above, in terms of accuracy, it is not easy to determine whether the TCM or TLM is more suitable for measuring or further predicting absorption ratios or transmission losses. To implement multilayer predictions in standing wave-duct systems, some problems encountered here need to be investigated further.

From Figs. 4 to 10, it can be seen that as the values of flow resistance increased, the available method for evaluating the absorption ratio and the transmission loss changed from TCM to TLM. For convenience in

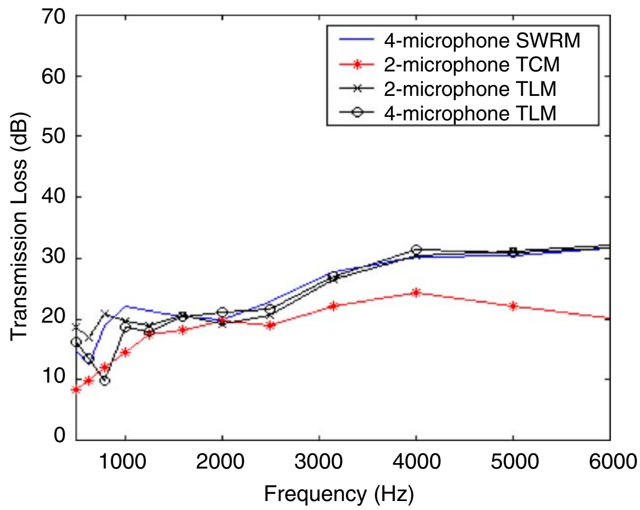

Fig. 10. Comparison of the transmission loss of a three-layered treatment of GW calculated from each discussed method.

application, the empirical conditions dictating the use of limitation were investigated here. It was found that only the absorption ratio measurement by TCM satisfied Delany's and Bazley's frequency interval, where $0.01 \leqslant f\rho/R \leqslant 1.0$; i.e. if the effective frequency range of a standing wave duct is $[f_1, f_2]$, the measurable materials should have flow resistance values as $R = [\rho f_2, 100\rho f_1]$. In the other cases, we could not find any theoretical algorithm or definition for giving definite limit values in the existing literature for either the absorption ratio or the transmission loss. However, based on the abundance of experimental data from the standing wave duct presented in this paper, we induced the empirical conditions of using limitation, referring to the flow resistivity range from 10^3–10^6 ralys/m of typical noise control porous materials used in engineering [19]. That is, for TCM,

$$\begin{aligned} &R \in [f_2\rho, 100 f_1\rho]\ \text{ralys/m} \quad &&\text{for AR}, \\ &R \in (0, 1 \times 10^4]\ \text{ralys/m} &&\text{for TL} \end{aligned} \tag{16a,b}$$

and for TLM,

$$\begin{aligned} &R \geqslant 10^6\ \text{ralys/m} &&\text{for AR}, \\ &R > 4 \times 10^4\ \text{ralys/m} \quad &&\text{for TL}, \end{aligned} \tag{17a,b}$$

where R, AR and TL are the flow resistance, absorption ratio, and transmission loss of the tested material, respectively. For the frequency range $[f_1, f_2] = [500, 6000]$ Hz, the flow resistances in Eq. (16a) become $R = [7200, 60\,000]$ ralys/m. This can be used to explain the low-frequency errors in Figs. 9 and 10, in which the total flow resistance of the three layers of GW $R = 1.622 \times 10^5$ ralys/m led to an invalid lower frequency limit value of 1351.7 Hz. Unfortunately, there are some "dead" ranges if one connects Eq. (16) with Eq. (17); the reasons for this are not entirely clear, and may serve as a useful research topic in the future.

4.2. Establishment of the element transfer matrix database

The acoustical properties of multilayered treatments of materials used in applications, such as automotive liners, generally cannot be directly measured by a one-time test in a standing wave duct. Therefore, we have to consider predicting them by the four-pole transfer matrix method. This method requires performing TCM or TLM for measuring the transfer matrix of each layer and calculating the total transfer matrix of the whole multilayered material by Eq. (13). The final predicted absorption ratios and transmission losses of the multilayered treatments strongly depend on the measured transfer matrix of each layer. Hence, before applying these methods, their limit conditions from Eqs. (16) and (17) must be taken into account.

