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Dynamic amplification factors in cable-stayed structures

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Abstract

The aim of this paper is to demonstrate that in cable-stayed structures dynamic amplification factors caused by the sudden breakage of cables can be larger than 2. This fact is extremely important since design guidelines for cable-stayed bridges indicate that the highest value for such factors is 2, whereas under certain circumstances that value could be considered unsafe. We set out the conditions that lead to that value being surpassed. We also show that the dynamic amplification factors related to deflections are lower than those related to bending moments and that the latter are in turn lower than those related to shear forces. Two examples are given: one involving the abrupt application of loads to a simply supported beam and the other the accidental breakage of a stay cable in a bridge with under-deck cable-staying.

1. Introduction

When an action is applied to a structure slowly, i.e. over a time that is more than twice as long as the main vibration period of the structure, the response of the structure is practically the same as its static response. However, if the action is applied more rapidly, the structure shows a dynamic response. In this case for the given structure and the given action we can define, for each section and for each movement or internal force, a dynamic amplification factor (DAF), which is the ratio of the maximum dynamic response to the static response.

Therefore, the maximum dynamic response in a given section to a given action can be evaluated in one of the following two ways: (1) we can make a dynamic calculation, or (2) we can make a static calculation, if we know the dynamic amplification factor appearing in this critical section of the structure for this response and this action.

Much research has focused on the establishment of dynamic amplification factor for bridges with the aim of supplying these values to engineers by means of design guidelines. However, some dynamic amplification factors have been established in guidelines without thorough research. This is the case of dynamic amplification factors for abrupt breakage of cables in cable-stayed bridges. In this case, all the guidelines limit the dynamic amplification factors to an upper bound of 2, since this is the maximum value for one degree of

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freedom (dof) systems under abrupt load application. But dynamic amplification factors may be larger than two in multi-dof systems.

The aims of this paper are to prove analytically and numerically that dynamic amplification factors for abrupt breakage of cables in cable-stayed bridges can be larger than 2, to explain what conditions must happen for that value be surpassed, and to corroborate the theory by two examples.

2. Research into dynamic amplification factors

By far the majority of studies have focused on evaluation of dynamic amplification factors as a result of traffic live loads of bridges, first for road bridges, and then for railway bridges.

In 1992, Paultre et al. [1–3] carried out a review of all the analytical and experimental studies that had previously been carried out. At the time, dynamic amplification factors were estimated on the basis of just one parameter, namely the span or the principal vibration frequency. The first analytical studies evaluated the maximum dynamic amplification factors relative to the deflection of simple beams and different boundary conditions at the supports [4,5]. In 1993, Humar and Kashif published a study in which they showed that dynamic amplification factors depend on the velocity and weight of the vehicle [6]; it criticised the continued evaluation of DAFs on the basis of a single parameter [6,7]. In 1997, Henchi et al. extended analytical studies to the subject of continuous bridges [8]. Since then, a number of studies have been made showing that dynamic amplification factors depend on various factors, namely the geometry of the bridge, the type of load, the velocity of the vehicles [9] and the roughness of the deck surface [10–12]. In addition to studying dynamic amplification factors for road bridges, studies were also made for railway bridges, since the consecutive application of loads with a given frequency can give raise to a considerable increase in DAFs [13,14]. In fact, clauses from codes for actions on road bridges provide the values of the actions that have been already amplified taking into account the dynamic amplification factors, while clauses from codes for actions on railway bridges provide the static values of the actions to be amplified using the DAFs that have to be expressly calculated for the bridge being designed. Likewise, many experimental studies have been made of existing bridges [15–18] with the aim of evaluating actual DAFs and comparing them to the design values provided in the codes.

Although most studies have focused on the action of traffic live loads, not all the existing studies refer to that action. Recently, some studies have been made and a number are underway on the subject of dynamic amplification factors related to the accidental breakage of stay cables in cable-stayed bridges. The first dynamic study to evaluate these factors was carried out by Välimäki [19,20] and established dynamic amplification factors of 1.80 in critical sections. In the doctoral thesis [21] written and supervised by the authors of this paper, among other aspects, the safety of under-deck cable-stayed bridges in the event of accidental breakage of stay cables was studied. One part of that study formed the basis for this paper. The second author of this paper is, at present, supervising another doctoral thesis (developed by C. Mozos) on the subject of accidental breakage of stay cables in conventional cable-stayed bridges. It is worth noting that, while there is abundant literature dealing with dynamic amplification factors related to traffic live loads of bridges, very few studies have been made of dynamic amplification factors related to the accidental breakage of stay cables in bridges or other cable-stayed structures.

3. Dynamic amplification factors due to accidental breakage of stay cables in design guidelines for cable-stayed bridges

The breakage of stay cables is an accidental event that must be taken into account in the design of a cable-stayed structure. It must be established that, in the event of this accidental breakage, the structure will not collapse.

In order to determine the response of the structure to an event of this kind, a dynamic analysis is required. This will involve obtaining the internal forces in any section and the movements of any given point of the structure at any given time. This allows one to establish the maximum internal forces or the maximum movements that occur in the course of the dynamic response of the structure before it is eventually damped. However, it is quite common for the maximum internal forces or maximum movements, caused by such an accident, to be obtained from a static analysis, with the static response being amplified by dynamic amplification factors.

There are various guidelines for the design of cable-stayed structures and they all include dynamic amplification factor values for determining the response of the structure to the accidental breakage of a stay cable. These values are invariably based on static calculations.

SETRA's guidelines for the design of cable-stayed bridges [22] indicate that the accidental breakage of a stay cable has to be considered. An equivalent static calculation must therefore be carried out, taking into account a dynamic amplification factor of between 1.5 and 2.0. These recommendations state that the dynamic amplification factor will depend on the origin of the breakage and on the structure. They note that 2.0 is a maximum value that corresponds to the sudden breakage of the whole stay cable section. Hence, taking into consideration that a whole stay cable section breakage is improbable, when compared to the occurrence of a partial breakage, they consequently recommend the standard use of a value of 1.5. The PTI [23] calls for a similar procedure, with a recommended amplification factor equal to 2 in this case.

In the draft of its recommendations for cable-stayed bridges [24], the ACHE, like the other organisations already mentioned, indicates the need to verify ultimate limit states in the event of the accidental breakage of a stay cable. Two alternatives are suggested: dynamic verification and static verification using a dynamic amplification factor of 2. A dynamic amplification factor equal to 1.50 is also being considered in the draft of Eurocode 3 Part 1.11 [25].

In our opinion, SETRA is right to claim in its recommendations that the dynamic amplification factor will depend on the type of occurrence that causes the breakage of the stay cable (vehicle collision, fire, corrosion, etc.). Thus, the time required for breakage due to fire is longer than for breakage due to vehicle collision. The shorter the time in which the action causing the breakage of the stay cable lasts, the greater the dynamic response of the structure and, consequently, the greater the dynamic amplification factors. Practically instantaneous breakage constitutes the worst case scenario. If we know the variation in the load function in the stay cable during the breakage period ($F(t)\neq 0$ where $t<t_{\text{breakage}}$ and $F(t)=0$ where $t\geqslant t_{\text{breakage}}$), we can make a more accurate calculation. For the duration of the breakage, the dynamic response will be in the form of forced vibrations and after that time the response will be in the form of free vibrations.

Nevertheless, it is important to bear in mind that in the case of sudden breakage of cables in cable-stayed bridges, the dynamic response is practically independent of the load function $F(t)$, since the breakage times to be taken into consideration (breakage times in the event of impact) are very small in comparison with the fundamental period of the structure. If we consider an instantaneous breakage, the response will be similar to what would occur in the event of breakage due to a collision, although slightly larger than in the latter case, but in any event on the safe side. In the case of cable-stayed structures with smaller vibration periods, such as certain roof structures, we will need to know the breakage function so as not to overestimate the response of the structure to this action.

