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An active vibration absorber for a flexible plate boundary-controlled by a linear motor

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Abstract

A long, narrow flexible plate subjected to cyclic disturbances at the midsection is regulated by a linear motor at the boundary. Oscillations at the midsection will be eliminated while the rest of the plate is allowed to swing in a way as to counteract the external force. The control design is based on a virtual passive approach without referring to the detailed mathematical model. A vibration absorber integrating the flexible plate with a combination of passive elements attached to the boundary is first devised. Rather than built with hard physical devices, these passive elements including mechanical springs, dampers, and masses are emulated by the linear motor with a suitable feedback law. The feedback signal is the boundary displacement from an LVDT sensor. Numerical simulations illustrate how a node is developed in the middle of the plate while the rest of the structure tends to a harmonic motion. Experimental results confirm the effectiveness of the control scheme.

1. Introduction

This paper reports the design and experimental results of a vibration control scheme on a long, narrow flexible plate subjected to cyclic disturbances. This flexible plate can be modeled as a laterally oscillating beam. The disturbances are imposed at the midsection of the flexible plate, while the actuator, a moving-magnet linear motor, is located at the boundary. The objective is to eliminate oscillations at the midsection. The system is generic to the class of flexible structures in which the source of disturbance is not collocated with the actuator, and therefore cannot be directly neutralized by the control actions.

The dynamic vibration absorber (DVA), first patented by Frahm in early last century [1] is a classic technique to neutralize a harmonic disturbance. A DVA is essentially a flexible substructure having a resonant frequency close to that of the disturbance. The original DVA was lightly damped and the system might be only marginally stable. As a refinement, a damped vibration absorber was introduced [2]. By adding a damper between the primary body and the additional structure, transient response was improved at a price: vibrations of the primary body could no longer be perfectly absorbed. Besides, the passive DVA also suffers a major drawback: its characteristic frequency is difficult to tune if the disturbance's frequency drifts over time.

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By contrast, active vibration control has the benefit of easily tunable parameters and can be adapted for a much wider range of operating frequency. Various kinds of actuators have been employed in active vibration control, including DC motors [3–5], electromagnetic actuators [6], piezoelectric materials [7], and linear voice-coil motors [8]. One common feature among these works is that the actuators are mounted directly on the locations where the vibrations are to be attenuated. In other words, the actuator is capable of directly counteracting the external disturbance with a proper control algorithm. However, there are situations where it is undesirable or impractical to mount a cumbersome physical device directly on the hot spot such as, for example, the end effector of a flexible arm. For such a system the disturbance must be indirectly compensated by an actuator located at a distance away. Recently in Ref. [9], procedures were developed using a chain of oscillators to passively impose nodes on a flexible structure. The nodes could be away from the oscillators and be located at any point of the structure, thereby quenching vibrations at the desired spot. Since no damping was considered in Ref. [9], the results were valid in the steady state but the transient times might be quite long.

In the control of flexible structures the so-called *spillover effect* [10] may occur, leading to system instability, if the sensor and the actuator are not collocated. This problem will be avoided in this research by mounting a displacement sensor (a linear variable differential transformer, LVDT) directly to the linear motor. That is, the actuator is collocated with the sensor, which measures the boundary displacements of the flexible plate. This is related to *boundary control* of flexible structures [11–13], where the control input is imposed at one end of a transversely vibrating beam or a stretched string moving axially. The major difference is that the disturbance concerned here stems from a non-decaying harmonic source that cannot be completely neutralized. Consequently, one has to settle for a solution that cancels out external forces in certain part of the plant and allows the rest of the system to oscillate.

In this paper, the controller will be developed from a virtual passive approach without referring to the detailed, sophisticated dynamic equations. We conceptually design a mechanical structure that makes physical sense by attaching a combination of passive elements at the boundary of the flexible plate, including mechanical springs, dampers, and masses. These elements integrate with the flexible plate to form a perfect dynamic vibration absorber with respect to the midsection. The conceptual device modifies the structure to have a node at the midsection and at the same time provides necessary damping to the structure. Such a conceptual device may be difficult to construct or install physically, but can be readily realized or emulated by the linear motor. In other words, the controller is primarily devised via physical reasoning; mathematical analysis is conducted at a later stage to verify performance. This is in contrast to the conventional approach, in which mathematical modeling is usually required for the development of the control algorithm, while physical interpretation, if any, is given later.

The notion of virtual vibration absorbers related to this research was first presented in Ref. [14], and was applied to a flexible beam in longitudinal motion in Ref. [15]. This paper tackles lateral vibrations of flexible plates and provides real-time testing results on an experimental prototype. Numerical simulations based on a finite number of flexible modes are conducted to examine the behavior of the flexible structure under the virtual passive controller. The key control parameter for a wide range of disturbance frequencies is also determined via numerical analysis.

The rest of paper is arranged as follows. Section 2 reviews the design of a novel vibration absorber for a simple mass–spring structure. This mechanism not only neutralizes a harmonic disturbance but also provides damping to the system. In Section 3 a vibration absorber integrating the flexible plate and a combination of passive elements at the boundary is developed in analogy to the method of Section 2. In this section the motions of the flexible plate are illustrated by numerical simulations. Parameters of the key passive elements are also determined. Section 4 shows how to emulate the passive elements by a linear actuator with a feedback law. Stability of the closed-loop system is then analyzed mathematically. Section 5 presents the experimental results on a testing apparatus, followed by concluding remarks in Section 6.

2. System description and review of vibration absorbers with inherent damping

The system under investigation is sketched in Fig. 1, where a strip of flexible plate is subjected to a cyclic disturbance at the middle. A linear actuator is mounted at the boundary. The plate is narrow in width so that it can be viewed as a flexible beam undergoing lateral oscillations. Equipped with a linear variable differential

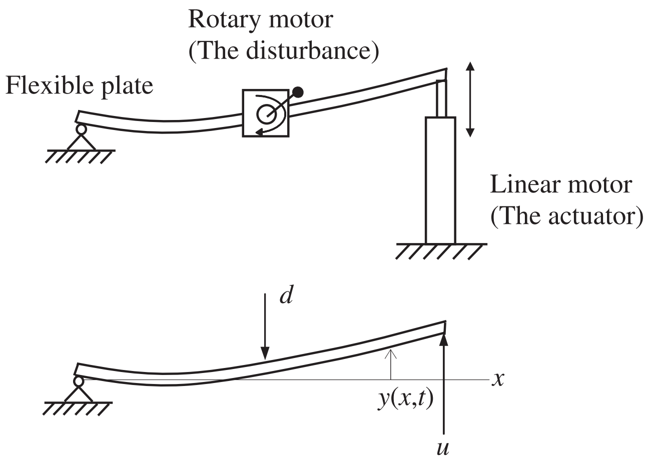

Fig. 1. A flexible plate subject to cyclic disturbances, and the simplified free-body diagram.

transformer, the actuator is capable of imposing a calculated force on the end of the plate according to the displacement of the boundary. The disturbances are generated by a rotary motor driving a block that moves along a linear guideway.

