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ScienceDirect
Journal of Sound and Vibration 300 (2007) 709–726
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Nonlinear vibrations and dynamic stability of viscoelastic orthotropic rectangular plates

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Received 2 February 2006; received in revised form 14 August 2006; accepted 23 August 2006
Available online 7 November 2006

Abstract

This paper describes the analyses of the nonlinear vibrations and dynamic stability of viscoelastic orthotropic plates. The models are based on the Kirchhoff–Love (K.L.) hypothesis and Reissner–Mindlin (R.M.) generalized theory (with the incorporation of shear deformation and rotatory inertia) in geometrically nonlinear statements. It provides justification for the choice of the weakly singular Koltunov–Rzhanitsyn type kernel, with three rheological parameters. In addition, the implication of each relaxation kernel parameter has been studied. To solve problems of viscoelastic systems with weakly singular kernels of relaxation, a numerical method has been used, based on quadrature formulae. With a combination of the Bubnov–Galerkin and the presented method, problems of nonlinear vibrations and dynamic stability in viscoelastic orthotropic rectangular plates have been solved, according to the K.L. and R.M. hypotheses. A comparison of the results obtained via these theories is also presented. In all problems, the convergence of the Bubnov–Galerkin method has been investigated. The implications of material viscoelasticity on vibration and dynamic stability are presented graphically.

1. Introduction

With increasing industrial development, the mechanics of composite materials has made remarkable progress. Interest in problems of deformation, durability, vibrations, and dynamic stability of plates and shells made of composite material is prompted by the fact that they are the main load-bearing elements in aeronautical and missile engineering, automobiles, pipelines, etc. The application of advanced composite materials in the engineering and designing of strong, light and reliable elements requires improvements in the mechanical models of body deformation and the development of mathematical models for its calculation, taking into account the actual properties of the construction materials. Therefore, the development of efficient algorithms for solving nonlinear problems on vibrations and dynamic stability of structures manufactured from composite materials constitutes an urgent issue. According to many experimental and fundamental investigations, the majority of composite materials feature distinct viscoelastic properties [1–6].

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.08.024

The strength of the structures made from modern composite materials is substantially determined by transverse shear deformations. The classical theory of shells neglects these effects, which now adds new impetus to the development of the general theory of shells. The main idea is to represent displacements or strains in a shell using a series: the approaches differ in the type of expansions. However, in practice, methods based on hypotheses on the distribution of shears or strains across the wall thickness have become popular. Together with the K.L. classical hypotheses, the R.M. model is widely recognized [7–10]. It should also be noted that, although the K.L. model allows accurate solutions to a number of practical problems, the solutions in most cases are not sufficiently comprehensive [11]. Viscoelastic plates made of composite materials having an anisotropic structure have been described previously [12–16].

Various studies [17–28] have been devoted to the solution of elastic structural (isotropic cases [17–26] and orthotropic cases [27,28]) problems, within these theories (K.L. [18–25] and R.M. [17,26–28]). Even if the problems were solved in the viscoelastic formulation, in many cases the viscoelastic characteristics of the material were only taken into account in a restricted context [5,6,12–16,29–38]. In these cases, the presence of viscoelastic properties of a material, or the Voight model, [15,29,30,38] was used, or for kernels of a relaxation exponential kernels were applied [5,6,12–14,16,31–37]. Actually, mathematical models of problems of viscoelastic systems based on these assumptions cannot describe real processes in constructions and the influence of dynamic loadings [3,4]. The lack of assumptions leads to incorrect approximation of relaxation and creep processes in the initial stage of deformation. The choice of assumptions should not be casually undertaken. In cases where a similar problem is solved in a linear statement, the decision is reduced to the application of various integrated transformations, e.g. Laplace, Laplace–Carson, etc. [12–14,16,31,37]. If they are solved in a nonlinear statement, it can be obtained as the solution of a system of integro-differential equations, where, by way of differentiation, it may be reduced to the solution of ordinary differential equations, which in most cases are solved by the known numerical method of Runge–Kutta [5,6,29,30,32]. At present, the existing methods do not allow the resolution of problems involving weakly singular kernels of the Koltunov, Rzhanitsyn, Abel and Rabotnov types, among others [3,4].

Thanks to the numerical method [39,40] developed by Kh. Eshmatov on the basis of quadrature rules, it is now possible to solve the system of nonlinear integro-differential equations with weakly singular kernels of the Koltunov–Rzhanitsyn, Abel and Rabotnov types. This method provides results of a reasonably high accuracy and is universal. It enables the resolution of a wide class of dynamic problems of the theory of viscoelasticity and is economical from the point of view of computer time [40]. Based on this method, a great number of numerical results have been obtained agreeing well with experimental predictions [41–44].

It is worth noting that, in the system of integro-differential equations, when we have to solve problems involving the dynamics of viscoelastic systems, only one kernel of relaxation with three various rheological parameters of viscosity is involved, whereas in the orthotropic case, within the K.L. hypotheses, five various kernels with 15 rheological parameters are involved. For the system of equations describing the process according to the R.M. theory, in the case of orthotropy, seven kernels with 21 rheological parameters are involved, leading to cumbersome calculations.

The purpose of this work is the study of nonlinear vibrations and the dynamic stability of viscoelastic orthotropic plates, via the various theories, and to determine the validity of these theories when used to solve realistic problems in the dynamics of viscoelastic systems.

2. Nonlinear vibration of viscoelastic orthotropic plate within the Kirchhoff–Love and Reissner–Mindlin hypotheses

To construct the mathematical model of a problem on the nonlinear vibration of a viscoelastic orthotropic plate in geometrically nonlinear formulation by means of the K.L. hypotheses, the constitutive equations are used, relating the stresses σ_x, σ_y, τ_{xy} with the strains ε_x, ε_y, γ_{xy} as [1,3]

$$\sigma_x = B_{11}\left(1 - R_{11}^*\right)\varepsilon_x + B_{12}\left(1 - R_{12}^*\right)\varepsilon_y, \quad (x \leftrightarrow y, 1 \leftrightarrow 2), \quad \tau_{xy} = 2B(1 - R^*)\gamma_{xy}, \tag{1}$$

where B_{ij} and B are elastic constants, R_{ij}^* and R^* are integral operators with kernels of relaxation $R_{ij}(t)$ and $R(t)$, respectively, $i = 1, 2$; $j = 1, 2$:

$$R^*\varphi = \int_0^t R(t-\tau)\varphi(\tau)\,\mathrm{d}\tau, \quad R_{ij}^*\varphi = \int_0^t R_{ij}(t-\tau)\varphi(\tau)\,\mathrm{d}\tau, \quad i,j = 1,2,$$
$$B_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}, \quad B_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}, \quad B_{12} = B_{21} = \mu_{12}B_{22} = \mu_{21}B_{11}, \quad B = \frac{G}{2}.$$

Here, E_1, E_2 are the moduli of elasticity in the direction of axes x and y; G is the shear modulus; μ_{12}, μ_{21} are Poisson's ratios; and hereafter, the symbol $(x \leftrightarrow y)$ shows that all the remaining equations, not explicitly written, are obtained by circular substitution of indices.

We shall take into account the relationship between the strains in the median surface ε_x, ε_y, γ_{xy} and displacements u, v, w in the directions x, y, z also including the geometric imperfections, written by considering the von Karman type of geometric nonlinearity, in the form [11,23,24]:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial w_0}{\partial x}\right)^2\right], \quad \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial w}{\partial y}\right)^2 - \left(\frac{\partial w_0}{\partial y}\right)^2\right],$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}, \tag{2}$$

where $w_0 = w_0(x, y)$ is associated with the initial geometric imperfection of the plate.

We express the bending and twisting moments of the element of the plate as follows [40–43]:

$$M_x = -\frac{h^3}{12}\left[B_{11}\left(1-R_{11}^*\right)\frac{\partial^2(w-w_0)}{\partial x^2} + B_{12}\left(1-R_{12}^*\right)\frac{\partial^2(w-w_0)}{\partial y^2}\right], \quad (x \leftrightarrow y, 1 \leftrightarrow 2),$$
$$H = -\frac{Bh^3}{3}(1-R^*)\frac{\partial^2(w-w_0)}{\partial x\,\partial y}, \tag{3}$$

where h is the thickness of a plate.

