Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 299 (2007) 645–655
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

# Dynamic behavior and sound transmission analysis of a fluid–structure coupled system using the direct-BEM/FEM

Z. Tong[a], Y. Zhang[b], Z. Zhang[a,*], H. Hua[a]

[a]*State Key Laboratory of Vibration, Shock and Noise, Shanghai Jiaotong University, Shanghai 200240, PR China*
[b]*Institute of Shanghai Marine Equipment Research, Shanghai 200031, PR China*

Received 26 April 2004; received in revised form 31 May 2006; accepted 12 June 2006
Available online 2 October 2006

**Abstract**

A direct-BEM/Fem method was proposed to analyze the vibration and acoustic radiation characteristics of a submerged structure. Model parameters of the structure and the fluid-structure interaction due to surrounding water were analyzed by using FEM and direct BEM. Vibration velocity of the outer hull surface and underwater sound pressure were computed through modal superposition technique. The direct-BEM/FEM method was first validated by analyzing a submerged cylindrical shell, then was used to analyze the vibro-acoustic behavior of a submarine stern structure. The results have demonstrated the direct-BEM/FEM method is more effective than FEM in computing the underwater sound radiation of the stern structure.


## 1. Introduction

As an elastic structure, the submarine stern vibrates in the presence of fluid dynamic loads. Especially, the natural frequencies of the stern structure will change during navigation due to the fluid–structure interaction. If one of the natural frequencies becomes close to the frequency of exciting forces, the submarine stern will vibrate intensively. Therefore, prior to a perfect design of the stern structure, the natural behavior should be accurately simulated by considering the fluid–structure interaction. The vibration induced sound transmission is a potential threat for submarines, and the acoustic property should be predicted as well during the process of design, which generally includes acoustic cavity, acoustic radiation and acoustic scatting from the submerged elastic structure.

An analytical solution to the fluid–structure interaction can be given when structures are in simple shapes (e.g. sphere or infinite cylinder) and lots of studies have been made in this field during the past decades [1]. But in real-world problems, structures are usually in complicated shapes. Therefore, it is very difficult to consider the effect of fluid–structure interaction only by analytic methods. Fortunately, these problems can be solved by numerous numerical techniques which model elastic structures with finite elements (including beams, plates, shells and solids). Compared to elastic structures, fluid can be modeled with finite elements,

*Corresponding author.
*E-mail addresses:* chychang@sjtu.edu.cn, , hhx@sjtu.edu.cn (Z. Zhang).

0022-460X/$ - see front matter 
doi:10.1016/j.jsv.2006.06.063

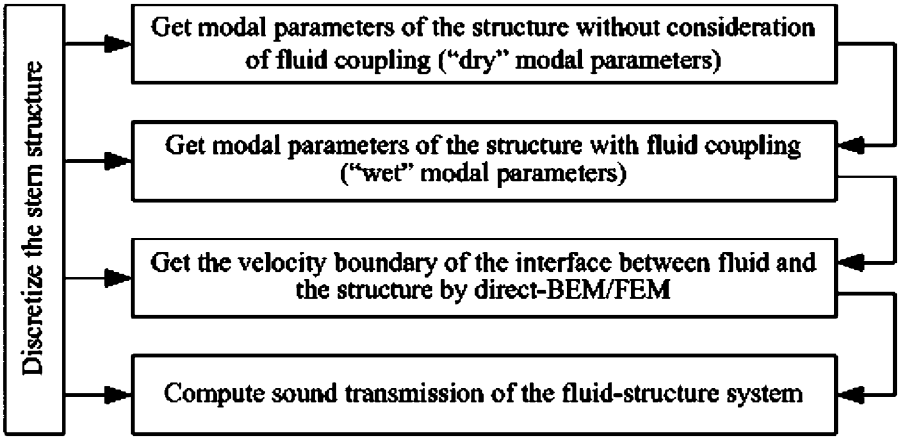


Fig. 1. The process of computation.

infinite elements or boundary elements. Combinations of these techniques have been used to deal with the fluid–structure interaction, for instance, the CFD can calculate viscous flow enclosed by deformable structures [2].

The first standard approach is FEM [3–8]. In some FEM software, fluid elements are supplied to solve the fluid–structure interaction problem. Hamdan [3] used two-dimensional, axisymmetric and three-dimensional finite elements for near-field fluid–structure interaction analysis which is based on the Lagrangian kinematical description of motion. In addition, only the fluid element can be directly used for an explicit fluid mesh and for the graphical display of wave motion through the fluid. The main disadvantages of using fluid elements in finite elements method (FEM) are the need for an approximate radiation boundary condition at the outer fluid boundary and limitations on mesh size and type (sometimes leading to frequency-dependent fluid meshes [8]), and the difficulty of generating the fluid mesh.