Accordingly, we measured all the transfer matrices of the material samples from Table 2 as the elements were ready for further multilayer prediction. We saved the measurements into a set of digital data files, referred to as the element transfer matrix database (ETMD) in this paper. The elements in the ETMD were used in the preceding multilayer predictions instead of the real material samples.

It should be noted that, for an in situ sample with either single- or multilayered materials, if its thickness is in the measurement range of the standing wave-duct system and the sound in the downstream section is loud enough, the two- and four-microphone SWRMs are undoubtedly the first selections for measuring the absorption ratio and the transmission loss, respectively. However, the SWRMs cannot be used for prediction purposes if there is no transfer matrix generated in their derivations (see Sections A.1 and A.2 in Appendix A.

4.3. Hybrid multilayer prediction

As mentioned, the flow resistance of each layer of the multilayered treatments used in engineering applications often lies in different ranges in Eqs. (16) and (17). For more precise predictions, the TCM and TLM and some plane wave-based theoretical methods were combined and applied in standing wave duct systems. In this paper, this procedure is referred to as the EHMP.

To verify the accuracy of this newly presented method, the absorption ratio and the transmission loss of the assumed two-layered treatment consisting of PET2 and RUB from Table 2, which can be measured directly by the SWRM approaches in the standing wave duct shown in Fig. 3, were predicted using the TCM, TLM and EHMP, respectively (Figs. 11 and 12). Here, the TCM or TLM calculates the total transfer matrix by performing the TCM or TLM to each layer of the treatment. For the EHMP, however, Eq. (18) was conducted in accordance with the empirical limit conditions from Eqs. (16) and (17):

$$[T] = [T_{\mathrm{PET}}]_{\mathrm{TCM}}[T_{\mathrm{RUB}}]_{\mathrm{TLM}}, \tag{18}$$

where $[T_{\mathrm{PET}}]$ and $[T_{\mathrm{RUB}}]$ are the transfer matrices of the PET2 and RUB layers, respectively. From Figs. 11 and 12, compared with the results of the TCM and TLM, the hybrid-predicted absorption ratio and transmission loss agreed well with the measured values. As such, the accuracy of the EHMP approach is adequate for the prediction of the acoustical properties of multilayered configurations. It should be clarified that, for proving the predicted results, the multilayered treatments used in Figs. 11 and 12 were chosen within the allowable measuring limitation of the SWRM approach in the standing wave duct. The element transfer matrices of samples from the ETMD were taken to represent samples from engineering applications. The absorption ratio and transmission loss of several multilayered treatments of materials, including some typical sandwich structures, were investigated using the EHMP, and the results are shown in Figs. 13–15, respectively.

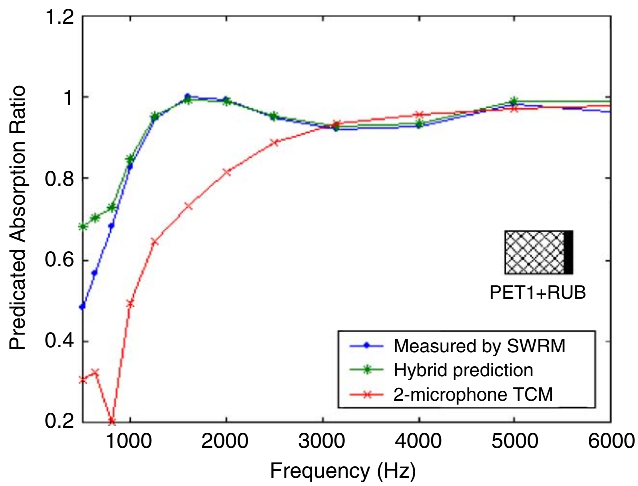

Fig. 11. Comparisons of the measured and predicted absorption ratios of a two-layered treatment of the PET2 and RUB.