It must be said that the value for the dynamic amplification factor given by the guidelines is questionable. In an undamped system with a single dof, the dynamic amplification factor in the event of instantaneous actions is always equal to 2. However, in systems with more dofs and therefore more vibrational modes, this is not always the case; indeed, in certain cases the dynamic amplification factor may be larger than 2, as we will show in this paper.

4. Upper limit of dynamic amplification factors related to deflections

The upper limit of the dynamic amplification factor related to deflections can be determined analytically. If we apply a load $q(x)$ to a structure, the structure is deformed and will begin to oscillate around a new position of equilibrium, which will be the position corresponding to the static deformation of the structure under load $q(x)$. The larger the application speed of load $q(x)$, the more pronounced the initial oscillations around that position will be. If the load is applied very slowly, the magnitude of those oscillations will be negligible and the dynamic amplification factor related to deflection will have a value of 1. To obtain the upper limit of the dynamic amplification factor, we must assume that load $q(x)$ is applied very quickly or even instantaneously.

The dynamic oscillations around the equilibrium position will gradually decrease in magnitude as the energy is dissipated by the damping effect of the structure. Once the structure has stopped moving, the deformation will be equal to the static deformation. First of all, in order to determine the upper limit, let us assume that it is an undamped structure. In this case, the dynamic response of the structure to load $q(x)$ applied instantaneously in $t = 0$ is

$$y_{\text{dynamic}}(x,t) = \sum_i v_i(x) A_i (1 - \cos(\omega_i t)) \tag{1}$$

and the static response of the structure to that same action is

$$y_{\text{static}}(x) = \sum_i v_i(x) A_i, \tag{2}$$

where $y_{\text{dynamic}}(x,t)$ is the deflection at the location x on the x-axis at time t, while $y_{\text{static}}(x)$ is the static deflection. The vibrational mode i is $v_i(x)$, the angular frequency is ω_i, and $A_i v_i(x)$ is the projection of the structural response on the vibrational mode $v_i(x)$, where A_i is a value that can be either positive or negative. In fact the static response of the structure, $y_{\text{static}}(x)$, is projected onto a vectorial space with an orthogonal base consisting of the vibrational modes of the structure, the coefficients A_{i} being the coordinates of the static response in that vectorial space.

The maximum dynamic response will occur at the instant when the functions $(1-\cos(\omega_i t))$ take the following values: 2, which is the maximum value, for all the functions associated with vibrational modes where $A_i v_i(x)$ has the same sign as $y_{\text{static}}(x)$ (i.e. the vibrational modes where the response of the structure has a positive projection); and 0, which is the minimum value, for all the functions associated with vibrational modes where $A_i v_i(x)$ has the sign opposite to $y_{\text{static}}(x)$ (i.e. the vibrational modes where the response of the structure has a negative projection). Due to being an undamped structure, from a numerical point of view, the instant in which these conditions are satisfied is always reached.

Consequently, the maximum dynamic response is given by the following expression:

$$y_{\text{dynamic,max}}(x) = \sum_i v_i(x) A_i 2\delta_i, \tag{3}$$

where

$$\delta_i = \begin{cases} 0 & \text{if } \dfrac{A_i v_i(x)}{y_{\text{static}}(x)} < 0, \\ 1 & \text{if } \dfrac{A_i v_i(x)}{y_{\text{static}}(x)} \geqslant 0, \end{cases} \tag{4}$$

Having defined the dynamic amplification factor related to deflections as the ratio of the maximum vertical deflection to the static vertical deflection, we can state that it is equal to

$$\text{DAF}_{\text{deflection}}(x) = \frac{y_{\text{dynamic,max}}(x)}{y_{\text{static}}(x)} = \frac{\sum_i v_i(x) A_i 2\delta_i}{y_{\text{static}}(x)}, \tag{5}$$

$$\text{DAF}_{\text{deflection}}(x) = \frac{\sum_i v_i(x) A_i 2 - \sum_i v_i(x) A_i 2(1-\delta_i)}{y_{\text{static}}(x)} = 2 - \frac{\sum_i v_i(x) A_i 2(1-\delta_i)}{y_{\text{static}}(x)} \tag{6}$$

and having defined the function δ^* as

$$\delta_i^* = \begin{cases} 1 & \text{if } \dfrac{A_i v_i(x)}{y_{\text{static}}(x)} < 0, \\ 0 & \text{if } \dfrac{A_i v_i(x)}{y_{\text{static}}(x)} \geqslant 0, \end{cases} \tag{7}$$

we can obtain a more concise expression of the dynamic amplification factor related to deflections, i.e.

$$\text{DAF}_{\text{deflection}}(x) = 2 + \sum_i \left| \frac{v_i(x) A_i}{y_{\text{static}}(x)} \right| 2\delta_i^*. \tag{8}$$

Consequently, in undamped systems, if there is no vibrational mode for which the projection of the structural response on the vibrational mode is negative (i.e. if there is no mode in which $\delta^* = 1$), the dynamic amplification factor will be exactly equal to 2. Conversely, if there is a vibrational mode for which the projection of the response on the vibrational mode is negative (i.e. if there is a mode in which $\delta^* = 1$), the dynamic amplification factor will be larger than 2. In the latter case, the larger the weight of the vibrational modes with negative projection in the structural response is, the larger the dynamic amplification factor will be.

In damped systems, the dynamic response of the structure to a load $q(x)$ applied instantaneously ($t = 0$) is

$$y_{\text{dynamic}}(x,t) = \sum_i v_i(x) A_i \left(1 - e^{-\eta_i \omega_i t}\left(\cos\left(\omega_i \sqrt{1-\eta_i^2}\, t\right) + \frac{\eta_i}{\sqrt{1-\eta_i^2}} \sin\left(\omega_i \sqrt{1-\eta_i^2}\, t\right)\right)\right), \tag{9}$$

where η_i is the damping ratio in relation to critical damping for the mode i.

Without losing accuracy from the practical point of view, Expression (9) can be simplified. Damping ratio values depend on several factors: the structure, the materials, the vibrational mode shape, the action applied, and so on. These values are very small (less than 10%) for common types of structures [26–28] and even smaller (less than 2%) for cable-stayed structures [29], so the second term in Expression (9) may be considered to be negligible, and then the equation becomes:

$$y_{\text{dynamic}}(x,t) \approx \sum_i v_i(x) A_i \left(1 - e^{-\eta_i \omega_i t} \cos\left(\omega_i \sqrt{1-\eta_i^2}\, t\right)\right). \tag{10}$$

With time, the oscillations are damped: the vibrational modes associated with high frequencies of vibration are damped first, while the low frequency vibrational modes, namely the primary vibrational modes of the structure, take longer to be damped.

When the maximum dynamic response is reached, a certain amount of time will have passed and part of the response will have been damped. Thus the DAF related to deflections will be less than it would have been if the structure was undamped. In this case the harmonic functions associated with vibrational modes with positive projections on the structural response are damped and their value is less than 2. Consequently, in damped systems, the DAF related to deflections can be larger than 2, but for this to be the case, it is necessary, but not sufficient, that there is at least one vibrational mode in which the projection of the response on that vibrational mode is negative. Otherwise, i.e. if there is no vibrational mode in which the projection of the response is negative, the DAF related to deflections will be less than 2 in damped systems.

Therefore, the DAF related to deflections of a damped structure will be less than the DAF of an undamped structure and will be given by the following expression:

$$\text{DAF}_{\text{deflection}}(x) < 2 + \sum_i \left|\frac{v_i(x) A_i}{y_{\text{static}}(x)}\right| 2\delta_i^*. \tag{11}$$

5. Dynamic amplification factors related to internal forces

Before obtaining the expressions for dynamic amplification factors related to internal forces (bending moments and shear forces), a simple example is presented. The analytical expressions obtained in the previous section for DAFs related to deflection (Expressions (8) and (11)) are valid as a general rule. However, the analytical expressions that will be obtained in this section are not valid on such a general level, although they help to draw some conclusions of a general nature.