The boundary element method (BEM) has been widely used to model fluid [9–13]. It is an attractive technique for the analysis of fluid–structure interaction since only the "wet" surfaces need to be discretized. For this class of problems, the doubly asymptotic approximation (DDA) is an alternative approach [10–11]. To obtain the pressure distribution of the fluid, the vibration velocity of the interface is taken as a boundary condition. However, the vibration velocity of the interface may vary due to the fluid–structure interaction. Therefore, it is difficult to obtain the velocity boundary condition, especially for irregularly shaped structures. In practice, the vibration velocity of structures free from fluid is taken as a boundary condition in the sound transmission computation. This FEM/BEM has not yet been completely satisfactory.

A compromise between the accuracy of boundary element analysis and the computational efficiency of finite element analysis is possibly made by fluid infinite elements [14–17]. The typical approach using infinite elements in structural acoustics is that the structure and a small portion of exterior fluid are modeled with finite elements and a layer of infinite fluid elements are added to the outer side of the fluid finite elements. But in practice, the finite element mesh is difficult to obtain when structures are in complicated shapes.

This work is concerned with an effective combination of direct-boundary element and finite element (direct—BEM/FEM) for the analysis of vibration and sound transmission of the submarine stern. The computation procedure is shown in Fig. 1. First, the stern is modeled by finite elements and the "dry" modal parameters of the structure (i.e. natural frequencies and mode shapes) are obtained by FEM. Next, the fluid domain is modeled by direct-boundary elements and the "wet" modal parameters of the stern are derived on the basis of the vibration response of the "wet" interface and the "dry" modal parameters. Then, the velocity response of the stern under external load is computed. Finally, the sound pressure is given by direct-BEM. In general, the combination of direct-BEM and FEM can reduce the number of elements.

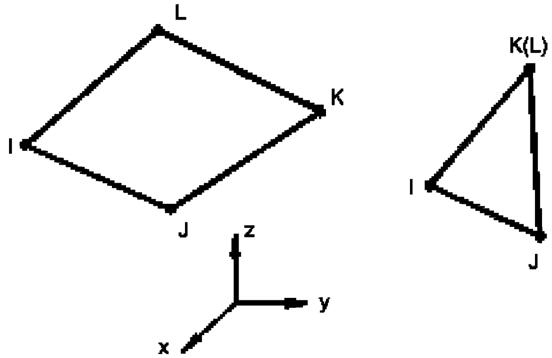


Fig. 2. The shell elements.

## 2. Numerical formulation

In this section, the numerical formulation of FEM and direct-BEM is briefly described.

### 2.1. Shape function of shell elements

The submarine stern is a complicated structure with irregularly shaped wing and rudder and modeled with beam elements, shell elements and solid elements. On the fluid–structure interface, the structure is modeled by shell elements which are covered by a layer of fluid boundary elements. The fluid boundary elements inherit all the necessary physics information from the shell elements on the interface.

Consider the shell element shown in Fig. 2 and suppose $u$, $v$, $w$, $\theta_x$, $\theta_y$ are displacements along the axes $x$, $y$, $z$ and rotation along the axis $x$, $y$, respectively.

Using barycentric coordinates, $u$, $v$, $w$, $\theta_x$, $\theta_y$ can be given in terms of the node displacements ($u_1$, $v_1$, $w_1$, $\theta_{x1}$, $\theta_{y1}$), $L$, ($u_4$, $v_4$, $w_4$, $\theta_{x4}$, $\theta_{y4}$), i.e. $\{u\ v\ w\ \theta_x\ \theta_y\}^{\mathrm{T}}[N]\{u\}^e = [\mathrm{N}_1\ L\ N_{20}]\{u_1\ v_1\ w_1\ \theta_{x1}\ \theta_{y1}\ L\ u_4\ v_4\ w_4\ \theta_{x4}\ \theta_{y4}\}^{\mathrm{T}}$, where $u^e$ denotes the nodal displacement.

The details on element properties, shape functions, coordinate transform and numerical integration are described in Ref. [18].

### 2.2. Finite element formulation

The motion equation of the finite element model of the structure is as follows:

$$\mathbf{M}_s\ddot{u}_s + \mathbf{C}_s\dot{u}_s + \mathbf{K}_s u_s = \mathbf{F}_s, \tag{1}$$

where

$$\mathbf{M}_s = \sum_e \mathbf{M}_s^e,\ \mathbf{K}_s = \sum_e \mathbf{K}_s^e,\ \mathbf{C}_s = \sum_e \mathbf{C}_s^e,\ \mathbf{F}_s = \sum_e \mathbf{F}_s^e,$$

$$\mathbf{M}_s^e = \int_A \mathbf{N}^{\mathrm{T}}\mathbf{N}\rho h\,\mathrm{d}A = \iint \mathbf{N}^{\mathrm{T}}\mathbf{N}\rho h|J|\mathrm{d}\eta\,\mathrm{d}\zeta,\ K_s^e = \int_A B^{\mathrm{T}}\mathbf{D}\mathbf{B}\,\mathrm{d}A = \iint \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}|J|\mathrm{d}\eta\,\mathrm{d}\zeta,$$

$$\mathbf{C}_s^e = \int_{V^e} \mu\mathbf{N}^{\mathrm{T}}\mathbf{N}\,\mathrm{d}V^e,\ \mathbf{F}_s^e = \int_{V^e} \mathbf{N}^{\mathrm{T}} f_V\,\mathrm{d}V^e + \int_{S^e} \mathbf{N}^{\mathrm{T}} f_S\,\mathrm{d}S^e.$$

