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Short Communication

A simple feedback control for a chaotic oscillator with limited power supply

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Abstract

We proposed a simple feedback control method to suppress chaotic behavior in oscillators with limited power supply. The small-amplitude controlling signal is applied directly to the power supply system, so as to alter the characteristic curve of the driving motor. Numerical results are presented showing the method efficiency for a wide range of control parameters. Moreover, we have found that, for some parameters, this kind of control may introduce coexisting periodic attractors with complex basins of attraction and, therefore, serious problems with predictability of the final state the system will asymptote to.

1. Introduction

Chaotic behavior is a very common dynamical regime identified, during the last decades, in many dynamical systems of interest in various fields like engineering [1,2], physics [3] and biochemistry [4], just to mention a few. A dynamical system is said to be chaotic whenever its evolution is aperiodic and depends sensitively on the initial condition. Due to these inherent difficulties for the prediction of the future state of the system, chaotic behavior has usually been regarded as undesirable and thus to be strongly avoided, mainly in mechanical systems designed for technological applications.

It turns out, however, that chaotic behavior, if properly handled, can be of practical interest in real-world applications, since there is an infinite number of unstable periodic motions embedded in a chaotic attractor. Among these infinite orbits, there may be some of them which, if properly stabilized, can yield an enhanced system performance. The utility of chaotic motion would be thus linked to the means of control chaotic motion in order to steer a trajectory toward such a periodic orbit.

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A major step toward the achievement of this goal occurred in 1990, when a chaos control procedure was proposed by Ott et al. [5]. This procedure is known nowadays as the OGY method and has had a great impact on nonlinear science. The OGY method consists on stabilizing a desired unstable periodic orbit embedded in a chaotic attractor by using only a tiny perturbation on an available control parameter. This is in marked contrast with usual control methods, as those used for periodic motions, for which tiny perturbations cause only small-size effects.

The procedure introduced by the OGY method is based on the ergodic properties shared by chaotic attractors. In fact, a chaotic attractor has an infinite number of unstable periodic orbits embedded in it and a typical trajectory in the attractor eventually visits the neighborhood of each periodic orbit, regardless of how small this neighborhood may be. Consequently, by controlling chaotic motion, the system can exhibit a flexible performance by switching the time asymptotic behavior from one periodic orbit to another. This kind of behavior can be desirable in a variety of applications, where one of these periodic orbits provides better performance than others in a particular situation.

Another interesting chaos control strategy was proposed by Pyragas [6]. The Pyragas method also considers the dynamical properties of a chaotic attractor to stabilize unstable periodic orbits. However, in this case the method implementation requires a delayed feedback signal. After the appearance of OGY and Pyragas methods, a wide variety of control chaos strategies was developed and verified experimental and numerically.

Recently, a new kind of feedback control method was proposed independently by Tereshko and co-workers [7] and Alvarez-Ramirez et al. [8]. This method consists in suppressing chaos by using a small-amplitude control signal, applied to alter the energy of a chaotic system so as to steer its trajectory to a stable periodic orbit. As an example, it was considered that the double-well Duffing oscillator equation with a feedback signal given by

$$\ddot{x} + 0.3\dot{x} - x + x^3 = 0.31\cos(1.2t) + f(\dot{x}),$$

where $f(x) = -0.06\tanh(2.0\dot{x})$ was chosen as the control function [7]. In a recent work we have applied this control technique to suppress chaotic behavior in a chaotic impact oscillator [9].

These methods have been applied for systems whose energy sources are described by a harmonic function. However, in several mechanical experiments the oscillator cannot be driven by systems whose amplitude and frequency are arbitrarily chosen, since the forcing source has a limited available energy supply. Such energy sources have been called non-ideal, and the corresponding system a non-ideal oscillator [10,11]. For this kind of oscillator, the driven system cannot be considered as given a priori, but it must be taken as a consequence of the dynamics of the whole system (oscillator and motor). In other words, a non-ideal oscillator is, in fact, the combined dynamical system resulting from the coupling of a *passive* oscillator and an *active* oscillator which serves as the driving source for the first one. The resulting motion will be thus the outcome of the dynamics for the combined system. Strictly speaking, any forced oscillator would be non-ideal, since the driving must come from a physical entity (as an external vibrator or an inbound unbalanced rotor fed by a motor) which has a limited energy supply. Previous works were devoted to the stabilization of non-ideal oscillators using impact dampers [12] and tuned liquid column dampers [13].

