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Short Communication

Simultaneous optimal structure and control design of flexible linkage mechanism for noise attenuation

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Abstract

Integrated structural and control design of linkage mechanism for noise attenuation is studied. Firstly, a new set of equations of motion is obtained by employing a set of wavenumber transformations to the original equations of motion of the flexible linkage mechanisms, in which the unknown variables can describe the structural acoustic radiation level of the mechanism directly. The new equation can be used to evaluate the structural acoustic radiation performance. Secondly, the performance index for simultaneous mechanism and noise control design is discussed. The weighted sum of structural mass and the energy of the controlled mechanism is selected as the performance index. Finally, numerical simulations are carried out on a four-bar linkage mechanism. Simulation results show that the acoustic radiation and the control inputs of the optimized mechanism are improved significantly.
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1. Introduction

Mechanisms are now being required to run at higher speeds while maintaining low noise radiation and high positioning accuracy [1,2]. As we know, the high-speed mechanisms have an aptitude to vibrate when they suffer exterior excitations, and strong acoustic radiation is produced. So it is necessary to look for effective methods to suppress the structural acoustic radiation. Many noise suppression methods such as absorption and insulation have been developed [3]. These methods use either absorbent material to dissipate the acoustical energy along the noise propagation path or barriers to obstruct the noise propagation, which may be referred to as passive control methods. However, the noise reduction capability of the acoustic absorbent material is effective only at middle and high frequencies. Active noise and vibration control technique serves as a useful alternative to passive control method, which provides a numerous advantages such as improving the low-frequency performance and programmable flexibility of design [3]. As for flexible linkage mechanisms, the mode frequencies are usually relatively low, and the active control technique will be an ideal choice.

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In recent years, considerable attention has been paid to the vibration and noise control for high-speed flexible mechanisms. Sung and Chen [4] attempted to control the elastodynamic response of a four-bar linkage mechanism with a flexible follower. Two patches of piezoceramic actuators were bonded to the surface of the follower at two locations that are symmetric with respect to its midpoint. Optimal linear quadratic regulator (LQR) is employed in the control system. Liao and Sung [5] gave a finite element model of the same mechanism, and studied the active vibration control problem both analytically and experimentally based on the linear quadratic Gaussian (LQG) theory. Choi et al. [6] studied the vibration control problem for a slider-crank mechanism, in which the connecting rod was considered as a flexible member. Sannah and Smaili [7] presented an experimental investigation for controlling the elastodynamic response of a four-bar mechanism with a flexible coupler, a slightly less flexible follower, and a rigid crank. A controller, which consists of a LQR and a Luenberger observer, was designed and implemented. Zhang et al. [8] studied the on-line vibration control problems of the flexible mechanisms based on the independent modal space control theory, in which all of the members were considered as flexible. Zhang et al. [9,10] developed a mathematical modeling approach for the high-speed flexible linkage mechanisms equipped with bonded piezoelectric sensing/actuating elements, and the active vibration control is discussed with reduced modal control method and H_∞ control method. Considering both geometric and inertial nonlinearities, Caracciolo and Trevisani [11] developed a control scheme for a flexible four-bar linkage in which the rigid-body motion and vibration control were considered simultaneously. Considering the position and vibration performances, Trevisani [12] presented a feedback control scheme for a four-bar linkage. Caracciolo et al. [13] further investigated the problem, and a mixed H_2/H_∞ control scheme was developed. Recently, Lu et al. [14] develop an active noise control method for flexible linkage mechanisms.

As stated above, active control is used to improve dynamic characteristics such as transient response and noise radiation. The strong coupling between structural dynamics and active control is a well-recognized challenge in the analysis of controlled flexible mechanisms. In the traditional design approach, mechanism parameters are optimized first and then an optimal controller is designed. The design of the flexible mechanisms and the control systems are usually carried out separately. The mathematical model for the mechanism optimum, or at least the final mechanism design, constitutes the plant for the control design. The control designer then proceeds to synthesize the optimal control system for the given plant. Uncertainties in mechanism modeling of the plant are often considered to ensure that the control system is robust with respect to plant errors. The couple between the mechanism and the control system becomes an essential factor in the design [15]. The integrated structural and control design problem was first considered by Hale et al. [16]. Up to now, the problem has received much attention in the field of flexible structural design [17,18]. But the research is mainly to focus on some of the simple systems such as beam and plate. The problem of integrated flexible mechanism design and noise control has not been extensively treated despite it is very important.