The dynamic equation of the system can be described by

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}(x,t)\right)+\rho A\frac{\partial^2 y}{\partial t^2}=0, \quad 0<x<\frac{\ell}{2},\ \frac{\ell}{2}<x<\ell. \tag{1}$$

The boundary condition on the left is

$$y(0,t)=0, \tag{2}$$

The continuous equation at the center is

$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 y}{\partial x^2}\right)\bigg|_{x=\ell^-/2}-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 y}{\partial x^2}\right)\bigg|_{x=\ell^+/2}=d, \tag{3}$$

where

$$d=\alpha\sin(\omega_0 t+\psi). \tag{4}$$

And the boundary condition on the right is

$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 y}{\partial x^2}\right)\bigg|_{x=\ell}=-u. \tag{5}$$

In the above equations, u is the control input and d is the harmonic disturbance of radian frequency ω_0. The length of the plate is ℓ and the disturbing force is imposed at $x=\ell/2$. The parameter EI denotes the bending stiffness and ρA is the mass per unit length. Note that while ω_0 is given, the amplitude α and phase ψ of the disturbance are uncertain constants. As mentioned in the introduction, initial controller design does not need the system equations. However, they are useful later in constructing a simulation model and verifying system stability and performance. Without loss of generality the inertia of the lumped mass at the midsection is neglected.

The objective of the research is to design a control algorithm with which oscillations at the midsection tend to zero while the rest of the structure swings in such a way as to cancel out the harmonic disturbance. Before exploring this distributed-parameter system, the development of vibration absorbers for a simple spring–mass structure is reviewed below. It will serve as an analogy to be followed in the next section.

An undamped vibration absorber: Fig. 2 shows a vibration absorber that completely neutralize the harmonic disturbance of radian frequency ω_0. The key is to mount a spring–mass pair on the primary body, and the

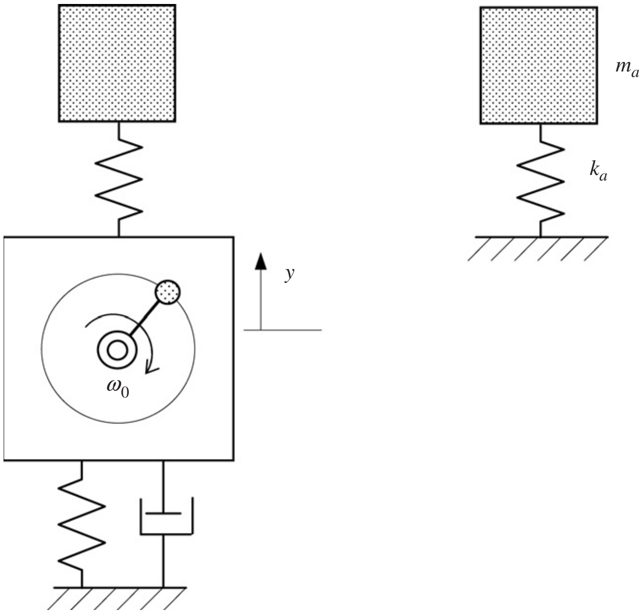

Fig. 2. Vibration absorber that perfectly counteracts a harmonic disturbance. The substructure with the primary body fixed (the output-fixed system) must have a resonant frequency equal to that of the harmonic disturbance ($\sqrt{k_a/m_a} = \omega_0$).

natural frequency of the spring–mass pair should match the frequency of the disturbance. In other words, the primary-body-fixed structure (or *output-fixed* system) should contain a characteristic frequency of ω_0.

A symmetric vibration absorber: If the primary body is lightly damped, the vibration absorber must contain some damping effect to allow for a decent stability margin. However, adding a damper to the absorber in parallel to the spring k_a in Fig. 2 also makes it impossible to perfectly neutralize the harmonic disturbance. Fig. 3 shows an alternative scheme in which the damper does not affect the characteristic frequency of the vibration absorber. The scheme is called a symmetric vibration absorber [14]. Note, however, the two spring–mass pairs connected by the dashpot need not be identical, as long as each pair has the same natural frequency of ω_0. Note also that while the mechanism is difficult to implement mechanically, it can be readily emulated by a servomotor [4].

3. Design of an integrated vibration absorber

In this section, a vibration absorber for the flexible plate will be devised in two steps following the analogy of the rigid-body system of Figs. 2 and 3. In contrast to the previous case, where the vibration absorber is distinct from the primary body, the flexible plate itself will serve as part of the vibration absorber.

Analogous to Fig. 2, the first step is devising a mechanism such that the output-fixed system (i.e., the midsection-stationary structure) has a natural frequency of ω_0. The simplest way in this case is attaching a spring of stiffness k_s to the boundary. The stiffness is tuned such that the midsection-hinged structure contains a natural frequency of ω_0, as shown in Fig. 4. Since there are infinite numbers of natural modes in the flexible structure, one may tune k_s to have the fundamental frequency match ω_0. Note that if the disturbance frequency ω_0 significantly exceeds the fundamental frequency of the structure, higher-frequency mode can be tuned to match ω_0.

The structure shown in Fig. 4 is not properly damped; the system is only marginally stable. The next step will be adding a damper to the system while preserving the natural frequency of ω_0. Fig. 5 shows one solution. Analogous to Fig. 3, an additional damper–mass–spring mechanism, with $\sqrt{k_a/m_a} = \omega_0$, provides needed damping to the system without affecting the fundamental natural frequency, since the flexible plate will oscillate synchronously with the mass m_a at the frequency ω_0 in the steady phase.