In this work, we employ the method of controlling chaos with a small-amplitude signal proposed in Refs. [8,7] by applying it to a non-ideal oscillator. In our proposed method, the feedback signal is applied to the power supply system instead of directly to the oscillator. In other words, the control function is not associated with a velocity of the cart, as in the Duffing oscillator example considered in Ref. [7]. In our case, the function is associated with a dynamical variable of the motor in such way that it represents a change in the a characteristic curve of driven motor.

This paper is structured as follows: in Section 2 we describe the model equations for the oscillator with limited power supply. Section 3 explores some aspects of the model dynamics from numerical simulations, emphasizing the performance of the control method. Our conclusion is presented in Section 4.

2. Theoretical model

In the following we will consider the one-dimensional motion of a cart of mass M connected to a fixed frame by a nonlinear spring and a dash-pot (viscous coefficient c) (Fig. 1). The nonlinear spring stiffness is given by

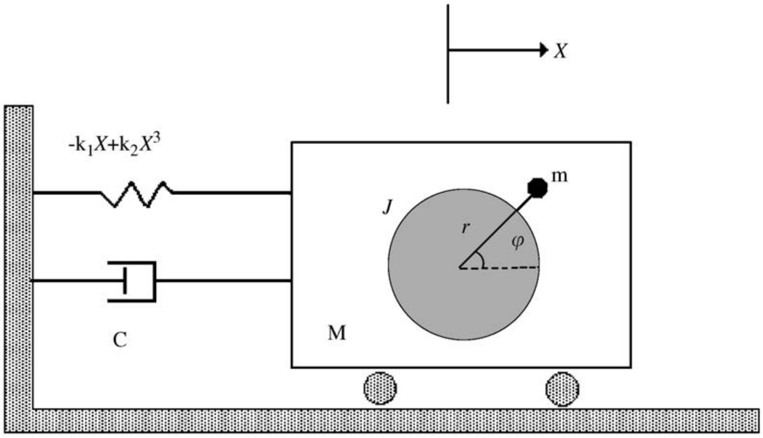

Fig. 1. Schematic model of a non-ideal system.

$k_1X - k_2X^3$, where X denotes the cart displacement with respect to some equilibrium position in the absolute reference frame, and k_1, k_2 are positive elastic constants. The motion of the cart is due to an in-board non-ideal motor with moment of inertia J and driving an unbalanced rotor. We denote by φ the angular displacement of the rotor, and model it as a particle of mass m attached to a massless rod of radius r with respect to the rotation axis. Here $\tilde{E}_1$ and $\tilde{E}_2$ are damping coefficients for the rotor, which can be estimated from the characteristic curve of the energy source (a DC motor) [12–14].

The motion of the cart is governed by the following equations [14]:

$$M\frac{\mathrm{d}^2X}{\mathrm{d}t^2} + c\frac{\mathrm{d}X}{\mathrm{d}t} - k_1X + k_2X^3 = mr\left[\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}\sin\varphi + \left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2\cos\varphi\right], \tag{1}$$

$$(J + mr^2)\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = mr\frac{\mathrm{d}^2X}{\mathrm{d}t^2}\sin\varphi + \tilde{E}_1 - \tilde{E}_2\frac{\mathrm{d}\varphi}{\mathrm{d}t} + U(\dot{\varphi}), \tag{2}$$

where $U(\dot{\varphi})$ is a controlling function which, in the spirit of Refs. [8,7], alters the oscillator energy to stabilize chaotic oscillations into a desired periodic orbit. The difference between the method we adopt in this paper and the OGY technique is that the periodic orbit which we aim to stabilize does not necessarily exist in the unperturbed system (i.e., the periodic orbit needs not to be embedded in the uncontrolled chaotic attractor).