The subscript "$s$" stands for the structure and the superscript "$e$" stands for elements. $\mathbf{M}_s \in R^{m\times m}$ is the structural mass matrix, $\mathbf{C}_s \in R^{m\times m}$ is the structural damping matrix, $\mathbf{K}_s \in R^{m\times m}$ is the structural stiffness matrix, $u_s \in R^{m\times 1}$ is the node displacement vector and $\mathbf{F}_s \in R^{m\times 1}$ is load vector. The structural mass and stiffness matrices are obtained by assembling individual element matrices. The damping matrix is used to give a constant damping ratio. $A$ is the mid-surface area of the element, $|J|$ is the determinant of Jacobin matrix, $\rho$ is the density of the shell and $h$ is the thickness of the element, $f_v$ is the distributed volume load, $f_S$ is the distributed surface load.

According to the vibration theory, the following property of modal orthogonality holds:

$$\Phi_s^{\mathrm{T}}\mathbf{M}_s\Phi_s = I_{m\times m},$$
$$\Phi_s^{\mathrm{T}}\mathbf{K}_s\Phi_s = \Omega_s, \tag{2}$$

where $\mathbf{I}_{m\times m}$ is the identity matrix, $\Phi_s = [\phi_1, \phi_2, \ldots, \phi_m]$, $\Omega_s = \mathrm{diag}(\omega_1^2, \ldots, \omega_m^2)$, $\phi_i, \omega_i (i = 1\sim m)$ are the $i$th mode vector and the $i$th natural frequency, respectively.

As indicated by Eq. (2), the stiffness and mass matrices can be directly computed if we have obtained mode vectors and natural frequencies.

### 2.3. Boundary element formulation

When harmonic acoustic waves are considered, the linearized wave equation reduces to Helmholtz equation. Consider the Helmholtz pressure Eq. (3) in pressure $P_f$

$$\nabla^2 P_f + k^2 P_f = 0 \tag{3}$$

with the pressure boundary condition

$$\frac{\partial P_f}{\partial n} = \rho\omega^2 u_f \quad \text{on the flexible boundary,} \tag{4}$$

where $\nabla^2$, $n$ and $\rho$ are the Laplacian operator, the unit outward normal at the surface point and the density of the fluid, respectively, $k = \omega/c_0$ is the wave number, $\omega$ is the angular frequency, $c_0$ is the speed of sound in water, and the subscript "$f$" stands for the fluid.

Suppose the fluid is incompressible, the Helmholtz Eq. (3) reduces to

$$\nabla^2 P_f = 0. \tag{5}$$

A weighting function $\bar{P}_f$ can be introduced such that $P_f$ has continuous first derivative within the fluid region $\Sigma$. The weighted residual description can be written as

$$\int_\Sigma (\nabla^2 P_f)\bar{P}_f \,\mathrm{d}\Sigma = \int_{\Gamma_2}\left(\frac{\partial P_f}{\partial n} - Q_0\right)\bar{P}_f \,\mathrm{d}\Gamma - \int_{\Gamma_1}(P_f - P_0)\bar{P}_f \,\mathrm{d}\Gamma \tag{6}$$

and the boundary condition is

$$P_f\big|_{\Gamma 1} = P_0, \quad \left.\frac{\partial P_f}{\partial n}\right|_{\Gamma 2} = Q_0,$$

where $\bar{P}_f$ can be the fundamental solution to the Laplace equation. Applying the Green's second identity theorem to Eq. (6) and using the standard boundary element procedure [9], we can get the boundary integral equation:

$$C_i P_{fi} + \int_\Gamma P_f \frac{\partial \bar{P}_f}{\partial n}\,\mathrm{d}\Gamma = \int_\Gamma \bar{P}_f \frac{\partial P_f}{\partial n}\,\mathrm{d}\Gamma, \tag{7}$$

where $C_i$ is the Green's constant which can be calculated by surrounding the boundary point $i$ with a small sphere of radius $\varepsilon$ and taking each term in Eq. (7) in the limit as $\varepsilon \to 0$.

In Eq. (7), there exist boundary integrals. For the submerged stern, the integration is computed on a series of internal cells created by dividing the fluid domain. Here, isoparameteric quadrilateral and triangular linear elements are used in the discritization of the boundary (Fig. 3).