This study is mainly to focus on how to reduce the structural acoustic radiation of the flexible mechanism. Other noise sources such as motors, transmissions, and aerodynamic noise are not taken into account, which may be studied separately. In this study, firstly, a new set of equations of motion is obtained by employing a set of wavenumber transformations on the original equations of motion of the flexible linkage mechanisms, in which the unknown variables can describe the structural acoustic radiation level of the mechanism directly. Secondly, the performance index for simultaneous mechanism and noise control design is discussed. The weighted sum of the structural mass, the vibration energy and the energy of the control system of the mechanism are selected as the performance index, an integrated optimal design model is developed. Finally, numerical simulations are presented to demonstrate the validity of the proposed method.

2. Formulation of control model

According to the Hamilton and finite element theory, the equations of motion of the flexible linkage mechanism can be expressed as [9]

$$\bar{\mathbf{M}}\ddot{\bar{\mathbf{q}}} + \bar{\mathbf{C}}\dot{\bar{\mathbf{q}}} + \bar{\mathbf{K}}\bar{\mathbf{q}} = \bar{\mathbf{F}}, \tag{1}$$

where $\bar{\mathbf{M}}$, $\bar{\mathbf{C}}$ and $\bar{\mathbf{K}}$ are the systematic mass, damping and stiffness matrices, respectively; $\bar{\mathbf{F}}$ is the vector of the disturbance forces; $\bar{\mathbf{q}}$, $\dot{\bar{\mathbf{q}}}$, $\ddot{\bar{\mathbf{q}}}$ are the generalized displacement, the velocity, and the acceleration vectors of the linkage mechanism, respectively. According to the acoustic theory, sound radiation is determined by the normal velocity of the structure surface. So it is necessary to extract the surface normal components of each link from the displacement vector $\bar{\mathbf{q}}$. By utilizing a series transformation [14], one obtains a new set of equations of motion as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}, \tag{2}$$

where $\mathbf{q}$ is the surface normal displacement vector of each link of the mechanism, and

$$\mathbf{M} = \mathbf{\Gamma}^{\mathrm{T}}\bar{\mathbf{M}}\mathbf{\Gamma}, \quad \mathbf{C} = \mathbf{\Gamma}^{\mathrm{T}}\bar{\mathbf{C}}\mathbf{\Gamma}, \quad \mathbf{K} = \mathbf{\Gamma}^{\mathrm{T}}\bar{\mathbf{K}}\mathbf{\Gamma}, \quad \mathbf{F} = \mathbf{\Gamma}^{\mathrm{T}}\mathbf{\Gamma}\mathbf{F}_n,$$

$\mathbf{\Gamma}$ is the transformation matrix; $\mathbf{F}_n$ is the disturbance force vector corresponding to vector $\mathbf{q}$. The equations of motion of the controlled mechanism system can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{D}\mathbf{u} + \mathbf{F}, \tag{3}$$

where $\mathbf{u}$ is the control force vector, $\mathbf{D}$ is the distribution matrix of the control force.

According to the expansion theorem, vector $\mathbf{q}$ can be rewritten as

$$\mathbf{q} = \sum_{i=1}^{N_m} \phi_i \eta_i = \mathbf{\Phi}\boldsymbol{\eta}, \tag{4}$$

where N_m is the number of the selected modes, $\mathbf{\Phi}$ is the eigenvector matrix consisting of N_m selected eigenvectors, $\boldsymbol{\eta}$ is the modal coordinate vector. Employing wavenumber transformation and inverse wavenumber transformation [14,19] on Eq. (4) yields

$$\mathbf{q} = \mathbf{W}\mathbf{p}, \tag{5}$$

where

$$\mathbf{W} = (\mathbf{\Psi}\mathbf{\Phi}^{\mathrm{T}}\mathbf{M})^{-1}, \tag{6}$$

$$\mathbf{p} = \sum_{i=1}^{N_m} \psi_i \eta_i = \mathbf{\Psi}\boldsymbol{\eta}, \tag{7}$$

$\mathbf{\Psi}$ is a matrix consisting of N_m vectors of $\boldsymbol{\psi}_i$ ($i = 1, 2, \ldots, N_m$), which can be expressed as

$$\psi_i = \int_{-k}^{k} \left(\int_{-\infty}^{\infty} \phi_i \mathrm{e}^{-\mathrm{j}k_x x} \, \mathrm{d}x \right) \mathrm{e}^{\mathrm{j}x k_x} \, \mathrm{d}k_x \quad (i = 1, 2, \ldots, N_m),$$

where k_x is the component of the acoustic wavenumber in x direction, and $k = \omega/c$, ω is angular frequency of the disturbance, c is the spread speed of sound in the air.