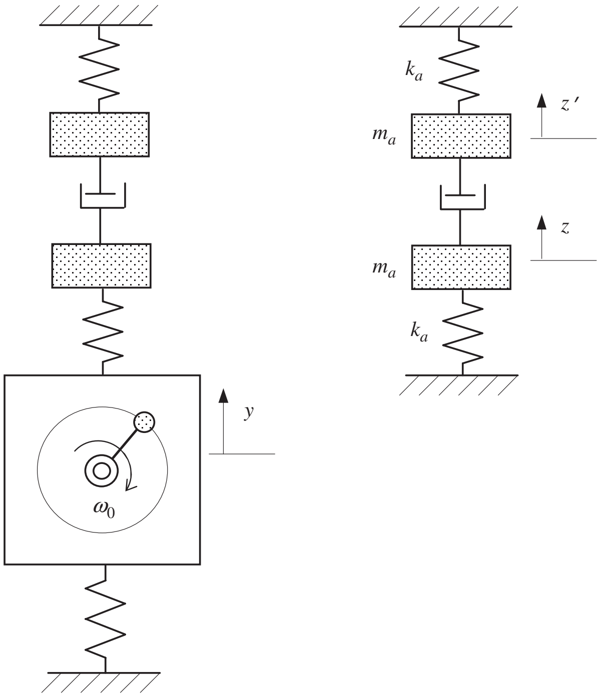

Fig. 3. A vibration absorber with inherent damping. With the damper, the absorber still has a resonant frequency at ω_0 if $\sqrt{k_a/m_a} = \omega_0$. In steady state the two masses swing synchronously in the same direction (no relative motions between z and z'). The damper is crucial in the transient phase so that the system is stabilized.

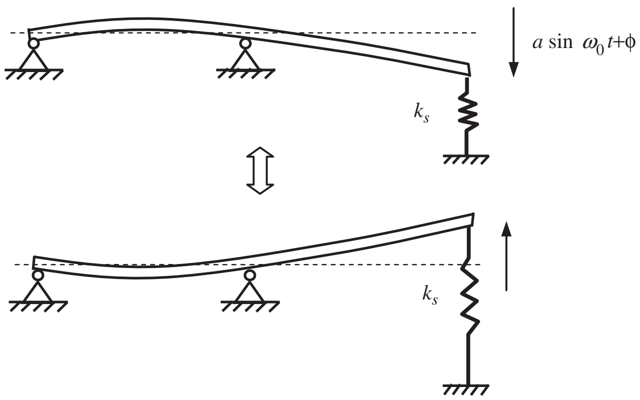

Fig. 4. Step 1—attaching a spring of stiffness k_s to the end of the flexible plate so that the midsection-hinged structure contains a natural frequency of ω_0.

The resulted system is shown in Fig. 6. Similar to the block in the system of Fig. 3, the midsection of the flexible plate tends to a standstill while the rest of the structure swings in a harmonic way at the frequency of ω_0. In other words, integrated with the passive elements, the flexible plate functions as a vibration absorber for the block at the midsection.

Illustrations: Numerical simulations are conducted to illustrate the behavior of the system of Fig. 6. The simulation model is derived using Lagrange equations based on five flexible modes for the flexible plate. The cyclic disturbance has a frequency of 1.5 Hz, and the spring k_s at the boundary is adjusted so that the

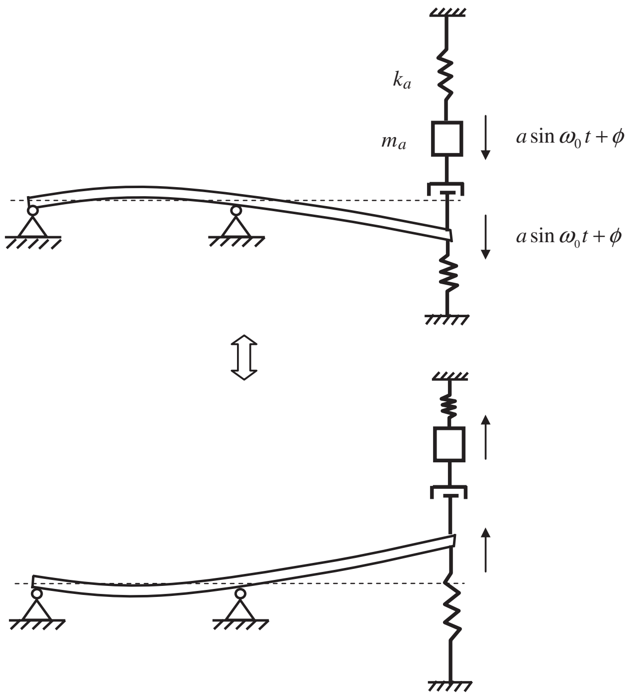

Fig. 5. Step 2—attaching an additional damper-mass-spring mechanism to the boundary, with $\sqrt{k_a/m_a} = \omega_0$. The natural frequency of ω_0 is preserved since the end of the plate oscillates synchronously with the mass m_a in the steady phase.

fundamental frequency of the midsection-hinged structure is calculated to be 1.5 Hz. Fig. 7 shows the motions of the flexible plate at various moments. It is seen that initially the midsection ($x/\ell = 0.5$) is vibrating significantly. The oscillations gradually decay. Eventually the point of concern comes to a standstill while the flexible plate keeps swinging with a node in the middle.

Note that the flexible modes used in the simulation are determined by first dividing the beam into 400 elements and calculating the mode shapes that are consistent with the boundary conditions. The first five modes are retained and then further refined by smoothing out the curves near the boundary using a cubic spline interpolation. Validity of using 5 modes in the simulation can be checked by examining the contribution of the individual modes in the time response. It turns out that the magnitudes of the last three modes are significantly lower than the first two modes, and that of the fifth mode is almost negligible. If the disturbance frequency is higher, however, a larger number of modes may be needed.

Determination of k_s: The fundamental natural frequency of the midsection-hinged plate is calculated against k_s. The results are normalized against the bending stiffness, length, and mass per unit length, as shown in Fig. 8. Using this figure one may determine the suitable k_s given the frequency of disturbance and other plant parameters. However, from this figure it is seen that there are limits to the achievable frequencies. If the disturbance's frequency is beyond these limits, a suitable k_s cannot be found. For higher frequencies one may try the second or third natural frequency, but for lower frequencies using a single spring at the boundary is not enough. The structure has to be "softened" in some way for a lower natural frequency.