In a simply supported isostatic beam of length L, vibrational modes $v_i^*(x)$ are given by the expression

$$v_i^*(x) = \sin\left(i\pi \frac{x}{L}\right). \tag{12}$$

Consequently, in an undamped system subjected to a load $q(x)$ that is applied instantaneously at time $t = 0$, the dynamic vertical movements can be obtained by substituting Expression (12) into Expression (1)

$$y_{\text{dynamic}}(x,t) = \sum_i \sin\left(i\pi \frac{x}{L}\right) A_i (1 - \cos(\omega_i t)). \tag{13}$$

Therefore, the bending moments will be

$$M_{\text{dynamic}}(x,t) = EI\frac{\partial^2 y_{\text{dynamic}}(x,t)}{\partial x^2}, \tag{14}$$

$$M_{\text{dynamic}}(x,t) = EI\sum_i \frac{\partial^2 v_i(x)}{\partial x^2} A_i(1-\cos(\omega_i t)) \tag{15}$$

and the shear forces will be given by

$$V_{\text{dynamic}}(x,t) = \frac{\partial M_{\text{dynamic}}(x,t)}{\partial x} = EI\frac{\partial^3 y_{\text{dynamic}}(x,t)}{\partial x^3}, \tag{16}$$

$$V_{\text{dynamic}}(x,t) = EI\sum_i \frac{\partial^3 v_i(x)}{\partial x^3} A_i(1-\cos(\omega_i t)). \tag{17}$$

For sake of simplicity, let us assume that vibrational mode j is the first to be excited by the application of load $q(x)$. In most cases j will be equal to 1, but because it depends on load $q(x)$, this will not always be the case. In this case, the maximum vertical deflection at the location x on the x-axis due to the first vibrational mode to be excited is

$$y_{j,\max}(x) = \sin\left(j\pi\frac{x}{L}\right)A_j 2 \tag{18}$$

and the maximum deflection due to vibrational mode i is

$$y_{i,\max}(x) = \sin\left(i\pi\frac{x}{L}\right)A_i 2. \tag{19}$$

Let us assume that for a section x, the maximum vertical movement due to the vibrational mode i is k_i times the maximum vertical movement due to vibrational mode j:

$$k_i(x) = \frac{y_{i,\max}(x)}{y_{j,\max}(x)} = \frac{\sin\left(i\pi\frac{x}{L}\right)A_i}{\sin\left(j\pi\frac{x}{L}\right)A_j}. \tag{20}$$

If for the same section x, the relation between the maximum bending moment due to vibrational mode i and the maximum bending moment due to vibrational mode j is calculated, it follows:

$$\frac{M_{i,\max}(x)}{M_{j,\max}(x)} = \frac{i^2}{j^2}k_i(x). \tag{21}$$

Moreover, if the relation between the maximum shear force due to vibrational mode i and the maximum shear force due to vibrational mode j is also calculated at the same section, this relation becomes:

$$\frac{V_{i,\max}(x)}{V_{j,\max}(x)} = \frac{i^3}{j^3}k_i(x). \tag{22}$$

The weight of the different vibrational modes in the dynamic response therefore depends on the type of movement or the internal force that are dealt with. The vibrational modes associated with high frequency vibrations have a larger weight in the shear forces than in the moments, and also larger than in the deflections.

We can also conclude on that basis that, in order to reach an accurate approximation to the dynamic response, the number of vibrational modes to be considered will depend on the type of response to be obtained. Thus, if we wished to obtain shear forces, it would be necessary to consider a larger number of vibrational modes than for bending moments, and a much larger number than for deflections.

If we then define the DAF related to bending moments as the ratio of the maximum bending moment to the static bending moment, it follows:

$$\mathrm{DAF}_{\text{bending moments}}(x) = 2 + \sum_i \left|\frac{M_{i,\max}(x)}{M_{\text{static}}(x)}\right| 2\delta_i^{**}. \tag{23}$$

The maximum bending moment will occur at the instant of the maximum values for the functions $(1-\cos(\omega_i t))$ associated with the vibrational modes for which the projection of the structural response (in this case, the structural response related to bending moments) is positive, while the functions $(1-\cos(\omega_i t))$ associated with the vibrational modes for which the projection of the structural response is negative are annulled. In such a way as δ_i^* was defined, δ_i^{**} can be set as follows:

$$\delta_i^{**} = \begin{cases} 1 & \text{if } \dfrac{M_{i,\max}(x)}{M_{\text{static}}(x)} < 0, \\ 0 & \text{if } \dfrac{M_{i,\max}(x)}{M_{\text{static}}(x)} \geqslant 0. \end{cases} \tag{24}$$

If the projection of the bending moment on this vibrational mode is positive, then $\delta_i^{**} = 0$, and if it is negative $\delta_i^{**} = 1$.

The vibrational modes associated with negative projections have a larger weight in the structural response for bending moments than for deflections, and consequently the dynamic amplification factors related to bending moments can be larger than those related to deflections.

In several previous studies [9,16], it has been noted that, under experimental conditions, the dynamic amplification factors related to bending moments are larger than those related to deflections, although no reasons were given for this fact.

We can obtain the dynamic amplification factor related to shear forces in the same way.

$$\mathrm{DAF}_{\text{shear forces}}(x) = 2 + \sum_i \left|\frac{V_{i,\max}(x)}{V_{\text{static}}(x)}\right| 2\delta_i^{***} \tag{25}$$

with

$$\delta_i^{***} = \begin{cases} 1 & \text{if } \dfrac{V_{i,\max}(x)}{V_{\text{static}}(x)} < 0, \\ 0 & \text{if } \dfrac{V_{i,\max}(x)}{V_{\text{static}}(x)} \geqslant 0. \end{cases} \tag{26}$$

If the projection of the shear force on this vibrational mode is positive, then $\delta_i^{***} = 0$ and if it is negative $\delta_i^{***} = 1$.

The vibrational modes associated with negative projections have a larger weight in the structural response for shear forces than for bending moments, and consequently the DAFs related to shear forces can be larger than the DAFs related to bending moments, which are in turn, larger than the DAFs related to deflections; but there must be at least one vibrational mode with negative projection onto the several responses (defections, rotations, bending moments, shear forces, etc.).

6. Application to an isostatic beam

Through an example, we will now determine the dynamic amplification factors for a simply supported beam in the case of two actions: (1) the application of a point load at mid-span, and (2) the application of two point loads at third-span (Fig. 1)

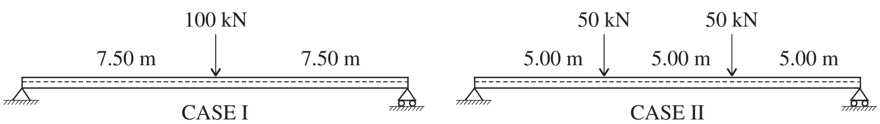

Fig. 1. Load cases. Cases I and II.