When deriving the equations of motion of the element of a viscoelastic plate, we shall proceed from equations [11,23,24]:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \rho\frac{\partial^2 u}{\partial t^2} = 0, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} - \rho\frac{\partial^2 v}{\partial t^2} = 0,$$
$$\frac{q}{h} + \frac{1}{h}\left(\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 H}{\partial x\,\partial y} + \frac{\partial^2 M_y}{\partial y^2}\right) + \frac{\partial}{\partial x}\left(\sigma_x\frac{\partial w}{\partial x} + \tau_{xy}\frac{\partial w}{\partial y}\right)$$
$$+ \frac{\partial}{\partial y}\left(\sigma_y\frac{\partial w}{\partial y} + \tau_{xy}\frac{\partial w}{\partial x}\right) - \rho\frac{\partial^2 w}{\partial t^2} = 0, \tag{4}$$

where q is the external shearing load, and ρ, the density of material structure.

Substituting Eqs. (1) and (3) into Eq. (4), we have

$$B_{11}\left(1-R_{11}^*\right)\frac{\partial \varepsilon_x}{\partial x} + B_{12}\left(1-R_{12}^*\right)\frac{\partial \varepsilon_y}{\partial x} + 2B(1-R^*)\frac{\partial \gamma_{xy}}{\partial y} - \rho\frac{\partial^2 u}{\partial t^2} = 0,$$
$$B_{22}\left(1-R_{22}^*\right)\frac{\partial \varepsilon_y}{\partial y} + B_{21}\left(1-R_{21}^*\right)\frac{\partial \varepsilon_x}{\partial y} + 2B(1-R^*)\frac{\partial \gamma_{xy}}{\partial x} - \rho\frac{\partial^2 v}{\partial t^2} = 0,$$
$$\frac{h^2}{12}\left\{B_{11}\left(1-R_{11}^*\right)\frac{\partial^4(w-w_0)}{\partial x^4} + \left[8B(1-R^*) + B_{12}\left(1-R_{12}^*\right) + B_{21}\left(1-R_{21}^*\right)\right]\right.$$
$$\left.\times\frac{\partial^4(w-w_0)}{\partial x^2\,\partial y^2} + B_{22}\left(1-R_{22}^*\right)\frac{\partial^4(w-w_0)}{\partial y^4}\right\}$$

$$
\begin{aligned}
&-\frac{\partial}{\partial x}\left\{\frac{\partial w}{\partial x}\left[B_{11}\left(1-R_{11}^{*}\right)\varepsilon_x+B_{12}\left(1-R_{12}^{*}\right)\varepsilon_y\right]+2B\frac{\partial w}{\partial y}(1-R^{*})\gamma_{xy}\right\}\\
&-\frac{\partial}{\partial y}\left\{\frac{\partial w}{\partial y}\left[B_{22}\left(1-R_{22}^{*}\right)\varepsilon_y+B_{21}\left(1-R_{21}^{*}\right)\varepsilon_x\right]+2B\frac{\partial w}{\partial x}(1-R^{*})\gamma_{xy}\right\}\\
&-\frac{q}{h}+\rho\frac{\partial^2 w}{\partial t^2}=0, \qquad (5)
\end{aligned}
$$

where ε_x, ε_y, γ_{xy} are provided by Eq. (2). This is the system of nonlinear integro-differential equations of motion of viscoelastic plates in the displacements u, v and w, from which one is able to derive equations of motion of viscoelastic plates made of an isotropic material.

Mathematical models for nonlinear vibrations of viscoelastic orthotropic plates, based on the K.L. hypothesis, have been described previously. This model allows reasonably accurate solutions to be obtained for a number of practical problems. In most cases, nevertheless, these models are not sufficiently comprehensive [11]. This is the case when considering viscoelastic plates made of composite material having an orthotropic structure. Thus, the necessity to identify the limits of application of the K.L. hypothesis for dynamic problems involving nonlinear vibrations and dynamic stability of viscoelastic orthotropic plates in geometrically nonlinear formulation is evident. Therefore, the mathematical model according to the R.M. generalized theory is considered, taking into account the shear deformations and rotatory inertia [7,8]. In this case, the constitutive relationships between the stresses σ_x, σ_y, τ_{xy}, τ_{xz}, τ_{yz} and strains ε_x, ε_y, γ_{xy}, γ_{xz}, γ_{yz} are expressed as

$$
\begin{aligned}
&\sigma_x=B_{11}\left(1-R_{11}^{*}\right)\varepsilon_x+B_{12}\left(1-R_{12}^{*}\right)\varepsilon_y, \quad (x\leftrightarrow y, 1\leftrightarrow 2),\\
&\tau_{xy}=2B(1-R^{*})\gamma_{xy}, \quad \tau_{xz}=2B_{13}\left(1-R_{13}^{*}\right)\gamma_{xz}, \quad (x\leftrightarrow y, 1\leftrightarrow 2), \qquad (6)
\end{aligned}
$$

where B_{ij} and B are elastic constants, and R_{ij}^{*} and R^{*} are integral operators with kernels of relaxation $R_{ij}(t)$ and $R(t)$, respectively, $i=1, 2; j=1, 2, 3$.

As for the kinematical relationships between the strains ε_x^z, ε_y^z, γ_{xy}^z and the angular displacements ψ_x, ψ_y, one should use [7,8]

$$
\varepsilon_x^z=\varepsilon_x+z\frac{\partial\psi_x}{\partial x}\ (x\leftrightarrow y), \quad \gamma_{xy}^z=\gamma_{xy}+z\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right). \qquad (7)
$$

In view of Eqs. (6) and (7), the bending and twisting moments M_x, M_y, H and shear forces Q_x, Q_y will be written as

$$
\begin{aligned}
&M_x=\frac{B_{11}h^3}{12}\left(1-R_{11}^{*}\right)\frac{\partial\psi_x}{\partial x}+\frac{B_{12}h^3}{12}\left(1-R_{12}^{*}\right)\frac{\partial\psi_y}{\partial y}, \quad (x\leftrightarrow y, 1\leftrightarrow 2),\\
&H=\frac{Bh^3}{6}(1-R^{*})\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right),\\
&Q_x=2K^2hB_{13}\left(1-R_{13}^{*}\right)\left[\frac{\partial(w-w_0)}{\partial x}+\psi_x\right]\ (x\leftrightarrow y, 1\leftrightarrow 2), \qquad (8)
\end{aligned}
$$

where the shear correction factor is equal to $K^2=5/6$ (Reissner) [7], $\pi^2/12$ (Mindlin) [8], and 2/3 (Uflyand) [9].

Substituting Eqs. (6) and (8) in the equations of motion [7,8]:

$$
\begin{aligned}
&\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}-\rho\frac{\partial^2 u}{\partial t^2}=0, \quad \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}-\rho\frac{\partial^2 v}{\partial t^2}=0,\\
&\frac{1}{h}\left(\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right)+\frac{\partial}{\partial x}\left(\sigma_x\frac{\partial w}{\partial x}+\tau_{xy}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial y}\left(\tau_{xy}\frac{\partial w}{\partial x}+\sigma_y\frac{\partial w}{\partial y}\right)+\frac{q}{h}-\rho\frac{\partial^2 w}{\partial t^2}=0,\\
&\frac{\partial M_x}{\partial x}+\frac{\partial H}{\partial y}-Q_x-\rho\frac{h^3}{12}\frac{\partial^2\psi_x}{\partial t^2}=0\ (x\leftrightarrow y), \qquad (9)
\end{aligned}
$$

one obtains the following system of integro-partial differential governing equations:

$$B_{11}\left(1-R_{11}^{*}\right)\frac{\partial\varepsilon_x}{\partial x}+B_{12}\left(1-R_{12}^{*}\right)\frac{\partial\varepsilon_y}{\partial x}+2B(1-R^{*})\frac{\partial\gamma_{xy}}{\partial y}-\rho\frac{\partial^2 u}{\partial t^2}=0,$$
$$B_{22}\left(1-R_{22}^{*}\right)\frac{\partial\varepsilon_y}{\partial y}+B_{21}\left(1-R_{21}^{*}\right)\frac{\partial\varepsilon_x}{\partial y}+2B(1-R^{*})\frac{\partial\gamma_{xy}}{\partial x}-\rho\frac{\partial^2 v}{\partial t^2}=0,$$

$$\begin{aligned}&-2K^2\left\{B_{13}\left(1-R_{13}^{*}\right)\left[\frac{\partial^2(w-w_0)}{\partial x^2}+\frac{\partial\psi_x}{\partial x}\right]+B_{23}\left(1-R_{23}^{*}\right)\left[\frac{\partial^2(w-w_0)}{\partial y^2}+\frac{\partial\psi_y}{\partial y}\right]\right\}\\&-\frac{\partial}{\partial x}\left\{\frac{\partial w}{\partial x}\left[B_{11}\left(1-R_{11}^{*}\right)\varepsilon_x+B_{12}\left(1-R_{12}^{*}\right)\varepsilon_y\right]+2B\frac{\partial w}{\partial y}(1-R^{*})\gamma_{xy}\right\}\\&-\frac{\partial}{\partial y}\left\{\frac{\partial w}{\partial y}\left[B_{22}\left(1-R_{22}^{*}\right)\varepsilon_y+B_{21}\left(1-R_{21}^{*}\right)\varepsilon_x\right]+2B\frac{\partial w}{\partial x}(1-R^{*})\gamma_{xy}\right\}-\frac{q}{h}+\rho\frac{\partial^2 w}{\partial t^2}=0,\end{aligned}$$