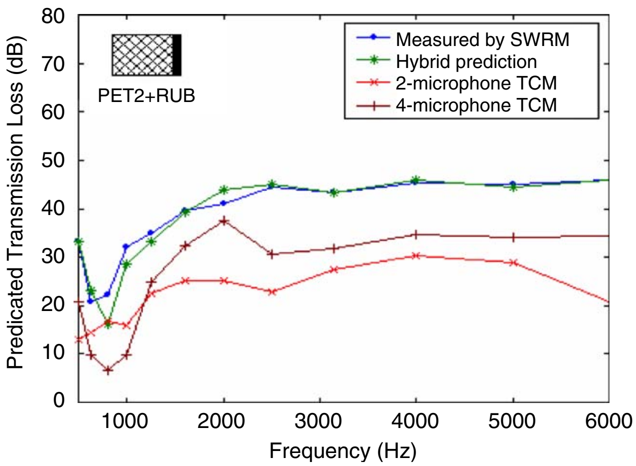

Fig. 12. Comparisons of the measured and predicted transmission losses of a two-layered treatment of the PET2 and RUB.

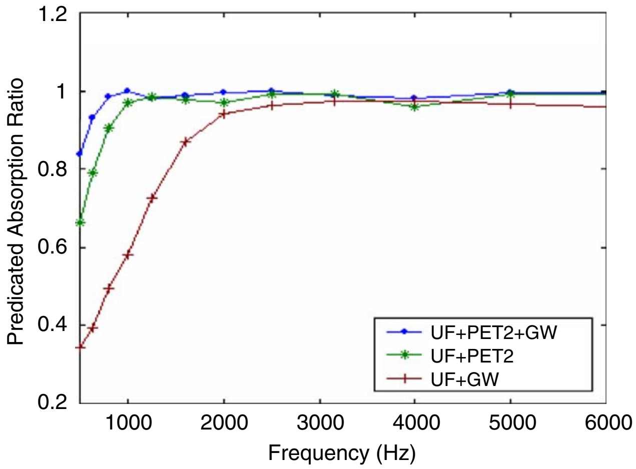

Fig. 13. Hybrid predictions for absorption ratio optimization of some multi-layered treatments of different materials.

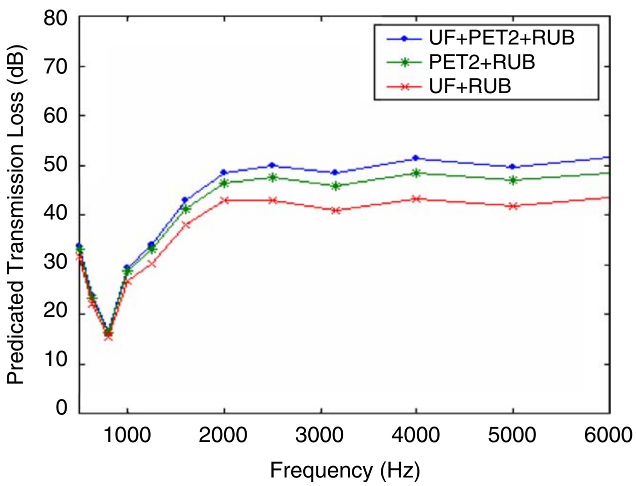

Fig. 14. Hybrid predictions for transmission loss optimization of some multi-layered treatments of different materials.

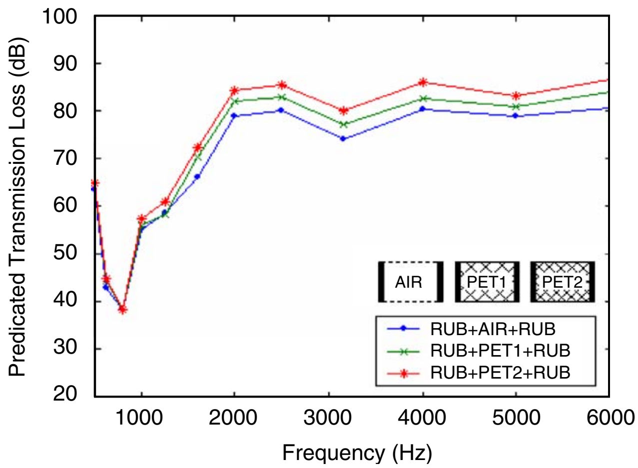

Fig. 15. Hybrid predicted transmission losses of three RUB-based sandwich structures with different area densities of the sandwiched materials.

