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Short Communication

Efficient methods for determining modal parameters of dynamic structures with large modifications

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Abstract

The paper presents efficient methods for determining the modified modal parameters (natural frequencies and mode shapes) in a structural dynamic modification analysis when structural modifications are relatively large. Based on the developed theory that can provide an exact relationship between the modifications of structural parameters (stiffness and mass) and the associated modal parameters, an efficient iterative computational procedure is proposed for determining the modified eigenvalues and the corresponding eigenvectors for complex structural systems. A high order approximation approach is further presented from the exact relationship and compared with the existing first-order approximation approach and the proposed iterative procedure. From the results for the given numerical example, it is shown that even in the cases with a large modification of structural parameters the proposed iterative procedure can provide exact predictions of the modified modal parameters after only a few iterations, and the high order approximation approach can give excellent estimates. The computation of the modified modal parameters does not require the knowledge of the original or modified structural parameters, and only a limited knowledge of the original modal data may be sufficient in a dynamic reanalysis for complex structures.

1. Introduction

Structural dynamic modification (SDM) techniques are very useful for rapidly analysing and applying the effect of structural changes on the dynamic response of structures. That is, given changes in structural parameters, such as stiffness or mass, SDM techniques can efficiently determine the corresponding changes in modal parameters, such as natural frequencies and mode shapes, without solving the generalised eigenvalue problem for the modified dynamic system. Therefore, SDM techniques, which often incorporate finite-element analysis techniques, can greatly reduce the computational effort and increase the efficiency of reanalysis during a structural optimisation process, especially in the cases where complex mechanical and structural systems are considered.

There are various SDM techniques available for the dynamic reanalyses of a structural system, such as sensitivity analyses based on eigenvalue and eigenvector derivatives [1,2] and Rayleigh quotient iteration [3].

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However, the efficiency of sensitivity analysis methods is limited because these methods are complicated and may be only suitable for small modifications of structural parameters [4]. It may be difficult to determine some high-frequency modes using Rayleigh quotient iteration because the predictions of those modes exceeded the limit bound of the Rayleigh quotient [2].

It should be pointed out that the first-order sensitivity analysis techniques and the truncated Taylor's expansion approximation approaches often used for estimating the modified modal parameters may perform properly when the changes in structural parameters from the initial model to the modified model are small. However, for the cases with relatively large modifications of structural parameters, the first order or the truncated Taylor's expansion approximations may be inaccurate. To avoid these shortcomings, a perturbation theory was developed [5], which can provide an exact relationship between the modifications of structural parameters and the associated modal parameters and can be applied to model updating and inverse structural damage identification [6,7]. Here, based on the developed perturbation theory, an efficient iterative computational procedure is proposed to provide exact predictions of the natural frequencies and the corresponding mode shapes for the modified dynamic system modelled with a large number of degrees of freedom (DOF). A high order approximation approach is also presented without iterative procedures required, which can give excellent estimates of the modified modal parameters. The results of a numerical example show that the iterative procedure presented here converges quickly for evaluating the modified modal parameters and finally gives the exact solution even when structural modifications are large. The proposed computational techniques successfully avoid adopting Taylor series expansion procedure and then the derivatives of modal parameters are not needed. Only limited information on the analytical or experimental modal data of the original structure is required in calculations. Therefore, the proposed methods are well suited for complex structures with a large number of DOFs, especially for the cases where the knowledge of the modified structural parameters is not available and the modified modal data cannot be obtained by solving the generalised eigenvalue problem.

2. Theory

2.1. Basic equations for SDM analyses

Assume that modal parameters λ_i and $\boldsymbol{\phi}_i$ are the ith eigenvalue and the corresponding mass normalised eigenvector of the original dynamic system with structural parameters of the global stiffness matrix $\mathbf{K}$ and the global mass matrix $\mathbf{M}$, where i ranges from 1 to N and N represents the total number of DOFs for the system. Then, the ith eigenvalue λ_i^* and the corresponding eigenvector $\boldsymbol{\phi}_i^*$ for the modified system can be given by

$$\lambda_i^* = \lambda_i + \Delta\lambda_i, \tag{1}$$

$$\boldsymbol{\phi}_i^* = \boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i, \tag{2}$$

where $\Delta\lambda_i$ and $\Delta\boldsymbol{\phi}_i$ represent the modifications of ith eigenvalue and corresponding eigenvector, which are caused by the modifications of stiffness matrix $\Delta\mathbf{K}$ and/or mass matrix $\Delta\mathbf{M}$. Suppose that the modified eigenvector $\boldsymbol{\phi}_i^*$ is normalised with respect to the original mass matrix in the form

$$\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{M}\boldsymbol{\phi}_i^* = 1. \tag{3}$$

