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An acoustic boundary integral formulation for open shells allowing different impedance conditions, top and bottom surfaces

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Abstract

In the automotive, aerospace industries as well as the underwater acoustics defense applications the addition of coatings to modify the response of the vibrating structure is commonplace. The formulation developed is an extension of the existing thin body formulation currently found in commercial noise prediction codes such as SYSNOISE in order to allow problems requiring different boundary condition on both sides of the thin shell to be solved. For instance, a generalised thin body acoustic formulation would allow for a local impedance boundary condition to be specified on the top surface and a hard Neumann boundary condition to be specified on the bottom surface. Such a formulation would allow for different local impedance coatings to be specified on the top and bottom surfaces.

Due to the lack of analytical solutions for such problems (even for the simplest geometries) it is difficult to validate such a formulation, however, three test cases are presented, two flat plate examples and a open cylinder with a point monopole source excitation at its centre.

1. Introduction

Initially the authors had developed a code that had the ability to model thin acoustic bodies in terms of a pressure difference formulation [1–7]. This formulation only, however, allows the imposition of the Neumann boundary condition. Furthermore, the normal displacement on the top surface of the body is constrained to be equal to the normal displacement on the bottom surface of the thin body.

The development of a formulation that permits the application of separate impedance on each side of the structural–acoustic interface allows a more realistic simulation of the boundary conditions. By altering the impedance values, one can depict a wide range of boundary conditions ranging from acoustically rigid to pressure release situations.

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A generalised thin body acoustic formulation must be applicable to problems where the impedance boundary condition on the top surface is arbitrarily different to that on the bottom surface. For instance, a problem may demand a local impedance boundary condition on the top surface and a hard Neumann boundary condition on the bottom surface. To this purpose a new formulation, as detailed in this paper, was developed, and then implemented within the current thin body numerical code. We have called this methodology, the full double wetted surface formulation. With this formulation it is possible to specify whether a boundary condition is to be applied to the top or bottom side of the wetted surface or indeed to both wetted surfaces. It should be noted that [8,9], also developed different integral equation formulations to cater for such generalised thin body problems.

Due to the lack of analytical solutions for such problems (even for the simplest geometries) it is difficult to validate such a formulation, however, three test cases are presented, two flat plate examples and a open cylinder with a point monopole source excitation at its centre.

2. Theoretical background

2.1. The pressure difference problem

Fig. 1 illustrates the pressure difference problem. The line in the figure represents a closed body comprising of the top and bottom surfaces of a shell. A pressure and displacement field is then defined on both the top and bottom surface of the shell. The pressure field is denoted by ϕ and the displacement field, $\mathbf{u}$, is defined as

$$\omega^2 \rho \mathbf{u} = \nabla \phi. \tag{1}$$

The fluid density is denoted by ρ and ω is the circular frequency of the problem. A harmonic time dependency is assumed with the form

$$\phi(\mathbf{r}, t) = \phi(\mathbf{r}) \mathrm{e}^{-\mathrm{i}\omega t}. \tag{2}$$

Note, the following definitions are valid:

$$\mathbf{n}^+(p^+) = -\mathbf{n}^-(p^-), \quad \frac{\partial \phi^+}{\partial \mathbf{n}^+} = \phi_n^+, \quad \frac{\partial \phi^-}{\partial \mathbf{n}^+} = \phi_n^-, \tag{3}$$

$$\frac{\partial \phi^+}{\partial \mathbf{n}^+} = -\frac{\partial \phi^+}{\partial \mathbf{n}^-}, \quad \frac{\partial \phi^-}{\partial \mathbf{n}^-} = -\frac{\partial \phi^-}{\partial \mathbf{n}^+}. \tag{4}$$

2.2. Observations

There is a surface discontinuity at the edge of the shell. At this point it will be assumed that,

$$\phi^+(p_\sigma) = \phi^-(p_\sigma), \tag{5}$$

$$\nabla \phi^+(p_\sigma) = \nabla \phi^-(p_\sigma). \tag{6}$$

Eq. (5) ensures continuity of pressure and Eq. (6) ensures continuity of displacement. Although the vector displacement field must be continuous, the normal displacement will be discontinuous due to the 180° phase

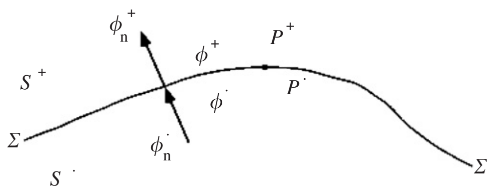

Fig. 1. Pressure difference problem.

discontinuity in the normal direction. At the edge, the normal displacement is undefined as strictly the normal on the edge is undefined. However, a normal displacement exists in the limit as the surface point approaches the edge. Hence, there is a discontinuity in the normal displacement at the edge of a thin body.