Softening the plate: The fundamental frequency of the midsection-hinged plate can be further reduced by attaching a spring–mass–spring mechanism instead of a single spring at the boundary, as shown in Fig. 9. In the fundamental mode the mass moves in the same direction as the boundary. The spring k_1 in the figure behaves like a negative spring with respect to the flexible plate: when the boundary is below (above) the equilibrium position, the spring k_1 pulls the plate downward (upward). The flexible plate is effectively softened by the spring–mass structure. Fig. 10 shows the range of achievable fundamental natural frequency with

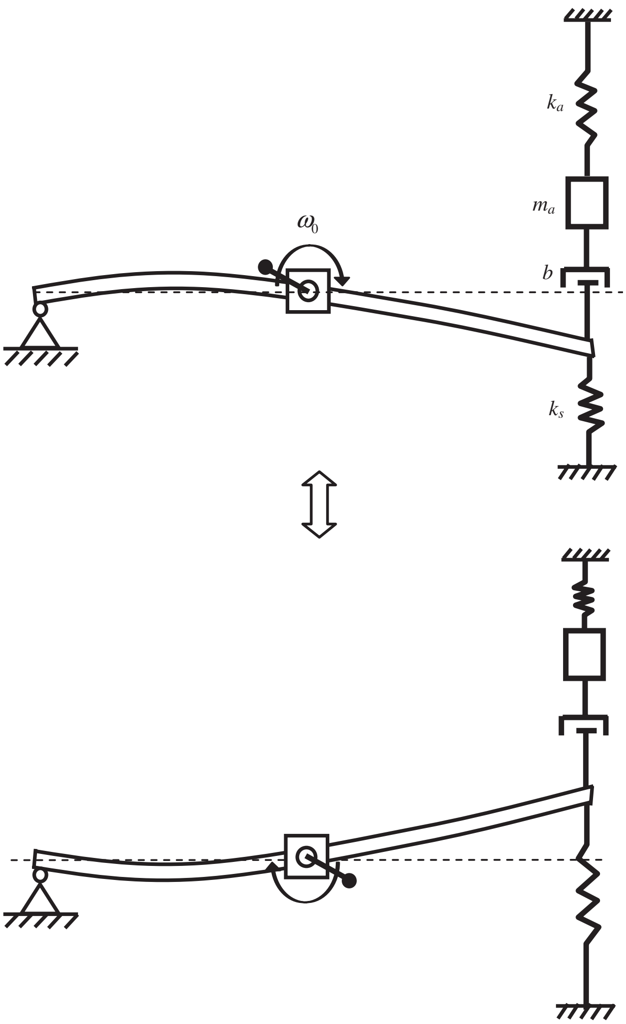

Fig. 6. Integrated with the passive elements at the boundary, the flexible plate functions as a vibration absorber to the block at the midsection.

different k_1's, given $k_2 = k_1$ and $m_s = \rho A\ell$. It is seen that the frequency can now be reduced all the way down to zero.

4. Realization by feedback control

The passive mechanisms devised in the previous section are simple to draw but difficult to implement with hard mechanical elements, especially if the parameters are to be accurately tuned. With the help of a linear motor, however, the passive elements at the boundary can be emulated by a feedback algorithm and easily tuned for various characteristic frequencies.

The following control law emulates the dynamics of the passive mechanism shown in Fig. 6:

$$u = -k_s y(\ell, t) + b(\dot{z} - \dot{y}(\ell, t)), \tag{6}$$

$$\ddot{z} = -\frac{k_a}{m_a} z - \frac{b}{m_a}(\dot{z} - \dot{y}(\ell, t)), \tag{7}$$

