Available online at www.sciencedirect.com

ScienceDirect

Journal of Sound and Vibration 301 (2007) 18–27

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Genetic algorithm-based optimum vehicle suspension design using minimum dynamic pavement load as a design criterion

Lu Sun[a,*], Ximing Cai[b], Jun Yang[a]

[a]Department of Civil Engineering, The Catholic University of America, Washington, DC 20064, USA
[b]Department of Civil and Environmental Engineering, The University of Illinois at Urbana-Champaign, IL, USA

Received 20 July 2004; received in revised form 31 August 2006; accepted 31 August 2006
Available online 28 November 2006

Abstract

In this paper, the design of a passive vehicle suspension system was handled in the framework of nonlinear optimization. The variance of the dynamic load resulting from the vibrating vehicle operating at a constant speed was used as the performance measure of a suspension system. Using a quarter-car model, the performance measure was derived as an integration of a complex function of several variables. A genetic algorithm is applied to solve the nonlinear optimization problem. It was found from the sensitivity analysis that appropriate mutation rate, crossover rate and population size are 1.0%, 25% and 100, respectively. The optimum design parameters of the suspension systems obtained are $k_s = 622{,}180\,\mathrm{N/m}$, $k_t = 1{,}705{,}449\,\mathrm{N/m}$ and $c_s = 26{,}582\,\mathrm{N\,s/m}$, respectively.

1. Introduction

The ride quality of a vehicle is significantly influenced by its suspension system, the road surface roughness, and the speed of the vehicle. A vehicle designer can do little to improve road surface roughness, so designing a good suspension system with optimum vibration performance under different road conditions becomes a prevailing philosophy in the automobile industry. Over the years, both passive and active suspension systems have been proposed to optimize a vehicle's ride quality [1–7]. Passive suspension systems use conventional dampers to absorb vibration energy, while active suspension systems use additional power to provide a response-dependent damper [8]. A passive suspension system does not require extra power, while an active suspension system is capable of producing an improved ride quality.

The primary performance measure of a suspension system is traditionally measured in terms of ride quality [9]. The two principal variables for the design and evaluation of the suspension system are sprung mass acceleration, which determines ride comfort, and suspension deflection, which indicates the limit of the vehicle body motion [10]. In the literature, the root mean square of vertical acceleration of the sprung mass is often taken as the performance measure (objective function) to be optimized [3,4].

*Corresponding author. Tel.: +1 202 319 6671.
E-mail addresses: sunl@cua.edu (L. Sun), xmcai@uiuc.edu (X. Cai), 02yang@cua.edu (J. Yang).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.08.040

Ride quality as the suspension system design criterion mainly stems from taking a human ride comfort perspective. On the other hand, from a cost-effectiveness of vehicle-infrastructure systems perspective, another performance measure is also of significant importance, that is, the dynamic tire load applied on the pavement [11–18]. It has been known for years that pavement damage is related to tire loads through a fourth power law [14]. In other words, a 10% increase in the magnitude of the tire load will lead to an almost 50% increase in pavement damage. Because of the huge maintenance cost associated with repairing transportation infrastructure networks, it would certainly benefit the whole society, in terms of the overall cost, if the dynamic load generated by vehicle vibration can be reduced to a minimum. To this end, a method for optimum vehicle suspension design using dynamic tire load as a design criterion has been developed.

Conventionally, vehicle suspension system design is carried out by using nonlinear optimization [19,20,4,6,7]. Classical nonlinear optimization algorithms such as the steepest-descent method and the Broyden–Fltcher–Godfarb–Shanno method require the gradient to be computed during the iterative search process [21]. The objective function used in the suspension system design is usually highly nonlinear and may possess multiple extrema. The optimal solution obtained by using classical optimization algorithms may often be trapped in a local minima. Genetic algorithms are randomized search techniques guided by the principles of evolution and natural genetics. They are effective, adaptive and robust search procedures, producing near global optimal solutions and having a large amount of implicit parallelism.

Genetic algorithms have been applied in a number of ways [22–25]. The idea utilized in a genetic algorithm is to use the optimization strategies nature uses successfully—known as Darwinian Evolution—and transform them for application in mathematical optimization theory to find the near global optimum. Chalasani [26], and others use classical optimization methods to study the problem of tire force minimization. Since the number of design parameters considered in these studies is only two, the optimal solution can even be obtained by inspection of a contour map of the objective function as the two design parameters vary. Baumal, McPhee and Calamai [3] applied genetic algorithm to vehicle suspension design, in which the road surface is assumed to be a deterministic sinusoidal function. This may limit the practical value of their optimum design result, as the actual road surface is a random field. To take advantage of genetic algorithms as well as to consider more realistic random field models for road surfaces, we implement genetic algorithms in this paper to solve the optimum vehicle suspension design problem.