Here the value of $\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{M}\boldsymbol{\phi}_i^*$ is assumed to be non-zero. Consequently, the modification of eigenvector $\Delta\boldsymbol{\phi}_i$ can be expressed as a linear combination of the independent original eigenvectors except the corresponding original one since the original stiffness and mass matrices are assumed to be symmetric [5], and is rewritten here as

$$\Delta\boldsymbol{\phi}_i = \sum_{k=1,k\neq i}^{N} C_{ik}\boldsymbol{\phi}_k\ ,\ \boldsymbol{\phi}_i^* = \boldsymbol{\phi}_i + \sum_{k=1,k\neq i}^{N} C_{ik}\boldsymbol{\phi}_k, \tag{4}$$

where C_{ik} are mode participation factors. It can be shown that Eq. (3) can be satisfied if the modified eigenvectors are computed from Eq. (4) since the original eigenvectors are assumed to be mass normalised as unity. Moreover, the orthogonality conditions of the modified mass matrix $\mathbf{M}^*$ with respect to the modified eigenvector $\boldsymbol{\phi}_i^*$ can be given by

$$\boldsymbol{\phi}_i^{*\mathrm{T}}\mathbf{M}^*\boldsymbol{\phi}_i^* = 1 + \sum_{m\neq i} C_{im}^2 + \sum_m \sum_l C_{im}C_{il}\boldsymbol{\phi}_m^{\mathrm{T}}\Delta\mathbf{M}\boldsymbol{\phi}_l. \tag{5}$$

Note that the obtained modified eigenvectors are usually not mass normalised as unit in this case. However, the mass normalised eigenvectors for the modified structure could be obtained without requiring the knowledge of the modified mass matrix.

Based on the above equations and considering the characteristic equations for the original and modified systems, the general theory for dynamic systems with structural modifications, which can provide an exact relationship between the modifications of structural parameters and the associated modal parameters, was developed and given in the paper by Chen [5]. The developed theory is now utilised for a SDM analysis, and the derived basic equations for determining the modification of eigenvalues $\Delta\lambda_i$ and the mode participation factors C_{ik} are rewritten here, respectively, as

$$\Delta\lambda_i = \frac{\boldsymbol{\phi}_i^{\mathrm{T}}(\Delta\mathbf{K} - \lambda_i\Delta\mathbf{M})(\boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i)}{1 + \boldsymbol{\phi}_i\Delta\mathbf{M}(\boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i)}, \tag{6}$$

$$C_{ik} = \frac{\boldsymbol{\phi}_k^{\mathrm{T}}(\Delta\mathbf{K} - \lambda_i^*\Delta\mathbf{M})\boldsymbol{\phi}_i + \sum_{l=1,l\neq i,k}^{N}\boldsymbol{\phi}_k^{\mathrm{T}}(\Delta\mathbf{K} - \lambda_i^*\Delta\mathbf{M})\boldsymbol{\phi}_l C_{il}}{\lambda_i^* - \lambda_k - \boldsymbol{\phi}_k^{\mathrm{T}}(\Delta\mathbf{K} - \lambda_i^*\Delta\mathbf{M})\boldsymbol{\phi}_k}. \tag{7}$$

Note that the computation of the modified natural frequencies and the corresponding mode shapes through the general equations, Eqs. (6) and (7), does not require the knowledge of the stiffness matrix and mass matrix of the original or modified system, which is very useful for the cases where structural parameters of the original and modified systems are not available. Consequently, the modal parameters of the modified system can be determined, provided that the natural frequencies and mode shapes of the original system obtained either analytically or experimentally, together with the modifications of structural parameters $\Delta\mathbf{K}$ and $\Delta\mathbf{M}$, are known.