$$\begin{aligned}&\frac{B_{11}h^2}{12}\left(1-R_{11}^{*}\right)\frac{\partial^2\psi_x}{\partial x^2}+\frac{B_{12}h^2}{12}\left(1-R_{12}^{*}\right)\frac{\partial^2\psi_y}{\partial x\,\partial y}+\frac{Bh^2}{6}(1-R^{*})\left(\frac{\partial^2\psi_x}{\partial y^2}+\frac{\partial^2\psi_y}{\partial x\,\partial y}\right)\\&-2K^2B_{13}\left(1-R_{13}^{*}\right)\left[\frac{\partial(w-w_0)}{\partial x}+\psi_x\right]-\frac{\rho h^2}{12}\frac{\partial^2\psi_x}{\partial t^2}=0\ (x\leftrightarrow y, 1\leftrightarrow 2),\end{aligned}\tag{10}$$

where ε_x, ε_y, γ_{xy} are expressed by Eq. (2).

The mathematical models obtained via the set of Eq. (10), with corresponding boundary and initial conditions, take into consideration the viscoelastic properties of a plate, as well as shear deformation and rotatory inertia.

System (10) is a set of nonlinear integro-partial differential equations of the Volterra type with seven different kernels of relaxation and 21 rheological parameters. Based on this system, one can obtain various mathematical models of elastic and viscoelastic systems [5,6,10,11].

To derive the equations of motion of an isotropic viscoelastic plate, according to the theories of K.L. and R.M., and taking into account the propagation of elastic waves, it is possible to use the systems of Eqs. (5) and (10), respectively. In this case, the elastic constants are $E_1=E_2=E$, and the kernels of relaxation are $R_{11}(t)=R_{12}(t)=R_{22}(t)=R_{21}(t)=R_{13}(t)=R_{23}(t)=R(t)$.

Here the problem of the nonlinear vibration of a viscoelastic orthotropic thin rectangular plate is considered for which the length is a in the x direction, the width is b in the y direction and the uniform thickness is h. The plate is simply supported, allowing a contour in the z direction under the influence of a constant external shearing load q. This problem, based on the K.L. and R.M. hypotheses, is described by the system of Eqs. (5) and (10), respectively.

To fulfill the boundary conditions, displacements are defined as w, w_0, u, v and the angular displacements as ψ_x, ψ_y, with the following definitions:

$$w(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$
$$w_0(x,y)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{0nm}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$
$$u(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}u_{nm}(t)\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$
$$v(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}v_{nm}(t)\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b},$$

$$\psi_x(x,y,t) = \sum_{n=1}^{N}\sum_{m=1}^{M}\psi_{xnm}(t)\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$
$$\psi_y(x,y,t) = \sum_{n=1}^{N}\sum_{m=1}^{M}\psi_{ynm}(t)\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}. \tag{11}$$

Substituting Eq. (11) into the systems of Eqs. (5) and (10), and applying the Bubnov–Galerkin procedure, using the dimensionless quantities w_{kl}/h, w_{0kl}/h, u_{kl}/h, v_{kl}/h, ωt, $qb^4/\sqrt{E_1E_2}h^4$, $R(t)/\omega$, $R_{ij}(t)/\omega$, $i,j = 1,2,3$, and, at the same time, maintaining the previous designations related to the dimensionless parameters u_{kl}, v_{kl}, w_{kl}, ψ_{xkl}, ψ_{ykl}, one obtains the system of nonlinear integro-differential equations, describing the process of nonlinear vibration of viscoelastic orthotropic plates according to the theories of K.L. and R.M., respectively.

The given systems of integro-differential equations will be solved by the numerical method, based on the use of the quadrature formulae presented by Eshmatov [39]. As kernels of relaxation, we will use the weakly singular Koltunov–Rzhanitsin kernels in the form:

$$R_{ij}(t) = A_{ij}e^{-\beta_{ij}t}t^{\alpha_{ij}-1}\ \left(0<\alpha_{ij}<1, \quad i=1,2; \quad j=1,2,3\right).$$

Figs. 1–3 show the results of the dynamic response obtained for a viscoelastic orthotropic plate according to the K.L. theory. In all considered cases, the numerical convergence of the Bubnov–Galerkin method has been studied. While determining the deflection time-history, the first five harmonics ($N = 5$, $M = 1$) were retained. The results show that a further increase in the number of components does not greatly influence the amplitude of vibration of the viscoelastic plate (Fig. 1). Here, curve 1 corresponds to $N = 1$, curve 2 to $N = 3$, curve 3 to $N = 5$ and curve 4 to $N = 7$.

The influence of material orthotropicity on plate vibration was investigated when the shearing load was absent. Parameter $\Delta(\Delta = E_1/E_2)$ defines the degree of orthotropy of the material. The curves in Fig. 2 correspond to various parameters Δ. Curve 1 corresponds to the case when $\Delta = 1$; curve 2 corresponds to $\Delta = 2$ and curve 3 to $\Delta = 3$. As the results reveal, an increase in parameter Δ yields a decrease in the oscillation.

Fig. 3 displays the results obtained for various kernels of relaxation (curve 1 corresponds to an exponential kernel; curve 2 to a weakly singular kernel of Koltunov–Rzhanitsyn). As can be seen in the figure, initially these kernels almost coincide, then, with time, a difference occurs and at time $t^* = 10$, the difference in the result is ~20%. With increasing time, the difference in the results continues to increase further.

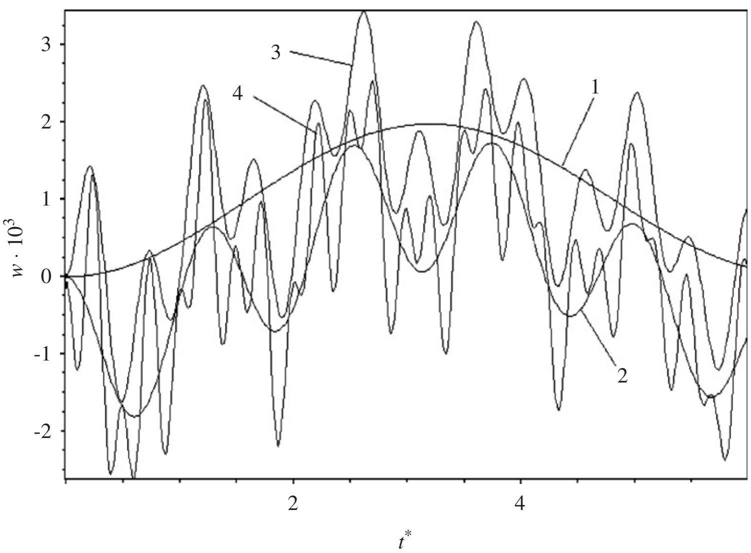

Fig. 1. The numerical convergence of the Bubnov–Galerkin method: $1 - N = 1$; $2 - N = 3$; $3 - N = 5$; $4 - N = 7$.

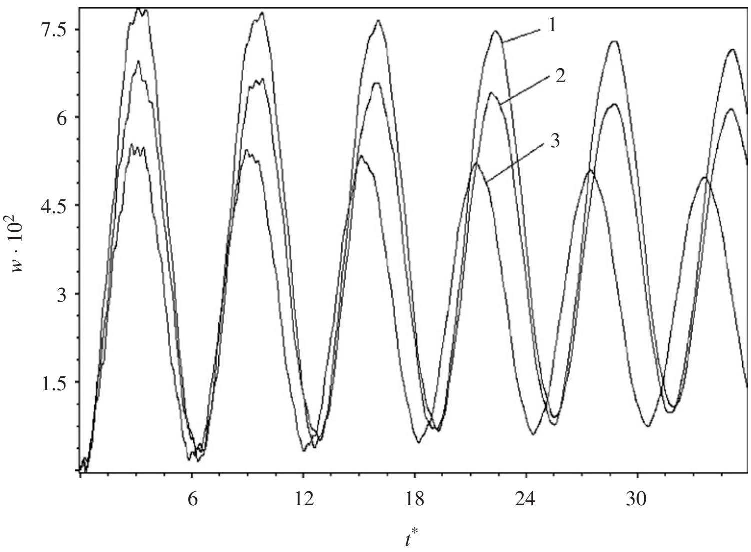

Fig. 2. The influence of the material orthotropicity: $1 - \Delta = 1$; $2 - \Delta = 2$; $3 - \Delta = 3$.