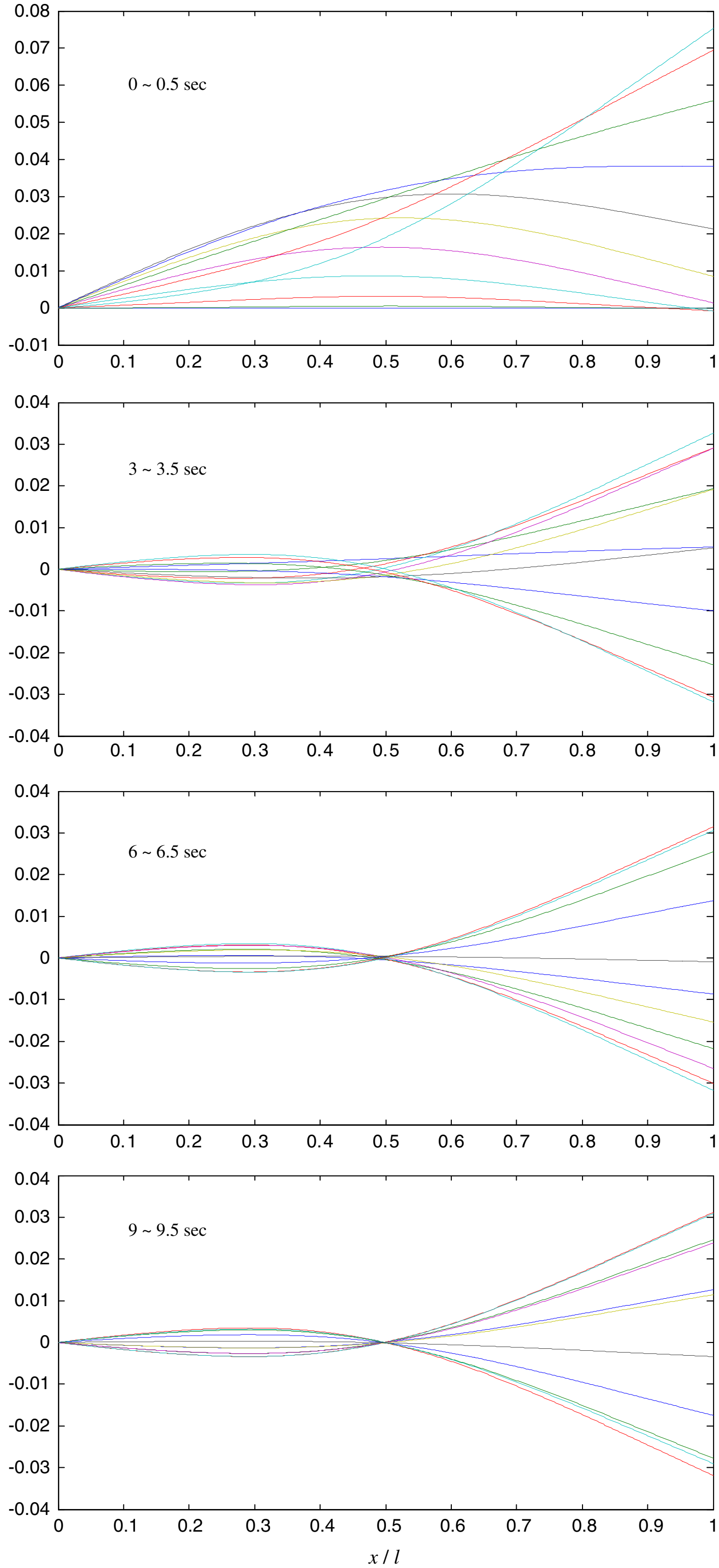

Fig. 7. Motions of the flexible plate as time elapses. Note how a node develops at the midsection.

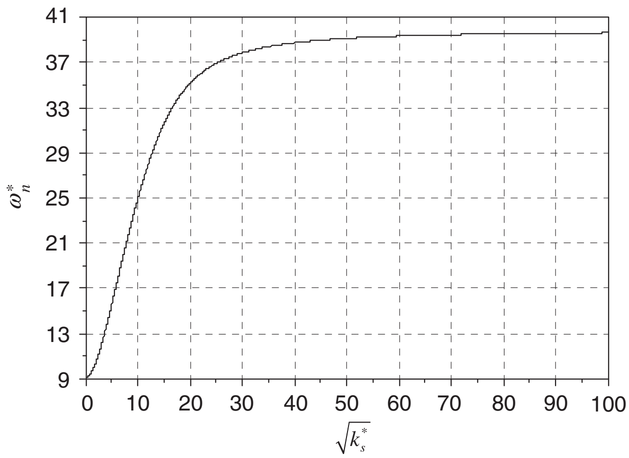

Fig. 8. Fundamental natural frequency ω_n of the midsection-hinged plate versus k_s. The parameters are normalized with $\omega_n^* = \omega_n/\sqrt{EI/\rho A\ell^4}$ and $k_s^* = k_s\ell^3/EI$.

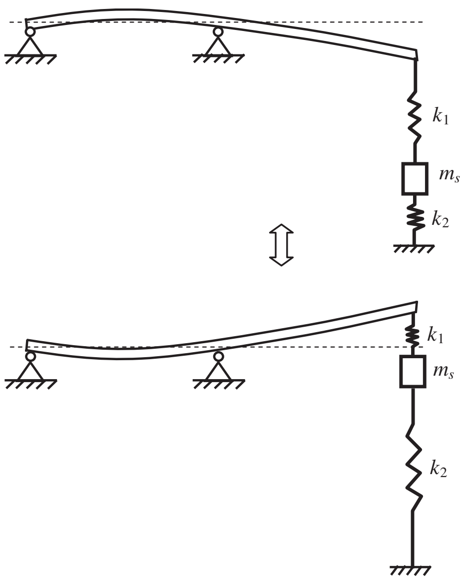

Fig. 9. Softening the structure: the fundamental frequency of the midsection-hinged structure can be lowered with an additional mass at the boundary.

where $\dot{y}(\ell, t) \equiv \partial y/\partial t(\ell, t)$; k_s, k_a, m_a, and b stand for the stiffness, inertia, and damping coefficient for the virtual passive elements, as defined before. The variable z represents the displacement of the virtual mass m_a. In implementation $y(\ell, t)$ is measured and $\dot{y}(\ell, t)$ is calculated by numerically differentiating $y(\ell, t)$; the variable z will be calculated and updated every sampling period in the computer.

Since the interaction between the lumped, passive elements and the flexible plate in Fig. 6 is governed by Eqs. (6) and (7) (as Fig. 11 illustrates), the closed-loop system of Eqs. (1)–(7) are mathematically equivalent to the structure of Fig. 6.

Stability of the unforced system: Stability of the closed-loop system, excluding the external disturbance, can be verified by choosing a Lyapunov function that represents the combined kinetic energy and potential energy

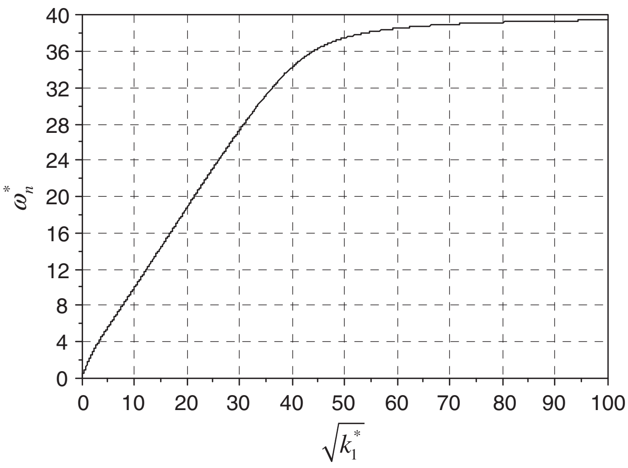

Fig. 10. ω_n^* versus k_1^* for the structure of Fig. 9 given $m_s = \rho A\ell$ and $k_2 = k_1$; $k_1^* = k_1\ell^3/EI$.