For a given position of the crank, the eigenvector matrix $\mathbf{\Phi}$ is known, so $\mathbf{\Psi}$ and $\mathbf{W}$ are also known. Substituting Eq. (5) into Eq. (3) and pre-multiplying Eq. (3) by $\mathbf{W}^{\mathrm{T}}$, yields

$$\mathbf{M}_e\ddot{\mathbf{p}} + \mathbf{C}_e\dot{\mathbf{p}} + \mathbf{K}_e\mathbf{p} = \mathbf{W}^{\mathrm{T}}\mathbf{D}\mathbf{u} + \mathbf{W}^{\mathrm{T}}\mathbf{F}, \tag{8}$$

where

$$\mathbf{M}_e = \mathbf{W}^{\mathrm{T}}\mathbf{M}\mathbf{W}, \ \ \mathbf{C}_e = \mathbf{W}^{\mathrm{T}}\mathbf{C}\mathbf{W}, \ \ \mathbf{K}_e = \mathbf{W}^{\mathrm{T}}\mathbf{K}\mathbf{W}.$$

Eq. (8) can be regarded as the equations of motion of a "virtual linkage mechanism", the displacement of the "virtual linkage mechanism" $\mathbf{p}$ directly reflect the acoustic radiation of the original mechanism. So it is more convenient to study the acoustic radiation control problem using Eq. (8).

Defining the state vector $\mathbf{x}$ as

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{bmatrix}, \tag{9}$$

then the state space form of Eq. (8) is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\bar{\mathbf{u}}, \tag{10}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_e^{-1}\mathbf{K}_e & -\mathbf{M}_e^{-1}\mathbf{C}_e \end{bmatrix}, \tag{11}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_e^{-1}\mathbf{D} \end{bmatrix}, \tag{12}$$

$$\bar{\mathbf{u}} = \mathbf{u} + \mathbf{D}^{+}\mathbf{F}, \tag{13}$$

and $\mathbf{D}^{+}$ is the general inverse matrix of $\mathbf{D}$.

3. Simultaneous optimal design of structure and control parameters

According to the optimal control theory, the optimal control forces can be calculated by

$$\bar{\mathbf{u}} = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{x}. \tag{14}$$

Matrix $\mathbf{P}$ can be obtained by solving the following Riccati equation [20]

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} = 0, \tag{15}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are weight matrices, which can be taken as

$$\mathbf{Q} = \begin{bmatrix} \boldsymbol{\alpha}\boldsymbol{\Omega}^2 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\beta}\mathbf{I} \end{bmatrix}, \tag{16}$$

$$\mathbf{R} = \lambda\mathbf{I}, \tag{17}$$

where $\mathbf{I}$ is the unit matrix; $\boldsymbol{\Omega}$ is the diagonal matrix of frequency of the mechanism, which is determined by the structural parameters; α, β, and λ are the weightings which can also be treated as design variables.

From the analysis above, one finds that the optimal control inputs are determined both by the structural parameter vector, $\boldsymbol{\xi}_s$, and the control parameter vector, $\boldsymbol{\xi}_c = [\alpha \;\; \beta \;\; \lambda]^T$. So the design variables for the whole mechanism can be expressed as

$$\boldsymbol{\chi} = (\boldsymbol{\xi}_s, \boldsymbol{\xi}_c)^{\mathrm{T}}. \tag{18}$$