2.2. Iterative computational procedure

An iterative procedure for calculating the modification of eigenvalues $\Delta\lambda_i$ and the mode participation factors C_{ik} is required because the two general equations, Eqs. (6) and (7), are coupled. An improved computational procedure is now developed for the cases of complex structures where a large number of DOFs may be present in order to efficiently determine the modal parameters for the modified dynamic system. To simplify the computation process, sensitivity coefficients associated with the original eigenvectors and the modifications of structural parameters, a_{ki}^K and a_{ki}^M, are defined in general forms as

$$a_{ki}^K = \boldsymbol{\phi}_k^{\mathrm{T}}\Delta\mathbf{K}\boldsymbol{\phi}_i, \quad a_{ki}^M = \boldsymbol{\phi}_k^{\mathrm{T}}\Delta\mathbf{M}\boldsymbol{\phi}_i. \tag{8}$$

Note that the sensitivity coefficients a_{ki}^K and a_{ki}^M can be determined in the case where only the elements of the original eigenvectors corresponding to the sites of the modifications of structural parameters are available. Eq. (6) for calculating the modification of eigenvalues, by utilising Eqs. (4) and (8), now can be rewritten as follows:

$$\Delta\lambda_i = \frac{(a_{ii}^K - \lambda_i a_{ii}^M) + \sum_{k=1,k\neq i}^{N}(a_{ki}^K - \lambda_i a_{ki}^M)C_{ik}}{1 + a_{ii}^M + \sum_{k=1,k\neq i}^{N} a_{ki}^M C_{ik}}. \tag{9}$$

Similarly, from Eq. (7) the mode participation factors C_{ik}, which are utilised for calculating the modification of eigenvectors, can be obtained from

$$C_{ik} = \frac{(a_{ki}^K - \lambda_i^* a_{ki}^M) + \sum_{l=1,l\neq i,k}^{N}(a_{kl}^K - \lambda_i^* a_{kl}^M)C_{il}}{(\lambda_i^* - \lambda_k) - (a_{kk}^K - \lambda_i^* a_{kk}^M)}. \tag{10}$$

The preceding formulation forms a basis for an iterative solution procedure. The procedure is initiated by assuming that the initial mode participation factors C_{ik} (where $k\neq i$) are equal to zero, that is $C_{ik}^{(0)} = 0$.

Physically, this implies that the initial modified eigenvalues are obtained from the assumption that the modified eigenvectors are identical to the original ones. A first approximation for the modification of eigenvalues is then calculated from Eq. (9). The next approximation for the mode participation factors, after substituting the currently obtained modified eigenvalue into Eq. (10), is then evaluated. The approximations for the modification of eigenvalues and the mode participation factors can be further improved by repeating the same process as described above. In general, the nth approximation for the modification of eigenvalues $\Delta\lambda_i^{(n)}$, by rewriting Eq. (9), can be expressed as

$$\Delta\lambda_i^{(n)} = \frac{(a_{ii}^K - \lambda_i a_{ii}^M) + \sum_{k=1,k\neq i}^{N}(a_{ki}^K - \lambda_i a_{ki}^M)C_{ik}^{(n-1)}}{1 + a_{ii}^M + \sum_{k=1,k\neq i}^{N} a_{ki}^M C_{ik}^{(n-1)}} \tag{11}$$

and the nth approximation for the mode participation factors $C_{ik}^{(n)}$, by adopting currently obtained $\lambda_i^{*(n)} = \lambda_i + \Delta\lambda_i^{(n)}$ and rewriting Eq. (10), can be evaluated from

$$C_{ik}^{(n)} = \frac{(a_{ki}^K - \lambda_i^{*(n)} a_{ki}^M) + \sum_{l=1,l\neq i}^{k-1}(a_{kl}^K - \lambda_i^{*(n)} a_{kl}^M)C_{il}^{(n)} + \sum_{l=k+1,l\neq i}^{N}(a_{kl}^K - \lambda_i^{*(n)} a_{kl}^M)C_{il}^{(n-1)}}{(\lambda_i^{*(n)} - \lambda_k) - (a_{kk}^K - \lambda_i^{*(n)} a_{kk}^M)}. \tag{12}$$

Note that $C_{il}^{(n)}$, where $l<k$ and k ranges from 1 to N in the numerator of Eq. (12), are already obtained when $C_{ik}^{(n)}$ are being calculated, as shown in Eq. (12), since $C_{ik}^{(n)}$ for the ith modified eigenvector are consecutively computed from $C_{il}^{(n)}$ to $C_{iN}^{(n)}$. Also, in the case when a significant number of DOFs are adopted for modelling the dynamic system, a subset of original eigenvectors, NC, may be utilised to replace the total number of all eigenvectors of the original system, N, in order to avoid the difficulty in computing all eigenvectors of the original system.