2.3. Integral formulation

The standard integral formulation of the Helmholtz equation gives the exterior pressure field in terms of an integral around the closed surface of a body,

$$\phi(P) = \int_S \left[\frac{\partial G(P,q)}{\partial n_q} \phi(q) - G(P,q)\phi_n(q) \right] \mathrm{d}S_q + \phi^i(P), \quad P \in E. \tag{7}$$

In Eq. (7), the integration is performed on both top and bottom surfaces of the thin body. The last term is the incident pressure field. This equation is valid for any point P in the fluid exterior to the thin body. It is possible to split Eq. (7) into integrations on the top and bottom surfaces,

$$\begin{aligned}\phi(P) = &\int_{S^+} \left[\frac{\partial G(P,q)}{\partial n_q^+} \phi^+(q) - G(P,q)\phi_n^+(q) \right] \mathrm{d}S_q^+ \\ &+ \int_{S^-} \left[\frac{\partial G(P,q)}{\partial n_q^-} \phi^-(q) + G(P,q)\phi_n^-(q) \right] \mathrm{d}S_q^- + \phi^i(P), \quad P \in E.\end{aligned} \tag{8}$$

From the description of the problem the following relationships hold:

$$-\frac{\partial G(P,q)}{\partial n_q^-} = \frac{\partial G(P,q)}{\partial n_q} = \frac{\partial G(P,q)}{\partial n_q^+}. \tag{9}$$

Defining,

$$\Delta\phi = [\phi^+(q) - \phi^-(q)], \quad \Delta\phi_n = [\phi_n^+(q) - \phi_n^-(q)], \tag{10}$$

the relationships in Eqs. (9), (8) can be written as an integral over the top part of the surface, in the following manner:

$$\phi(P) = \int_{S^+} \left[\frac{\partial G(P,q)}{\partial n_q} \Delta\phi - G(P,q)\Delta\phi_n \right] \mathrm{d}S_q^+ + \phi^i(P), \quad P \in E. \tag{11}$$

The limit of Eq. (11) is taken as it goes onto the upper surface of the shell from the exterior region. The first term in the integral is discontinuous. This limit is taken for all field points not on the edge of the shell:

$$\phi^+(p) = \int_{S^+} \left[\frac{\partial G(p,q)}{\partial n_q} \Delta\phi - G(p,q)\Delta\phi_n \right] \mathrm{d}S_q^+ + \frac{1}{2}\Delta\phi + \phi^i(p), \quad p \in S^+,\ p \notin \Sigma. \tag{12}$$

The following definitions will clarify the remaining analysis for fluid variables not on the edge, where

$$\overline{\phi} = \tfrac{1}{2}(\phi^+ + \phi^-), \quad \overline{\phi}_n = \tfrac{1}{2}(\phi_n^+ + \phi_n^-).$$

Eq. (12) can be rearranged in the form,

$$\overline{\phi}(p) = \int_{S^+} \left[\frac{\partial G(p,q)}{\partial n_q} \Delta\phi - G(p,q)\Delta\phi_n \right] \mathrm{d}S_q^+ + \phi^i(p), \quad p \in S^+,\ p \notin \Sigma. \tag{13}$$

This equation can be differentiated with respect to the normal on the upper surface at the field point p to give,

$$\overline{\phi}_n = \int_{S^+} \left[\frac{\partial^2 G(p,q)}{\partial n_p \partial n_q} \Delta\phi - \frac{\partial G(p,q)}{\partial n_p} \Delta\phi_n \right] \mathrm{d}S_q^+ + \phi_n^i(p), \quad p \in S^+,\ p \notin \Sigma. \tag{14}$$