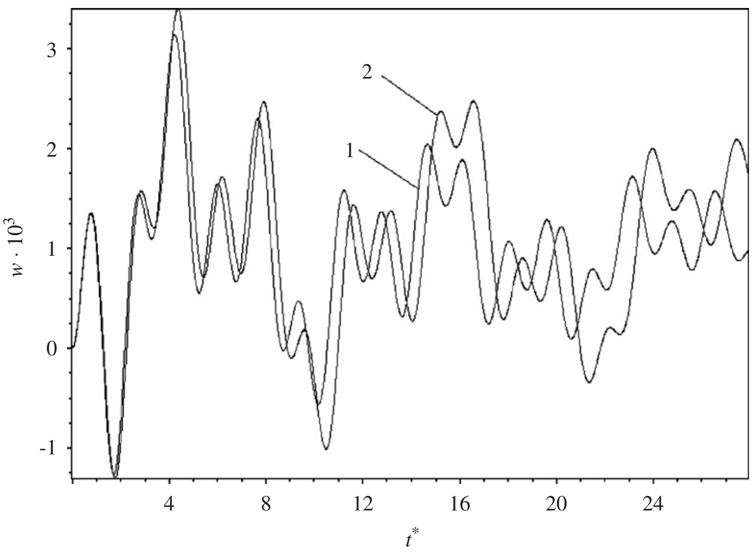

Fig. 3. Dependence of the deflection on time for various kernels of relaxation: 1—exponential kernel; 2—weakly singular kernel of Koltunov–Rzhanitsyn.

Figs. 4–9 present the deflection time-history of a viscoelastic orthotropic plate, oscillating according to the generalized theory of Reissner–Mindlin, at various physical and geometrical parameters.

The influence of the initial geometric imperfections of the plate on frequency and oscillation amplitude was studied (Fig. 4, curve 1 corresponds to $w_0 = 0.3 \times 10^{-3}$, curve 2 to $w_0 = 0.7 \times 10^{-3}$ and curve 3 to $w_0 = 10^{-3}$). Note that a change in w_0 does not significantly influence the frequency of vibrations. However, with an increase in the value of the initial geometric imperfections, the amplitude of oscillations increases proportionally.

The implications of the aspect ratio λ ($\lambda = a/b$) were also investigated. In the case of elongated plates ($\lambda = 2$ and 3), in contrast to square ones ($\lambda = 1$), the frequency of oscillation decays and a considerable phase shift is observed (Fig. 5, curve 1 is $\lambda = 1$; curve 2 is $\lambda = 2$; curve 3 is $\lambda = 3$).

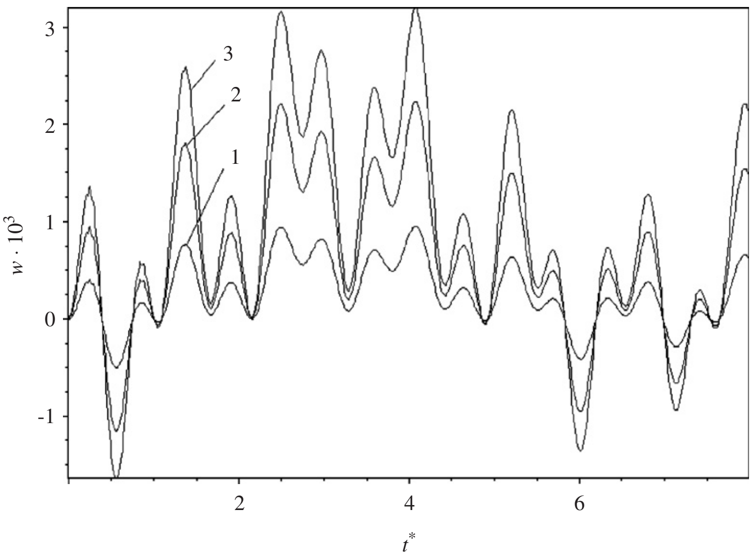

Fig. 4. The influence of the initial imperfections: $1 - w_0 = 0.3 \times 10^{-3}$; $2 - w_0 = 0.7 \times 10^{-3}$; $3 - w_0 = 10^{-3}$.

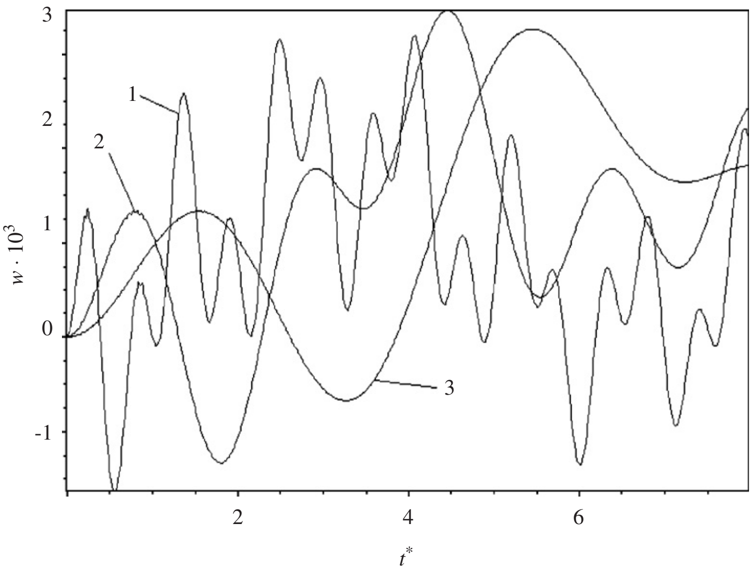

Fig. 5. Dependence of the deflection on time for various values of λ: $1 - \lambda = 1$; $2 - \lambda = 2$; $3 - \lambda = 3$.

In addition, the influence of the static external shearing load q on the behaviour of the viscoelastic plate was investigated (Fig. 6), where curve 1 corresponds to $q = 0$; curve 2 to $q = 0.01$ and curve 3 to $q = 0.02$. Initially, the vibrational amplitude does not change substantially, but as time unfolds, with an increase in load parameter, a proportional increase in the amplitude of vibrations is observed, similar to the previous case. High values of the static load q correspond to high values of the amplitude of oscillation, and vice-versa.

The influence of the nonlinear properties of the plate material upon its behaviour has also been studied. Fig. 7 presents diagrams of the deflection time-history for the linear case (curve 1 corresponding to $\lambda = 1$; $q = 1$; $w_0 = 10^{-1}$, curve 3 to $\lambda = 1$; $q = 6$; $w_0 = 10^{-4}$ and curve 5 to $\lambda = 6$; $q = 1$; $w_0 = 10^{-4}$) and nonlinear case (curve 2 corresponding to $\lambda = 1$; $q = 1$; $w_0 = 10^{-1}$ and curve 4 to $\lambda = 1$; $q = 6$; $w_0 = 10^{-4}$ and curve 6 to $\lambda = 6$; $q = 1$; $w_0 = 10^{-4}$). The results demonstrate that, when the vibrations of the viscoelastic square plate are considered without external loads and initial geometric imperfections, the response corresponding to the linear

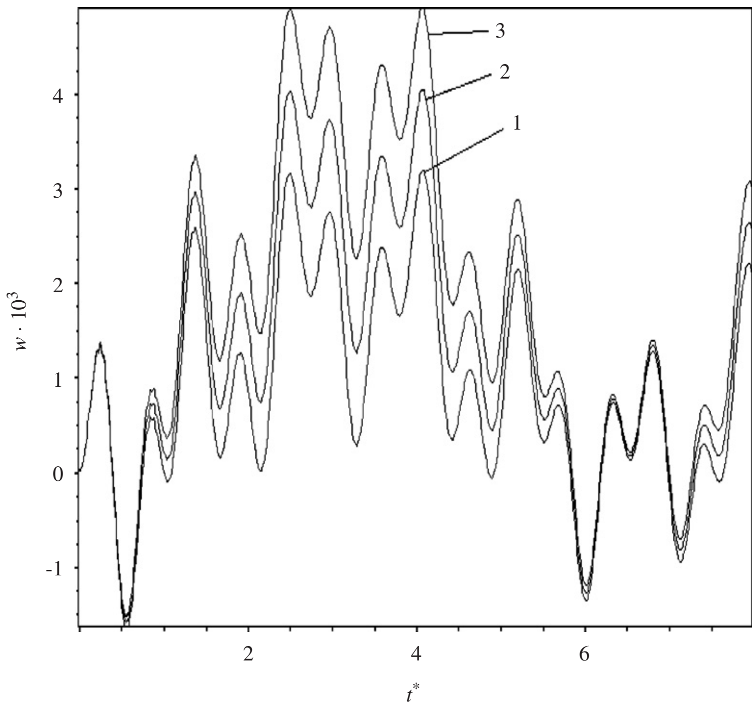

Fig. 6. The influence of the static shearing edge load q: $1 - q = 0$; $2 - q = 0.01$; $3 - q = 0.02$.