2. Vehicle model

Two approaches can be adopted to study dynamic vehicle loads. One approach relies on experiments and field tests, while the other approach utilizes computer modeling and simulation to conduct a numerical study. The former is expensive but indispensable for validating the computer models. Over the years the second approach has gained increasing popularity in research and design because of its low cost and powerful capability for testing various scenarios. Using this approach, a vehicle needs to be simplified to develop a vehicle model for simulating the real operation conditions. Based on the vehicle model, the dynamic response at any position in the vehicle can be approximated numerically.

Commonly used vehicle models include quarter-, half- and full-vehicle models [27–32,15]. It is known that a quarter-vehicle model can be used to predict ride quality and pavement loading very well [33–36]. In this paper a quarter vehicle model is adopted to characterize vehicle dynamics. Such a quarter vehicle model is based on the following assumptions: constant vehicle velocity, no vehicle body or axle roll, rigid vehicle bodies, linear suspension and tire characteristics, point tire to road contact, and small pitch angles. In Fig. 1, a quarter-vehicle model with two degrees of freedom moving on a rough pavement, in which a parallel spring and damper with constant coefficients are used to model the tire is shown.

According to the D'Alembert's principle, the vehicle's suspension system is governed by

$$m_t\ddot{y}_t + c_s(\dot{y}_t - \dot{y}_s) + c_t(\dot{y}_t - \dot{\xi}) + k_s(y_t - y_s) + k_t(y_t - \xi) = 0, \tag{1}$$

$$m_s\ddot{y}_s - c_s(\dot{y}_t - \dot{y}_s) - k_s(y_t - y_s) = 0, \tag{2}$$

where m_s is the vehicle sprung mass, m_t is unsprung mass, k_s is suspension spring constant, k_t is the tire spring constant, c_s is suspension damping constant, and c_t is tire damping constant. Here, pavement roughness ξ is

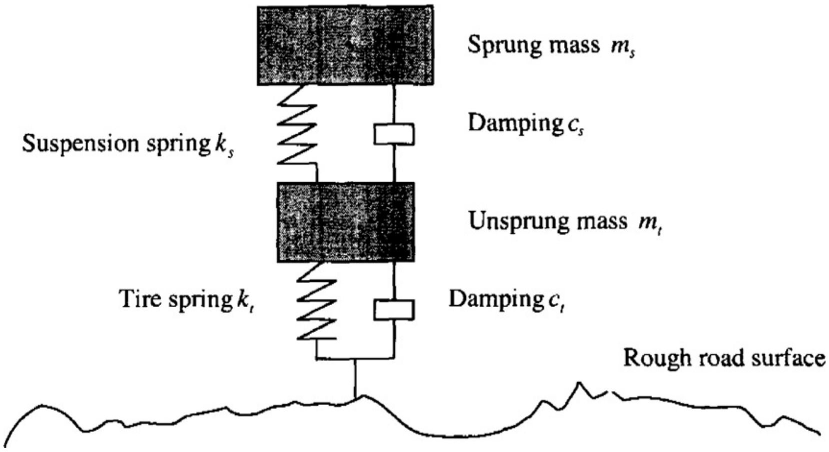

Fig. 1. A quarter vehicle suspension system.

modeled as a one-dimensional random field, and y_s and y_t are absolute displacements of the sprung mass and the unsprung mass, respectively. One and two dots above a symbol indicate, respectively, first and second derivative with respect to time.

Let $z_t(t)$ and $z_s(t)$ be the respective relative displacements of the sprung mass and the unsprung mass, namely, $z_t(t) = \xi - y_i(t)$ and $z_s(t) = y_t(t) - y_s(t)$. $H_t(\omega)$ and $H_s(\omega)$ denote the frequency response functions of the unsprung mass and the sprung mass, respectively. Specifically, they relate the steady-state response of the unsprung mass and the sprung mass to a sinusoidal excitation $\xi = e^{i\omega t}$. Frequency response function of the quarter vehicle model are determined from [15]:

$$\begin{bmatrix} \omega^2 - i\omega c_t/m_t - k_t/m_t & i\omega c_s/m_t + k_s/m_t \\ \omega^2 & \omega^2 + i\omega c_s/m_s + k_s/m_s \end{bmatrix} \begin{bmatrix} H_t(\omega) \\ H_s(\omega) \end{bmatrix} = \begin{bmatrix} \omega^2 \\ \omega^2 \end{bmatrix}.$$