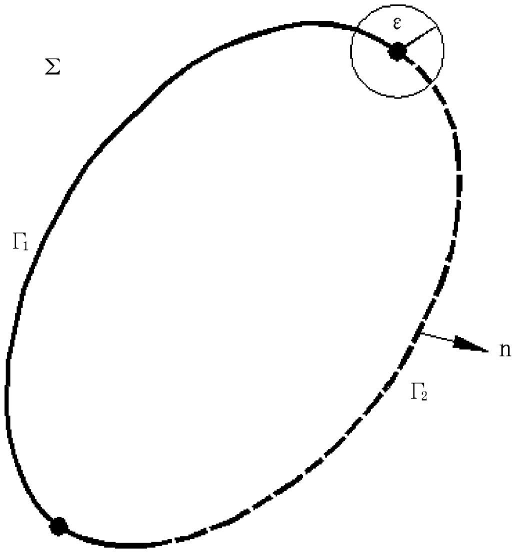


Fig. 3. Boundary in BEM.

Assume that the boundary of the domain is discretized into $L$ isoparameteric quadrilateral and triangular linear elements with $L$ nodes, Eq. (7) can be discretized and expressed in matrix form as

$$C_i\delta_{ij}P_{fi} + \sum_{j=1}^{L} \hat{H}_{ij}P_{fj} = \sum_{j=1}^{L} G_{ij}\left(\frac{\partial P_f}{\partial n}\right)_j. \tag{8}$$

Combining the constant term $C$ with $\hat{\mathbf{H}}$ matrix, Eq. (8) takes the form

$$\mathbf{H}P_f = \mathbf{G}\frac{\partial P_f}{\partial n}, \tag{9}$$

where

$$\mathbf{H} = \sum_{j=1}^{L} C_i\delta_{ij} + \hat{\mathbf{H}}_{ij},\ \delta_{ij} = \begin{cases} 1 & i = j, \\ 0 & i \neq j, \end{cases}$$

$$G = \sum_{j=1}^{L} G_{ij},\ \hat{\mathbf{H}}_{ij} = \int_{\Gamma i} \frac{\partial \bar{P}_f}{\partial n}\,\mathrm{d}\Gamma,\ \ G_{ij} = \int_{\Gamma i} \bar{P}_f\,\mathrm{d}\Gamma.$$

Combining Eqs. (4) and (9), we can get

$$\mathbf{H}P_f = \mathbf{G}\rho\omega^2 u_f. \tag{10}$$

### 2.4. Direct-BEM/FEM formulation

In Fig. 4, $\Sigma_1$ represents the structure, $\Sigma_2$ is the fluid domain inside and outside the structure, $\Gamma$ is the boundary of the fluid domain.

To couple the boundary elements and the finite elements at the fluid–structure interface, the following compatibility and equilibrium conditions are imposed:

$$u_f = u_s, \quad \dot{u}_f = \dot{u}_s, \tag{11}$$

$$P_s + P_f = 0, \tag{12}$$

where the subscript "$f$" stands for boundary element nodes of the fluid and "$s$" stands for shell element nodes of the structure at the fluid–structure interface.

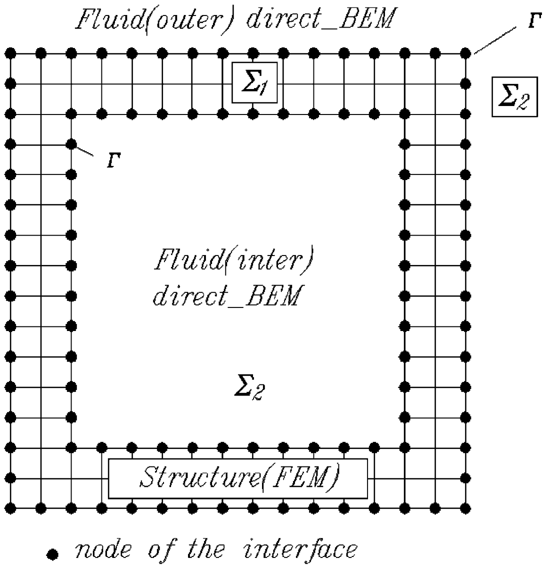


Fig. 4. The model of direct-BEM/FEM.