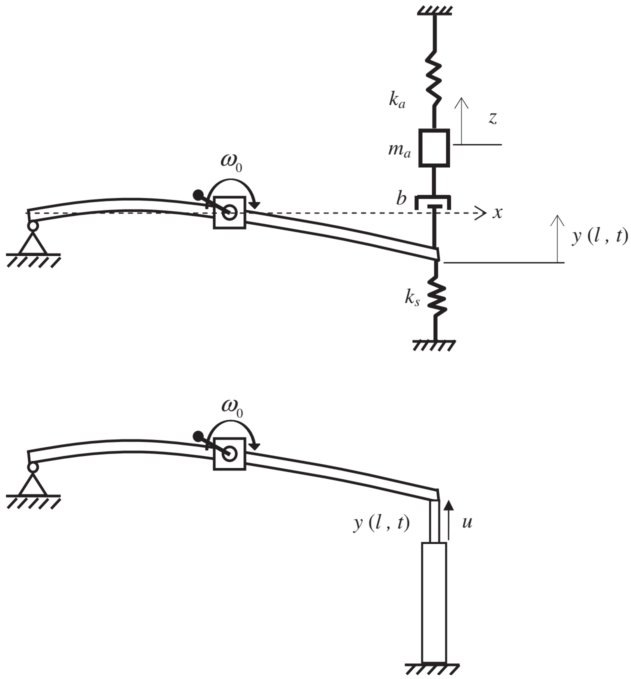

Fig. 11. Passive mechanism emulated by a linear actuator: under the feedback law of Eqs. (6) and (7), the actively controlled system (below) is equivalent to the passive mechanical structure (above).

of the equivalent mechanical structure:

$$V = \frac{1}{2}\left[\int_0^\ell EI\left(\frac{\partial^2 y}{\partial x^2}\right)^2 \mathrm{d}x + \int_0^\ell \rho A\left(\frac{\partial y}{\partial t}\right)^2 \mathrm{d}x + k_s y(\ell,t)^2 + m_a\dot{z}^2 + k_a z^2\right]. \tag{8}$$

Using Eqs. (1)–(7) (dropping the disturbance, i.e., $d = 0$), it can be shown that

$$\dot{V} = -b(\dot{z} - \dot{y}(\ell,t))^2, \tag{9}$$

which is the power dissipated by the damper. Since V is lower bounded and $\dot{V}$ is negative semi-definite, $\dot{V}$ must tend to zero (from the "Lyapunov-like lemma" [16]); therefore $\dot{z} \to \dot{y}(\ell,t)$. Hence the interactions between the mass m_a and the flexible plate vanish so that eventually the two substructures either oscillate *synchronously* at their respective natural frequencies, or the amplitudes vanish. However, since ω_0 (the natural frequency of the m_a–k_a pair) is also the fundamental frequency of the midsection-*hinged* structure, it must be higher than the fundamental frequency of this midsection-*free* plate, meaning the two substructures cannot move synchronously. Therefore the only possible solution is $z \to 0$ and $y(x,t) \to 0$.

Zeros at $\pm i\omega_0$: In the previous section, the fact that the system output, namely $y(\ell/2, t)$ tends to zero under the harmonic disturbance has been explained from the viewpoint of a dynamic vibration absorber. From the control perspective, it can also be explained by the zero dynamics, which is defined to be the internal dynamics of the system when the output is rendered identically zero by the input [17]. (Here the disturbance d is regarded as the input.) In linear systems the eigenvalues of the zero dynamics coincide with the zeros of the transfer function. Since ω_0 is a natural frequency of the midsection-hinged structure, it is one of the characteristic frequencies of the system's zero dynamics. In other words, if the closed-loop system is discretized with a large but finite number of modes, $\pm i\omega_0$ will be a pair of zeros in the transfer function from d to $y(\ell/2, t)$. The steady-state response of the output is therefore zero for a sinusoidal disturbance of frequency ω_0. Refer to Ref. [15] for a formal treatment of zero output response on a longitudinally moving beam.

Note that when a virtual mass at the boundary is required to deal with low-frequency disturbance (Fig. 9), the control law should be modified to be

$$u = -k_1(y(\ell,t) - z_2) + b(\dot{z}_1 - \dot{y}(\ell,t)), \tag{10}$$

$$m_a\ddot{z}_1 = -k_a z_1 - b(\dot{z}_1 - \dot{y}(\ell,t)), \tag{11}$$

$$m_s\ddot{z}_2 = -(k_1 + k_2)z_2 + k_1 y(\ell,t), \tag{12}$$

where z_1 and z_2 represent the displacements of the virtual mass m_a and the virtual mass m_s, respectively.

5. Experimental results

Fig. 12 shows the picture of the experimental apparatus and its functional sketch. The flexible plate is made of copper alloy; its dimensions are 800 mm long, 100 mm wide, and 1.1 mm thick. The bending stiffness (EI) is

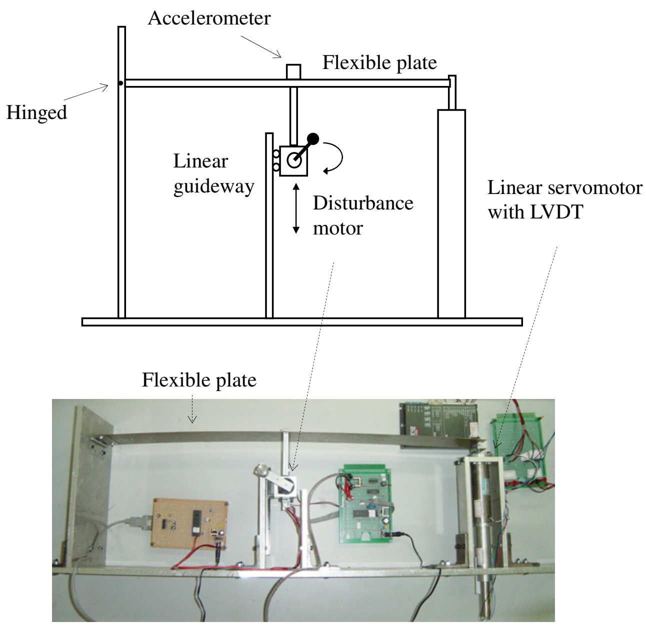

Fig. 12. Photo of the experimental apparatus and its functional sketch.

measured to be about 2.83 $N\,m^2$. The moving-magnet linear motor is made by H2W Technology, Inc.; it has a maximum stroke of 38.1 mm, and a maximum continuous force of 14.11 N. The force is linearly proportional to the applied current so that a driver in current mode is used. A linear variable differential transformer is mounted at the end of the linear motor to measure the boundary displacements. The displacement signals are fed back through a 12-bit A/D converter. The control algorithm is implemented using a PC running in MS-DOS mode. (The algorithm is programmed in C.) The disturbance is generated by a rotary DC servomotor driven by a microcontroller (PIC16F877). To monitor the response at the point of concern, a 1G accelerometer is glued to the middle of the flexible plate. However, only the displacement signal from the LVDT is used in the control loop: the actuator and the sensor are collocated as mentioned earlier. The accelerations of the midsection are monitored and recorded by a separate PC. Fig. 13 shows the experimental setup.