The above recursive process for evaluating the approximations for λ_i^* and C_{ik} is repeated until the following convergence criterion, where ε is the convergence tolerance, is satisfied:

$$\delta\lambda_i = \frac{|\Delta\lambda_i^{(n)} - \Delta\lambda_i^{(n-1)}|}{|\lambda_i + \Delta\lambda_i^{(n)}|} \prec \varepsilon. \tag{13}$$

The modified eigenvectors then can be calculated using the obtained C_{ik} from Eqs. (4), and therefore Eq. 3 is satisfied. The pairing of the eigenmodes for the original and the modified structural dynamic systems can be checked using the Modal Assurance Criterion (MAC) factors, defined as follows:

$$\mathrm{MAC}(k,i) = \frac{|\boldsymbol{\phi}_k^{\mathrm{T}}\boldsymbol{\phi}_i^*|^2}{|\boldsymbol{\phi}_k^{\mathrm{T}}\boldsymbol{\phi}_k||\boldsymbol{\phi}_i^{*\mathrm{T}}\boldsymbol{\phi}_i^*|}. \tag{14}$$

The highest MAC(k, i) factors indicate the most likely pairings of the original mode $\boldsymbol{\phi}_k$ and the modified mode $\boldsymbol{\phi}_i^*$.

2.3. High order approximation

Based on the derived basic equations, approximate approaches are proposed for estimating the eigenvalues and the corresponding eigenvectors for the modified dynamic system to avoid iterative computational procedures. First, assume that no change of eigenvectors exists between the modified and original systems, that is, $\Delta\boldsymbol{\phi}_i = 0$. From Eq. (6), the first-order approximation of the modified eigenvalues λ'_i can be obtained:

$$\lambda'_i = \frac{\lambda_i + \boldsymbol{\phi}_i^{\mathrm{T}}\Delta\mathbf{K}\boldsymbol{\phi}_i}{1 + \boldsymbol{\phi}_i^{\mathrm{T}}\Delta\mathbf{M}\boldsymbol{\phi}_i}. \tag{15}$$

Note that the first-order approximation is identical to the Rayleigh quotient approximation to the modified eigenvalues based on the original eigenvectors $\boldsymbol{\phi}_i$, and is also equivalent to the first approximation from Eq. (11), where $n = 1$, in the proposed iterative procedure.

Furthermore, replace λ_i^* in Eq. (7) with the first-order approximation λ'_i given by Eq. (15) and ignore the mode participation factors in the numerator in Eq. (7), i.e. $C_{il} = 0$. Considering Eq. (15), then the estimate of mode participation factors C'_{ik} can be expressed as

$$C'_{ik} = \frac{\boldsymbol{\phi}_k^{\mathrm{T}}(\Delta K - \lambda'_i \Delta \mathbf{M})\boldsymbol{\phi}_i}{(\lambda'_i - \lambda'_k)(1 + \boldsymbol{\phi}_k^{\mathrm{T}} \Delta \mathbf{M} \boldsymbol{\phi}_k)}. \tag{16}$$

Consequently, the estimates for the modification of eigenvectors and modified eigenvectors can be calculated from Eq. (4). Substituting the obtained modification of eigenvectors into Eq. (6), a higher order approximation for the modification of eigenvalues can be given by

$$\Delta\lambda_i = \frac{\boldsymbol{\phi}_i^{\mathrm{T}}(\Delta \mathbf{K} - \lambda_i \Delta \mathbf{M})\boldsymbol{\phi}_i + \sum_{k=1,k\neq i}^{N} C'_{ik} \boldsymbol{\phi}_k^{\mathrm{T}}(\Delta \mathbf{K} - \lambda_i \Delta \mathbf{M})\boldsymbol{\phi}_i}{1 + \boldsymbol{\phi}_i^{\mathrm{T}} \Delta \mathbf{M} \boldsymbol{\phi}_i + \sum_{k=1,k\neq i}^{N} C'_{ik} \boldsymbol{\phi}_k^{\mathrm{T}} \Delta \mathbf{M} \boldsymbol{\phi}_i}. \tag{17}$$