The solution of the above equation gives

$$H_t(\omega) = \frac{\omega^4 - i\alpha_s(1+\hbar)\omega^3 - \beta_s(1+\hbar)\omega^2}{\omega^4 + i(\alpha_t - \hbar\alpha_s - \alpha_s)\omega^3 + (\alpha_t\alpha_s + \beta_t - \beta_s - \hbar\beta_s)\omega^2 + i(\alpha_t\beta_s + \alpha_s\beta_s)\omega - \beta_t\beta_s} \tag{3}$$

and

$$H_s(\omega) = \frac{-i\alpha_t\omega^3 - \beta_t\omega^2}{\omega^4 + i(\alpha_t - \hbar\alpha_s - \alpha_s)\omega^3 - (\alpha_t\alpha_s + \beta_t + \beta_s + \hbar\beta_s)\omega^2 + i(\alpha_t\beta_s + \alpha_s\beta_t)\omega + \beta_t\beta_s}, \tag{4}$$

where $i = \sqrt{-1}$, $\hbar = m_s/m_t$, $\alpha_s = c_s/m_s$, $\alpha_t = c_t/m_t$, $\beta_s = k_s/m_s$ and $\beta_t = k_t/m_t$.

3. Roughness excitation and vehicle response

Road surface roughness can be modeled as a zero mean Gaussian isotropic random field. Let random variable $\xi(x)$ be the elevation of road surface profile along the wheelpath [37–41,15]. Clearly $\xi(x)$ is a function of spatial distance x along the road. From Wienner–Khintchine theory, the following forms constitute a Fourier transform couple [15]

$$S_\xi(\Omega) = \int_{-\infty}^{\infty} R_\xi(X)\, e^{-i2\pi\Omega X}\, dX, \tag{5a}$$

$$R_\xi(X) = \int_{-\infty}^{\infty} S_\xi(\Omega)\, e^{i2\pi\Omega X}\, d\Omega, \tag{5b}$$

where X is the distance of two points along the road, $S_\xi(\Omega)$ is the power spectral density in terms of wavenumber Ω, and $R_\xi(X)$ is the spatial auto-correlation function. In engineering practice the single-sided power spectral densities, denoted by, $G_\xi(\Omega)$, are most often used.

These are defined by

$$G_\xi(\Omega) = \begin{cases} 2S_\xi(\Omega) & \text{for } \Omega \geqslant 0, \\ 0 & \text{for } \Omega < 0. \end{cases} \tag{6}$$

The random field describing the road surface in the space domain produces a stochastic excitation process in the time domain when the vehicle moves along the road. Let $S_\xi(\omega)$ be the power spectral density of the surface roughness as a function of angular frequency ω. It can be shown that at a constant speed the wavenumber and angular frequency power spectral densities are related through [15]

$$S_\xi(\omega) = \frac{1}{4\pi v} G_\xi(\Omega), \tag{7}$$

where $\omega = 2\pi v\Omega$ and v is the velocity.

Road surface roughness has been the subject of considerable research [42]. Over the years many power spectral density functions of roughness have been developed for a variety of surface types, such as bridge pavement surface [43], rail-track irregularities [44], and highway surface profiles [42,45]. It is found that many road profiles have similar power spectral densities [43,46]. Dodds and Robson [45] presented an approximate power spectral density function covering the entire range of spatial frequencies which is a piecewise fit in a form of the split power law [47]:

$$G_\xi(\Omega) = \begin{cases} G_{\mathrm{sp}}\Omega_1^{-w1} & \text{for } 0 \leqslant \Omega \leqslant \Omega_1, \\ C_{\mathrm{sp}}\Omega_1^{-w2} & \text{for } \Omega_1 \leqslant \Omega, \end{cases} \tag{8}$$

where $G_\xi(\Omega)$ is the single-sided power spectral density of roughness, coefficient C_{sp} and exponents w_1 and w_2 relate to conditions of the pavement surface [47]. The road profile described by Eq. (8) will be used in the following sections when developing the optimum design for the vehicle suspension system.