In the experiment the frequency of the cyclic disturbance is set at 4 rev/s (4 Hz). Following the design procedure described in Section 3, we first attach a virtual spring to the boundary. This is achieved by using a simple proportional feedback law for the linear motor, i.e.,

$$u = -k_s y(\ell, t). \tag{13}$$

The stiffness k_s is tuned such that the fundamental natural frequency of the midsection-hinged structure is equal to 4 Hz. This is verified by the spectrum of the impact response of the boundary (with the midsection held still) as shown in Fig. 14, where k_s is tuned to be 0.123 (N/m). Note that with the proportional feedback alone (Eq. (13)), an impact on the boundary leads to sustained oscillations because the system is only marginally stable.

An additional set of virtual spring, mass, and damper is then added to the control action, as governed by Eqs. (6) and (7), where k_a, m_a, b are, respectively, set to be 631.5 (N/m), 1(kg), 0.24 (N s/m). Note that $\sqrt{k_a/m_a} = 25.13$ rad/s, or 4 Hz as required.

The experimental results of the closed-loop system are shown in Fig. 15, where the accelerations of the midsection before and after the control activation are recorded. It is seen that vibrations at the midsection are drastically reduced thanks to the control action. In fact, there are no visible oscillations at the midsection

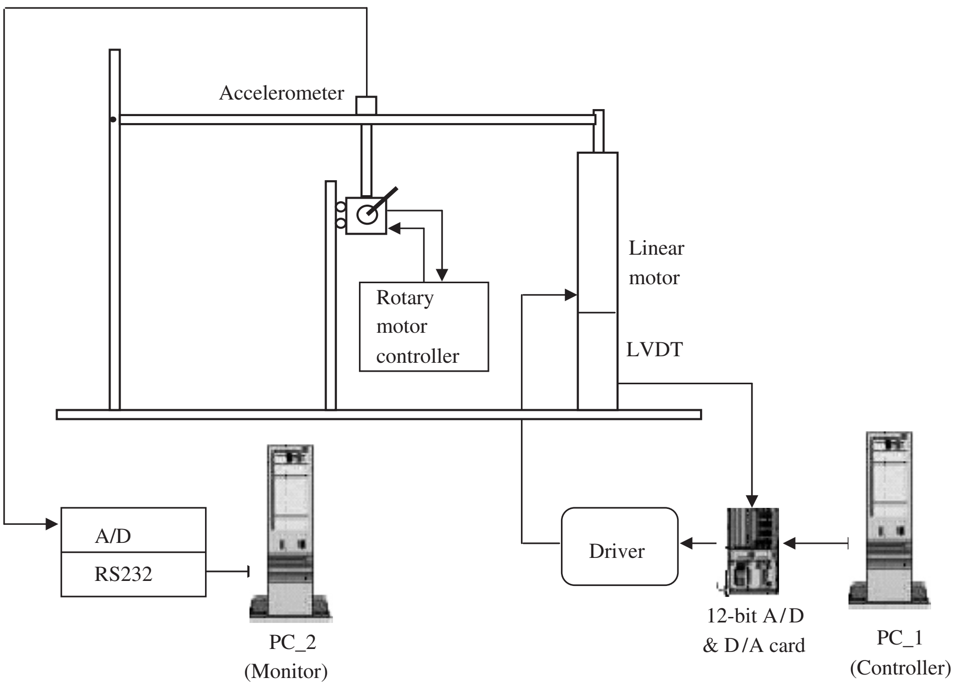

Fig. 13. Experimental setup.

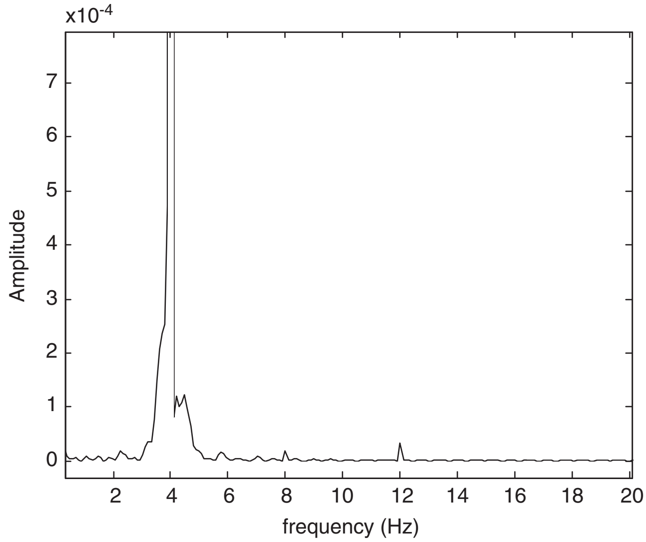

Fig. 14. Spectrum of the impact response of the boundary with a simple proportional feedback control (Eq. (13)) and with the midsection held still. Note that the fundamental frequency is tuned to 4 Hz by the proportional gain (k_s).

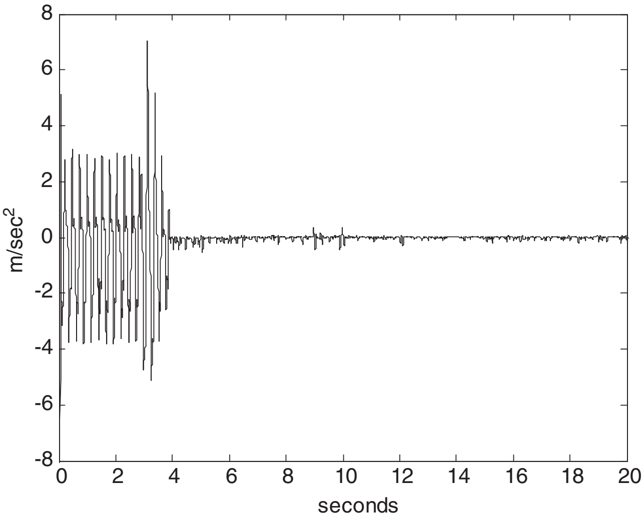

Fig. 15. Time response (accelerations) of the midsection under the control law of Eqs. (6) and (7). The controller is activated after 3 s.

relative to the steel base. Fig. 16 shows the displacements of the boundary measured by the LVDT. As expected, the boundary tends to a harmonic motion of 4 Hz. Note that the spectrum is calculated with a FFT algorithm using the recorded data over the interval of 10–20 s.

