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ScienceDirect
Journal of Sound and Vibration 299 (2007) 388–396
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

Response of an open-plane frame to multiple support horizontal seismic excitations with soil–structure interaction

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Received 20 April 2006; received in revised form 10 July 2006; accepted 18 July 2006
Available online 11 September 2006

Abstract

The problem of seismic structural design reduces ultimately to estimating the response of the structure to an assumed forced motion imposed on the ground. For multiple supported structures, in most cases, it is generally sufficient to assume that the arrival time of each component of the base motion is the same for each support point, making the transmission time zero (i.e., uniform or rigid base excitation). The inappropriateness of this assumption has been established for long structures like bridge spans. In the current study, the effect of wave passage on the response of an open-plane frame building structure on isolated column bases has been examined for a few selected horizontal accelerograms. Soil–structure interaction has also been considered. The results indicated that a multiple supported excitation approach yields significantly different peak column shear compared to uniform base excitation. Further, the peak column shear mobilized is affected by soil–structure interaction. The pseudo-static contribution to the peak response was seen to be very significant (>90%) particularly for low wave velocities even though the span was only 6.0 m for the non-interactive structure. When soil–structure interaction was considered, the pseudostatic contribution was found to be (for certain accelerograms depending on the ground displacement record) in excess of 25% for the structure founded on hard soil. These results suggest that is prudent to consider wave passage effects when determining the response to seismic excitations even of open plane frames with short spans.

1. Introduction

The problem of seismic structural design reduces ultimately to estimating the response of the structure to an assumed forced motion, which may be either deterministic or stochastic, imposed on the ground. In perhaps the majority of aseismic design problems it is doubtless satisfactory to consider the base of the structure as a single point and to consider the assumed earthquake process as a disturbance imposed at this single point. If the structure has more than one point of attachment to the ground, the inference is made that the arrival time of each component of the base motion is the same for each support point, making the transmission time zero (i.e., uniform or rigid base excitation). It is not always advisable to dismiss the possible effect of ground transmission time upon the behaviour and safety of the structure.

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.07.008

Observations have clearly demonstrated that seismic ground motion can vary significantly over distances of the same order of magnitude as the dimensions of some extended structures such as bridges. Three phenomena are responsible for these variations [1]: (1) the difference in the arrival times of waves at different stations, denoted as the '*wave passage*' effect, (2) the loss of coherency of the motion due to reflections and refractions of the waves in the heterogeneous medium of the ground, as well as due to the difference in the manner of superposition of waves arriving from an extended source at various stations, denoted as the 'incoherence effect'; and (3) the difference in the local soil conditions at each station and the manner in which they influence the amplitude and frequency content of the bedrock motion, denoted as the 'local' effect.

Spatial variability of the strong ground motion can significantly influence the internal forces induced in structures with multiple supports, such as bridges and viaducts. The variability in the support motion usually tends to reduce the inertia generated forces within the structure, as compared to the forces generated in the same structure when the supports move uniformly. However, differential support motions generate additional forces known as pseudo-static forces which are absent when the structure is subjected to uniform support motions. The resultants of the two sets of forces can exceed the level of forces generated in the structure with uniform support excitations, particularly when the structure is stiff [1].

The study of a simplified bridge subjected to travelling disturbance consisting of packets of damped oscillatory waves with random amplitudes, frequencies, arrival times, phases and velocities of propagation indicated that a finite transmission time for a seismic disturbance can significantly reduce the probability of survival of the structure [2].

It has been reported by Kuireghian and Neuenhofer [3] that the cross-correlation coefficient between ground displacement at one station and oscillator response at another station very rapidly diminishes from unity with increase in the oscillator frequency for ground acceleration represented by a white noise passed through the twin filters of the Kanai–Tajimi model described by Clough and Penzien [4]. While the study does not account for soil–structure interaction at the multiple supports, the results of an example application on a two-span continuous beam (both spans of 50 m) show that totally uncorrelated support excitations could result in mid-span deflections exceeding those corresponding to the case of totally correlated support excitations.