3. Numerical formulation

Eqs. (13) and (14) can be combined in a matrix form in the following way:

$$\int_{S^+}\begin{bmatrix}\dfrac{\partial^2 G(p,q)}{\partial n_p\,\partial n_q} & -\dfrac{\partial G(p,q)}{\partial n_p}\\ \dfrac{\partial G(p,q)}{\partial n_q} & -G(p,q)\end{bmatrix}\begin{bmatrix}\Delta\phi\\ \Delta\phi_n\end{bmatrix}\mathrm{d}S_q^+ - \begin{bmatrix}\overline{\phi}_n(p)\\ \overline{\phi}(p)\end{bmatrix} = -\begin{bmatrix}\phi_n^i(p)\\ \phi^i(p)\end{bmatrix},\quad p\in S^+,\ p\notin\Sigma. \tag{15}$$

In order to integrate any formulation into the numerical code, it is more convenient to pose the problem in terms of the unconstructed fluid variables,

$$\int_{S^+}\begin{bmatrix}\dfrac{\partial^2 G(p,q)}{\partial n_p\,\partial n_q} & -\dfrac{\partial^2 G(p,q)}{\partial n_p\,\partial n_q}\\ \dfrac{\partial G(p,q)}{\partial n_q}-\dfrac{1}{2}\delta(p-q) & -\dfrac{\partial G(p,q)}{\partial n_q}-\dfrac{1}{2}\delta(p-q)\end{bmatrix}\begin{bmatrix}\phi^+(p)\\ \phi^-(p)\end{bmatrix}\mathrm{d}S_q^+$$
$$-\int_{S^+}\begin{bmatrix}\dfrac{\partial G(p,q)}{\partial n_p}+\dfrac{1}{2}\delta(p-q) & -\dfrac{\partial G(p,q)}{\partial n_p}+\dfrac{1}{2}\delta(p-q)\\ G(p,q) & -G(p,q)\end{bmatrix}\begin{bmatrix}\phi_n^+(p)\\ \phi_n^-(p)\end{bmatrix}\mathrm{d}S_q^+ = -\begin{bmatrix}\phi_n^i(p)\\ \phi^i(p)\end{bmatrix},\quad p\in S^+,\ p\notin\Sigma. \tag{16}$$

The fluid variables are to be supported on a surface discretisation, by element shape functions,

$$\phi(p) = \{N(p)\}^{\mathrm{T}}\{\phi\},\quad \phi_n(p) = \{N(p)\}^{\mathrm{T}}\{\phi_n\}.$$

The vector N is the vector of shape functions. The vectors ϕ and ϕ_n are the vectors of nodal values. The integral operators in Eq. (16) can be numerically implemented and the following notation is used.

Using the following notation:

$$\mathbf{L}_{ij} = \int_{S^+}[G(p_i,q)N_j(q)]\mathrm{d}S_q^+,\quad \mathbf{M}_{ij} = \int_{S^+}\left[\frac{\partial G(p_i,q)}{\partial n_q}N_j(q)\right]\mathrm{d}S_q^+,$$
$$\mathbf{M}_{ij}^{\mathrm{T}} = \int_{S^+}\left[\frac{\partial G(p_i,q)}{\partial n_p}N_j(q)\right]\mathrm{d}S_q^+,\quad \mathbf{N}_{ij} = \int_{S^+}\left[\frac{\partial^2 G(p_i,q)}{\partial n_p\,\partial n_q}N_j(q)\right]\mathrm{d}S_q^+, \tag{17}$$

the integral operators in Eq. (16) can be constructed numerically.

The Dirac functions in Eq. (16) also have the following simple numerical form:

$$\int_{S^+}[\delta(p_i-q)N_j(q)]\,\mathrm{d}S_q^+ = \delta_{ij}. \tag{18}$$

Therefore, the numerical formulation of the full thin body acoustic formulation can now be written in matrix form as

$$\begin{bmatrix}\mathbf{N}_{ij} & -\mathbf{N}_{ij}\\ \mathbf{M}_{ij}^{\mathrm{T}}-\frac{1}{2}\delta_{ij} & -\mathbf{M}_{ij}^{\mathrm{T}}-\frac{1}{2}\delta_{ij}\end{bmatrix}\begin{bmatrix}\phi^+(p)\\ \phi^-(p)\end{bmatrix}$$
$$-\begin{bmatrix}\mathbf{M}_{ij}^{\mathrm{T}}+\frac{1}{2}\delta_{ij} & -\mathbf{M}_{ij}^{\mathrm{T}}+\frac{1}{2}\delta_{ij}\\ \mathbf{L}_{ij} & -\mathbf{L}_{ij}\end{bmatrix}\begin{bmatrix}\phi_n^+(p)\\ \phi_n^-(p)\end{bmatrix} = -\begin{bmatrix}\phi_n^i(p)\\ \phi^i(p)\end{bmatrix},\quad p_j,\ p_i\notin\Sigma. \tag{19}$$