Note that the estimate of mode participation factors C'_{ik} can be directly determined from Eq. (16), provided that the original modal data, λ_i and $\boldsymbol{\phi}_i$, and the modifications of structural parameters, $\Delta\mathbf{K}$ and $\Delta\mathbf{M}$, are known. From the sensitivity coefficients defined in Eq. (8), Eqs. (16) and (17) can be rewritten here as, respectively:

$$C'_{ik} = \frac{a_{ki}^K - \lambda'_i a_{ki}^M}{(\lambda'_i - \lambda'_k)(1 + a_{kk}^M)}, \tag{18}$$

$$\Delta\lambda_i = \frac{(a_{ii}^K - \lambda_i a_{ii}^M) + \sum_{k=1,k\neq i}^{N} C'_{ik}(a_{ki}^K - \lambda_i a_{ki}^M)}{1 + a_{ii}^M + \sum_{k=1,k\neq i}^{N} C'_{ik} a_{ki}^M}. \tag{19}$$

Then, the high order approximations for the modified eigenvalues and the corresponding eigenvectors can be calculated from Eqs. (1) and (4), respectively.

3. Numerical example

A rectangular thin plate model shown in Fig. 1 is utilised to demonstrate the effectiveness of the proposed techniques for calculating the modal data for the modified dynamic system by introducing various levels of structural modifications and comparing different computational approaches. The rectangular aluminium plate is 1000 mm long, 600 mm wide and 10 mm thick, with material properties of Young's modulus $E = 6.89 \times 10^{10}\,\mathrm{N/m^2}$, Poisson's ratio $\upsilon = 0.30$ and density $\rho = 2796\,\mathrm{kg/m^3}$. The thin plate is modelled as a plate bending problem in a free–free boundary condition. A finite-element analysis is performed for the plate model using eight-node isoparametric plate bending elements with three DOFs for each node. The full plate model contains 60 elements, 213 nodes and 639 DOFs, and is reduced to 15 master DOFs by using Guyan reduction [8] as shown in Fig. 1 and marked with (●).

The convergence performance of the proposed iterative procedure for determining the modified modal data is demonstrated in Table 1, where the reduced model of 15 master DOFs is considered. In this case, the thickness of the plate model is modified by an increase of 50% to a thickness of 15 mm over the shaded areas

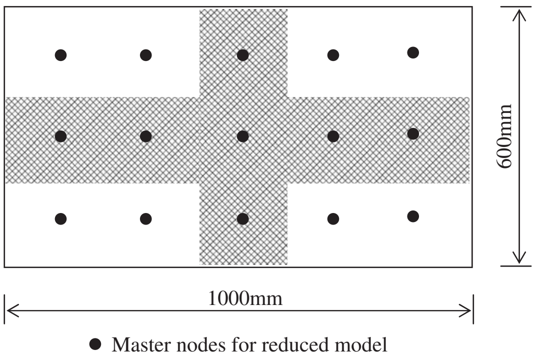

Fig. 1. Thin plate bending model problem.

Table 1
Predicted natural frequencies (Hz) at different iteration numbers, reduced model adopted and thickness increased locally by 50%

Original	Modified (exact)	Exact $\Delta\omega$	First iteration		Second iteration		Sixth iteration		MAC diagonal value
			Predicted ω	Predicted $\Delta\omega$	Predicted ω	Predicted $\Delta\omega$	Predicted ω	Predicted $\Delta\omega$	
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8832
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8836
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9986
51.6344	68.7107	17.0763	71.5041	19.8697	68.9640	17.3296	68.7107	17.0763	0.9950
52.8160	76.5300	23.7139	76.7156	23.8996	76.5299	23.7139	76.5300	23.7139	0.9996
121.314	154.710	33.3958	155.349	34.0344	154.766	33.4520	154.710	33.3958	0.9849
146.979	179.721	32.7425	180.674	33.6956	179.888	32.9091	179.721	32.7425	0.9945
154.525	220.730	66.2054	219.987	65.4620	220.844	66.3189	220.730	66.2054	0.9689
196.020	260.571	64.5510	262.487	66.4669	260.445	64.4247	260.571	64.5510	0.9526
229.281	293.331	64.0498	293.308	64.0267	293.331	64.0498	293.331	64.0498	0.9998
291.120	335.422	44.3020	334.314	43.1935	335.241	44.1208	335.422	44.3020	0.9723
342.325	460.052	117.726	455.787	113.462	459.491	117.166	460.052	117.726	0.9585
431.170	556.723	125.553	540.861	109.691	557.363	126.193	556.723	125.553	0.9573
455.201	577.831	122.630	563.201	108.000	578.303	123.102	577.831	122.630	0.9591
722.647	915.551	192.904	892.587	169.940	914.016	191.369	915.551	192.904	0.9691
$\Sigma\delta\lambda_i$[a]			3.62E−01		3.88E−02		1.07E−06		