There are occasions, such as multi-storey buildings founded on soft soil, when it becomes necessary to consider the effects of deformability of the foundation. It has been well established in the literature [5–7] that reduction in the natural frequencies of the system occurs when foundation flexibility is introduced in the non-interactive system. In particular, for open plane frames on isolated foundations, soil–structure interaction lowers the fundamental frequency—the reduction being severe for softer soils (lower soil shear modulus, G_s, values), stiffer superstructures and smaller footing sizes [8,9]. It has been determined that the fundamental mode shape almost entirely determines the response of both non-interactive as well as structures with soil–structure interaction to seismic excitations [8]. It has also been reported that when the seismic excitation is represented by a fully coherent white noise acting at all the supports, soil–structure interaction is always beneficial in case of realistically proportioned superstructure frames on isolated footings [9].

The existing literature offers little information on the response of open-plane frames on isolated footing to multiple support excitations. Compared to bridge spans the spacing of footings is small and the soil characteristics under different footings are not likely to vary significantly. Consequently, wave passage effects may be more important than incoherency in the ground motion at different supports arising from differing soil conditions at the supports (local effect) and the 'incoherency effect.'

The appropriateness of considering multiple support excitations for long structures is now well recognized. No study is present in the literature which examines the relative importance of pseudo-static forces in determining the overall structural response. The relevance of a multiple support excitation approach in the case of shorter spans, i.e., open-plane frame building structures, has not been explored. Further, these structures are borne on isolated column bases so that soil–structure interaction needs to be considered in the dynamic analysis.

In this paper the dynamic response of an open-plane frame subjected to seismic horizontal excitation is examined accounting for wave passage effect and soil–structure interaction.

2. Equations of motion for a structure subjected to multiple support excitations with soil–structure interaction

For the open-plane frame on isolated footings shown in Fig. 1, the equations of motion for the n-degree of freedom system subjected to m support excitations can be expressed as

$$\begin{bmatrix} m_s & m_{sf} \\ m_{fs} & m_f \end{bmatrix} \begin{Bmatrix} \ddot{v}_s^t \\ \ddot{v}_f^t \end{Bmatrix} + \begin{bmatrix} c_s & c_{sf} \\ c_{fs} & c_f \end{bmatrix} \begin{Bmatrix} \dot{v}_s^t \\ \dot{v}_f^t \end{Bmatrix} + \begin{bmatrix} k_s & k_{sf} \\ k_{fs} & k_f \end{bmatrix} \begin{Bmatrix} v_s^t \\ v_f^t \end{Bmatrix} = \begin{Bmatrix} 0 \\ R \end{Bmatrix}, \tag{1}$$

where the suffixes s and f refer to degrees of freedom pertaining to nodes in the superstructure and in the foundation, respectively. The suffixes sf and fs refer to degrees of freedom pertaining to nodes common to superstructure and foundation elements. The total displacement vector $\{v^t\}$ can be separated into a vector of displacements induced by the dynamic forces $\{v^d\}$ and that of the ground motion $\{v^p\}$.

$$\begin{Bmatrix} v_s^t \\ v_f^t \end{Bmatrix} = \begin{Bmatrix} v_s^d \\ v_f^d \end{Bmatrix} + \begin{Bmatrix} v_s^p \\ v_g \end{Bmatrix}. \tag{2}$$

For the non-interactive system, $\left\{v_f^d\right\} = \{0\}$ since the dynamic base displacements are not permitted. The vector $\{R\}$ of forces on the foundation nodes can be separated into components

$$\{R\} = \{R^d\} + \{R^p\}, \tag{3}$$

where $\{R^d\}$ are the support reactions produced by the dynamic forces and $\{R^p\}$ are the support reactions produced by the motion of the supports. For rigid base excitation, the entire frame translates so that no reactions are developed at the supports, i.e., $\{R^p\} = \{0\}$. For non-uniform translation of the supports, the entire structure undergoes distortions, which may be called pseudo-static displacements, as a result of which support reactions are developed. That is, $\{R^p\} \neq \{0\}$.