Under the equilibrium condition (12), the motion Eq. (1) of undamped structures can be given in the frequency domain:

$$(\mathbf{K}_S - \omega^2\mathbf{M}_S)\{u_S\} = \{\mathbf{F}\}, \tag{13}$$

where

$$\mathbf{F} = -\mathbf{A}_s^{\mathrm{T}} P_s = \mathbf{A}_s^{\mathrm{T}} P_f,\ \mathbf{A}_S = \sum_e \int_\Gamma \mathbf{N}\mathbf{N}^{\mathrm{T}}\,\mathrm{d}\Gamma.$$

In light of Eqs. (10) and (13), the governing equation of the coupled fluid–structure system can be expressed as

$$\begin{bmatrix} \mathbf{K}_S - \omega^2\mathbf{M}_S & -\mathbf{A}_s^{\mathrm{T}} \\ -\mathbf{G}\rho\omega^2 & \mathbf{H} \end{bmatrix} \begin{Bmatrix} u_f \\ P_f \end{Bmatrix} = 0. \tag{14}$$

The eigenvalues of the coupled fluid–structure system can be determined from Eq. (14). Given the "dry" natural frequencies and mode vectors of the structure, the corresponding "wet" natural frequencies and mode vectors of the coupled system can be computed from the following equation:

$$\begin{bmatrix} \mathbf{\Omega}_s - \omega^2\mathbf{I}_{m\times m} & -\mathbf{\Phi}_s^{\mathrm{T}}\mathbf{A}_s^{\mathrm{T}} \\ -\rho\omega^2\mathbf{G}\mathbf{\Phi}_s & \mathbf{H} \end{bmatrix} \begin{bmatrix} \xi_s \\ P_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{15}$$

where $u_s = \Phi_s\xi_s$. Suppose only the first $mr(mr \ll m)$ modes of the structure are considered, then $u_s$ can be approximately expressed as $u_s \approx \tilde{\Phi}_s\tilde{\xi}_s$, where $\tilde{\Phi}_s = [\phi_1, \phi_2, \ldots, \phi_{mr}]$ composed of the first $mr$ mode vectors, and Eq. (15) reduces to

$$\begin{bmatrix} \tilde{\mathbf{\Omega}}_s - \omega^2\mathbf{I}_{mr\times mr} & -\tilde{\mathbf{\Phi}}_s^{\mathrm{T}}\mathbf{A}_s^{\mathrm{T}} \\ -\rho\omega^2\mathbf{G}\tilde{\mathbf{\Phi}}_s & \mathbf{H} \end{bmatrix} \begin{bmatrix} \tilde{\xi}_s \\ P_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{16}$$

where $\tilde{\Omega}_s = \mathrm{diag}\,(\omega_1^2, \ldots, \omega_{mr}^2)$. The stable state response of the coupled fluid–structure system under periodical load $F_{\mathrm{load}}$ satisfies

$$\begin{bmatrix} \tilde{\mathbf{\Omega}}_s - \omega^2\mathbf{I}_{mr\times mr} & -\tilde{\mathbf{\Phi}}_s^{\mathrm{T}}\mathbf{A}_s^{\mathrm{T}} \\ -\rho\omega^2\mathbf{G}\tilde{\mathbf{\Phi}}_s & \mathbf{H} \end{bmatrix} \begin{bmatrix} \tilde{\xi}_s \\ P_f \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{\Phi}}_s^{\mathrm{T}} F_{\mathrm{load}} \\ 0 \end{bmatrix}. \tag{17}$$

Therefore, the stable state response can be approximately computed from Eq. (17).

### 2.5. Sound transmission formulation

There are two fundamental methods used in the BEM analysis. One is the direct BEM known as collocation, and the other is indirect BEM known as variation. The difference between the two methods is that the former uses velocity whereas the latter uses the jump of velocity as the primary unknowns. Here, we consider the collocation method to obtain sound transmission. The equation for the description of sound transmission is

$$\mathbf{HP} = \mathbf{G}\rho\omega\dot{u}_f, \tag{18}$$

where $\dot{u}_f = \omega u_s$ is the vibration velocity of the fluid at the interface.

## 3. Numerical example

### 3.1. Numerical results and discussion

Fig. 5 is a submerged cylindrical model with flat end closures. Computation of the fluid loaded resonances of the model using the direct- BEM/FEM is illustrated in this example. Parameters involved in the illustration are follows:

| | |
|---|---|
| $R = 5\,\text{m}$ | mean shell radius |
| $L = 60\,\text{m}$ | shell length |
| $H = 0.05\,\text{m}$ | shell/end plate thickness |
| $E = 196\,\text{GPa}$ | Young's modulus |
| $\mu = 0.3$ | Poisson's ratio |
| $\rho_s = 7900\,\text{kg}\,\text{m}^{-3}$ | shell density |
| $\rho_f = 1000\,\text{kg}\,\text{m}^{-3}$ | fluid density |
| $c = 1500\,\text{m}\,\text{s}^{-1}$ | fluid speed of sound |

In the direct-BEM/FEM model, the interface between the cylindrical shell and fluid consists of 1984 isoparametric linear shell elements. Fourteen natural frequencies of the cylindrical shell in-vacuo and in-fluid