In terms of the flow resistances of the samples in Table 2, all the absorption ratios in Fig. 13 were computed by applying the TCM approach to each layer, but the transmission losses in Fig. 14 were obtained by integrating the TCM for the UF and PET2 with the four-microphone TLM for the RUB shown in Fig. 13,

$$[T] = \prod_{i=1}^{n}([T_{\mathrm{UPG}}]_{\mathrm{TCM}})_i \tag{19}$$

and in Fig. 14,

$$[T] = \prod_{i=1}^{n-1}([T_{\mathrm{UPG}}]_{\mathrm{TCM}})_i[T_{\mathrm{RUB}}]_{\mathrm{TLM}}, \tag{20}$$

where $[T_{\mathrm{UPG}}]$ denotes the transfer matrix of the UF, PET2, or GW in Table 2, and n is the total number of layers in a treatment. It can be concluded from Fig. 13 that the PET2 has good absorption characteristics from 1000 to 2000 Hz, and the three layers consisting of the UF, PET2, and GW were an almost ideal treatment for absorbing any sound in the frequency range from 1000 to 6000 Hz. Compared to the RUB's transmission loss in Fig. 8, the transmission losses of the multilayered combinations shown in Fig. 14 were mainly dependent on the RUB, and slightly increased above the frequency of 1000 Hz by adding the accessional layers. Fig. 15 shows the hybrid-predicted transmission losses of three commonly seen sandwich structures, which cannot be directly measured in the standing wave-duct system due to the aforementioned limitations. The total transfer matrix of the three layers may be expressed as

$$[T] = [T_{\mathrm{RUB}}]_{\mathrm{TLM}}[T_{\mathrm{sandwich}}]_{\mathrm{TCM}}[T_{\mathrm{RUB}}]_{\mathrm{TLM}}. \tag{21}$$

Here, $[T_{\mathrm{sandwich}}]$ represents the transfer matrix of the sandwiched AIR, PET1 or PET2. The AIR thickness is assumed to be the same as those of the PET1 and PET2, and based on the plane wave propagation equations, its transfer matrix can be expressed in the same form as Eq. (12) by setting $k_m = k$, $Z_c = Z_0$ and $x_3 = 50$ mm (area density: 60.5 g/m^2), where k and Z_0 are the wavenumber and air characteristic impedance, respectively. The results showed that when the size of the sandwich was held constant while the density (actually the flow resistance) of the packing material was increased, the transmission loss was improved (almost parallel in Fig. 15).

Generally, for a multilayered configuration, more attached material layers or higher total flow resistance (only for transmission loss) yielded improved acoustical properties of absorption and isolation. For optimization, replacing certain layers with higher-quality materials or a sub-multilayer configuration, for instance, substituting the UF with the PET2, is a good way to improve the acoustical properties of a multilayered system. Through this method, the absorption ratio in Fig. 13 was greatly improved below 2500 Hz, a nearly 5 dB increase was seen in the transmission loss >1500 Hz in Fig. 14, and a 3–5 dB benefit was realized by changing the PET1 to PET2 in Fig. 15. The property improvements shown in the low-frequency range for the absorption ratio but in the high-frequency range for transmission loss implies that with more material layers or greater area density of the absorptive elements, more noise energy will be dissipated by the multilayered treatments.

The above hybrid prediction procedures also apply to a sample with a thickness that exceeds the measurement range of the standing wave duct, which have to be cut into pieces and treated as a multilayer sample consisting of the same materials. During the tests for the element (each layer) transfer matrices, it was found that TLM was more cumbersome to apply than the TCM because of its complex calibration and measurement procedures. As to the number of microphones in the experiments, the two-microphone configuration was convenient for determining the absorption ratio, but usually cannot provide reasonable estimates of transmission loss, and the four-microphone configuration is the essential choice for determining transmission loss. Due to the need for accuracy and easy operation, this newly presented EHMP could be regarded as a substitute for the present theoretical methods for predicting the acoustical properties of multilayered configurations. Note that the EHMP extends the measurement scope of standing wave-duct systems so much that, if the aforehand ETMD is big and all round enough including transfer matrices of sub-element in noise control configurations, one can evaluate and predict the acoustical properties of any material sample (or configuration) combined by one or more elements in the ETMD, regardless of the element material density, thickness, number of layers, and element types in the configurations. Moreover, the goal of optimizing the absorption and transmission characteristics can be fulfilled.