According to random vibration theory, we have [37,15]

$$S_{Z_t}(\omega) = |H_t(\omega)|^2 S_\xi(\omega),$$

$$S_{Z_s}(\omega) = |H_s(\omega)|^2 S_\xi(\omega), \tag{9}$$

where $S_{Z_t}(\omega)$ and $S_{Z_s}(\omega)$ are the power spectral densities of the displacement responses of the unsprung mass and the sprung mass, respectively. The power spectral density of pavement load as a function of power spectral density of the road roughness profile is given by Sun and Deng [15]:

$$S_P(\omega) = |(k_t + \mathrm{i}c_t\omega)H_t(\omega)|^2 S_\xi(\omega) = A(\omega)S_\xi(\omega) = \left|\frac{X_A(\omega) + \mathrm{i}X_B(\omega)}{X_C(\omega) + \mathrm{i}X_D(\omega)}\right|^2 S_\xi(\omega), \tag{10}$$

where

$$X_A(\omega) = \left(k_t + \frac{c_t c_s}{m_t} + \frac{c_t c_s}{m_s}\right)\omega^4 - \left(\frac{k_t k_s}{m_t} + \frac{k_s}{m_t}\right)\omega^2,$$

$$X_B(\omega) = c_t\omega^5 - \left(\frac{k_t c_s}{m_t} + \frac{k_t c_s}{m_s} - \frac{c_t k_s}{m_t} - \frac{c_t k_s}{m_s}\right)\omega^3,$$

$$X_C(\omega) = \omega^4 - \left(\frac{c_t c_s}{m_t m_s} + \frac{k_t}{m_t} + \frac{k_s}{m_s} + \frac{k_s}{m_s}\right)\omega^2 + \frac{k_t k_s}{m_t m_s},$$

and

$$X_D(\omega) = -\left(\frac{c_t}{m_t} + \frac{c_s}{m_t} + \frac{c_s}{m_s}\right)\omega^3 + \left(\frac{c_t k_s}{m_t m_s} + \frac{c_s k_t}{m_t m_s}\right)\omega.$$

Let σ_P^2 be the variance of dynamic vehicle load. The variance σ_P^2 is a measure reflecting the fluctuation of the magnitude of dynamic vehicle load and is defined by

$$\sigma_P^2 = \int_{-\infty}^{\infty} S_P(\omega)\,\mathrm{d}\omega = \int_{-\infty}^{\infty} A(\omega)S_\xi(\omega)\,\mathrm{d}\omega$$
$$= C_{\mathrm{sp}}\left[\int_0^{2\pi v\Omega_1} \frac{A(\omega)\Omega_1^{-w_1}}{2\pi v}\,\mathrm{d}\omega + \int_{2\pi v\Omega_1}^{2\pi v\Omega_2} \frac{A(\omega)}{2\pi v}\left(\frac{\omega}{2\pi v}\right)^{-w_2}\mathrm{d}\omega\right]. \tag{11}$$

The suspension deflection is measured by σ_{Zs}^2, which is calculated as

$$\sigma_{Zs}^2 = \int_{-\infty}^{\infty} S_{Zs}(\omega)\,\mathrm{d}\omega = \int_{-\infty}^{\infty} |H_s(\omega)^2|S_\xi(\omega)\,\mathrm{d}\omega$$
$$= C_{\mathrm{sp}}\left[\int_0^{2\pi v\Omega_1} \frac{|H_s(\omega)^2|\Omega_1^{-w_1}}{2\pi v}\,\mathrm{d}\omega + \int_{2\pi v\Omega_1}^{2\pi v\Omega_2} \frac{|H_s(\omega)^2|}{2\pi v}\left(\frac{\omega}{2\pi v}\right)^{-w_2}\mathrm{d}\omega\right]. \tag{12}$$

4. Optimization via genetic algorithms

Genetic algorithms were invented in the 1960s [22,23] and developed by many others [48,49]. The metaphor underlying genetic algorithms is that of natural evolution. In evolution, the problem each species faces is one of searching for beneficial adaptations to a complicated and changing environment. The "knowledge" that each species has gained is embodied in the makeup of the genes of its members. In general, genetic algorithms can be perceived as a search procedure through a space of potential solutions. Classical optimization methods work well for searching in small spaces. However, for larger spaces artificial intelligence techniques are more effective and genetic algorithms are among such techniques. Genetic algorithms are stochastic algorithms whose search motivations model natural evolution: genetic inheritance and Darwinian strife for survival. Genetic algorithms are stochastic algorithms because random numbers that are generated during the operation of the algorithm determine the search result. This means that if a genetic algorithm is deployed to optimize the same problem twice in exactly the same way, usually two different answers will be obtained, though they may be very close to each other.