The general idea of the control method developed in Refs. [8,7] is that the transition to chaotic attractors, as a system parameter is varied, increases the oscillator energy, averaged over a characteristic period. Many chaotic attractors arise from period-doubling bifurcation cascades, and periodic orbits have higher averaged energy the higher are their periods. Hence, the control strategy is to modify the oscillator energy by using the controlling function $U(\dot{\varphi})$ so as to stabilize higher (lower)-energy period orbits by increasing (decreasing) the averaged oscillator energy.

It is convenient to work with dimensionless positions and time, according to the following definitions:

$$X \to x \equiv \frac{X}{r}, \tag{3}$$

$$Y \to y \equiv \frac{Y}{r}, \tag{4}$$

$$t \to \tau \equiv t\sqrt{\frac{k_1}{M}}, \tag{5}$$

in such a way that Eqs. (1) and (2) are rewritten in the following form:

$$\ddot{x} + \beta\dot{x} - x + \delta x^3 = \varepsilon_1(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi), \tag{6}$$

$$\ddot{\varphi} = \varepsilon_2 \ddot{x} \sin\varphi + E_1 - E_2\dot{\varphi} + u(\dot{\varphi}), \tag{7}$$

where the dots stand for differentiation with respect to the scaled time τ, and the following abbreviations were introduced:

$$\beta \equiv \frac{c}{\sqrt{k_1 M}}, \quad \delta \equiv \frac{k_2}{k_1} r^2, \quad \varepsilon_1 \equiv \frac{m}{M}, \tag{8}$$

$$\varepsilon_2 \equiv \frac{mr^2}{J + mr^2}, \quad E_1 \equiv \frac{\tilde{E}_1 M}{k_1(J + mr^2)}, \quad E_2 \equiv \frac{\tilde{E}_2 M}{J + mr^2}\sqrt{\frac{M}{k_1}}, \tag{9}$$

and the controlling function is given by

$$u(\dot{\varphi}) = -k \tanh(\eta(\dot{\varphi} - \xi)), \tag{10}$$

where k, η, and ξ are parameters characterizing the control through modification in the oscillator energy.

This choice of controlling function is not unique, but it must be compatible with dynamical requirements. Firstly, in order to avoid undesirable instabilities stemming from the new dynamics, the control should be represented by a bounded function of the velocities. We also require that $u(\dot{\varphi})$ be an odd function of its arguments. Finally, we choose u such that the control effect vanishes a for specific velocity. The tanh function of Eq. (10) has been suggested in Ref. [7]. The control amplitude parameter k is varied so as to change the averaged oscillator energy to values corresponding to desired stable periodic orbits.

3. Dynamical analysis of the non-ideal system with controller

For numerical simulations, we fix the scaled parameter values as $\beta = 0.02$, $\varepsilon_1 = 0.05$, $\delta = 1.0$, $\varepsilon_2 = 0.25$, $E_1 = 2.1$, and $E_2 = 1.6$. Fig. 2 shows phase portraits for the cart motion (velocity versus displacement of the cart). When there is no control acting on the system, we found for this set of parameter values two coexisting attractors in the phase plane: a periodic (C_1) and a chaotic one (C_2). When the control is activated, the change in the formerly existent periodic attractor (C_1) is practically not noticeable (this new periodic attractor is not shown in Fig. 2). Nevertheless, the chaotic attractor is replaced by one of the two newborn periodic attractors (represented in Fig. 2 by the black curves immersed in the chaotic attractor depicted in gray), named as D_1 and D_2, respectively.