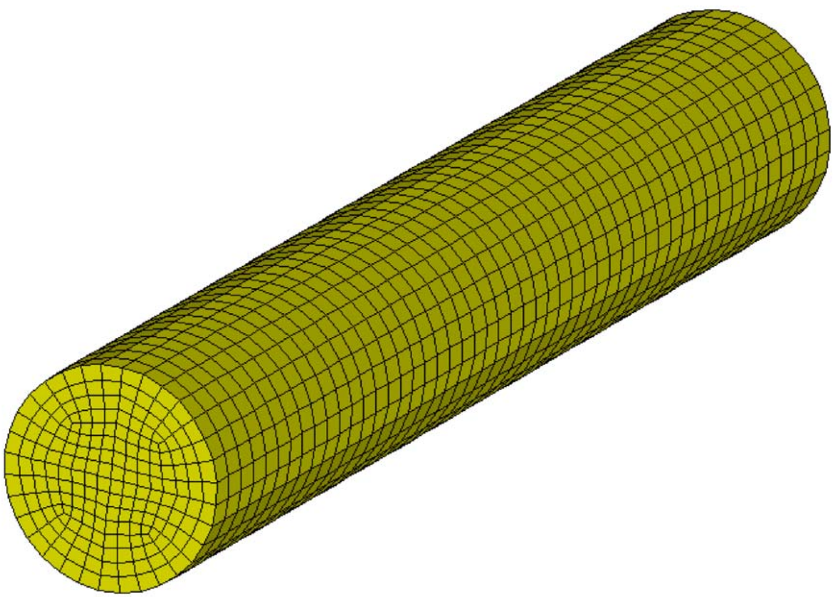

Fig. 5. Cylindrical shell with flat and closures.

Table 1
In-vacuo and in-fluid natural frequencies of the cylindrical shell with flat end plate

| No. | In vacuo | In fluid | | No. | In vacuo | In fluid | |
|---|---|---|---|---|---|---|---|
| | | Direct-BEM/FEM | Approx. theory | | | Direct-BEM/FEM | Approx. theory |
| 1 | 2.711 | 1.13 | 1.11 | 8 | 11.298 | 6.77 | 6.18 |
| 2 | 3.846 | 1.82 | 1.77 | 9 | 12.125 | 6.64 | 6.67 |
| 3 | 4.338 | 1.43 | 1.38 | 10 | 15.646 | 8.70 | 8.65 |
| 4 | 7.039 | 3.84 | 3.57 | 11 | 16.749 | 8.91 | 8.66 |
| 5 | 9.218 | 4.79 | 4.70 | 12 | 18.484 | 8.14 | 7.75 |
| 6 | 9.534 | 4.27 | 4.22 | 13 | 22.525 | 12.83 | 12.6 |
| 7 | 10.335 | 4.94 | 4.82 | 14 | 27.973 | 15.13 | 14.7 |

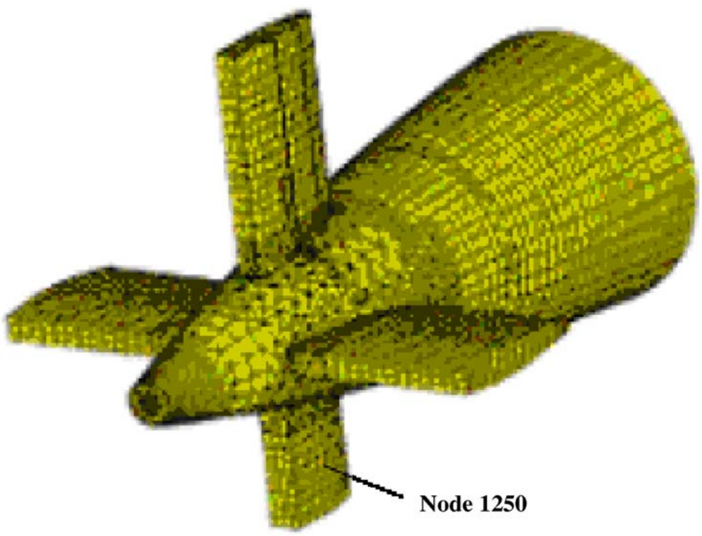


Fig. 6. The stern model.

are computed by FEM, the direct-BEM/FEM and approximate theory (Refs. [1,19–20]), respectively, and given in Table 1. Results in this table show that the added mass effect has been predicted by the direct-BEM/FEM and the approximate theory, and the two methods are in good agreement for these fluid–structure interaction modes. Therefore, in view of the similarity between the prediction of the direct-BEM/FEM and the approximate theory, the direct-BEM/FEM is reliable.

### *3.2. Numerical results and discussion*

Fig. 6 is the stern model, which is discretized with isoparametric linear shell elements, beam elements and solid elements. The adjacent infinite fluid domain is modeled with isoparameteric quadrilateral and triangular linear boundary elements. The total number of boundary elements is 26588.

Fig. 7 is the first 150 ''wet'' natural frequencies of the stern. It is shown that the ''wet'' frequencies are lower than the corresponding ''dry'' frequencies but the two curves become closer as the frequency increases, which is due to the decreased mass effect of fluid in the fluid–structure interaction. Figs. 8 and 9 are, respectively, the surface velocity of the submerged stern and the corresponding sound pressure at Node 1250.