[a] $\Sigma\delta\lambda_i$ represents the sum of $\delta\lambda_i$ defined in Eq. (13) over the total number of eigenvalues considered.

shown in Fig. 1. Therefore, changes in both stiffness and mass matrices are present. The natural frequency estimates of all 15 modes for the reduced model, including three rigid body modes, through a succession of iterations are listed in Table 1 along with the exact solution. It can be seen that the proposed iterative procedure achieves convergence after only a few iterations, and the results given at the first iteration are identical to those from the Rayleigh quotient approximation. The parings of the original modes and the modified modes are assured by using MAC values as defined in Eq. (14). The MAC diagonal values given in Table 1 show that the modified modes match the corresponding original modes very well.

Different computational approaches are now utilised to predict the modal data for the modified system. Here, the reduced model of 15 master DOFs is considered and the structural modification with local changes in plate thickness is introduced as given earlier. From the results shown in Table 2, the iterative procedure can give the exact values of the modified natural frequencies and the high order approximation approach can give excellent predictions, even for the case where relatively large structural modifications exist. The values of the modified natural frequencies obtained from the high order approximation approach are much improved, compared with the results obtained using the first-order approximation approach.

The results in Table 3 demonstrate the effectiveness of different computational approaches with respect to various levels of structural modifications. It is assumed that the thin plate model is modified by increasing the thickness over the shaded areas by 10%, 40% and 100%, respectively. Note that some coefficients of the stiffness and mass matrices could be increased by factors of approximately 700% and 100%, respectively, in the case with the thickness increased by 100%. The results indicate that the proposed iterative procedure can give exact predictions for the modified natural frequencies, even in the case where significant structural modifications exist. The high order approximation approach can provide satisfactory estimates for all cases, while the first-order approximation approach can give reasonable estimates only in the case where relatively small structural modifications exist.

In order to investigate the effect of noise in the original modal data on the predictions of the modified natural frequencies, random errors of up to 1% in the eigenvalues of the original system and up to 20% in its eigenvectors are introduced, resulting in the values shown in Table 4. The iterative procedure is utilised and the plate model is modified by an increase of 50% in thickness over the shaded areas. From the results, it can be seen that the proposed technique is insensitive to random errors existing in the original modal data. Therefore, satisfactory predictions of the modified modal parameters could be obtained from the modal data

Table 2
Predicted natural frequencies (Hz) from different computational approaches, reduced model adopted and thickness increased locally by 50%

Modified (exact)	Exact $\Delta\omega^E$	First order[a]			High order[a]			Iterative procedure[a]		
		Predicted ω	Predicted $\Delta\omega^P$	$\delta\omega/\Delta\omega^E$ (%)	Predicted ω	Predicted $\Delta\omega^P$	$\delta\omega/\Delta\omega^E$ (%)	Predicted ω	Predicted $\Delta\omega^P$	$\delta\omega/\Delta\omega^E$ (%)
0.0000	—	0.0000	—	—	0.0000	—	—	0.0000	—	—
0.0000	—	0.0000	—	—	0.0000	—	—	0.0000	—	—
0.0000	—	0.0000	—	—	0.0000	—	—	0.0000	—	—
68.7107	17.0763	71.5041	19.8697	16.4	69.0996	17.4652	2.3	68.7107	17.0763	0.0
76.5300	23.7139	76.7156	23.8996	0.8	76.5299	23.7139	0.0	76.5300	23.7139	0.0
154.710	33.3958	155.349	34.0344	1.9	154.820	33.5050	0.3	154.710	33.3958	0.0
179.721	32.7425	180.674	33.6956	2.9	179.980	33.0010	0.8	179.721	32.7425	0.0
220.730	66.2054	219.987	65.4620	−1.1	220.888	66.3631	0.2	220.730	66.2054	0.0
260.571	64.5510	262.487	66.4669	3.0	260.345	64.3251	−0.3	260.571	64.5510	0.0
293.331	64.0498	293.308	64.0267	0.0	293.331	64.0498	0.0	293.331	64.0498	0.0
335.422	44.3020	334.314	43.1935	−2.5	335.050	43.9297	−0.8	335.422	44.3020	0.0
460.052	117.726	455.787	113.462	−3.6	458.998	116.673	−0.9	460.052	117.726	0.0
556.723	125.553	540.861	109.691	−12.6	558.379	127.209	1.3	556.723	125.553	0.0
577.831	122.630	563.201	108.000	−11.9	578.993	123.792	0.9	577.831	122.630	0.0
915.551	192.904	892.587	169.940	−11.9	913.501	190.855	−1.1	915.551	192.904	0.0

[a] $\delta\omega = \Delta\omega^P - \Delta\omega^E$.