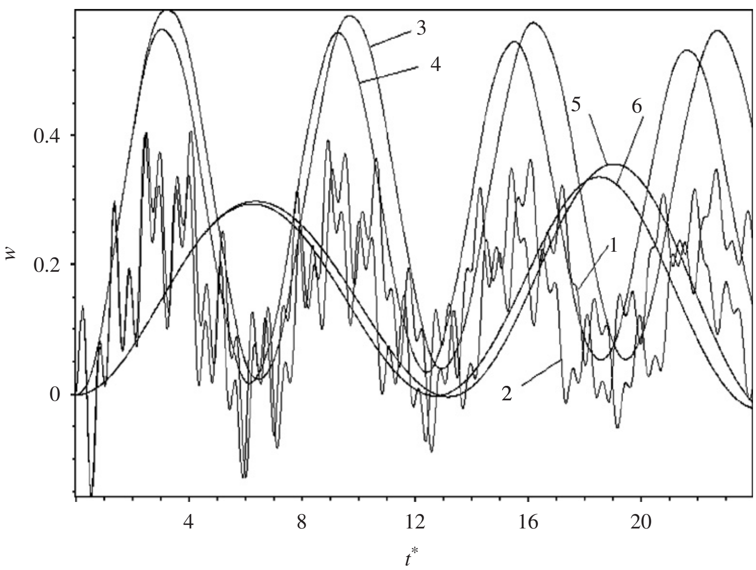

Fig. 7. The influence of the nonlinear properties of the plate material: $1 - \lambda = 1$; $q = 1$; $w_0 = 10^{-1}$; $3 - \lambda = 1$; $q = 6$; $w_0 = 10^{-4}$; $5 - \lambda = 6$; $q = 1$; $w_0 = 10^{-4}$—linear case; $2 - \lambda = 1$; $q = 1$; $w_0 = 10^{-1}$; $4 - \lambda = 1$; $q = 6$; $w_0 = 10^{-4}$; $6 - \lambda = 6$; $q = 1$; $w_0 = 10^{-4}$—nonlinear case.

and nonlinear problems are rather close, and therefore, the problems can be solved via a linear formulation. However, with the increase in parameter λ, and considering the external static transverse load q and the initial geometric imperfections w_0, a discrepancy in the results for the oscillation amplitudes is observed. This implies that a nonlinear approach should be applied.

The curves in Fig. 8 correspond to different viscoelastic properties of the material: curve 1 corresponds to the viscoelastic properties of a material in transverse shear directions ($A = A_{13} = A_{23} = 0.05$; $A_{ij} = 0$, $i, j = 1, 2$);

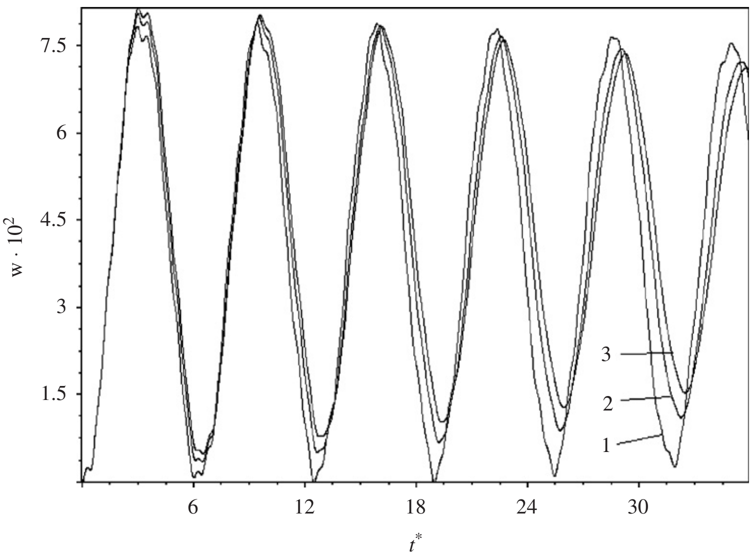

Fig. 8. The influence different viscoelastic properties of the material: $1 - A = A_{13} = A_{23} = 0.05$, $A_{ij} = 0$, $i, j = 1, 2$; $2 - A = A_{ij} = 0.05$, $i = 1, 2, j = 1, 2, 3$; $3 - A = 0.05$, $A_{11} = 0.06$, $A_{12} = 0.07$, $A_{21} = 0.08$, $A_{22} = 0.09$, $A_{13} = 0.1$, $A_{23} = 0.11$.

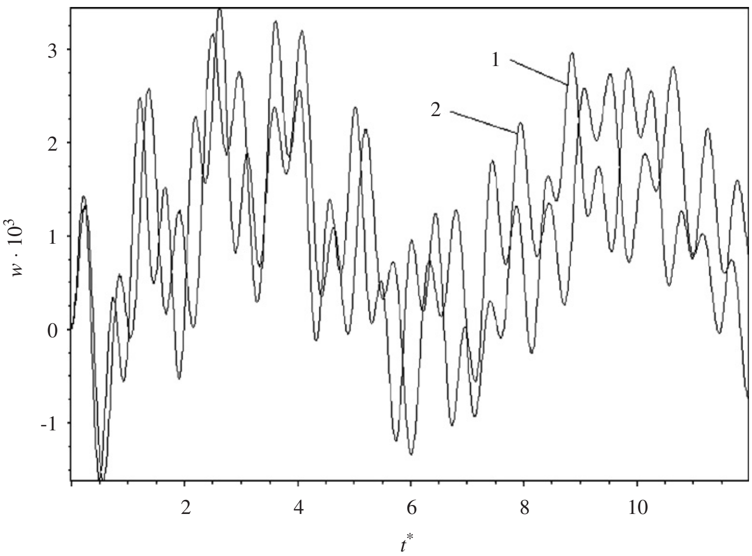

Fig. 9. Dependence of the deflection on time for various theories: 1—Kirchhoff–Love; 2—Reissner–Mindlin.

curve 2 to viscoelastic properties of the material in all directions equally ($A = A_{ij} = 0.05$, $i = 1, 2, j = 1, 2, 3$—an isotropic case); curve 3 corresponds to the case when the material viscoelastic properties are different in all directions ($A = 0.05$, $A_{11} = 0.06$, $A_{12} = 0.07$, $A_{21} = 0.08$, $A_{22} = 0.09$, $A_{13} = 0.1$, $A_{23} = 0.11$—an orthotropic case). Taking the viscoelastic properties of a material into consideration simultaneously in all directions leads to a larger decay in vibrational amplitude and to a shift of the phases to the right. Moreover, in the case of viscoelastic properties taken in the transverse shear directions, the results are close to those obtained in the purely elastic case. This fact reveals that considering the viscoelastic properties of a material only in transverse shear is not enough to describe the real processes taking place in constructions with uniform viscoelastic properties. Therefore, in such a case it is necessary to consider the viscoelastic properties of the material in all directions.

Fig. 9 shows the results obtained within the K.L. (curve 1) and R.M. one (curve 2) theories for a viscoelastic orthotropic plate. As the results reveal, both quantitative and qualitative differences in the responses obtained, as per these two theories, are emerging.

3. Dynamic stability of viscoelastic orthotropic plates within the Kirchhoff–Love and Reissner–Mindlin hypotheses

Regarding the problem of dynamic stability of viscoelastic plates subjected to the normal P_x, P_y and the shear P_{xy} edge loadings (Fig. 10) in the governing Eqs. (5), (10), one should include these by replacing $P_x(t)(\partial^2 w/\partial x^2)$, $P_y(t)(\partial^2 w/\partial y^2)$ and $P_{xy}(t)(\partial^2 w/\partial x\,\partial y)$, according to Refs. [10,11,23,24].