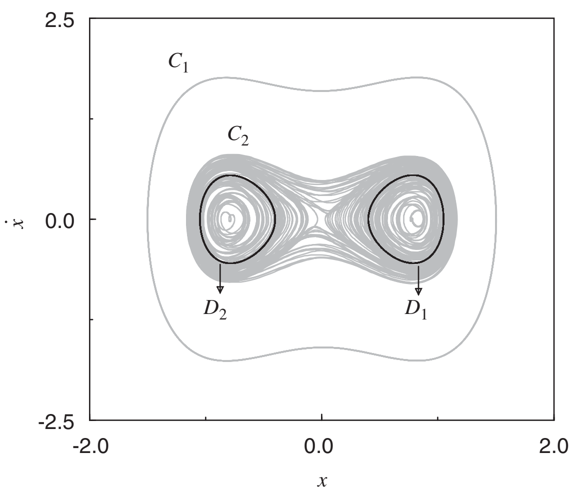

Fig. 2. Velocity versus displacement of the cart showing a chaotic attractor (C_2) and three periodic attractors (C_1, D_1, D_2). Gray curves represent the situation without control, whereas black curves refer to the application of a control function with parameters $k = 0.2$, $\eta = 50.0$ and $\xi = 1.3$.

In order to find appropriate values of the parameter ξ in Eq. (10) that specify the phase of our control function, we plot in Fig. 3(a) the evolution of variable $\dot{\varphi}$ for the chaotic attractor C_2 (shown in Fig. 2). From our numerical tests we estimated that the parameter ξ should take on a value about 1.3, in order to assure that the control function be effective. Thus, with this value of ξ, the energy source function, defined as

$$\mathscr{M}(\dot{\varphi}) = E_1 - E_2\dot{\varphi} - k\tanh[\eta(\dot{\varphi} - \xi)], \tag{11}$$

can increase or decrease as required to keep the system under control.

In Fig. 3(b), we note the alteration of characteristic curve of the energy source (DC motor) due to control. Without the control the motor oscillation can be described by a characteristic curve given by the evolution of the energy source function $\mathscr{M}$. This characteristic curve is changed by the feedback control, as shown in Fig. 3(b). This change depends on the chosen parameters k, η, and ξ.

In order to characterize quantitatively the attractors involved in this study, we computed the Lyapunov exponents by using Gram–Schmidt orthonormalization, as found in the Wolf–Swift–Swinney–Vastano algorithm [15]. Fig. 4 shows the time evolution of the three largest Lyapunov exponents for the uncontrolled chaotic attractor C_2 (Fig. 4(a)) and the controlled periodic attractor D_1 (Fig. 4(b)). As expected, for the chaotic attractor, one of the Lyapunov exponents is a positive number, whereas for the periodic attractor, there are no positive exponents. Table 1 contains all the stationary values of the Lyapunov exponents for these four attractors.

Fig. 5(a) exhibits the basins of attraction of the uncontrolled attractors C_1 and C_2 shown in Fig. 2. The basin of the periodic attractor, C_1, is depicted in gray and the chaotic attractor, C_2, in white, for a grid of

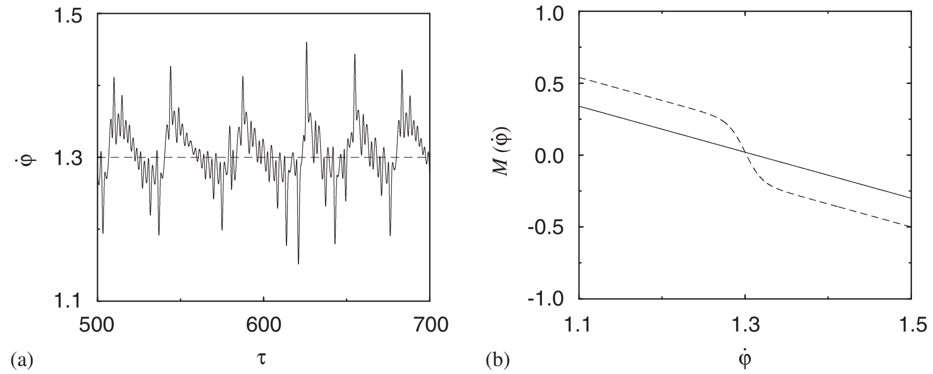

Fig. 3. (a) Time evolution of the rotor velocity for the chaotic attractor shown in Fig. 2; (b) energy source function $\mathscr{M}(\dot{\varphi})$ for the uncontrolled (solid line) and the controlled case (dashed line) for the same parameters of the previous figure.