The dimensions of the matrix system in Eq. (19) are $2(N-N_e)$; the integral equations are satisfied at all node points except those on the edge and the solution variables are the fluid nodal values at all points except those on the edge.

3.1. Impedance boundary conditions

In general there will be a impedance boundary condition that relates the surface pressures to the surface displacements. This boundary condition could be derived from the elastic behaviour of the body or it could be a simple imposed boundary condition. Such boundary conditions can be expressed in the following general way:

$$[Y]\begin{Bmatrix} \phi^+ \\ \phi^- \end{Bmatrix} + [Z]\begin{Bmatrix} \phi_n^+ \\ \phi_n^- \end{Bmatrix} = \begin{Bmatrix} a \\ b \end{Bmatrix}, \quad p_j,\ p_i \in S^+. \tag{20}$$

Depending on the form of this boundary condition, either pressure or normal displacement will be the correct unknown, the remaining variable will be reconstructed from the boundary condition. Particularly for finite element boundary conditions it may be the case that the assumption of continuity of the fluid variables at the edge, is violated by the impedance boundary condition. By constraining the pressure and displacement difference to be zero at the edges, it is not necessary to constrain the impedance boundary condition to obey the same constraint. Any impedance boundary condition can be factored into the acoustic formulation and then the assumption of continuity is enforced on the coupled equation set.

Consider for instance the case of a thin body which has two different impedance boundary conditions on the top and bottom surfaces. Clearly from the boundary conditions there cannot be continuity of both pressure and normal displacement. The discontinuity of boundary condition cannot be represented in the formulation. Satisfaction of such a discontinuous boundary condition can only be represented in an over determined way by collocating at the edge points for the top and bottom pressures and displacements. The current strategy implies that any contribution to the surface impedance, arising from the edge nodes is discarded from the coupled formulation.

4. Validation

The sophistication of the new methodology presented in this paper means that in the absence of experimental results the code is difficult to validate. This is especially true for acoustic problems requiring the analysis of unbaffled thin bodies. However, in the following examples we will apply the new methodology to a number of problems and then compare the results to available asymptotic solutions. The final application test case, is the calculation of the insertion loss of a point source in an open ended cylinder. This example is only open to qualitative validation.

4.1. Terai's scattering problem

The full thin body formulation can be applied to a problem with hard boundary conditions both sides. It must be remembered however that this class of problem can also be solved using the existing thin shell formulation. In a paper by Terai [10] there are experimental results for scattering of sound by a finite unbaffled rigid plate. Fig. 2 shows the experimental layout of the problem. A rigid rectangular plate with dimensions

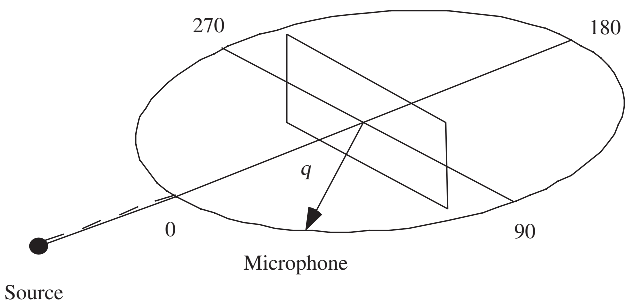

Fig. 2. Experimental set-up for Terai results.

30 cm × 20 cm is isonified by a point source. The sound pressure is then measured around the plate in the manner as indicated in Fig. 2. The ratio of measured to direct sound levels are then plotted at a wavenumber of 18.44. The results of the numerical code calculation are shown in Fig. 3, compared to the experimental results. There is good agreement further validating the formulation.