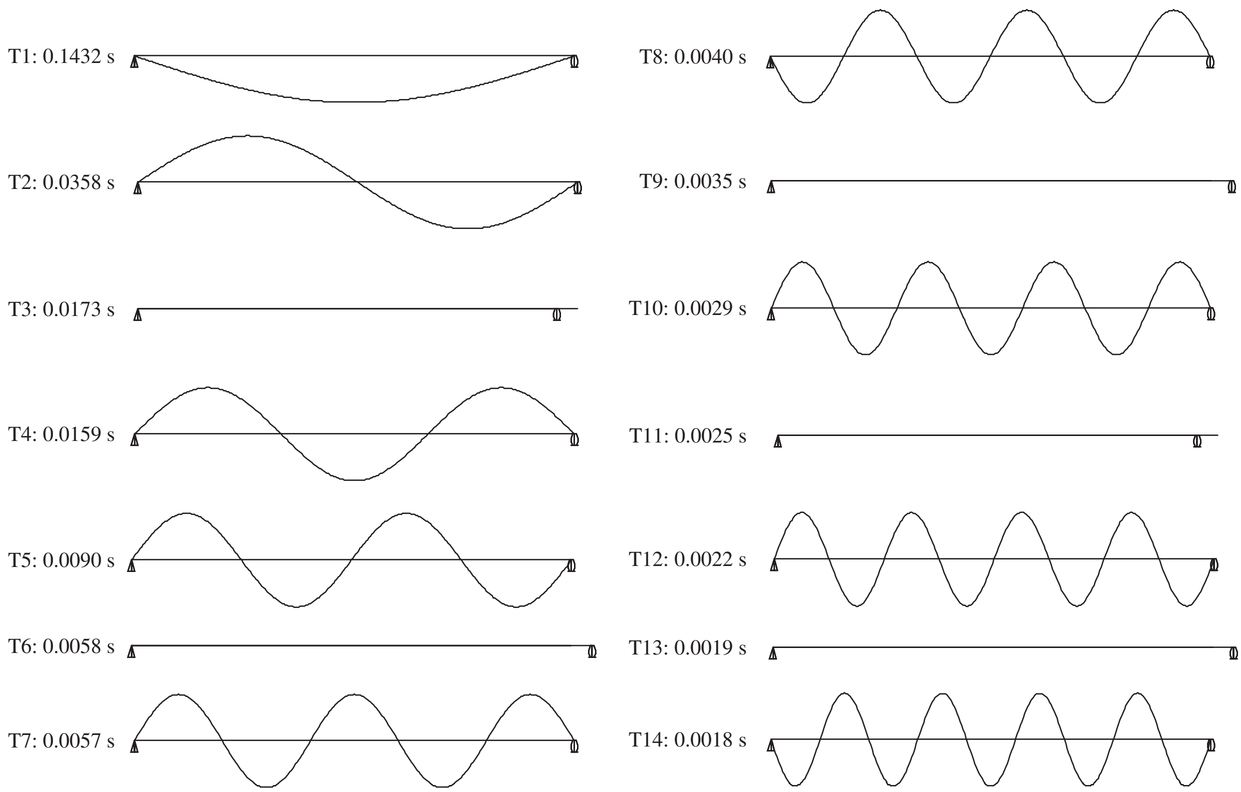

Fig. 2. Vibrational modes considered and the associated periods (Cases I and II).

A 15 m long beam, with a 0.60 m^2 cross section and 0.05 m^4 moment of inertia, is considered. The structural material is concrete with a Young's modulus of 30 GPa and a unit weight of 25 kN/m^3.

In both cases we assume that the structure is not damped and that the loads are applied instantaneously. We obtain the dynamic response by means of modal superposition, taking the first 14 vibrational modes of the structure, as shown in Fig. 2. The response is integrated during the first 10 s, using time increments of 10^{-4} s.

6.1. Case 1. Load applied at mid-span

In the first case, when the load is applied at mid-span, only the symmetrical vibrational modes (modes 1, 4, 7, 10 and 14) are activated. The projection of the dynamic response on the non-symmetrical vibrational modes (modes 2, 5, 8 and 12) and on the modes related to the axial response (modes 3, 6, 9, 11 and 13) is zero.

Table 1 shows the projection of the structural response on the different vibrational modes. In the cases of deflections and bending moments at mid-span, the projection of the static response on all the vibrational modes that have been considered is positive, so the maximum deflection and the maximum moment are both twice the static value. Consequently, in this case the DAF related to deflection and the DAF related to moments are exactly equal to 2.

Table 2 shows how the first five active vibrational modes (modes 1, 4, 7, 10 and 14) can be used to represent 100% of the response in deflections and 96.3% of the response in moments.

Let us now consider the energy. Since there is no damping, the total energy of the system is conserved. The total energy of the system is the sum of the kinetic energy, the deformation energy and the potential energy of the external forces. We calculate an energy balance for determining how each energy varies over time. At the initial time the structure starts from a position with no deformation and no velocity, therefore the deformation energy, the kinetic energy, the potential energy of the external forces and the total energy of the system are all equal to zero. Since there is no damping, the total energy of the system will remain zero over time. As the

Table 1
Response at mid-span (Case I)

Mode	Deflection at mid-span (mm)		Bending Moment at mid-span (MN m)	
	Static	Dynamic	Static	Dynamic
1	−4.620	−9.239	0.304	0.609
4	−0.057	−0.114	0.034	0.068
7	−0.007	−0.015	0.012	0.025
10	−0.002	−0.004	0.006	0.013
14	−0.001	−0.001	0.004	0.008
$\sum$(5 first active modes)	−4.687	−9.373	0.361	0.723
Maximum		−9.373		0.723
DAF	2.000		2.000	

Table 2
Accuracy of the calculated response (Case I)

Static response	Deflection at mid-span (mm)	Bending Moment at mid-span (MN m)
With 5 first active modes	−4.687	0.361
Exact	−4.688	0.375
Percentage represented (%)	100.0	96.3

Table 3
Energy balance at five different instants (Case I)

	Deformation energy (kJ)	Kinetic energy (kJ)	Potential energy of external forces (kJ)	Total energy (kJ)
(1) Maximum deflection at mid-span	0.93735	0.00000	−0.93735	0.00000
(2) Maximum bending moment at mid-span	0.93735	0.00000	−0.93735	0.00000
(3) Maximum deformation energy	0.93735	0.00000	−0.93735	0.00000
(4) Maximum kinetic energy	0.23409	0.23434	−0.46843	0.00000
(5) Maximum negative potential energy of external forces	0.93735	0.00000	−0.93735	0.00000
Static equilibrium	0.23434	0	−0.46868	−0.23434

structure is deformed, the potential energy of the external forces will decrease and the deformation energy and the kinetic energy will increase. When the equilibrium position, corresponding to the static deformation, is crossed, the deformation energy will continue to increase, the potential energy of the external forces will continue to decrease, and the kinetic energy will begin to decrease. When the kinetic energy reaches zero, the deformation energy is equal to the potential energy of the external forces. Table 3 shows the values for each of these energies at five different instants: (1) when the maximum deflection is reached at mid-span, (2) when the maximum moment is reached at mid-span, (3) when the maximum deformation energy is reached, (4) when the maximum kinetic energy is reached and (5) when the potential energy of the external forces reaches its minimum value. The values of the energies at static equilibrium are also shown.

At static equilibrium, the value of the deformation energy (U_e) is equal in absolute value to half the potential energy of the external forces ($-2U_e$), and to the total energy ($-U_e$). When the load is applied abruptly, the system has a total energy of zero, i.e. it has an extra-energy of U_e in comparison with its energy in the case of static equilibrium. This is the energy that permits the movement of the structure and that is dissipated by damping in the case of a damped structure.

When all the harmonic functions associated with the vibrational modes take a value of 1, the structure crosses the equilibrium position and therefore its deformation energy is equal to U_e and the potential energy of the external forces is $-2U_e$. Since the total energy must be zero, at that instant the kinetic energy will be equal to U_e, i.e. all the extra-energy will be in the form of kinetic energy.

When all the harmonic functions associated with the vibrational modes take a value of 2, the structure will have a deformation that will double the deformation corresponding to the equilibrium position. The deformation energy will be four times the energy corresponding to the equilibrium position ($4U_e$) and the potential energy of the external forces will be double the energy corresponding to the equilibrium position ($-4U_e$). Since the total energy is equal to zero, the kinetic energy will also be equal to zero.

Since in this case the structural response has a positive projection on all the vibrational modes, the maximum deflection is produced when the associated harmonic functions have a value of 2. Therefore, the maximum deformation energy and the minimum potential energy of the external forces are also produced at that instant, as seen in Table 3.

6.2. Case 2. Loads applied at third-span

In the second case, where the loads are applied at third-span, only some symmetrical vibrational modes (modes 1, 7 and 10) are activated. Certain symmetrical vibrational modes (modes 4 and 14) are not activated because in these modes there is bending moment at mid-span but none at third-span, and since the static response has a constant moment in the middle third of the beam, the projection on these two vibrational modes is zero. The projection of the dynamic response on the non-symmetrical vibrational modes (modes 2, 5, 8 and 12) and on the modes that mobilise the axial response (modes 3, 6, 9, 11 and 13) is also zero.