Table 3
Predicted natural frequencies (Hz) for various local thickness modifications, reduced model adopted and different computational approaches utilised

Thickness increased by 10%				Thickness increased by 40%				Thickness increased by 100%			
Modified (exact) ω	Exact/iterative $\Delta\omega$	First order $\Delta\omega$	High order $\Delta\omega$	Modified (exact) ω	Exact/iterative $\Delta\omega$	First order $\Delta\omega$	High order $\Delta\omega$	Modified (exact) ω	Exact/iterative $\Delta\omega$	First-order $\Delta\omega$	High order $\Delta\omega$
0.0000	—	—	—	0.0000	—	—	—	0.0000	—	—	—
0.0000	—	—	—	0.0000	—	—	—	0.0000	—	—	—
0.0000	—	—	—	0.0000	—	—	—	0.0000	—	—	—
55.172	3.538	3.913	3.543	65.333	13.698	18.569	13.928	86.258	34.623	55.974	36.198
57.163	4.347	4.365	4.347	71.436	18.620	18.945	18.620	103.56	50.743	53.303	50.741
128.05	6.730	7.021	6.731	148.07	26.756	30.775	26.819	187.33	66.013	85.597	66.503
152.64	5.660	5.910	5.665	172.27	25.295	29.273	25.462	222.61	75.629	98.792	75.825
169.38	14.859	15.847	14.860	209.17	54.641	68.274	54.734	265.66	111.14	178.11	111.19
211.45	15.425	16.511	15.422	250.30	54.281	69.841	54.149	294.34	98.323	172.25	97.761
241.24	11.962	11.848	11.962	279.65	50.364	48.906	50.364	366.23	136.95	130.53	136.95
300.11	8.986	9.945	8.976	326.78	35.663	50.504	35.387	378.63	87.505	167.12	88.092
364.46	22.136	23.193	22.131	435.65	93.321	109.61	92.808	580.51	238.18	326.38	229.14
455.40	24.230	24.480	24.245	531.27	100.10	104.87	100.98	676.58	245.41	275.57	256.13
479.24	24.036	24.896	24.048	553.13	97.928	112.29	98.564	698.24	243.04	325.99	249.16
759.99	37.342	39.130	37.313	876.92	154.27	185.62	153.04	1092.0	369.31	565.56	364.77

of the original system with a certain level of noise, which may be measured from laboratory or full scale model testing, if the proposed techniques are adopted.

The effectiveness of the proposed iterative procedure for determining the modified modal data with respect to the number of original eigenvectors adopted in calculations is now investigated, as shown in Table 5. A full finite-element analysis model with 639 DOFs is considered, and again it is assumed that structural parameters of the plate model are modified by an increase of 50% in thickness over the shaded areas. The natural frequency estimates of the first 15 modes including three rigid body modes are presented. It is found that only

Table 4
Predicted natural frequencies (Hz) for various random noise levels in original eigenvetors, reduced model adopted and thickness increased locally by 50%

Original (exact)	Modified (exact)	Random noise level in original eigenvetors				
		At 1%	At 2%	At 5%	At 10%	At 20%
0.0000	0.0000	—	—	—	—	—
0.0000	0.0000	—	—	—	—	—
0.0000	0.0000	—	—	—	—	—
51.6344	68.7107	68.7358	68.7648	68.8753	69.1338	69.9373
52.8160	76.5300	76.5085	76.4913	76.4659	76.5144	77.0455
121.314	154.710	154.770	154.831	155.022	155.368	156.152
146.979	179.721	179.711	179.702	179.678	179.653	179.652
154.525	220.730	220.731	220.734	220.759	220.855	221.251
196.020	260.571	260.401	260.234	259.750	259.000	257.712
229.281	293.331	293.333	293.336	293.344	293.364	293.437
291.120	335.422	335.432	335.441	335.462	335.475	335.431
342.325	460.052	460.173	460.288	460.598	460.998	461.403
431.170	556.723	557.083	557.440	558.520	560.401	564.630
455.201	577.831	577.769	577.705	577.494	577.083	576.055
722.647	915.551	915.530	915.506	915.407	915.157	914.340