4.2. Kirchhoff comparison

It is very difficult to test the full thin body formulation with respect to a thin body with edges and non-symmetric boundary conditions. In this test case, the high-frequency bistatic target strength of a 1 m square plate is evaluated, for a plate with a ρc local impedance boundary condition on one side and a soft, pressure release boundary condition on the other side. The results are compared to the Kirchhoff formulation also encoded in the code. The results are calculated at a frequency of 7 kHz. The numerical results are calculated using one quarter of the plate, discretised into 144 and 100 quadrilateral nine noded isoparametric elements. The Kirchhoff approximation is calculated using 400 quadrilateral nine noded isoparametric elements. Two bistatic far-field results are calculated corresponding to a plane incident wave impinging on either boundary condition surface. The results are shown in Figs. 4 and 5. The first comment to make about these results is that there seems to be convergence in the numerical results for the two mesh discretisation. Secondly the agreement

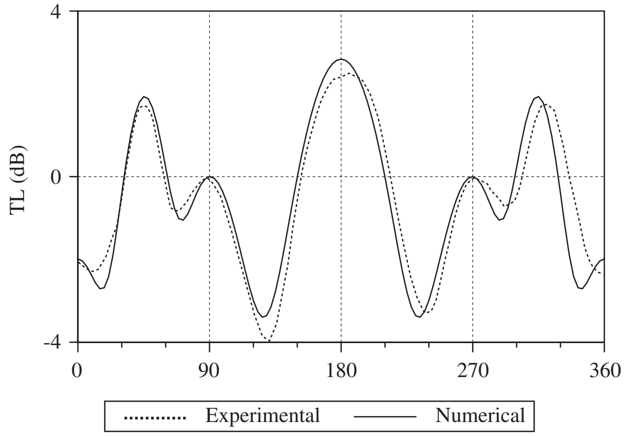

Fig. 3. Terai results—experimental versus numerical.

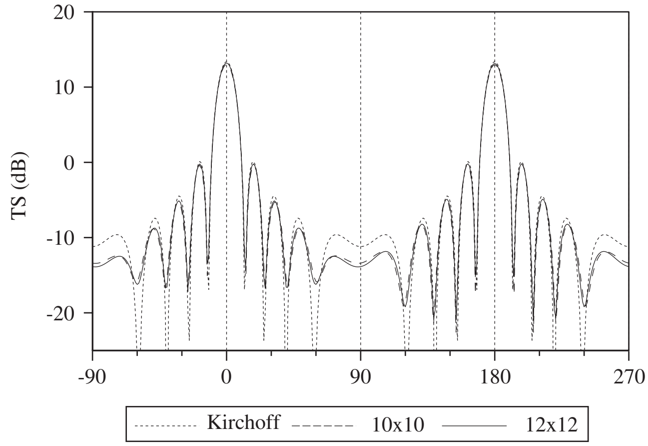

Fig. 4. Bistatic target strength of plate with 7 kHz plane wave incident on soft side. Back side impedance is ρc.

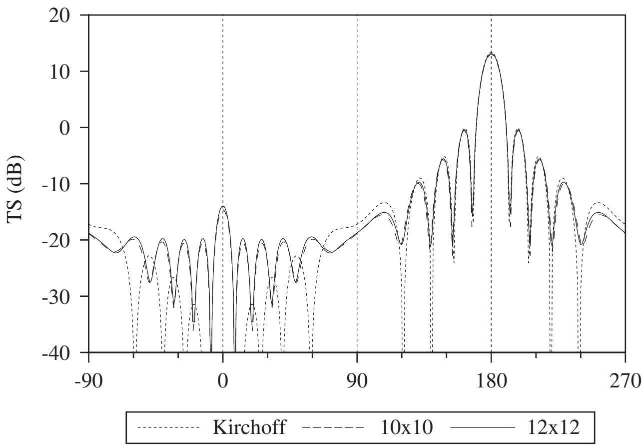

Fig. 5. Bistatic target strength of plate with 7 kHz plane wave incident on ρc side. Back side impedance is soft.