Assume that the rectangular plate ($a \times b$) is subjected to a dynamic edge load $P(t) = vt$ (v—loading speed) along the y-direction (Fig. 10). In this case, one should use the systems of Eqs. (5) and (10) with corresponding initial and boundary conditions. Assuming that the plate is simply supported all over the contour, one should find the solutions to Eqs. (5) and (10) by representing displacements w, w_0, u, v and the angular displacements ψ_x, ψ_y according to Eq. (11). Substituting Eq. (11) in the systems of Eqs. (5) and (10), and applying the Bubnov–Galerkin procedure, using the dimensionless quantities:

$$\frac{w_{kl}}{h},\ \frac{w_{0kl}}{h},\ \frac{u_{kl}}{h},\ \frac{v_{kl}}{h},\ P^* = \frac{P}{\sqrt{E_1E_2}}\left(\frac{b}{h}\right)^2,\ q^* = \frac{q}{\sqrt{E_1E_2}}\left(\frac{b}{h}\right)^4,\ t^* = \frac{P}{P_{\mathrm{cr}}} = \frac{vt}{P_{\mathrm{cr}}} = \frac{\omega t}{\sqrt{S}} = \frac{P^*}{P^*_{cr}},$$
$$S = P^{*3}_{\mathrm{cr}}\left(\frac{\pi c\sqrt{E_1E_2}h^3}{vb^4}\right)^2,\ P^*_{\mathrm{cr}} = \frac{P_{\mathrm{cr}}}{\sqrt{E_1E_2}}\left(\frac{b}{h}\right)^2 = \frac{\pi^2}{6(1-\mu_{12}\mu_{21})}\eta,\ \frac{\sqrt{S}}{\omega}R(t),\ \frac{\sqrt{S}}{\omega}R_{ij}(t),\ i,j = 1,2,$$

where $c = \sqrt{\sqrt{E_1E_2}/\rho}$; $\omega = \sqrt{\pi^2\sqrt{E_1E_2}h^2P^*_{\mathrm{cr}}/(\rho b^4)}$ and, at the same time, maintaining the previous designations related to the dimensionless u_{kl}, v_{kl}, w_{kl}, ψ_{xkl}, ψ_{ykl}, one obtains the system of nonlinear integro-differential equations, describing the process of dynamical stability of viscoelastic orthotropic plates according to the theories of K.L. and R.M., respectively.

Figs. 11–16 show results of the dynamic response obtained for a viscoelastic orthotropic plate according to the K.L. theory. Here, as in Ref. [11], the criterion for determining the critical time and the critical load was the condition that the deflection should be smaller than the plate thickness that is used. To determine the stability of the plate, we use the dynamic amplification factor K_D, which is equal to the ratio of the dynamic critical load to the Euler static load (i.e. the dynamic critical load being K_D times greater than the Euler static load).

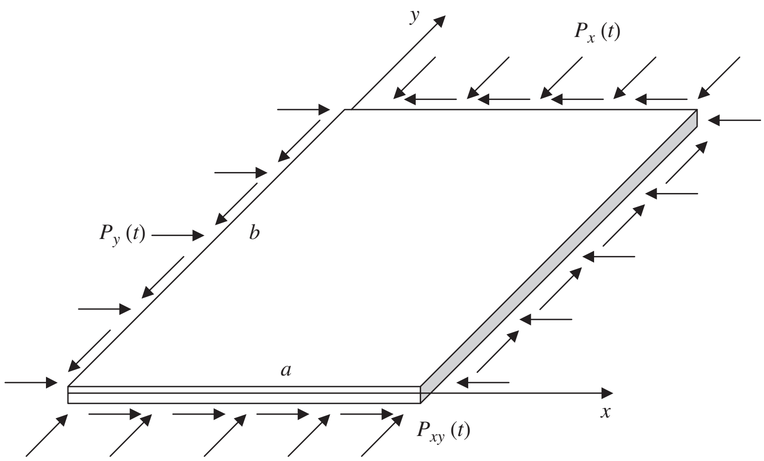

Fig. 10. Model geometry and loading.

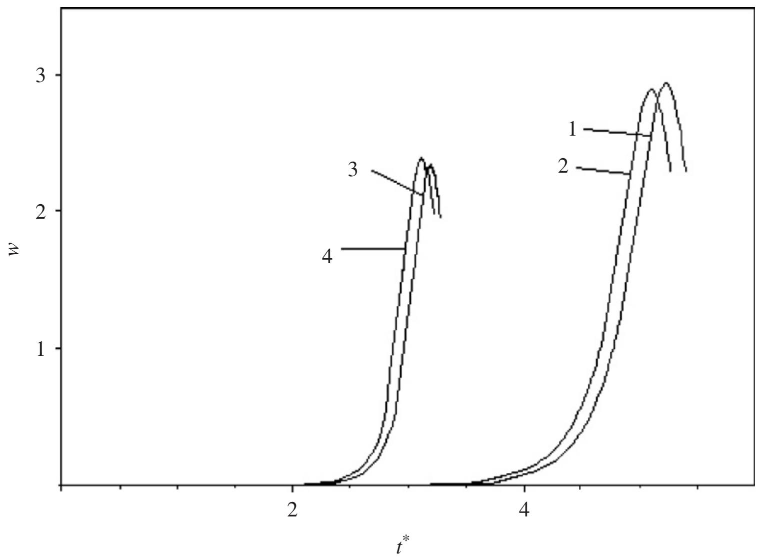

Fig. 11. The influence of viscoelastic properties of material: $1 - S = 1$; $3 - S = 10$—elastic case; $2 - S = 1$; $4 - S = 10$—viscoelastic case.

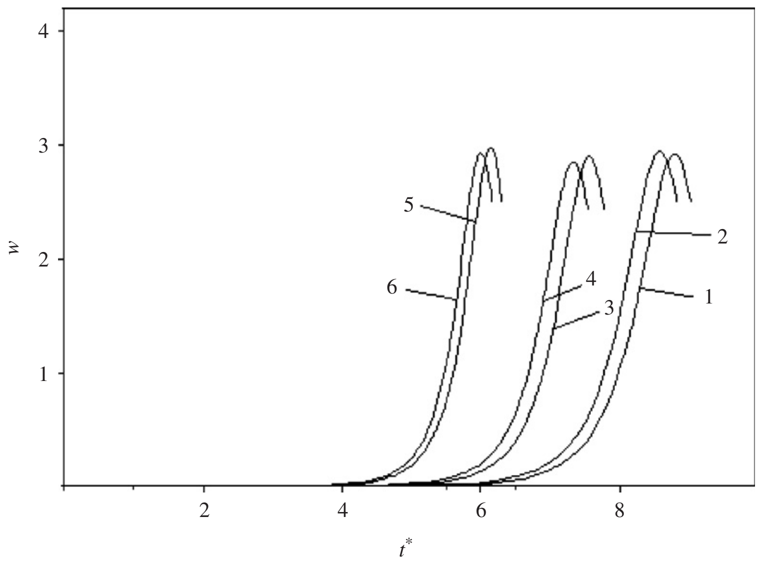

Fig. 12. Dependence of the deflection on time for various values of loading speed: $1 - S = 0.1$; $3 - S = 0.2$; $5 - S = 0.5$—elastic case; $2 - S = 0.1$; $4 - S = 0.2$; $6 - S = 0.5$—viscoelastic case.

Fig. 11 shows a square plate ($\lambda = 1$) with the initial geometric imperfection $w_0 = 10^{-4}$. Assuming that a load q is absent, curves 1 and 2, corresponding to ($S = 1$) and curves 3 and 4 to ($S = 10$), are associated with the elastic and viscoelastic cases, respectively. The abscissa corresponds to the dimensionless parameter t^*, previously defined, and the ordinate axis is the dimensionless deflection w. The amplification factor K_D in the elastic and viscoelastic cases, when $S = 1$, is equal to $K_D = 4.8$ and 4.6, respectively, and when $S = 10$, $K_D = 3$ and 2.85, respectively. The result obtained shows that considering the viscoelastic characteristics of a material leads to a decrease in the critical load.

Fig. 12 shows an example of the w function for various values S. When $S = 0.1$, 0.2 and 0.5, the "critical" values of K_D in the viscoelastic case will be 7.7, 7.6 and 5.4, respectively (curves 2,4,6), and for the elastic case, 8.0, 7.9 and 5.6 (curves 1,3,5), respectively. Note that the parameter S is inversely proportional to v. The diagram shows that as the loading speed v increases, the value of the K_D factor increases.

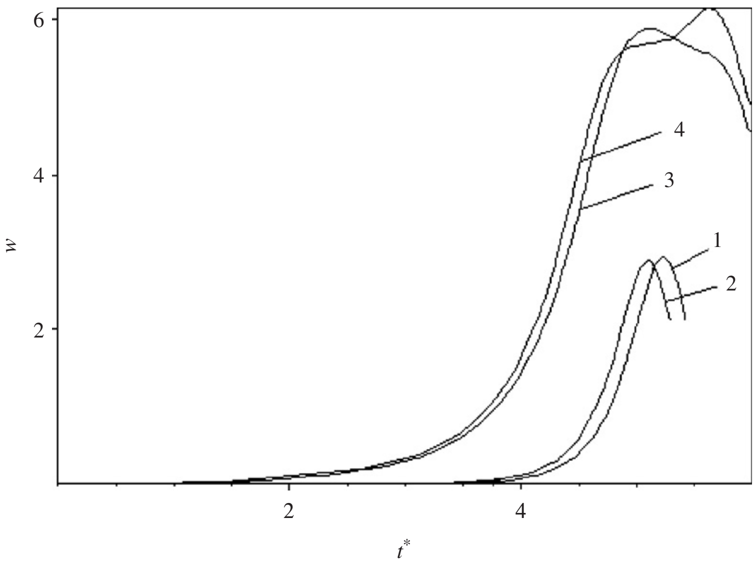

Fig. 13. The influence of additional statical shear load q: $1 - q = 0$; $3 - q = 1$—elastic case; $2 - q = 0$; $4 - q = 1$—viscoelastic case.