Table 5
Predicted natural frequencies (Hz) from different number of original eigenvectors adopted, full model adopted and thickness increased locally by 50%

Original	Modified (exact)	Number of original eigenvectors adopted, NC				
		$NC = 30$	$NC = 60$	$NC = 90$	$NC = 120$	MAC diagonal value
0.0000	0.0000	—	—	—	—	0.9963
0.0000	0.0000	—	—	—	—	0.8553
0.0000	0.0000	—	—	—	—	1.0000
51.5320	68.5281	69.0971	68.7339	68.7041	68.6660	0.9932
52.7087	76.2357	76.8165	76.4922	76.4116	76.3907	0.9919
119.990	152.349	153.704	153.004	152.836	152.793	0.9696
143.471	175.263	177.860	175.687	175.548	175.506	0.9904
149.812	209.666	212.831	211.187	211.077	210.713	0.9611
187.990	243.071	247.193	244.753	244.670	244.294	0.9422
218.920	276.604	286.074	278.082	277.322	277.257	0.9684
267.536	307.647	313.012	309.104	308.742	308.601	0.9719
302.254	387.541	392.661	389.374	389.162	388.908	0.9416
361.768	442.721	457.486	445.820	444.650	444.392	0.9514
395.437	484.138	500.217	489.215	486.292	486.085	0.9468
431.549	483.933	533.826	487.899	485.039	484.790	0.8926

a limited knowledge of the original eigenvectors is required. Even a total number of 60 original modes (approximately 10% of all original modes) is sufficient to give good predictions of the modified natural frequencies for the case with relatively large modifications of structural parameters. As expected, excellent results can be obtained when 120 original modes (approximately 20% of all original modes) are adopted in the calculations.

4. Conclusions

An improved iterative procedure is proposed for efficiently determining the eigenvalues and the corresponding eigenvectors for a dynamic system with large modifications of structural parameters and a large number of DOFs present. A high order approximation approach is also presented without iterative procedures involved. From the results for the given thin plate bending model problem, it has been shown that the convergence of the proposed iterative procedure can be achieved rapidly, leading to exact solutions to SDM analyses even in the cases where large structural modifications exist. Only a limited knowledge of original modes is required to provide correct predictions of the modified modal parameters, and the knowledge of the original or modified stiffness and mass matrices may not be needed. The proposed techniques are insensitive to random errors existing in the original modal data, and therefore satisfactory estimates for the modified modal parameters could be obtained from the original modal data that may be measured in laboratory or full scale model testing in practice. Furthermore, it is found that the proposed high order approximation approach can give good predictions of the modified modal parameters even in the cases where relatively large modifications of structural parameters are present, whereas the first-order approximation approaches may not be sufficient.

References

[1] R.L. Fox, M.P. Kapoor, Rates of changes of eigenvalues and eigenvectors, *AIAA Journal* 6 (12) (1968) 2426–2429.

[2] A.D. Belegundu, B.G. Yoon, Iterative methods for design sensitivity analysis, *AIAA Journal* 26 (11) (1988) 1413–1415.

[3] W.M. To, D.J. Ewins, Structural modification analysis using Rayleigh quotient iteration, *International Journal of Mechanical Sciences* 32 (3) (1990) 169–179.

[4] J. Wang, W. Heylen, P. Sas, Accuracy of structural modification techniques, *Proceedings of the Fifth International Modal Analysis Conference, Union College*, 1987, pp. 65–75.

[5] H.P. Chen, Nonlinear perturbation theory for structural dynamic systems, *AIAA Journal* 43 (11) (2005) 2412–2421.

[6] N. Bicanic, H.P. Chen, Damage identification in framed structures using natural frequencies, *International Journal for Numerical Methods in Engineering* 40 (23) (1997) 4451–4468.

[7] H.P. Chen, N. Bicanic, Inverse damage prediction in structures using nonlinear dynamic perturbation theory, *Computational Mechanics* 37 (5) (2006) 455–467.

[8] R. Guyan, Reduction of stiffness and mass matrices, *AIAA Journal* 3 (2) (1965) 380–387.