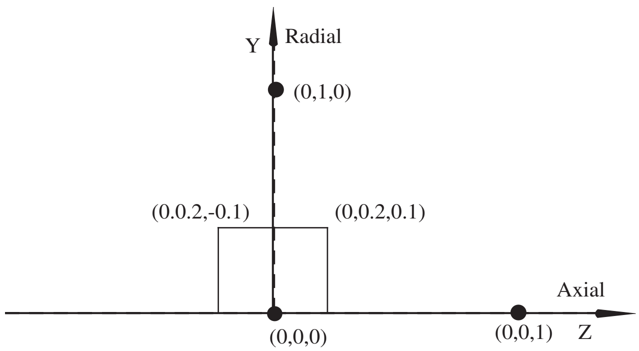

Fig. 6. Geometry of the open cylinder problem.

between the numerical results for the case where the plane wave hits the soft side are very good. The agreement for the case where the plane wave hits the ρc side are also reasonable within the limitations of the Kirchhoff model.

4.3. Open ended cylinder with point source

This final test case consists of an open ended cylinder with a point monopole source at its centre. The insertion loss due to the cylinder is measured at 1 m from the point source, along the axis of the cylinder and radially from the cylinder. This insertion loss is measured as a function of frequency. The geometry of the problem is shown in Fig. 6. The results of two sets of boundary conditions are shown. In the first, the cylinder body is assumed to be rigid. In the second, the inside (or bottom) surface of the cylinder is assumed to have a near ρc local impedance boundary condition. The outside surface is assumed to be rigid. The results of these two analyses are shown in Figs. 7 and 8. In the absence of experimental results, a qualitative analysis of these results is outside the scope of the current report. However the characteristic lossy resonance, shown particularly in the radial insertion loss would be expected from the semi-closed geometry of the open ended cylinder for the hard boundary condition. The introduction of a ρc boundary condition in the interior of the cylinder increases dramatically the insertion loss in the radial direction and removes resonance effects in both radial and axial directions.

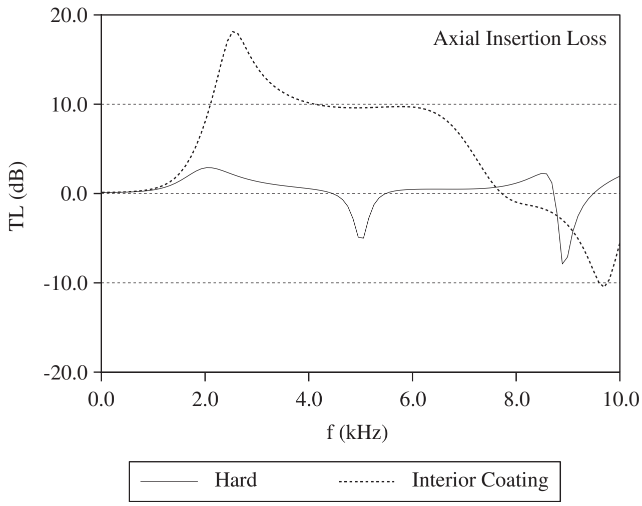

Fig. 7. Axial insertion loss of hard and coated open cylinder.

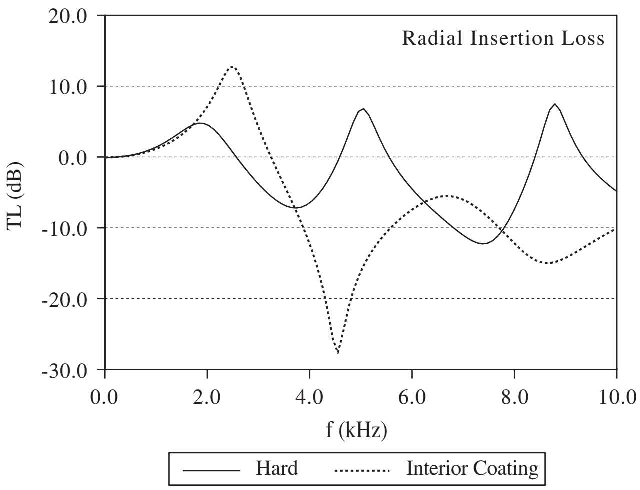

Fig. 8. Radial insertion loss of hard and coated open cylinder.

5. Summary

The theoretical content of the new formulation implemented within an analysis package named CScat has been given. The aim of this paper was to show the derivation and implementation of an acoustic formulation, applicable to thin bodies with different boundary conditions on top and bottom surfaces. This model is a generalisation of an existing thin body formulation [1], that had already been implemented and coded.

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