Large magnitude dynamic vehicle loads may cause more damage on highway pavements. To mitigate load-induced damage, the variance (σ_P^2) of the dynamic load should be minimized. For an ordinary quarter vehicle model, the vehicle parameters used in the optimum design are $m_t = 550\,\mathrm{kg}$, $m_s = 4450\,\mathrm{kg}$, $v = 20\,\mathrm{m/s}$ and $c_t = 0\,\mathrm{N\,s/m}$, and road roughness parameters are chosen to be $C_{\mathrm{sp}} = 1\times10^{-2}, \Omega_l = 0.1\,\mathrm{cycle/m}$, $w_1 = w_2 = 2.0$. Specifically, having k_s and c_s as the design variables, the optimum suspension design is formulated as

minimization of σ_p^2 subject to the constraints:

$$\begin{aligned}
&1\times10^5\,\mathrm{N/m} \leqslant k_s \leqslant 3\times10^6\,\mathrm{N/m},\\
&1.5\times10^6\,\mathrm{N/m} \leqslant k_t \leqslant 2\times10^6\,\mathrm{N/m},\\
&0 \leqslant c_s \leqslant 3\times10^5\,\mathrm{N\,s/m},\\
&\sigma_{Zs}^2 \leqslant 0.2.
\end{aligned} \tag{13}$$

The last constraint is used to restrict the suspension deflection.

Clearly, the above optimization is a nonlinear programming problem. The classical approach to optimization in this setting is to use a derivative-based approach. Examples are Newton–Raphson and conjugate gradient techniques. The scheme of these algorithms is (1) holding one solution at a time; (2) looking locally to see what direction to move in (via the gradient of the function at the current solution); and (3) selecting the new current solution after deciding how far to move along that path. For problems where the objective function is not differentiate or multiple local optima exist, genetic algorithms offer advantages over the classical optimization algorithms [24]. In Fig. 2, the necessary components of a genetic algorithm are shown.

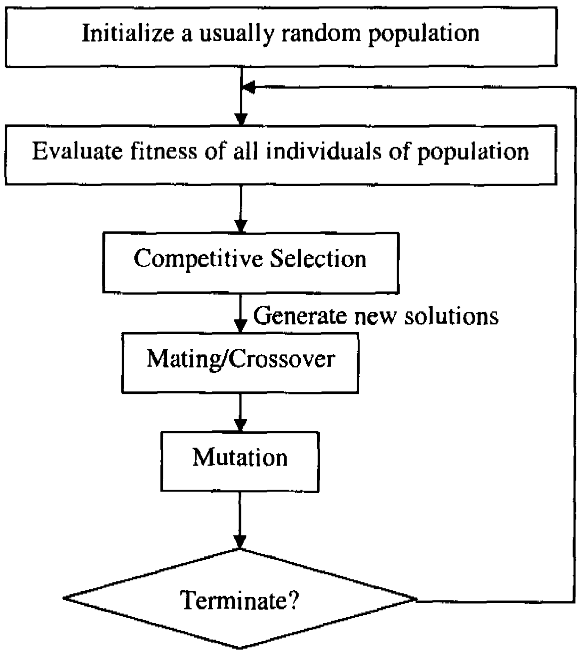

Fig. 2. Schematic diagram of a genetic algorithm.

To solve an optimization problem using genetic algorithms, the problem needs to be represented in the framework used by genetic algorithms [50,24]. An initial population of solutions is created by using random samples. The population is then coded into chromosomes, a binary representation of a solution (consisting of the components of the decision variables known as genes in the genetic algorithm). The whole population of chromosomes represents a generation. An evaluation function rates solutions in terms of their fitness. Here, fitness is a numerical value describing the probability for a solution (genome) to survive and reproduce (i.e., genome copies itself into the next generation). Only a portion of the population (survivors or solutions with higher fitness values) is selected for creating a new population (offspring production). This new population is created by using a crossover operator.

Crossover is a procedure for exchanging pieces of choromosome data with one another. It includes single-point crossover: sniping both strings at the same location and interchange the pieces, and multiple-point crossover, involving recombination of the strings at several locations. Crossover allows genes that generate good fitness to be preserved and enlarged in a new generation of the population. Mutation is a genetic operator and it randomly flips the bits of an offspring's genotype. This is equivalent to perturbing the mated (i.e., after crossover operation) population stochastically. Mutation prevents the population from homogenizing in a particular set of genes such that any gene in a generation has a certain probability (determined by the mutation rate) of being mutated in future generations. The crossover rate and mutation rate are the proportion of the chromosomes involved in crossover or mutation with respect to the whole population. The new population is being mixed up to bring some new information into this set of genes, and this needs to happen in a well-balanced way. Once the new generation is created, the aforementioned steps are repeated until some convergence criteria are satisfied, such as running time or fitness.