Considering the equilibrium of the system subjected to a static set of support displacements, $\{v_g\}$, the static equivalent of Eq. (1) yields

$$\begin{bmatrix} k_s & k_{sf} \\ k_{fs} & k_f \end{bmatrix} \begin{Bmatrix} v_s^p \\ v_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ R^p \end{Bmatrix}. \tag{4}$$

So that

$$\{v_s^p\} = -[k_s]^{-1}[k_{sf}]\{v_g\} = [\bar{K}]\{v_g\}. \tag{5}$$

Further, for the system when soil–structure interaction is considered, the dynamic support reactions $\{R^d\}$ can be related to the soil impedance at the supports. If the soil impedance is considered to be frequency independent,

$$\{R^d\} = -[c_{ss}]\{\dot{v}_f^d\} - [k_{ss}]\{v_f^d\}.$$

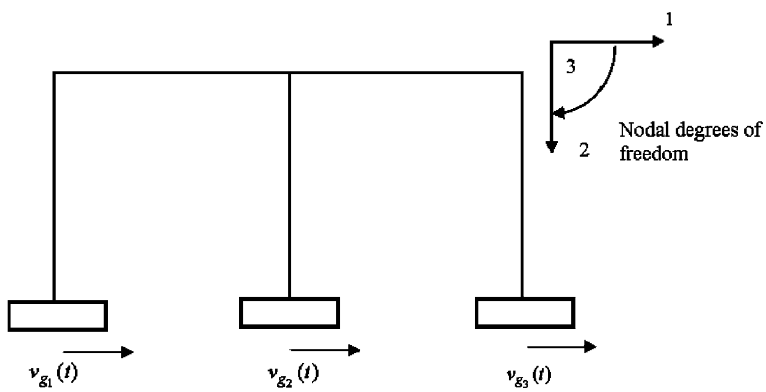

Fig. 1. Open-plane frame subjected to multiple support excitation.

Eq. (1) now takes the form, assuming lumped mass formulation and superstructure damping as being proportional to the superstructure mass and stiffness matrices and also neglecting the contribution of the damping matrix to the load vector.

$$\begin{bmatrix} m_s & 0 \\ 0 & m_f \end{bmatrix}\begin{Bmatrix} \ddot{v}_s^d \\ \ddot{v}_f^d \end{Bmatrix} + \begin{bmatrix} c_s & c_{sf} \\ c_{fs} & c_f + c_{ss} \end{bmatrix}\begin{Bmatrix} \dot{v}_s^d \\ \dot{v}_f^d \end{Bmatrix} + \begin{bmatrix} k_s & k_{sf} \\ k_{fs} & k_f + k_{ss} \end{bmatrix}\begin{Bmatrix} v_s^d \\ v_f^d \end{Bmatrix} = -\begin{bmatrix} m_s & 0 \\ 0 & m_f \end{bmatrix}\begin{Bmatrix} \ddot{v}_s^p \\ \ddot{v}_g \end{Bmatrix}. \tag{6}$$

For the system in the absence of soil–structure interaction, the **C** matrix can be proportional to the mass and stiffness matrices. When soil–structure occurs, the $[c_s]$ matrix can be generated for the superstructure after assigning values to the damping ratios of the different modes of vibration of the superstructure. In this approach, the matrices $[c_{sf}]$ $[c_{fs}]$ and $[c_f]$ are null matrices and the $[c_{ss}]$ matrix contains the elements of soil damping (possibly frequency independent) [10,11]. The entire $[C]$ matrix can also be treated as proportional to the mass and stiffness matrices of the interactive structure [12].

3. Frames adopted and range of soil properties

A single-storey 1-bay frame of flat slab construction were adopted for the study. Figs. 2(a) and (b) show the plan and transverse section of the frame which has a bay span of 6 m. The slab is 0.3 m thick (Fig. 2(b)). In the transverse section (which is also the direction of the ground motion), the slab with columns constitutes a flexible frame as shown shaded in Fig. 2(a) with an inter-frame spacing of 4 m. The column section is 0.2×0.5 m and the larger column cross-section direction is in the direction of the ground motion and in the plane of the frame as seen in Fig. 2(a). The fundamental undamped frequency of the frame in the absence of soil–structure interaction is 41.98 rad/s.