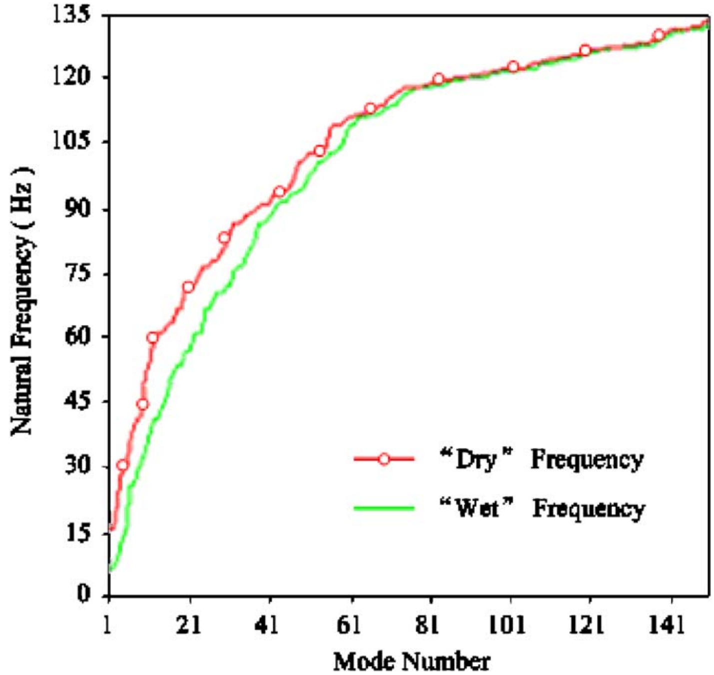


Fig. 7. The ‘‘dry’’ and ‘‘wet’’ frequencies of the stern.

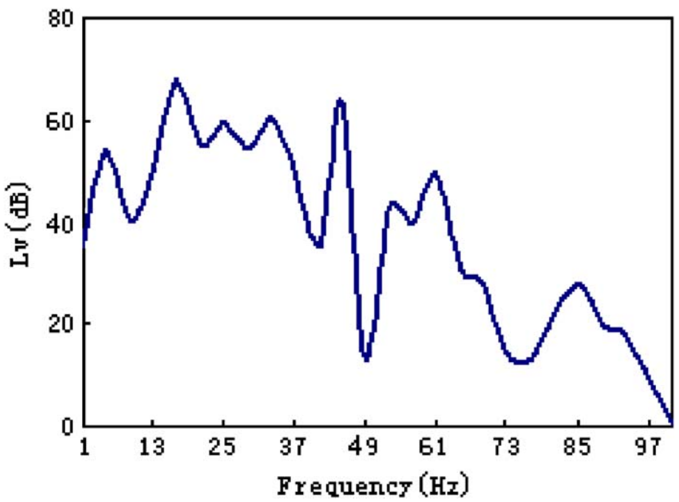


Fig. 8. Predicted velocity at node 1250.

## 4. Comparison of FEM and direct-BEM/FEM

In Table 2, the ‘‘wet’’ frequencies of Model are obtained by FEM and direct-BEM/FEM, respectively. From this table, we can see that the number of elements has been drastically reduced in the direct-BEM/FEM, and FEM has not given all the ‘‘wet’’ natural frequencies of the fluid–structure system as the direct-BEM/FEM. Furthermore, the computation time of FEM is almost 4 times that of the direct-BEM/FEM, but this comparison is made only on a PC with single Intel P4 CPU (2.0 GHz).

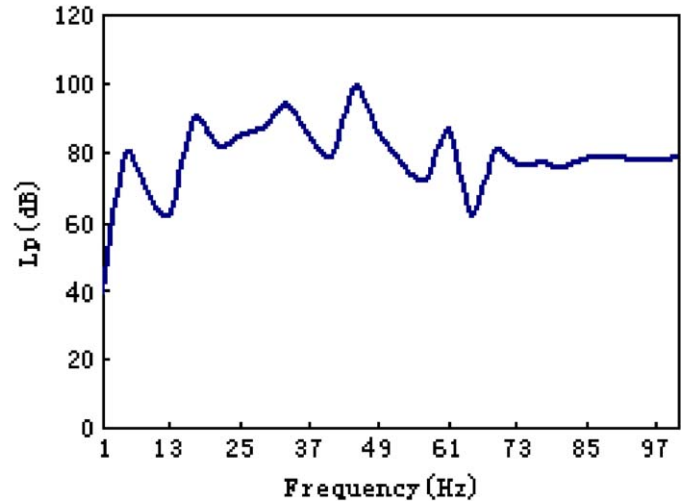


Fig. 9. Predicted sound pressure near node 1250.