To implement the genetic algorithm for the optimum suspension design problem formulated above, we set the required precision to be zeroth decimal place for each design variable. The domain of variable k_s, has length 2,900,000. The precision requirement implies that the range [100,000; 3,000,000] needs to be divided into at least 2,900,000 equal size ranges. This means that 22 bits are required for the first part of the gene: $2^{21} = 2{,}097{,}152 < 2{,}900{,}000 < 2^{22} = 4{,}194{,}304$. The domain of variable c_s has length 300,000. The precision requirement implies that the range [0; 300,000] should be divided into at least 300,000 equal size ranges. Hence, 19 bits are required as the third part of the gene: $2^{18} = 262{,}144 < 300{,}000 < 2^{19} = 524{,}288$. Similarly, 18 bits should be assigned to k_t. Hence, the total length of a gene is $22+18+19=59$ bits, consisting of the first 22 bits of the gene code for k_s, the second 18 bits for k_t and the last 19 bits for c_s. This 59 bits number represents a chromosome which contains a solution to the problem consisting of 3 decision variables. For instance, if the 19 bits for coding c_s are 0010001001011010000, they represent $c_s = 0 + \text{decimal}\ (0010001001011010000_2)\times$

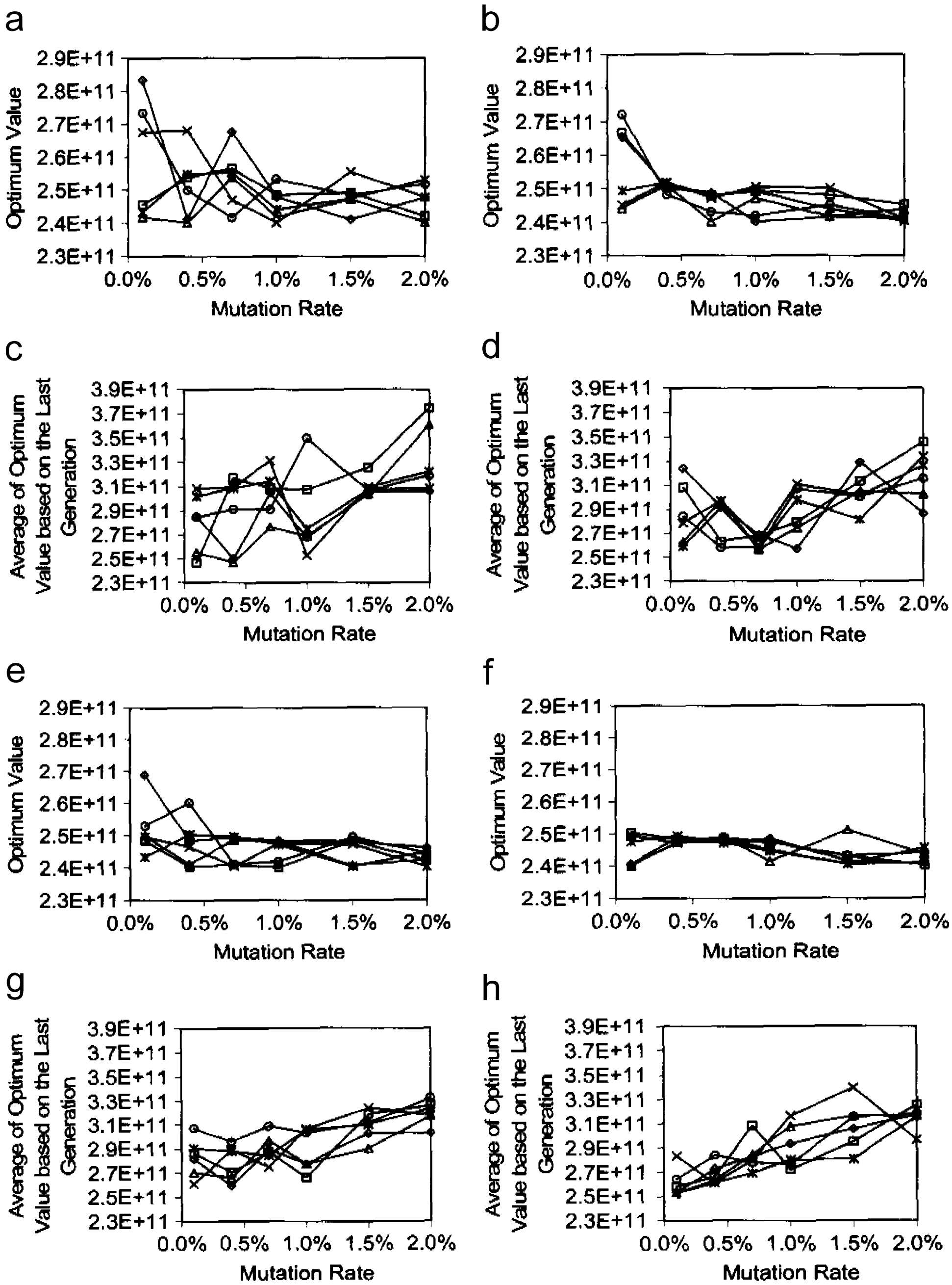

Fig. 3. Sensitivity analysis of probabilities of mutation and crossover: (a) total generation 50, population 50; (b) total generation 100, population 50; (c) total generation 50, population 50; (d) total generation 100, population 50; (e) total generation 50, population 100; (f) total generation 100, population 100; (g) total generation 50, population 100; and (h) total generation 100, population 100. - ◇ - Cross rate = 5%. -□- Cross rate = 15%. -△-Cross rate = 25%. -×- Cross rate = 35%. -*- Cross rate = 45%. -○- Cross rate = 55%.

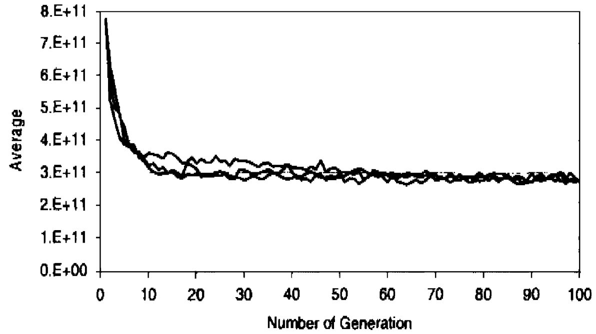

Fig. 4. Average values of 100 populations at each generation.

$300{,}000/(2^{19}-1) = 0 + 70{,}352 \times 300{,}000/524{,}287 \approx 40{,}256$. The fitness of a chromosome as a candidate solution is a function of these genes and is obtained by evaluating the objective function.