Table 4 shows the static and dynamic response in deflection and in bending moments of the section located at third-span. All the vibrational modes considered have positive projection on the static response and the dynamic amplification factors related to deflection and to bending moments take a value of 2.

Table 5 shows the static and dynamic response in deflections and in bending moments at mid-span. Two of the three vibrational modes considered (modes 7 and 10) have negative projections on the static response. Thus, the maximum deflection and maximum moment occur when the harmonic function associated with vibrational mode 1 takes its maximum value, i.e. 2, and the harmonic functions associated with vibrational modes 7 and 10 are zero. Since there are vibrational modes with negative projections on the static response, the dynamic amplification factors are larger than 2. These modes (modes 7 and 10) have larger weight in the bending response than in the deflection response, and so the DAF related to bending moments (2.133) is larger than the DAF related to deflection (2.004).

Table 5 shows that the model fails to capture the instant when the maximum deflection and maximum moment occur at mid-span. Thus it would be necessary to calculate the response over a longer period and with smaller time increments. In any case, the values provided by the model do give DAFs larger than 2.

With the 14 vibrational modes, it is possible to represent 100% of the response in deflection both at the section located at third-span and at the section located at mid-span, 97% of response in bending moments at third-span and 99% of the response in bending moments at mid-span, as seen in Table 6.

Table 4
Response at third-span (Case II)

Mode	Deflection at third-span (mm)		Bending moment at third-span (MN m)	
	Static	Dynamic	Static	Dynamic
1	−3.465	−6.930	0.228	0.456
7	−0.006	−0.011	0.009	0.019
10	−0.001	−0.003	0.005	0.010
$\sum$(3 first active modes)	−3.472	−6.944	0.242	0.485
Maximum		−6.944		0.485
DAF	2.000		2.000	

Table 5
Response at mid-span (Case II)

Mode	Deflection at mid-span			Bending moment at mid-span		
	Static (mm)	Dynamic (mm)	Weights (%)	Static (MN m)	Dynamic (MN m)	Weights (%)
1	−4.001	−8.002	100.2	0.264	0.527	106.6
7	0.006	0.013	−0.2	−0.011	−0.022	−4.4
10	0.002	0.003	0.0	−0.006	−0.011	−2.3
∑(3 first active modes)	−3.993	−7.986		0.247	0.494	
Actual Maximum		−8.002			0.527	
Calculated Maximum		−7.990			0.526	
DAF		2.004			2.133	

Table 6
Accuracy of the calculated response (Case II)

Static response	At third-span		At mid-span	
	Deflection (mm)	Bending moment (MN m)	Deflection (mm)	Bending moment (MN m)
With 3 first active modes	−3.472	0.242	−3.993	0.247
Exact	−3.472	0.250	−3.993	0.250
Percentage represented (%)	100.0	97.0	100.0	98.8

Let us consider again what happens as regards energy. At the initial moment the structure starts from a position with no deformation and zero velocity, so that the deformation energy, the kinetic energy, the potential energy of the external forces and the total energy of the system are all equal to zero. Since there is no damping, the total energy of the system will remain zero over time. This is exactly the same as in Case I.

In the case of deflection and bending moments at third-span, the response of the structure has a positive projection on all the vibrational modes considered. Consequently, when the harmonic functions associated with all the vibrational modes take a value of 2, the maximum dynamic response (maximum deflection and maximum bending moments) at third-span are reached. At this instant the deformation energy will be maximum ($4U_e$), the potential energy of the external forces will be minimum ($-4U_e$) and the kinetic energy will be zero.

The responses of the structure in deflections and bending moments at mid-span have negative projections on some vibrational modes, so the maximum response will occur when the harmonic functions associated with modes with a positive projection take a value of 2, while the harmonic functions associated with modes with a negative projection are equal to zero. At this instant the kinetic energy of the system will also be zero, since none of the vibrational modes will have a velocity component. At this instant, since not all the harmonic functions take a value of 2, the deflection at the point where the load is applied (third-span) will not be maximum. Consequently, the potential energy of the external forces will not take its minimum value. Given that the kinetic energy is equal to zero, the deformation energy will not take its maximum value either. This is compatible with the fact that the deflections at mid-span are higher than twice the static values, since at many other points of the structure the deflections will be less than twice the static values; therefore, the deformation energy is less than the maximum deformation energy (which occurs when the dynamic deflection at all the points in the structure is twice the value of the static deflection). The total energy remains conserved.

As shown in Table 7, with the introduced model, the maximum deflection at mid-span (−7.990 mm; Table 5) does not occur at the same instant as the maximum moment at mid-span (0.526 MN m; Table 5), since the energies associated with these two instants are different and the kinetic energy is not equal to zero. If the model had captured the instant when the maximum deflection (−8.002 mm; Table 5) and the maximum moment (0.527 MN m; Table 5) occur at mid-span, the energies would coincide, since both occur at the same

Table 7
Energy balance at seven different instants (Case II)

	Deformation energy (kJ)	Kinetic energy (kJ)	Potential energy of the external forces (kJ)	Total energy (kJ)
(1) Maximum deflection at third-span	0.69435	0.00000	−0.69435	0.00000
(2) Maximum bending moment at third-span	0.69435	0.00000	−0.69435	0.00000
(3) Maximum deflection at mid-span	0.69353	0.00025	−0.69378	0.00000
(4) Maximum bending moment at mid-span	0.68905	0.00197	−0.69102	0.00000
(5) Maximum deformation energy	0.69435	0.00000	−0.69435	0.00000
(6) Maximum kinetic energy	0.17343	0.17359	−0.34701	0.00000
(7) Maximum negative potential of the external forces	0.69435	0.00000	−0.69435	0.00000
Static equilibrium	0.17359	0.00000	−0.34718	−0.17359

Table 8
Dynamic amplification factors obtained by means of direct integration (Case II)

	At third-span		At mid-span	
	Deflection (mm)	Bending moment (MN m)	Deflection (mm)	Bending moment (MN m)
Maximum value	−6.944	0.4965	−8.002	0.5342
Static value	−3.472	0.25	−3.993	0.25
DAF	2.000	1.986	2.004	2.137

instant (when the harmonic function associated with vibrational mode 1 is equal to 2 and the harmonic functions associated with vibrational modes 7 and 10 are equal to zero).

For this second case of loading, if a dynamic integration over time rather than a modal superposition is carried out, over 10 s with an integral increment of 10^{-5} s, we are able to represent 100% of the static response, both in deflection and in bending moments. Direct integration is equivalent to considering all the active vibrational modes rather than just the first three. Table 8 shows the obtained values of the dynamic amplification factors, which are practically equal to those obtained by means of modal superposition.

It should be noted that the model does not capture the maximum dynamic moment at third-span, which is 0.50 MN m. This would require calculation of the response over a longer period of time with smaller time increments than those used.

In any event, the aspect that we wish to stress is that the dynamic amplification factors are larger than 2.

Having completed this numerical analysis, we believe that it was to be expected that, in this second case of loading, the dynamic amplification factors would be larger than 2 in the section located at mid-span of the beam. In fact, we already knew this before carrying out any calculations. Let us explain. As Fig. 3 shows, in a simply supported beam with a span L, when two loads Q are applied at third-span a static bending moments diagram is produced that grows linearly up the third-spans (remaining constant in the central third part of the span) whose value is $QL/3$. Considering only one vibrational mode, we would obtain a bending moments diagram that would give a larger moment at mid-span than at third-span, and we would need at least a second vibrational mode with negative projection on the response of the structure at mid-span to give a constant bending moments diagram throughout the middle third part of the beam. Consequently, since there is a vibrational mode with negative projection on the response of the structure, the dynamic amplification factor will be larger than 2.

Similarly, we can see how in the first case of loading, when a point load is applied at mid-span, there are sections near the supports where the dynamic amplification factor will be larger than 2. In this case, this fact is less important than in the previous case, since the factors that are larger than 2 occur in sections that are not critical.