The objective of this research is to minimize the acoustic radiation of the mechanism. So the acoustic radiation performance of the mechanism is one of the most important factors to be considered. Furthermore, the mass of the mechanism and the energy cost by the control system are also should be considered. As the energy of the structural acoustic radiation of the original linkage mechanism is equivalent to that of the vibration energy of the "virtual linkage mechanism". So one can select the energy of the vibration and the consumption of the control system of the "virtual linkage mechanism" in one cycle of operation as the performance index. Summary above, the objective function can be written as

$$J = w_1 f_m + w_2 f_e = w_1 W(\boldsymbol{\xi}_s) + w_2 \int_{t_0}^{t_0+T} \left[\mathbf{x}(\boldsymbol{\xi}_s, \boldsymbol{\xi}_c)^{\mathrm{T}}\mathbf{Q}\mathrm{x}(\boldsymbol{\xi}_s, \boldsymbol{\xi}_c) + \mathbf{u}(\boldsymbol{\xi}_c)^{\mathrm{T}}\mathbf{R}\mathbf{u}(\boldsymbol{\xi}_c)\right] \mathrm{d}t, \tag{19}$$

where f_m, f_e is the structural mass and the existing energy, respectively, w_1, w_2 are the weighting coefficients, which can be determined by

$$w_1 = \left|\frac{\partial f_e}{\partial \|\boldsymbol{\chi}\|}\right|_{\chi^{(0)}}, \quad w_2 = \left|\frac{\partial f_m}{\partial \|\boldsymbol{\chi}\|}\right|_{\chi^{(0)}}, \tag{20}$$

where $\boldsymbol{\chi}^{(0)}$ is the initial value of $\boldsymbol{\chi}$.

In accordance with analysis above, the simultaneous mechanism and control optimization problem can be stated as follows:

Find the design variables $\boldsymbol{\chi} = (\xi_s, \xi_c)^{\mathrm{T}}$ that minimize the objective

$$\text{Min} \left(w_1 W(\xi_s) + w_2 \int_{t_0}^{t_0+T} \left[\mathbf{x}(\boldsymbol{\chi})^{\mathrm{T}} \mathbf{Q} \mathbf{x}(\boldsymbol{\chi}) + \mathbf{u}(\xi_c)^{\mathrm{T}} \mathbf{R} \mathbf{u}(\xi_c) \right] \mathrm{d}t \right). \tag{21a}$$

The constraints may include a number of compatible behavior constraints such as frequencies, stresses, etc. In this study, we just consider the stress and the control force constrains. Considering the design variables should be positive, the whole constraints can be expressed as

$$\boldsymbol{\sigma}_{\max} \leqslant [\boldsymbol{\sigma}], \tag{21b}$$

$$\mathbf{u}_{\max} \leqslant [\mathbf{u}], \tag{21c}$$

$$\chi_i > 0, \quad i = 1, 2, \ldots, n_d, \tag{21d}$$

where $[\boldsymbol{\sigma}]$ and $[\mathbf{u}]$ are the allowed stress and control force, respectively, n_d is the number of the design variables, and

$$\boldsymbol{\sigma}_{\max} = E \mathbf{B}_{1i}(\boldsymbol{\chi}) \mathbf{R}_T \mathbf{B}_{2i} \bar{\mathbf{q}}, \tag{22}$$

where E is the Young's modulus, $\mathbf{B}_{1i}$ is the strain matrix of the element having the largest stress, $\mathbf{R}_{\mathrm{T}}$ is the transformation matrix between the reference coordinate and to the element coordinate [16]; $\mathbf{B}_{2i}$ is the concerted matrix of the coordinates, which element is 1 or 0.

There are many algorithms available to solve this kind of nonlinear constrained problem. The updated constrained variable metric method [21] is used in this paper. The variable metric part comes from looking at the global topology of the problem in addition to the local information given by the derivatives. This algorithm is both robust and efficient. The flowchart of integrated optimal design of flexible mechanism and noise control is illustrated in Fig. 1.

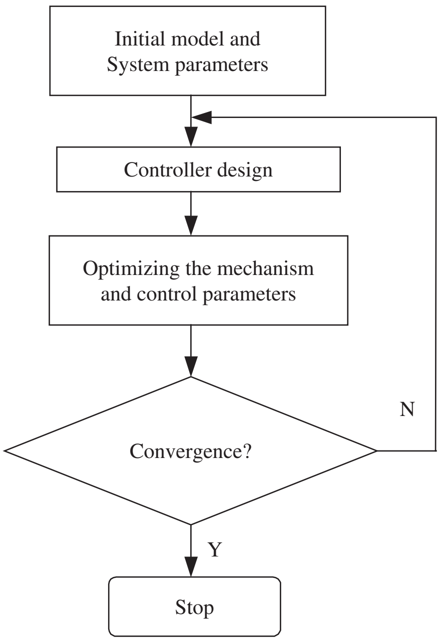

Fig. 1. Integrated control and mechanism optimization.