5. Conclusion

This paper presented an EHMP method for evaluating the acoustical properties of noise control materials using standing wave-duct systems. Firstly, an accuracy comparison among six approaches for absorption ratios and transmission losses, including two SWRMs, one TCM, two TLMs and a theoretical semi-empirical method, were implemented using five types of typical noise control materials. Based on the experimental results, some empirical conditions of using limits for the absorption ratio and transmission loss of the TCM and TLM, with respect to flow resistance values, were discussed and established for standing wave-duct systems. These limit conditions must be considered in element transfer matrix determination for a multilayer system evaluation. According to the flow resistance of the samples in Table 2, an ETMD, herein referred to as the ETMD, was readily built up for further prediction of the multilayered systems.

Finally, instead of measuring the real sample materials, the newly proposed EHMP was performed to evaluate absorption and transmission characteristics of a set of multilayered treatments consisting of different materials by directly taking the corresponding element transfer matrix from the ETMD. It can be concluded that the EHMP method is accurate and credible enough for the prediction of acoustical properties of multilayered noise control configurations. Acoustical property optimizations of a multilayered system may be obtained by adding more layers or by substituting certain layers with higher-quality material treatments. The EHMP extends the measurement scope of standing wave-duct systems and is capable of predicting and optimizing the acoustical properties of any material samples used in absorption and transmission characteristic design.

The work presented in this paper might be extended to study other types of noise control configurations or equipment, such as to measure, predict, or optimize the acoustical performance of automotive mufflers.

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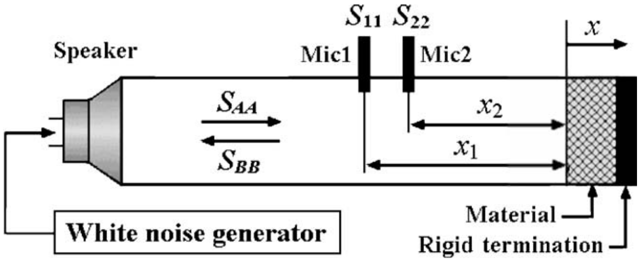

Fig. A.1. Experimental setup for evaluation of the absorptive characteristics of a material sample in a standing wave duct with the two-microphone SWRM method.

Appendix A. Standing wave ratio methods

A.1. Two-microphone method for absorption ratio

The two-microphone method, as the basis for the SWRMs, needs to be briefly described here. Fig. A.1 shows its schematic diagram. During measuring, the material sample is backed by a rigid termination, and the autospectral densities S_{11} and S_{22} of the signals at two microphone locations and their cross-spectral density $S_{12} = C_{12} + jQ_{12}$ are captured and used to calculate the separated auto-spectra S_{AA} and S_{BB} of the incident and reflected sound waves in the upstream section of the duct

$$\begin{bmatrix} S_{AA} \\ S_{BB} \\ C_{AB} \\ Q_{AB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2\cos 2kx_1 & 2\sin 2kx_1 \\ 1 & 1 & 2\cos 2kx_2 & 2\sin 2kx_2 \\ \cos kn & \cos kn & 2\cos km & 2\sin km \\ -\sin kn & \sin kn & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} S_{11} \\ S_{22} \\ C_{12} \\ Q_{12} \end{bmatrix}, \tag{A.1}$$

where $S_{AB} = C_{AB} + \mathrm{j}Q_{AB}$ is the cross-spectrum between the incident and reflected waves, x_1 and x_2 represent the distances from the front surface of the sample to microphones 1 and 2, respectively, and also $n = x_1 - x_2$, $m = x_1 + x_2$.

According to the definition of absorption ratio

$$\alpha = 1 - \frac{S_{BB}}{S_{AA}}. \tag{A.2}$$

A.2. Four-microphone method for transmission loss

Discriminatingly, the four-microphone method discussed here is different from that in the following text because it is also based on the standing wave separation. Therefore, the above Eq. (A.1) can be used to calculate the autospectum S_{AA} of the incident sound, using the measured data of the first pair of microphones in the upstream section (Fig. 1). Similarly, the autospectum S_{CC} in the downstream section may be obtained by imposing the same computations on the second pair of microphones. Note that the S_{DD} should approach zero due to the anechoic termination. The transmission loss TL can be shown to be

$$\mathrm{TL} = 10\log_{10}\left(\frac{S_{AA}}{S_{CC}}\right). \tag{A.3}$$

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