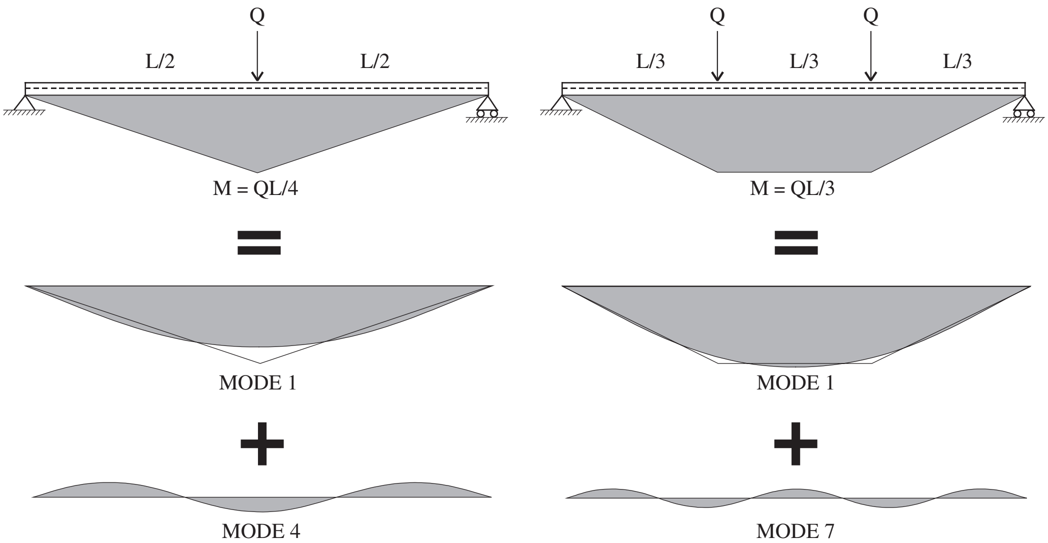

Fig. 3. Estimation of the first two active vibrational modes (Cases I and II).

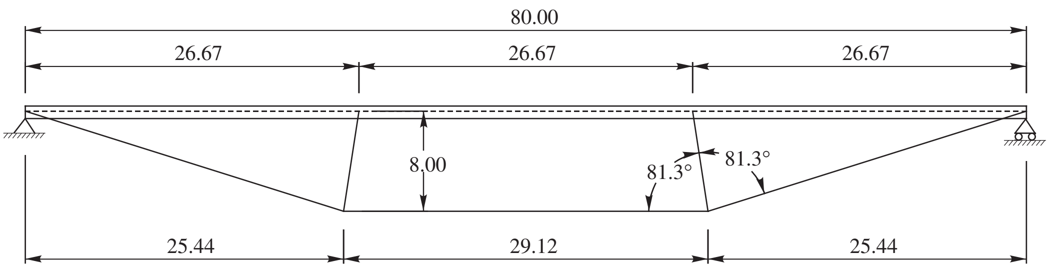

Fig. 4. Schematic elevation of the bridge.

7. Application to a bridge with under-deck stay cables

In the previous section, it has been shown by means of a simple example that dynamic amplification factors can be larger than 2 when an abrupt load is applied. This fact is very important in the design of cable-stayed bridges, since the abrupt action due to the accidental breakage of stay cables must be considered. Through this new example, it will be shown that dynamic amplification factors may be larger than 2 for cable-stayed bridges.

The dynamic response of a bridge with under-deck stay cables will be studied in the event of the accidental breakage of a stay cable.

The bridge has a span of 80 m. The deck is made of a hollow concrete slab with a characteristic strength of 40 MPa. Fig. 4 shows an elevation of the bridge and Fig. 5 shows the cross section used in the analysis. We used this cross section to represent a different type of cross section that is more appropriate from a construction point of view, as shown in Fig. 6. The weight of the deck is 188.25 kN/m and the dead load is 43.10 kN/m.

The under-deck cable-staying system is made up of five stay cables with a total cross section comprising 258 strands, of 140 mm^2 each one of them. The characteristic tensile strength of these cables is 1860 MPa. The under-deck cable-staying system is deviated by means of two struts that divide the span into equal portions. The stay cables are blocked where they cross through the deviator. The eccentricity of the under-deck cable-staying system at mid-span is 8 m.

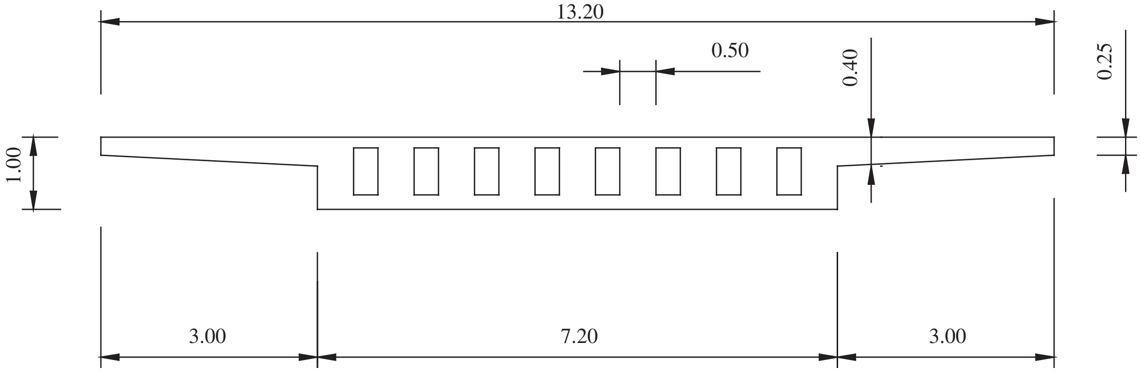

Fig. 5. Cross section of the bridge used in the analysis stage.

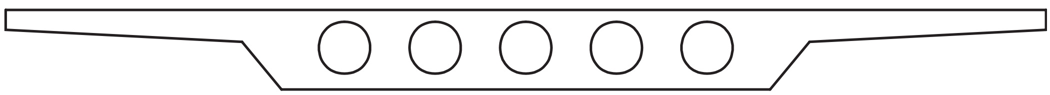

Fig. 6. Real cross section of the bridge.

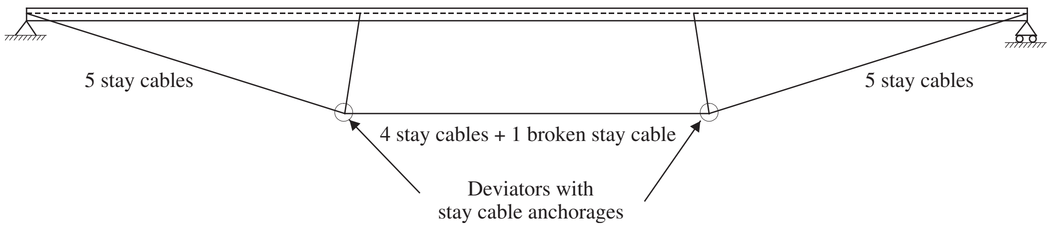

Fig. 7. Scheme of breakage of a stay cable between struts.

We obtained the dynamic response of the bridge due to the sudden accidental breakage of one of the five stay cables located in the central part of the bridge between the deviators, located on the bottom part of the struts (Fig. 7).

To obtain the dynamic response, the stay cable concerned by the breakage is eliminated from the model and the forces applied by this stay cable before breakage are applied at the anchor points. At the initial time, the breakage of the eliminated stay cable occurs and, as a result, the force that had been acting upon the anchor points disappears abruptly. Therefore, at the same instant an equal and opposite force is applied. The structure is not in equilibrium, so it begins to oscillate around the final equilibrium position, which is reached once the accumulated energy has been dissipated due to the damping of the structure. A damping ratio equal to 2% has been adopted for all the vibrational modes (similar values have been measured in Glacis Bridge [30] and Takehana Bridge [31], both of which are under-deck cable-stayed bridges).

During the time necessary to stop the movement, we analysed the records of the internal forces acting on the deck and on the stay cables. We compared these internal forces to the static internal forces caused by the action, which allowed us to evaluate the dynamic amplification factors related to the different internal forces and movements.