The material properties of structural members used for the linear analysis of the frame are modulus of elasticity of concrete $E_c = 22$ GPa and mass density of concrete $\rho_c = 2400\,\text{kg/m}^3$.

To permit soil–structure interaction the columns end in rigid bases of concrete of size $1.0\,\text{m} \times 0.5\,\text{m}$ and 0.3 m thick which are considered as adequate in medium-to-hard soils. The larger plan dimension of the footing (1.0 m) is in the direction of the ground motion.

For the non-interactive system, a constant modal damping ratio of 5% was adopted. For the interactive system, frequency-independent soil stiffness coefficients used by Pais and Kausal [13] have been used. For simplicity, a modal damping ratio of 0.05 in all modes of vibration was adopted for the interactive structure.

To render the results of the interactive study realistic, the shear modulus of soil G_s has been varied from 10 to 500 MPa so that the results are representative of medium-to-hard soils where isolated footings are used to support light-to-medium structures. As a result, the time delay between excitation of extreme end footing varies from 0.0108 to 0.073 s. A value of 0.3 was adopted for the Poisson's ratio of soil, μ_s. A constant value of $1500\,\text{kg}\,\text{m}^{-3}$ has been adopted to represent the mass density of soil over the entire G_s range.

Eq. (6) has been directly solved in the frequency domain using the fast Fourier transform technique. The eigenvalues and eigenvectors of the undamped system necessary for computing the damping matrix of Eq. (6) have been obtained by the Jacobi method.

4. Details of dynamic loads selected and the analysis

The first 30 s of the horizontal acceleration components of five earthquakes were used as seismic loading for evaluating the effect of soil–structure interaction over the range of G_s adopted. The details of these excitations are listed in Table 1. There is wide variation in their intensity as defined by Housner's response spectrum intensity (SI), evaluated for 5% critical damping ratio (ξ). The period of the damped simple oscillator which yields the maximum pseudo-spectral velocity for each excitation is also indicated in the table.

5. Results and discussion

In Table 2 are indicated the six lowest natural frequencies of the frame over a range of soil shear modulus G_s values. Also included are the six natural frequencies when soil–structure interaction is not permitted.