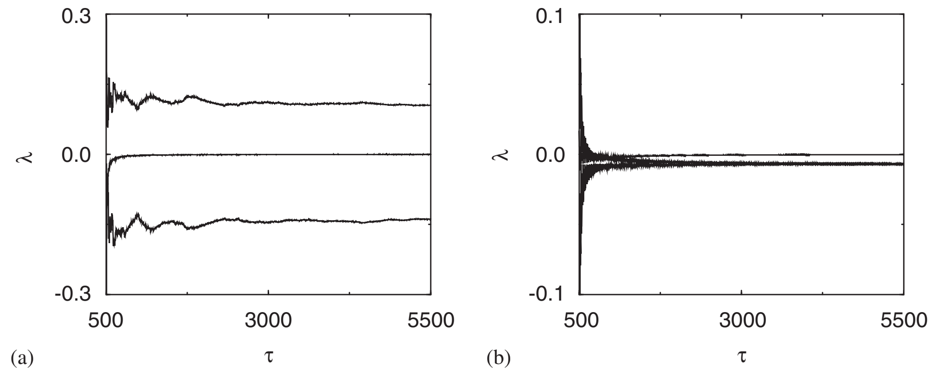

Fig. 4. Time evolution of the three largest Lyapunov exponents for the attractors shown in Fig. 2, C_2 (a) and D_1 (b).

Table 1
Lyapunov exponents for the non-ideal system

C_1	C_2	D_1	D_2
0.0005	0.1053	−0.0003	−0.0005
−0.1033	−0.0003	−0.0072	−0.0069
−0.1035	−0.1396	−0.0066	−0.0067
−1.4244	−1.5958	−10.9327	−10.9299

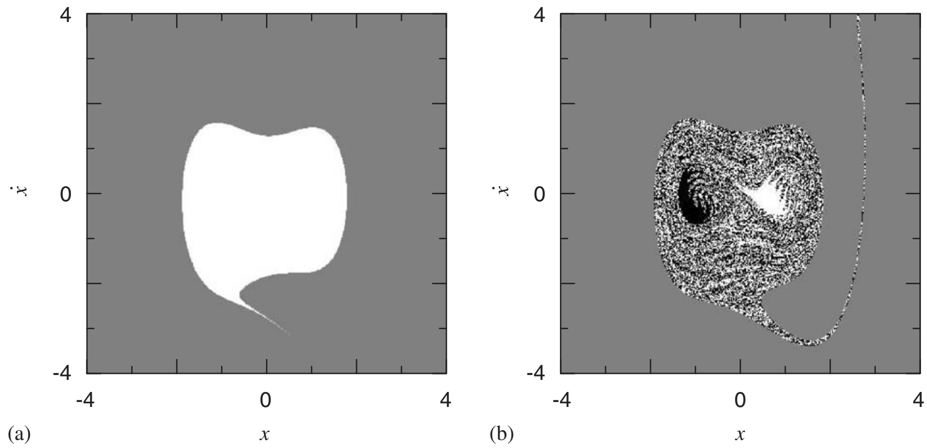

Fig. 5. Basins of attractors for the attractors shown in Fig. 2: (a) C_1 (gray) and C_2 (white); (b) C_1 (gray), D_1 (white) and D_2 (black).

400×400 pixels. In this case, there are two distinct regions with a smooth boundary. When we applied the control method, as mentioned before, the chaotic regime has been replaced by either one of the two previously existent periodic regimes (D_1 or D_2, shown in Fig. 2), the periodic regime, C_1, being kept practically unchanged. Therefore, for the controlled system, there are three periodic attractors (D_1, D_2 and the modified C_1) whose basins of attractions are presented in Fig. 5(b).

The gray region depicted in Fig. 5(b) represents the basin of the modified attractor C_1, and it turns out that its overall aspect is similar to that attractor for the case without control. However, the filamentary extension in the right side of Fig. 5(b) becomes larger than in the uncontrolled case, for which it is not visible in Fig. 5(a) due to insufficient graphical resolution. In addition, as can be seen in Fig. 6, where a magnification of a small box selected inside of Fig. 5(b), the basin structures of the periodic attractors, D_1 and D_2, are very complex with a fractal boundary [16]. Consequently, there is an undesirable effect of the final-state sensitivity in the phase space.