The dynamic response was obtained from a modal analysis in which a total of 12 vibrational modes were taken into account. Fig. 8 shows the seven modes activated in the structural response.

Fig. 9 shows the DAFs related to the bending moments in the deck. In many areas of the deck, these DAFs are larger than 2, even in the critical mid-span section. Therefore, if we had obtained the maximum dynamic response on the basis of the static response multiplied by a dynamic amplification factor equal to 2, we would have obtained internal forces weaker than the actual internal forces, leaving us on the unsafe side. In this

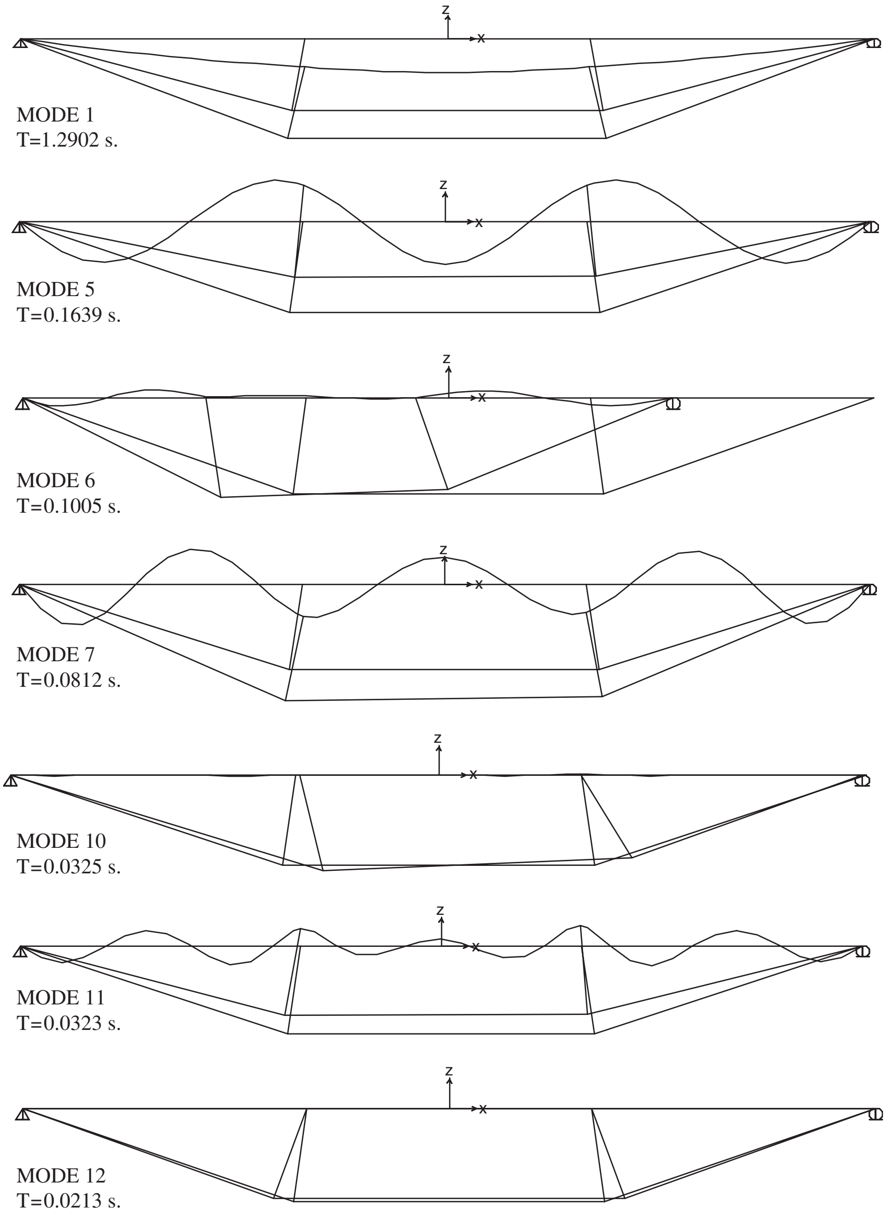

Fig. 8. First vibrational modes excited in the dynamic response due to the breakage of a stay cable.

example, we see once again that the dynamic amplification factors in the event of accidental breakage of stay cables can be larger than 2. This should lead to reconsider the calculation procedure proposed in some guidelines for cable-stayed bridges, which advise an equivalent static calculation adopting a DAF equal to 2, on the assumption that it is an upper limit; however, the results obtained in that way will be unsafe.

We will now go on to describe the DAFs related to bending moments in three sections of the deck: one located at a distance of 4 m from the support (DAF = 4.22), another located above the struts (DAF = 1.42) and a third at the mid-span section (DAF = 2.79).

Table 9 shows the maximum bending moment in the section located at a distance of 4 metres from the support and its projection on the different vibrational modes. It also shows the maximum amplitude of the projection on each vibrational mode and the value of the projection after damping of the response. Thus,

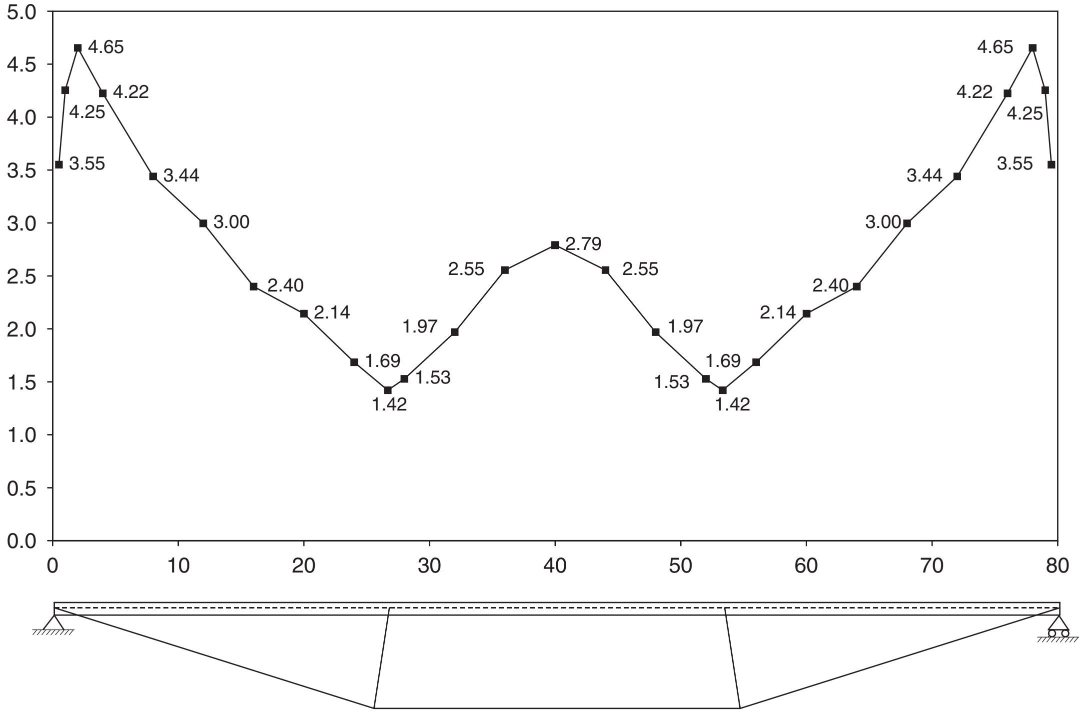

Fig. 9. DAFs related to the bending moments of the deck.