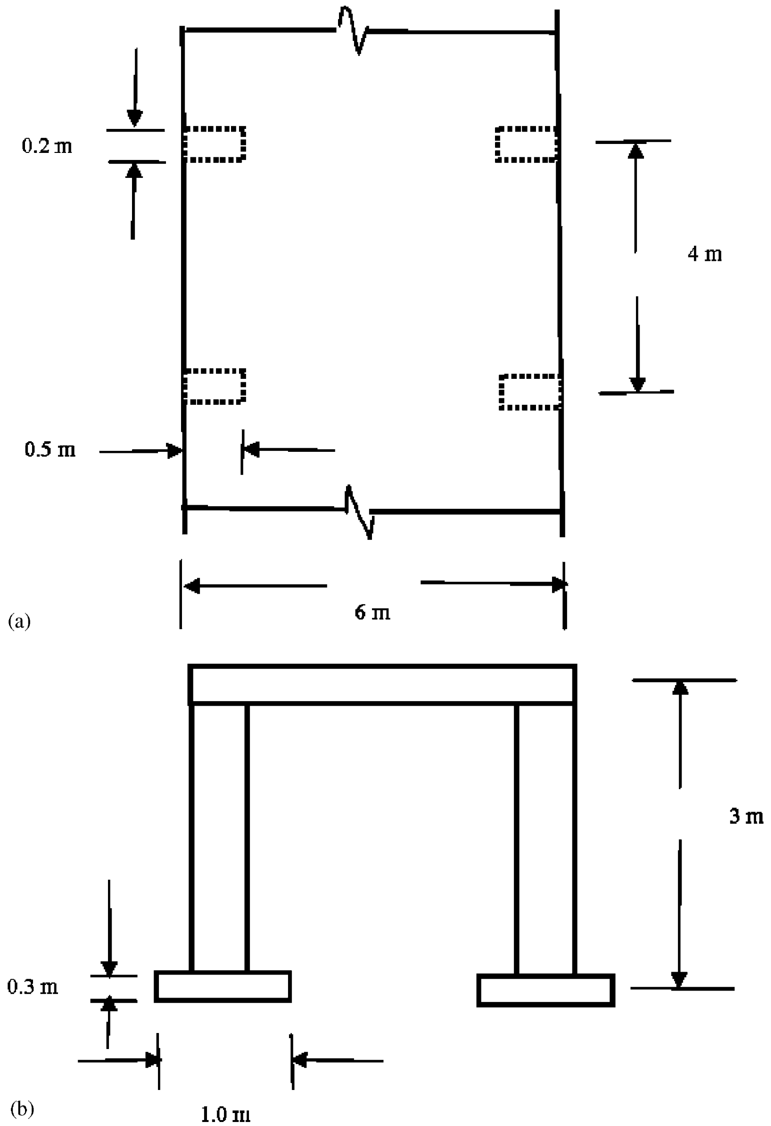

Fig. 2. (a and b) Typical open-plane frame on isolated footings.

Table 3 presents the peak absolute column shear and the instant of seismic excitation at which this occurs for non-interactive frame when subjected to uniform base excitation as well as when there is delay in application of the excitation at the farther support. When the excitation acts simultaneously at both columns, the column shear then arises only from the dynamic displacements induced in the superstructure. When a time delay occurs in application of the seismic excitation at the farther column, the elastic forces in the frame depend both on the induced dynamic displacements in the structure as well as on the pseudo-static displacements in the superstructure produced by the differential ground motions applied at the supports. The pseudo-static contribution to the peak absolute column shear expressed as a percentage is also indicated in the table. It is seen that, in general, the peak column shear increases with increase in time delay for the seismic excitations used and so also does the contribution of the pseudo-static displacements. For the largest time delay of 0.07348 s (presumed to occur when the space between the two supports of the non-interactive frame is occupied by soil with $G_s = 10\,\text{MPa}$), the peak column shear with time delay (which greatly exceeds that for uniform support excitation) largely arises from the differential support displacements. From Table 3 it is seen

Table 1
Details of earthquakes selected

Description of earthquakes		Maximum acceleration		Spectral pseudo-velocity for $\xi = 5\%$		Response spectrum intensity SI for $\xi = 5\%$ (m)
Description	Symbol	Value (m/s^2)	Time (s)	Max. S_v (m/s)	Period (s)	
Uttarakashi 1991 (N15W)	Q1	−2.372	6.22	0.464	0.249	0.432
UK (Abhat) 1991 (N85E)	Q2	2.484	4.26	0.541	0.887	0.697
Eurake 1954 (N46W)	Q3	1.973	7.10	0.697	1.413	1.038
El Centro 1940 (S90W)	Q4	2.101	11.44	0.724	2.067	1.119
El Centro 1940 (S00E)	Q5	3.417	2.12	0.802	0.990	1.356

Table 2
Natural frequencies of the interactive frame

Shear modulus of soil, G_s (MPa)							Fixed base
10	50	90	120	150	300	500	
19.311	27.256	30.593	32.211	33.417	36.661	38.435	41.976
47.487	65.313	66.219	66.650	66.974	67.879	68.402	69.541
49.337	82.497	90.452	92.928	94.223	97.082	98.337	100.595
62.016	104.618	133.710	149.262	161.649	199.580	224.188	285.500
106.546	126.107	147.143	160.362	171.419	206.815	230.427	290.128
178.953	362.449	475.983	541.678	596.012	779.807	949.744	989.972

Table 3
Effect of multiple support excitation on peak column shears for non-interactive frame