6. Conclusions

The control technique developed from physical reasoning was shown to be capable of transforming a boundary-controlled flexible structure into a dynamic vibration absorber. Numerical simulations demonstrated how a node was developed in the point of concern. The experimental results confirmed the effectiveness of the proposed method. Because the source of disturbance was not directly neutralized by the actuator,

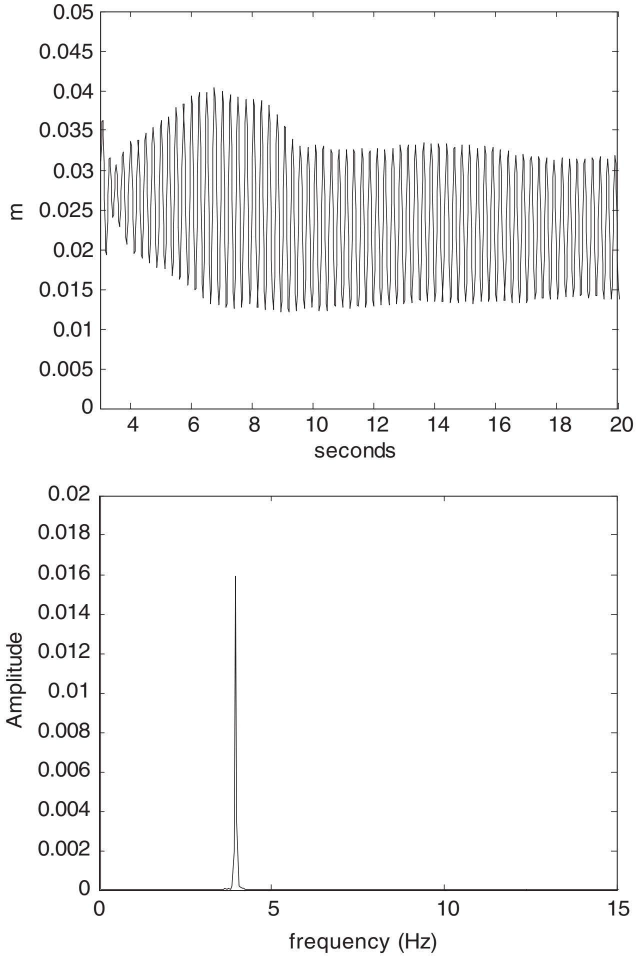

Fig. 16. Time response at the boundary and its spectrum.

oscillations could be eliminated only in certain part of the structure. However, it is possible to shift the output of concern from the middle to other locations by adjusting k_s in such a way that a node is developed at the location of interest. Moreover, since the control design is independent of the detailed mathematical model of the plant, it could be applied to a flexible structure of different shapes.

Acknowledgments

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References

[1] H. Frahm, Device for damping vibration of bodies, U.S. Patent No. 989,958, 1911.
[2] J.P. Hartog, *Mechanical Vibrations*, fourth ed., McGraw-Hill, New York, 1956.
[3] A. Alleyne, M. Tharayil, Semi active internal model control for passive disturbance rejection, *Proceedings of the American Control Conference*, Arlington, 2001, pp. 1438–1443.
[4] S.-T. Wu, Y.-C. Chuang, Output regulation of robot manipulators with a constantly revolving arm, *IEEE Transactions on Robotics and Automation* 19 (6) (2003) 1002–1006.
[5] S. Huang, R. Lian, A dynamic absorber with active vibration control, *Journal of Sound and Vibration* 178 (3) (1994) 323–335.

[6] H. Elmali, M. Renzulli, N. Olgac, Experimental comparison of delayed resonator and PD controlled vibration absorbers using electromagnetic actuators, *ASME Journal of Dynamic Systems, Measurement, and Control* 122 (2000) 514–520.
[7] N.W. Hagood, A. von Flowtow, Damping of structural vibrations with piezoelectric materials and passive electrical networks, *Journal of Sound and Vibration* 146 (1991) 243–268.
[8] Y.-D. Chen, C.-C. Fuh, P.-C. Tung, Application of voice coil motors in active dynamic vibration absorbers, *IEEE Transactions on Magnetics* 41 (3) (2005) 1149–1154.
[9] P.D. Cha, G. Ren, Inverse problem of imposing nodes to suppress vibration for a structure subjected to multiple harmonic excitations, *Journal of Sound and Vibration* 290 (1–2) (2006) 425–447.
[10] M.J. Balas, Feedback control of flexible systems, *IEEE Transactions on Automatic Control* 23 (1978) 673–679.
[11] R.-F. Fung, J.-H. Chou, Y.-L. Kuo, Optimal boundary control of an axially moving material system, *Journal of Dynamic Systems, Measurement, and Control* 124 (1) (2002) 55–61.
[12] M.P. Fard, S.I. Sagatun, Exponential stabilization of a transversely vibrating beam by boundary control via Lyapunov's direct method, *Journal of Dynamic Systems, Measurement, and Control* 123 (2) (2001) 195–200.
[13] S.M. Shahrutz, C.A. Narashima, Suppression of vibration in stretched strings by the boundary control, *Journal of Sound Vibration* 195 (1) (1997) 835–840.
[14] S.-T. Wu, Virtual vibration absorbers with inherent damping, *AIAA Journal of Guidance, Dynamics, and Control* 25 (4) (2002) 644–650.
[15] S.-T. Wu, Remote vibration control for flexible beams subject to harmonic disturbances, *Journal of Dynamic Systems, Measurement, and Control* 126 (1) (2004) 198–201.
[16] J.-J.E. Slotine, W. Li, *Applied Nonlinear Control*, Prentice-Hall, New Jersey, 1991 pp. 125–127.
[17] A. Isidori, *Nonlinear Control Systems*, second ed., Springer, Berlin, 1989 pp. 174–176.