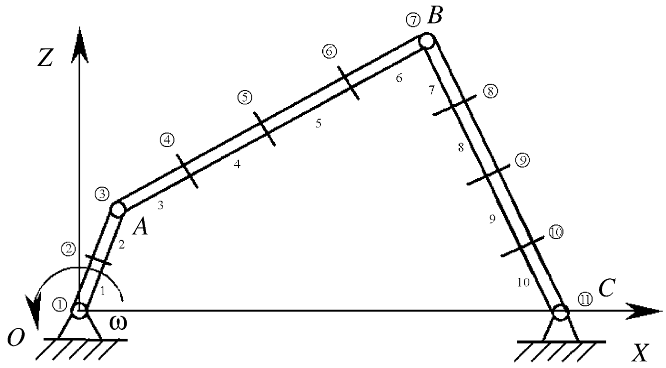

Fig. 2. Four-bar linkage mechanism.

Table 1
Results of the optimization

Design variable vector χ		Objective function		Number of iteration
Initial values	Optimal results	Initial value	Final value	
0.0100	0.0101	6.1781E−4	4.8642E−4	16
0.0100	0.0120			
0.0100	0.0202			
1.0000	5.0382			
1.0000	3.1711			
1.0000	4.5881			

4. Numerical example and discussion

In order to verify the validity of the proposed methodology, a computer simulative analysis is carried out on a four bar linkage mechanism shown in Fig. 2. It was constructed of aluminum strips having the following dimensions: $l_{OA} = 0.1\,\mathrm{m}$, $l_{AB} = 0.48\,\mathrm{m}$, $l_{BC} = l_{OC} = 0.4\,\mathrm{m}$, the width of each link is 0.048 m. The mass density and the Young's modulus of the material are $2712\,\mathrm{kg/m^3}$and $7.102 \times 10^{10}\,\mathrm{N/m^2}$, respectively. The crank speed is 600 rpm. The crank is modeled by two finite elements, and the coupler and follower are both modeled by four elements.

In order to simplify the process of the calculation, only the thickness of the links are taken as the structural design parameters. Then the design variable vector of the whole system is given by

$$\chi = (\xi_s, \xi_c) = \begin{bmatrix} h_1 & h_2 & h_3 & \alpha & \beta & \lambda \end{bmatrix}^{\mathrm{T}},$$

where h_1, h_2, h_3 are the thickness of the crank, coupler, and follower, respectively. The initial values of the design variables are chosen as: $\chi = [0.01,\ 0.01,\ 0.01,\ 1.0,\ 1.0,\ 1.0]^{\mathrm{T}}$. The initial values of the structural parameter are chosen randomly. The initial values of the control parameter are all chosen as 1 which means that the initial control scheme is identical to the conventional control scheme. After 16 iterations, the optimal results are obtained, which are shown in Table 1.

In this study, piezoelectric elements shaped in rectangle chips are used as actuators. As shown in Fig. 3, the actuators are bonded on the links of the mechanism conveniently. The strain of the piezoelectric element in x direction can be written as [8]

$$\varepsilon = \frac{1}{E_P}\sigma_P + d_{31}\frac{V}{h_P}, \tag{24}$$

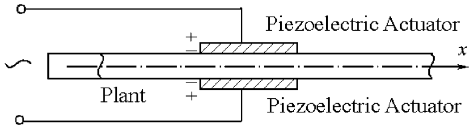

Fig. 3. Piezoelectric actuator element.

Table 2
Structural and material parameters of the actuator

Length (mm)	60
Width (mm)	15.0
Thickness (mm)	0.8
Density (kg/m^3)	7500
Young's modulus (N/m^2)	1.17×10^{11}
Poisson's ratio	0.25
Piezoelectric constant (m/V)	1.85×10^{-10}