Table 9
Dynamic response at a section located 4 m from the support

Mode	Bending moments at the point located at a distance of 4 m from the support		
	Bending moment at $t = 0.68$ s (MN m)	Maximum bending moment of each mode (MN m)	Bending moment at $t = \infty$ (MN m)
1	+0.801	+0.807	+0.416
5	−0.300	−0.914	−0.471
6	−0.027	−0.054	−0.028
7	+0.516	+0.814	+0.420
10	−0.013	−0.028	−0.014
11	−0.084	−0.175	−0.090
12	−0.025	−0.050	−0.026
$\sum$(7 first active modes)	0.868		+0.207
All modes	0.866		+0.205
DAF		4.2	

the DAF related to moments in this section will be given by the expression:

$$\mathrm{DAF}_{\text{bending moment}} = \frac{0.801 - 0.300 - 0.027 + 0.516 - 0.013 - 0.084 - 0.025}{0.416 - 0.471 - 0.028 + 0.420 - 0.014 - 0.090 - 0.026} = 4.2. \quad (27)$$

There are several modes (modes 5, 6, 10, 11 and 12) for which the projection of the response on the vibrational mode is negative. In other words, the projection on the mode has the opposite sign to the value of the total response at infinity, which is just the static response.

Table 10
Dynamic response at a section located at third-span, over the strut

Mode	Bending Moments at the point located over the strut		
	Bending moment at $t = 0.115$ s (MN m)	Maximum bending moment of each mode (MN m)	Bending moment at $t = \infty$ (MN m)
1	+0.021	+0.273	+0.141
5	+0.572	+0.859	+0.443
6	−0.014	−0.060	−0.031
7	+0.726	+0.822	+0.424
10	+0.049	+0.058	+0.030
11	+0.418	+0.056	+0.261
12	+0.159	+0.217	+0.112
$\sum$(7 first active modes)	+1.931		+1.380
All modes	+1.940		+1.366
DAF		1.4	

Table 11
Dynamic response at mid-span

Mode	Bending Moments at mid-span		
	Bending moment at $t = 0.654$ s (MN m)	Maximum bending moment of each mode (MN m)	Bending moment at $t = \infty$ (MN m)
1	+4.104	+4.106	+2.116
5	−0.255	−1.247	−0.643
6	0	0	0
7	−0.132	−0.393	−0.203
10	+0.010	+0.020	+0.010
11	+0.062	+0.122	+0.063
12	+0.012	+0.024	+0.012
$\sum$(7 first active modes)	+3.801		+1.355
All modes	+3.811		+1.366
DAF		2.8	

Table 10 shows the bending moments in the section of the deck located over the strut. Thus, the DAF related to bending moments at this section will be given by the expression:

$$\mathrm{DAF}_{\text{bending moment}} = \frac{0.021 + 0.572 - 0.014 + 0.726 + 0.049 + 0.418 + 0.159}{0.141 + 0.443 - 0.031 + 0.424 + 0.030 + 0.261 + 0.112} = 1.4. \tag{28}$$

However, in this section there is only one vibrational mode (mode 6) on which the response of the structure has a negative projection, but the weight of that mode in the response is very small (2%). The fact that the weight of this mode is very small and the fact that the structure is damped give rise to a substantial reduction of the dynamic amplification factor.

Table 11 shows the bending moments in the section of the deck at mid-span. Thus, the DAF related to bending moments in this section will be given by the expression:

$$\mathrm{DAF}_{\text{bending moment}} = \frac{4.104 - 0.255 - 0.132 + 0.010 + 0.062 + 0.012}{2.116 - 0.643 - 0.203 + 0.010 + 0.063 + 0.012} = 2.8. \tag{29}$$

In this section there are once again several vibrational modes on which the projection of the response of the structure is negative (modes 5 and 7), and these are modes with substantial weight in the responses (47% and 15% respectively), which considerably increase the value of the dynamic amplification factor.

In addition, a direct integration was performed confirming the accuracy of the solution calculated by means of modal superposition.

We obtained the DAFs related to bending moments, we also obtained the DAFs related to deflections and the DAFs related to axial forces. In the deck at mid-span, the DAF related to deflection is 1.97; the DAF related to moments is 2.80 and the DAF related to axial forces is 28.53. As the vibrational modes associated with high frequencies gain in weight, given the existence of modes with negative projection, the DAFs increase and can reach values that are much larger than 2.

If we consider once again the form of the static bending moments diagram in the case of the breakage of a stay cable, we see that it resembles the bending moments diagram in the second case of loading in Section 6 (Fig. 3). Therefore, using a similar approach, we could have predicted values for the dynamic amplification factors at mid-span larger than 2.

8. Conclusions

(i) Dynamic amplification factors for sudden applied loads to systems with several dofs can be larger than 2, but in order for this to be the case there must be at least one mode on which the projection of the structural response is negative with significant weight. The larger the weight of these modes is, the larger the dynamic amplification factors will be.

(ii) Dynamic amplification factors are reduced with an increase in the damping of the structure, attaining a value of 1 for critical damping.

(iii) If there is not any vibrational mode on which the projection of the structural response is negative, the dynamic amplification factor for sudden applied loads will be exactly 2 in undamped systems and less than 2 in damped systems.

(iv) The weight of the different vibrational modes in the response of the structure depends on the type of internal force or movement involved. The weight of the vibrational modes associated with high frequencies is larger in the response in bending moments than in the response in deflections. The weight of vibrational modes associated with high frequencies increases progressively from deflections to shear forces in the order: deflections → rotations → bending moments → shear forces. Consequently, if there are vibrational modes with negative projection in the several responses (deflections, rotations, bending moments, shear forces, etc.), the dynamic amplification factors also increase respectively in the same order.

(v) The dynamic amplification factors are specific to each section, to each response (movement or internal force), to each action considered and to the structure.

(vi) Given that a DAF equal to 2 is not an upper limit in the case of actions applied abruptly to a structure, it is advisable to evaluate maximum internal forces by means of a dynamic analysis since there is a lack of research studies which establish the order of magnitude of these factors in conventional structures.

(vii) In cable-stayed structures, the DAFs related to internal forces as a result of the abrupt breakage of a stay cable can be larger than 2. Thus, following the guidelines for cable-stayed bridges and carrying out a static calculation in which the forces are amplified by a DAF of 2 can underestimate loads even when the maximum internal forces in critical sections of the structure are assessed.

(viii) We must revise the guidelines for cable-stayed bridges [22–24] as well as the draft of the Eurocode 3–Part 1.11 [25] in connection with the accidental action caused by the abrupt breakage of stay cables. It would be particularly advisable to carry out dynamic analysis when it is known that vibrational modes can exist with negative projection on the response of the structure, because in such cases, the DAFs can attain values larger than 2. Due to such an accidental action we can predict DAFs related to bending moments larger than 2 in critical sections not only in bridges with under-deck cable-staying, but also in conventional cable-stayed bridges (e.g. in the event of the simultaneous breakage of two consecutive stay cables).

(ix) Given the calculation tools currently available for assessing structures, we believe that a dynamic analysis should always be made for large structures.

(x) It would be advisable to carry out a research project to evaluate the DAFs related to the different internal forces in conventional cable-stayed structures with the aim of establishing some values for guidance to be taken into account in the pre-dimensional stage of the structure design.

(xi) From a practical point of view, the shape of the functions $F(t)$ for a breakage of the stay cables ($F(t)\neq 0$ if $t < t_{\text{breakage}}$; and $F(t) = 0$ if $t \geqslant t_{\text{breakage}}$) does not affect the values of the maximum internal forces if the breakage time is negligible in comparison to the fundamental period of the structure ($t_{\text{breakage}} \ll T_1$). In that case, the assumption $t_{\text{breakage}} = 0$ can be made, obtaining an upper limit of the DAF which is almost equal to the real value. The shorter the breakage time in comparison with the fundamental period of the structure, the larger dynamic amplification factors.

(xii) In cable-stayed bridges, the assumption $t_{\text{breakage}} = 0$ can be made in the event of an abrupt breakage of stay cables due to impacts. Although there is no experimental data on the breakage time of stay cables due to collisions, these values will be negligible in comparison to the fundamental period of the structure ($t_{\text{breakage}} \ll T_1$). However, the assumption $t_{\text{breakage}} = 0$ is too conservative in the event of a slower breakage of stay cables due to another reason. In other types of cable-stayed structures with smaller vibration periods (as in roof structures), this assumption is too conservative even in the case of an abrupt breakage of stay cables due to impact, and consequently new lines of research should be undertaken in this respect.

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