Figs. 7(a) and (b) present bifurcation diagrams in which the asymptotic dynamical state of the cart, after a transient regime, is plotted against the parameters of the control function k and η. As we can see, the control function used in this paper works efficiently to suppress chaotic motion and steer the system dynamics to a stable low-period attractor for wide ranges of the control function parameters.

4. Conclusions

We presented a procedure to suppress chaotic behavior in non-ideal oscillators using a small-amplitude signal associated with the power supply in such a way that the control signal alters the characteristic curve of the motor. This method is based in the fact that, altering the averaged energy of the oscillator, one can steer the system trajectories from a chaotic attractor to a periodic orbit which would give improved system

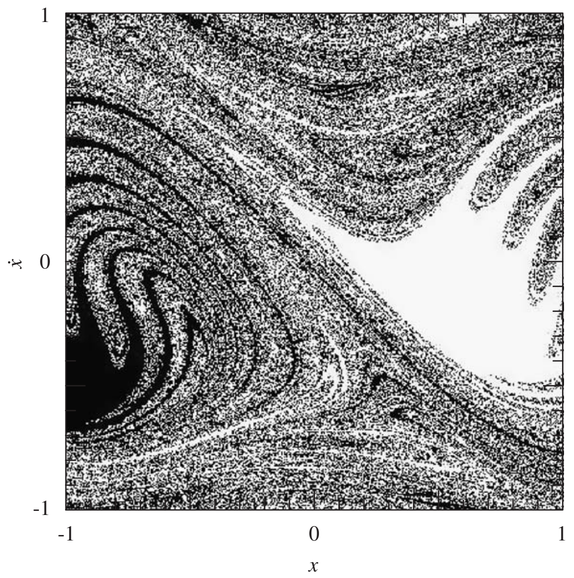

Fig. 6. Magnification of a portion of the Fig. 5(b) showing finer details of the basin structure.

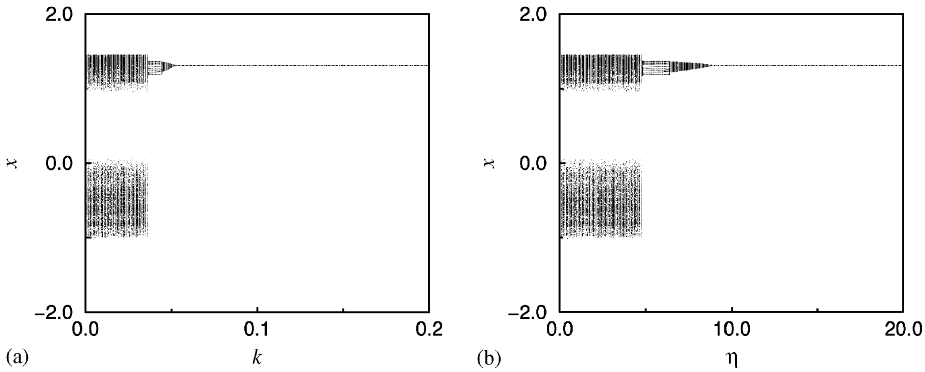

Fig. 7. Bifurcation diagram of the cart displacement in terms of the control parameters (a) k, for $\eta = 50$; (b) η, for $k = 0.2$.

performance. In the application presented, the proposed method works efficiently for a large range of control parameters.

However, as in other feedback controls, the control strategy used in this work may introduce coexisting attractors whose basins of attraction form a complex structure. These complex basins introduce a certain degree of unpredictability on the final controlled state. This may be a serious problem if one of the attractors corresponds to an undesired behavior. To avoid that, the dynamics could be further explored to determine the most appropriated phase space region, outside the complex basin structure, to start applying the feedback control. Knowing this region would allow us to use a targeting method to improve the control efficiency. For example, the control could be applied whenever the trajectory approaches the desired periodic attractor. In the system considered in this work, this particular procedure could be easily followed.

Acknowledgments

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