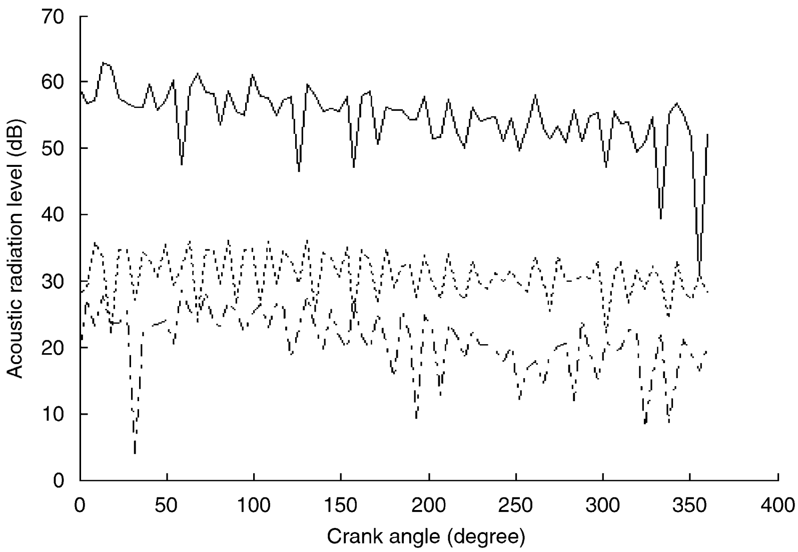

Fig. 4. The acoustic radiation power level before and after optimization. —, initial mechanism without control; · · · · · ·, initial mechanism with control; – · –, integrated optimal mechanism with control.

where σ_P is the stress of the piezoelectric actuator; E_P is the Young's modulus; d_{31} is the piezoelectric constant; h_P is the thickness of the piezoelectric element; V is the control input voltage.

When two uniform piezoelectric elements are bonded on the structural plant symmetrically as shown in Fig. 3, the actuate moment of the jth actuator can be expressed as

$$m_a^j = g_j V_j, \tag{25}$$

where

$$g_j = E_P d_{31}(h_b + h_p^j) b_p^j, \tag{26}$$

b_p^j and V_j are the width and the input voltage of the jth actuator, respectively; h_b is the thickness of the plant. The piezoelectric actuators and strain gauge sensors are bonded on the midpoint of each link. The size and material parameters of the piezoelectric actuators are shown in Table 2.

Fig. 4 shows the acoustic radiation power of the initial mechanism without control, the original mechanism with control, and the integrated optimal mechanism with control, respectively. The acoustic radiation power level is calculated by [22]

$$L_w = 10\lg \frac{W}{W_0}, \tag{27}$$

where W is the sound power of the mechanism; W_0 is the reference sound power, which is equal to 10^{-12}. From Fig. 4, one finds that the acoustic radiation performance is improved significantly after the control is employed. The mean value of the acoustic radiation level in one cycle of operation decreases from 53.26 to 30.91 dB. The acoustic radiation performance is further improved after integrated optimization. The mean value of the acoustic radiation power level in one cycle of operation decreases from 30.91 to 20.96 dB.

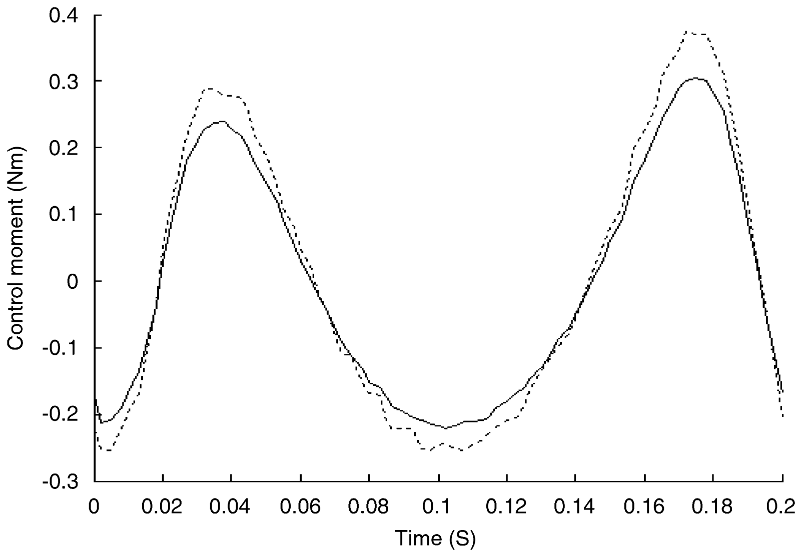

Fig. 5. Control forces of the crank, —, initial mechanism with control; · · · · · ·, integrated optimal mechanism.