To minimize the objective function using a genetic algorithm, the 59 bits in all chromosomes are initialized randomly. During the evaluation phase each chromosome is decoded and the parameter value calculated for the objective function. Some chromosomes are better in terms of reducing the value of objective function than others, and these are then given more opportunities (large weights) to be involved in producing the next generation. The individual solution in the new generation is either the same as some solution in present generation or produced by crossover or mutation. Crossover is done by exchanging the same segment of two chromosomes, say, the last 10 bits, while mutation is done by changing a bit in a chromosome from 0 to 1 or from 1 to 0 [3,24,50]. After the new generation is obtained, the previous steps are repeated until convergence criterion is met. In general, lower values of the objective function will be achieved through iteration of generations. The best chromosome corresponds to the minimum objective function.

To choose appropriate probabilities of mutation and crossover, four scenarios are created that corresponds to different combinations of the number of generation (50 and 100) and the population size (50 and 100). Specifically, for the preliminary study, the number of generation and the population size are fixed and only the mutation rate and crossover rate (CR in Fig. 3) are allowed to change. The average values of the last generation and the optimization values from the simulation are plotted in Fig. 3. The average value of the last generation oscillates as the mutation rate changes, which is less informative than the minimum value. From these two figures it can be seen that 1% may be a reasonable choice for the mutation rate since there is good convergence to the minimum value of the objective function for a variety of crossover rate. Also, a population size of 100 and a 25% crossover rate seem to yield robust performance in terms of minimizing the objective function.

As a result of the preliminary analysis, 1%, 25% and 100 are chosen as the appropriate mutation rate, crossover rate and population size, respectively, and the only changeable quantity for this problem is the number of generations, which will be determined by using the convergence criterion during the iteration. In Fig. 4 is shown the average value of the objective function at each generation. In the figure, the results of three simulation runs are plotted. The convergence is evaluated using the average value of the population. The design variables corresponding to the optimum value of the objective function are $k_s = 622{,}180\,\mathrm{N/m}, k_t = 1{,}705{,}449\,\mathrm{N/m}$ and $c_s = 26{,}582\,\mathrm{N\,s/m}$, respectively.