Delay (s)	0.0	0.01039	0.01342	0.01897	0.02121	0.02449	0.03286	0.07348
Seismic excitation Q1								
Peak column shear (kN)	78.858	77.659	76.855	75.529	74.385	75.330	78.435	88.907
	(4.015)	(4.02)	(4.02)	(4.03)	(4.03)	(5.62)	(5.63)	(5.66)
Pseudo-static component (%)	0.0	0.41	0.70	4.14	4.22	36.64	46.59	94.26
Seismic excitation Q2								
Peak column shear (kN)	48.649	52.577	53.111	54.103	54.669	54.949	56.782	80.108
	(4.263)	(4.263)	(4.263)	(4.265)	(4.265)	(4.265)	(4.265)	(3.953)
Pseudo-static component (%)	0.0	8.48	10.54	13.81	16.01	18.74	28.78	96.51
Seismic excitation Q3								
Peak column shear (kN)	34.027	42.678	45.665	51.815	55.227	57.865	71.613	129.820
	(7.112)	(6.968)	(6.968)	(6.970)	(6.970)	(6.970)	(6.973)	(6.907)
Pseudo-static component (%)	0.0	43.84	48.12	55.27	58.73	61.78	71.494	98.42
Seismic excitation Q4								
Peak column shear (kN)	31.397	51.180	60.021	76.080	82.515	91.937	115.67	231.60
	(4.188)	(2.10)	(2.103)	(2.105)	(2.107)	(2.11)	(2.118)	(2.16)
Pseudo-static component (%)	0.0	59.47	66.61	74.43	76.94	79.93	85.72	98.32
Seismic excitation Q5								
Peak column shear (kN)	50.213	59.421	64.954	74.938	78.802	84.599	98.20	173.90
	(3.547)	(2.248)	(2.248)	(2.250)	(2.25)	(2.250)	(2.252)	(2.19)
Pseudo-static component (%)	0.0	30.19	36.38	44.94	48.09	52.48	60.97	91.77

The values in brackets denote the instant of excitation at which the peak response occurred.

that for any time delay, the pseudo-static contribution to the peak column shear is different for different excitations (particularly for small magnitudes of delay) whose individual ground displacement records differ.

Except for the seismic excitation Q1, it is also seen that consideration of significant time delay results is much larger peak column shear values with the pseudo-static contribution being very significant. This suggests that a static frame analysis with supports subjected to a likely value of relative displacement can replace a dynamic analysis for non-interactive open-plane frames in situations of wave velocities lower than 100 m/s.

Table 4 pertains to the condition when soil–structure interaction is permitted. For the selected seismic excitations, the absolute peak column shear for rigid base excitation and when time delay occurs is also indicated along the respective instant of occurrence for a soil G_s ranging from 10 to 500 MPa. For rigid base excitation, it is seen on comparing the values in Table 4 with the values indicated in Table 3 for zero time delay that soil–structure interaction affects the peak absolute column shear. However, there is no clear trend with respect to G_s for any of the five seismic excitations used. It has been brought out in an earlier study (Babu and Allam, 2002) that soil–structure interaction reduces the fundamental frequency (which contributes the bulk of the system's response to seismic excitation) and modifies the modes shapes by introducing footing displacements and rotations which tend to attenuate the elastic forces produced in the columns by the dynamic

Table 4
Effect of multiple support excitation on peak column shears for frame with soil–structure interaction