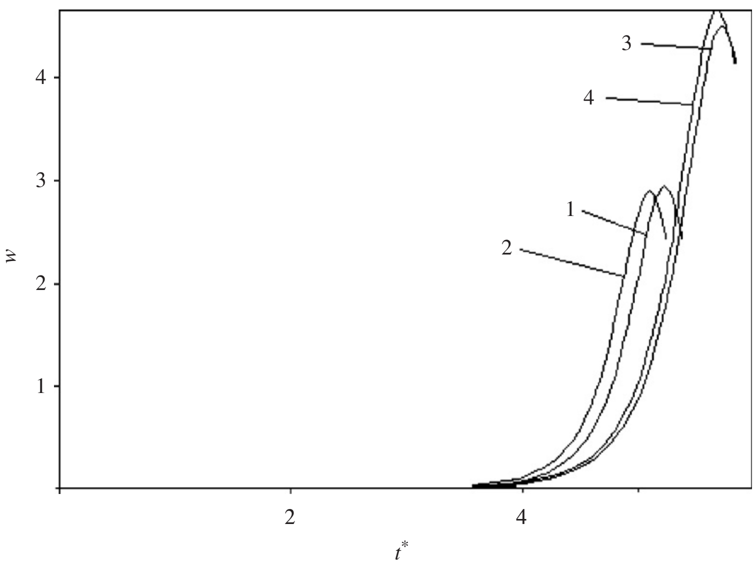

Fig. 14. The diagram depicting the w curves for the cases of a viscoelastic square and elongated plate $1 - \lambda = 1$; $3 - \lambda = 2$—elastic case; $2 - \lambda = 1$; $4 - \lambda = 2$—viscoelastic case.

The influence of an additional statical shear load on the behavior of a square plate is presented in Fig. 13. At $q = 1$, the value of K_D is 3.9 in the elastic case (curve 3), and 3.8 in the viscoelastic case (curve 4), while at $q = 0$ it is equal to 4.8 (curve 1) and 4.6 (curve 2), respectively.

Fig. 14 shows diagrams of the w curves for cases of a viscoelastic square and elongated plate. The results show that for $\lambda = 2$, the corresponding values of K_D are equal to 5 (curve 3), in the elastic case, and 4.9 in the viscoelastic case (curve 4). For $\lambda = 1$, K_D corresponds to 4.8 (curve 1) and 4.6 (curve 2), respectively. When comparing these figures for $\lambda = 1$ and 2, we can conclude that the K_D values determined for a square plate in the viscoelastic case could be carried over, with insignificant error, to plates of a different configuration.

Fig. 15 shows the influence of the initial geometric imperfection w_0. Diagrams for w, corresponding to $w_0 = 10^{-6}$, 10^{-4} and 10^{-2}, are displayed. For $w_0 = 10^{-6}$, 10^{-4} and 10^{-2}, the values of K_D in the viscoelastic

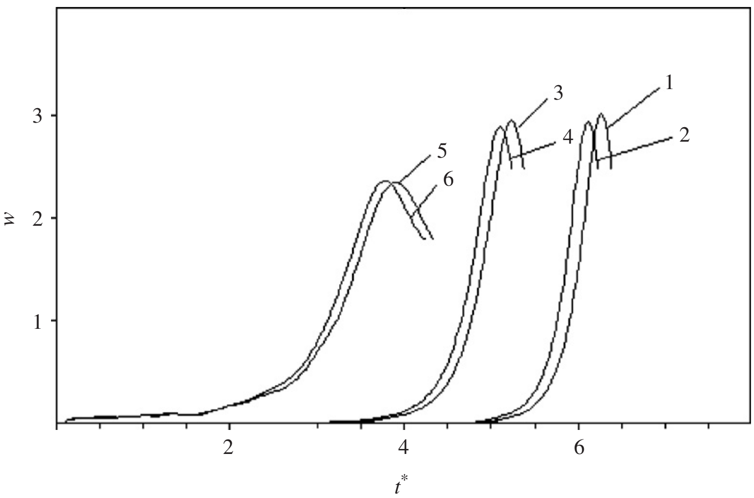

Fig. 15. The influence of the initial geometric imperfection: $1 - w_0 = 10^{-6}$; $3 - w_0 = 10^{-4}$; $5 - w_0 = 10^{-2}$—elastic case; $2 - w_0 = 10^{-6}$; $4 - w_0 = 10^{-4}$; $6 - w_0 = 10^{-2}$—viscoelastic case.

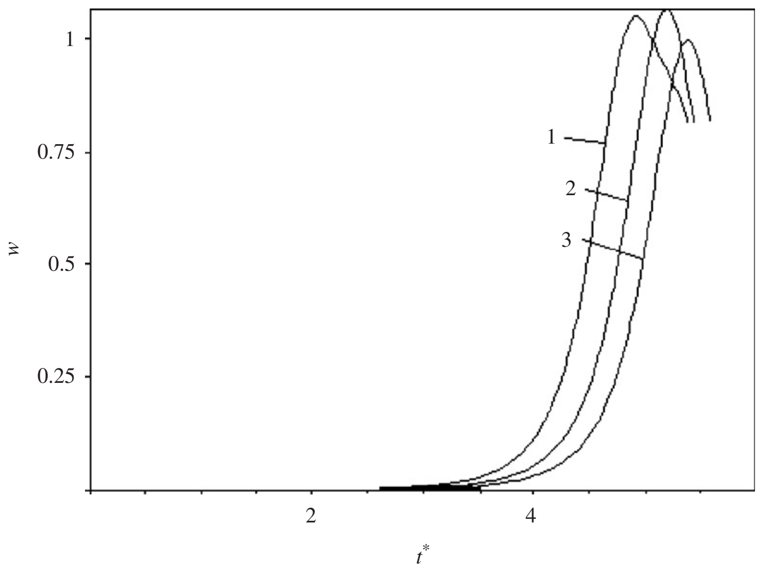

Fig. 16. The influence of the orthotropy of the material: $1 - \Delta = 1$; $2 - \Delta = 1.5$; $3 - \Delta = 2$.

case equal 5.7, 4.65 and 3.1 (curves 2,4,6), respectively. In the elastic case, K_D equals 5.9; 4.8; 3.25, respectively (curves 1,3,5).

The influence of orthotropy of the material on the stability of the plate is presented in Fig. 16. An increase in Δ, which determines the degree of orthotropy (curve 1 corresponding to $\Delta = 1$; curve 2 to $\Delta = 1.5$ and curve 3 to $\Delta = 2$), yields a delayed increase in deflection and, correspondingly, to an increase in the critical value K_D. Similar results were observed in experiments carried out with structures manufactured from composite materials [11], which again confirms the validity of the chosen method and, correspondingly of the obtained results.

Figs. 17–20 present the deflection time-history for a dynamical stability of a viscoelastic orthotropic plate according to the generalized theory of Reissner–Mindlin at various physical and geometrical parameters.

Fig. 17 represents the dynamic response of a square plate ($\lambda = 1$) with the initial geometric imperfection amplitude $w_0 = 10^{-4}$ when load q is absent. Curve 1 in this diagram corresponds to the case when viscoelastic

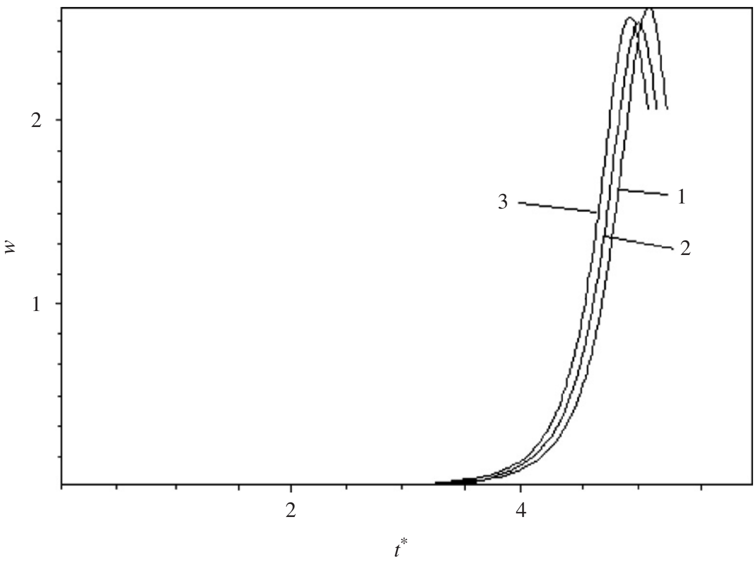

Fig. 17. Dependence of the deflection on time for various kernels of relaxation: 1—elastic case; 2—exponential kernel; 3—weakly singular kernel of Koltunov–Rzhanitsyn.