Table 2
Computation results of the two numerical methods

| | | FEM | Direct-BEM/FEM |
|---|---|---|---|
| Number of elements | | 215876 | 26588 |
| Number of nodes | | 93438 | 8345 |
| Natural frequencies (Hz) | 1 | 5.7 | 5.8 |
| | 2 | — | 6.9 |
| | 3 | — | 7.4 |
| | 4 | — | 9.6 |
| | 5 | 12.6 | 12.3 |
| | 6 | — | 14.8 |
| | 7 | 19.1 | 18.7 |
| | 8 | — | 23.9 |
| | 9 | 31 | 30.1 |
| Computation time | | ≈49 h | ≈12 h |

## 5. Conclusions

To predict the sound radiation of irregularly shaped structures in the presence of fluid as well as to reduce the time of numerical computation, the direct-BEM/FEM technique is introduced. In this technique, modal parameters of the structure without fluid interaction or "dry" parameters are first obtained and the "wet" modal parameters of the fluid–structure coupled system and the sound radiation are subsequently computed. As compared with the method using FEM only, this approach has vastly reduced the number of elements and nodes as well as the time of numerical computation, and modal parameters are completely obtained. Therefore, the effectiveness of the direct-BEM/FEM technique has been numerically demonstrated in the analysis of fluid–structure coupled systems.

## References

[1] M.C. Junger, D. Feit, *Sound, Structures, and Their Interaction*, second ed., MIT Press, Cambridge, MA, 1986 460pp.

[2] P. Le Tallec, J. Mouro, Fluid structure interaction with large structural displacements, *Computer Methods in Applied Mechanics and Engineering* 190 (2001) 3039–3067.

[3] F.H. Hamdan, Near-field fluid–structure interaction using Lagrangian fluid finite elements, *Computers & Structures* 71 (1999) 123–141.
[4] M.N. Guddati, J.L. Tassoulas, Space–time finite elements for the analysis of transient wave propagation in unbounded layered media, *International Journal of Solids and Structures* 36 (1999) 4699–4723.
[5] Xinsen, R.W. Clough, *A Hybrid Substructure Approach for Fluid– Structure Interaction in Ship Vibration, Numerical Methods for Coupled Problems*, PINE RIDGE Press, 1981.
[6] G.C. Everstine, Structural–acoustic finite element analysis, with application to scattering, *Proceedings of the Sixth Invitational Symposium on the Unification of Finite Elements, Finite Differences, and Calculus of Variations*, University of Connecticut, Storrs, CT 1982, pp. 102–122.
[7] H.C. Chen, R.L. Taylor, Vibration analysis of fluid–solid systems using a finite element displacement formulation, *International Journal of Numerical Methods in Engineering* 29 (1990) 683–693.
[8] A.J. Kalinowski, C.W. Nebelung, Media–structure interaction computations employing frequency-dependent mesh size with the finite element method, *Shock Vibration Bulletin* 51 (1981) 173–193.
[9] C.A. Brebbia, J.C.F. Telles, L.C. Wrobel, *Boundary Element Techniques Theory and Applications in Engineering*, Springer, New York, 1983.
[10] T.L. Geers, Resident potential and approximation methods for three dimensional fluid–structure interaction problems, *Journal of the Acoustical Society of America* 49 (1971) 1505–1510.
[11] C.A. Felippa, Top–down derivation of doubly asymptotic approximations for fluid–structure interaction analyses, in: R.P. Shaw (Ed.), *Innvoative Numerical Analysis for Applied Engineering Sciences*, University of Virginia Press, Virginea, 1980, pp. 79–88.
[12] G.C. Everstine, Finite element formulations of structural acoustics problems, *Computers & Structures* 65 (1997) 307–321.
[13] G.X. Wu, R. Eatock Taylor, The coupled finite element and boundary element analysis of nonlinear interactions between waves and bodies, *Ocean Engineering* 30 (2003) 387–400.
[14] P. Bettess, Infinite elements, *International Journal of Numerical Methods in Engineering* 11 (1977) 53–64.
[15] P. Bettess, More on infinite elements, *International Journal of Numerical Methods in Engineering* 15 (1977) 1613–1626.
[16] E.T. Moyer Jr., Performance of mapped infinite elements for exterior wave scattering applications, *Communications in Applied Numerical Methods* 8 (1992) 27–39.
[17] G.C. Everstine, F.M. Henderson, Coupled finite element/boundary element approach for fluid–structure interaction, *Journal of the Acoustical Society of America* 87 (1990) 1938–1947.
[18] M. Wang, M. Sao, *The Basis Theory and Numerical Method of the Finite Element*, second ed., Tsinghua Unversity Publisher, 1997.
[19] G.C. Everstine, Prediction of low frequency vibrational frequencies of submerged structures, *Journal of Vibration and Acoustics* 13 (1991) 187–191.
[20] R.D. Blevins, *Formulas for Natural Frequency and Modes Shape*, Van Nostrand Reinhold Company, New York, 1979.