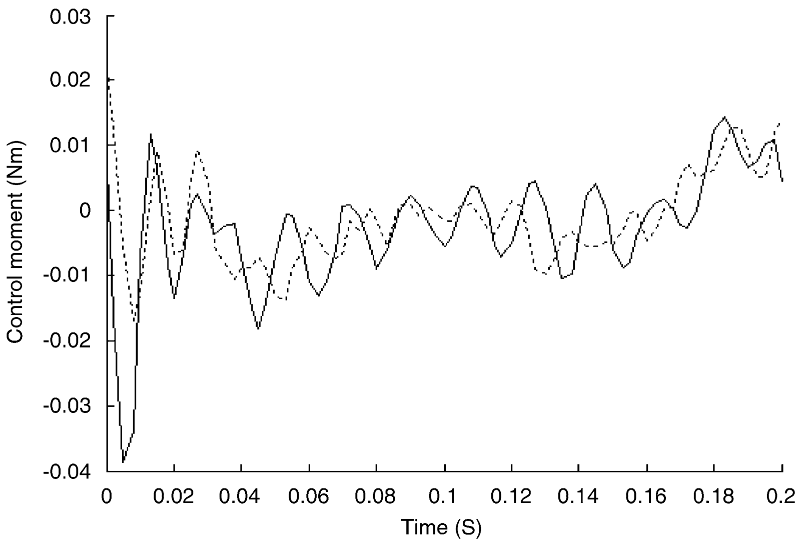

Fig. 6. Control forces of the coupler, —, initial mechanism with control; · · · · · ·, integrated optimal mechanism.

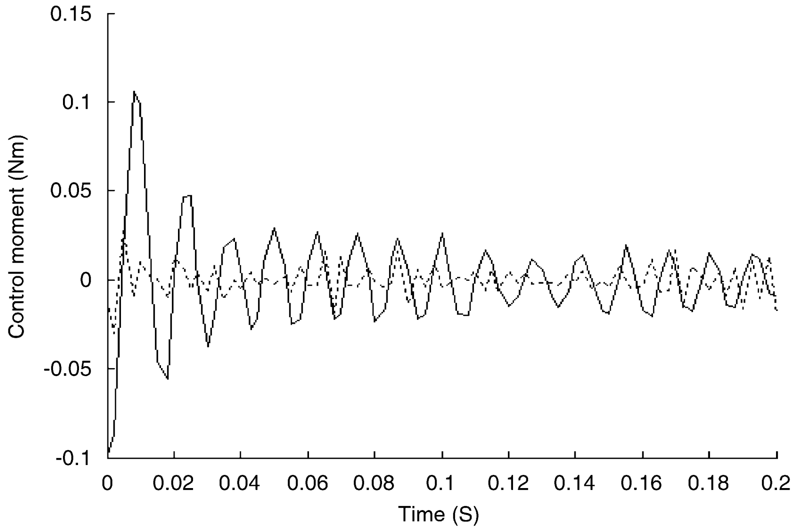

Fig. 7. Control forces of the follower, —, initial mechanism with control; · · · · · ·, integrated optimal mechanism.

Figs. 5–7 show the control moments of the crank, the coupler, and the follower, respectively, versus the operating time. The dot line indicates the control moment of the initial mechanism. The solid line represents the control moment of the integrated optimized mechanism. It is seen that the maximum input control moment of the crank is somewhat increased, while the maximum optimal control moments of the coupler and follower is decreased significantly for the case of integrated optimization. From Figs. 4–7, one finds that though the acoustic radiation performance is improved by utilizing the integrated optimal method, the control cost does not increase so much.

5. Conclusion

Integrated structural and control design of linkage mechanism for noise attenuation is studied in this paper. A new set of equations of motion is obtained by employing a set of wavenumber transformations on the original equations of motion of the flexible linkage mechanisms. The new equations of motion can be used to evaluate the structural acoustic radiation performance. The performance index for simultaneous mechanism and noise control design is discussed. The weighted sum of structural mass and the energy of the controlled mechanism is selected as the performance index. Finally, numerical simulations are carried out on a four-bar linkage mechanism, in which the thickness of the links and the parameters of control mechanism are taken as the design variables. Simulation results show that the acoustic radiation performance and the control inputs are improved significantly for the integrated optimized mechanism.

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