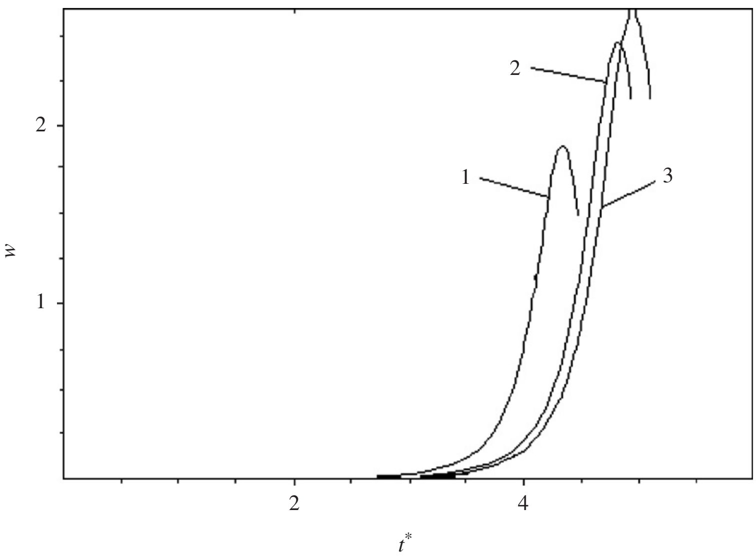

Fig. 18. Dependence of the deflection on time for various values of the geometric parameter δ: $1 - \delta = 10$; $2 - \delta = 20$; $3 - \delta = 30$.

features are not taken into consideration (purely elastic case). Curve 2 corresponds to the results obtained when an exponential kernel of relaxation is considered, while curve 3 is the case with a Koltunov–Rzhanitsyn kernel. Analysis shows that, although there is no great difference in K_D in these cases, a qualitative difference is, however, observed.

The influence of parameter β_{ij} $(i = 1, 2; j = 1, 2, 3)$ on the behaviour of the plate has been also analysed. The results (not displayed) here reveal that, as in case of vibration, β_{ij} within the interval $0 < \beta_{ij} < 1$ does not greatly affect the critical time and load.

Fig. 18 shows the deflection time-history w of the viscoelastic plate for various values of the geometric parameter δ (defined as the ratio between width and thickness of the plate). When $\delta = 10$, 20 and 30 (curves 1, 2, 3,

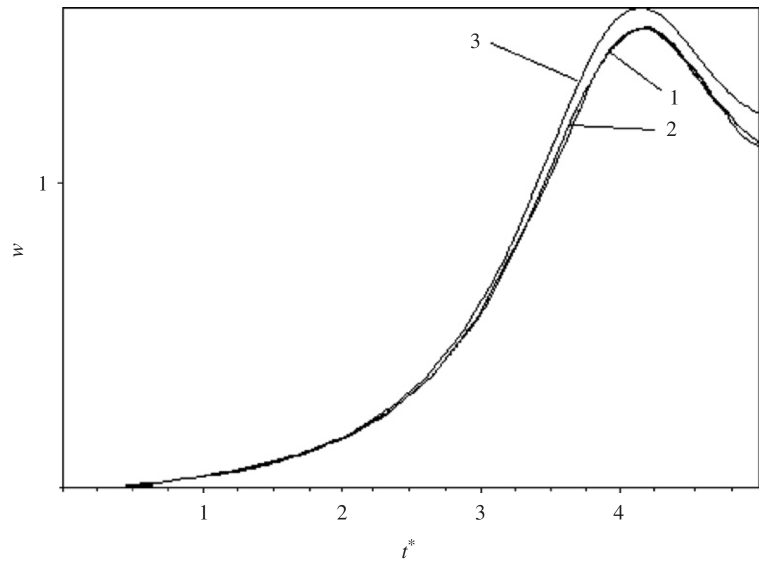

Fig. 19. The influence different viscoelastic properties of the material: $1 - A = A_{ij} = 0,\ i = 1, 2,\ j = 1, 2, 3;\ 2 - A = A_{13} = A_{23} = 0.1,$ $A_{ij} = 0,\ i,j = 1, 2;\ 3 - A = A_{ij} = 0.1,\ i = 1, 2, j = 1, 2, 3.$

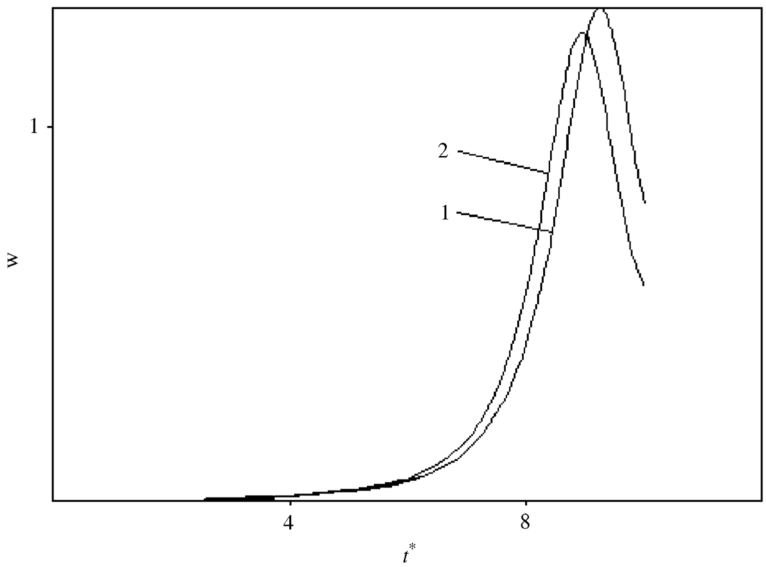

Fig. 20. Dependence of the deflection on time for various theories: 1—Kirchhoff–Love; 2—Reissner–Mindlin.

respectively), the ‘‘critical’’ value K_D is 4.1, 4.4 and 4.5, respectively. Note that an increase in δ leads to a shift of all curves to the right, i.e. to a large value of t^*, hence, to an increase in the amplification factor.

In Fig. 19, curve 1 corresponds to the case where the viscoelastic properties of the material are not taken into consideration ($A = A_{ij} = 0$, $i = 1, 2$; $j = 1, 2, 3$—purely elastic case), curve 2 to the case (frequently used by many researchers [7,8] in studying viscoelastic orthotropic constructions $A_{ij} = 0$, $A = A_{13} = A_{23} = 0.1$, $i,j = 1, 2$) when viscoelastic properties of the material are taken into consideration only in transverse shear, and curve 3 relates to the case when viscoelastic properties of materials are taken into account equally in all directions ($A = A_{ij} = 0.1$, $i = 1, 2$; $j = 1, 2, 3$). As can be clearly seen, the results for the purely elastic case almost coincide with those obtained for viscoelastic properties considered only in transverse shear.

Considering viscoelastic properties in all direction yields an earlier increase in deflections and, hence, a decrease in the critical values of K_D.

Fig. 20 compares the results obtained through various theories: K.L. model (curve 1) and M.R. theory (curve 2). The "critical" value of K_D, corresponding to the K.L. models, equal 8.8. Considering this problem via the M.R. theory, the critical value is 8.5.

4. Conclusions

The study of the nonlinear behaviour in vibrations and stability of orthotropic viscoelastic plates reveals a number of facts.

For a more comprehensive investigation of viscoelastic structures, it is necessary to take into account the viscoelastic properties of material, not only in transverse shear, but in all other directions.

For the kernels of relaxation, it is necessary to use Koltunov–Rzhanitsyn type kernels containing a sufficient number of rheological parameters, validated via experiments.

Depending on the geometrical and physical parameters of the plates, it is necessary to use appropriate theories (both in linear and nonlinear formulations), i.e. the classical K.L. and R.M. theories, which agree with experimental predictions. According to the results, the Reissner–Mindlin theory is more acceptable as it takes into account both transverse shear deformation and rotatory inertia.

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