	Shear modulus, G_s (MPa)						
	10	50	90	120	150	300	500
Seismic excitation Q1							
Peak column shear with no time delay (kN)	55.053 (4.74)	88.456 (6.25)	68.501 (6.103)	62.870 (6.085)	59.312 (4.063)	89.742 (4.043)	91.004 (4.032)
Peak column shear with time delay (kN)	52.395 (4.773)	82.997 (6.145)	68.228 (6.117)	61.875 (5.985)	59.464 (4.075)	89.894 (4.05)	90.667 (4.037)
P–S comp. (%)	25.75	11.36	7.21	5.88	6.93	4.16	2.23
Seismic excitation Q2							
Peak column shear with no time delay (kN)	38.662 (7.317)	49.922 (4.317)	47.009 (7.453)	57.317 (4.30)	62.35 (4.29)	60.545 (4.708)	56.73 (4.695)
Peak column shear with time delay (kN)	37.351 (7.20)	49.332 (4.338)	48.314 (7.465)	55.301 (4.313)	59.604 (4.303)	65.706 (4.715)	61.097 (4.702)
P–S comp. (%)	40.11	72.88	10.62	2.14	0.63	12.0	10.22
Seismic excitation Q3							
Peak column shear with no time delay (kN)	25.34 (7.13)	25.750 (8.613)	30.983 (7.015)	31.352 (7.00)	33.732 (7.17)	35.734 (7.14)	35.672 (7.13)
Peak column shear with time delay (kN)	40.990 (6.918)	42.170 (6.768)	46.413 (7.023)	49.071 (7.008)	49.561 (7.00)	43.435 (6.982)	41.687 (6.975)
P–S comp. (%)	96.89	50.75	38.33	39.76	39.01	36.23	36.32
Seismic excitation Q4							
Peak column shear with no time delay (kN)	37.691 (4.64)	47.141 (2.938)	49.346 (2.898)	40.912 (4.612)	41.399 (4.603)	34.913 (11.42)	34.636 (3.607)
Peak column shear with time delay (kN)	81.052 (2.413)	73.090 (2.738)	73.312 (2.473)	73.495 (2.458)	71.010 (2.442)	54.461 (2.325)	48.997 (2.317)
P–S comp. (%)	84.50	48.99	52.23	50.88	51.99	60.62	56.33
Seismic excitation Q5							
Peak column shear with no time delay (kN)	61.696 (2.635)	65.637 (2.518)	57.386 (2.502)	55.901 (2.68)	57.686 (2.265)	62.197 (4.93)	51.054 (2.24)
Peak column shear with time delay (kN)	78.577 (2.195)	78.025 (2.533)	73.008 (2.513)	71.691 (2.275)	76.121 (2.27)	79.680 (2.252)	68.664 (2.243)
P–S comp. (%)	63.15	23.37	24.41	29.90	27.41	24.07	24.74

displacements induced in the frame. The response is governed by the frequency content of the seismic excitation used. The interactive structure may develop larger peak column shears than the non-interactive structure.

When time delay is considered it is seen from Table 4 that larger peak column shears are developed by the interactive structure for the seismic excitations Q3, Q4 and Q5 through out the G_s range studied compared to the values obtaining when rigid base excitation was applied. For the excitations Q1 and Q2 consideration of time delay yields larger peak forces in only five cases. Further, where ever time delay yields larger peak forces, the pseudo-static component is very significant and can be as much as 97% of the total response. In effect, the dynamic component of the total peak response decreases when time delay occurs in support excitation but the pseudo-static contribution can result in the total response being larger than that obtained for uniform support excitation. As seen from Table 4, the pseudo-static contribution to the peak column shear for any G_s value varies for the different excitations being a function of the ground displacement record.

From Tables 3 and 4 it is also noted that for any time delay the non-interactive frame yields larger peak column shear values compared to the corresponding interactive frame where the soil modulus G_s yields the same delay interval. This can be attributed to the fact that in a non-interactive frame a given support displacement yields larger support reactions at the restrained degrees of freedom and consequently larger internal stresses in the members compared to the interactive frames where in supports are restrained elastically by the soil.

6. Conclusions

(1) It is observed, based on the peak response of an open-plane frame subjected to some selected horizontal accelerograms, that a multiple support excitation approach can yield significantly different (and frequently larger) peak column shears compared to a uniform support approach indicating that wave passage phenomenon should not be overlooked even in short-span structures in the absence of soil–structure interaction.
(2) The pseudo-static contribution to the peak response can be very significant. For the non-interactive frame studied this was found to be $>90\%$ for low wave velocities even though the frame had a span of just 6.0 m.
(3) When soil–structure interaction is included in the analysis, wave passage effect yielded larger peak columns shears than those obtained for uniform support excitations for several of the accelerograms over a large range of shear modulus. The pseudo-static contribution (for certain accelerograms depending on the ground displacement record) was in excess of 25% for the frame founded on hard soil.
(4) These results suggest that while determining the response of open-plane frames (with or without consideration of soil–structure interaction) to horizontal seismic excitations, it might be prudent to consider wave passage effects.

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