5. Conclusions

In this paper genetic algorithms are demonstrated to be an effective searching algorithm for optimum design of passive vehicle suspension systems. The vehicle model used in the optimization is a quarter-truck model, a simplification of a real truck. Three design variables were considered in this study: the suspension spring constant, the suspension damping and the tire stiffness. Instead of using ride quality as the performance measure of suspension systems, in this paper the dynamic load generated by vehicle–pavement interaction is used as the objective function to be minimized. The optimum suspension design problem treated here is a constrained nonlinear optimization with three decision variables (design parameters), which can also be solved using classic gradient-based searching algorithms. The constraints are the practical ranges for the suspension parameters and the restriction of the suspension deflection. It is found from the sensitivity analysis that the appropriate mutation rate, crossover rate, and population size are 1%, 25% and 100, respectively. The optimum design parameters for the suspension systems were found to be: $k_s = 622{,}180\,\mathrm{N/m}$, $k_t = 1{,}705{,}449\,\mathrm{N/m}$ and $c_s = 26{,}582\,\mathrm{N\,s/m}$. When a more realistic truck model is adopted in the optimum suspension design, the number of design parameters will be more than three, and there may exist multiple local extrema. In this situation, there will be difficulties using the traditional optimization methods while genetic algorithms will still be an effective approach.

References

[1] D. Karnopp, Analytical results for optimum actively damped suspension under random excitation, *Journal of Acoustic, Stress and Reliability in Design* 111 (1989) 278–283.

[2] D. Bestle, Optimization of automotive systems, in: E.J. Huang (Ed.), *Concurrent Engineering: Tools and Technologies for Mechanical System Design*, Springer, Berlin, 1993.

[3] A.E. Baumal, J.J. McPhee, P.H. Calamai, Application of genetic algorithms to the design optimization of an active vehicle suspension system, *Computational Methods in Applied Mechanics Engineering* 163 (1998) 87–94.

[4] J.A. Tamboli, S.G. Joshi, Optimum design of a passive suspension system of a vehicle subjected to a actual random road excitations, *Journal of Sound and Vibration* 219 (1999) 193–205.

[5] A. Giua, C. Seatzu, G. Usai, A mixed suspension system for a half-car vehicle model, *Dynamics and Control* 10 (2000) 375–397.

[6] L. Sun, T.W. Kennedy, Spectral analysis and parametric study of stochastic pavement loads, *Journal of Engineering Mechanics ASCE* 128 (2002) 318–327.

[7] L. Sun, Optimum design of 'road-friendly' vehicle suspension systems subjected to rough pavement surfaces, *Applied Mathematical Modeling* 26 (2002) 635–652.

[8] T. Sireteanu, N. Stoia, Damping optimization of passive and semi-active vehicle suspension by numerical simulation, *Proceedings of the Romanian Academy* 4 (2) (2003) 121–127.

[9] L. Sun, On human perception and evaluation to road surfaces, *Journal of Sound and Vibration* 247 (2001) 547–560.

[10] M. Sunwoo, K.C. Cheok, N.J. Huang, Model reference adaptive control for vehicle active suspension systems, *Institute of Electrical and Electronics Engineers Transactions on Industrial Electronics* 38 (1991) 217–222.

[11] A.N. Heath, M.G. Good, Heavy vehicle design parameters and dynamic pavement loading, *Australian Road Research* 15 (1985) 249–263.

[12] Woodrooffe, P.A. LeBlanc, The influence of suspension variations on dynamic wheel loads of heavy vehicles, Society of Automotive Engineers Technical Paper No. 861973, 1986.

[13] D. Cebon, Interaction between heavy vehicles and roads, Society of Automotive Engineers, SAE Technical Paper, No. 93001, 1993.

[14] T.D. Gillespie, Effects of heavy vehicle characteristics on pavement response and performance. National Corporative Highway Research Program, Report 353, Transportation Research Board, 1993.

[15] L. Sun, X. Deng, Predicting vertical dynamic loads caused by vehicle–pavement interaction, *American Society of Civil Engineers, Journal of Transportation Engineering* 126 (1998) 470–478.

[16] L. Sun, An explicit representation of steady state response of a beam resting on an elastic foundation to moving harmonic line loads, *International Journal for Numerical and Analytical Methods in Geomechanics* 27 (2003) 69–84.

[17] L. Sun, Dynamics of plate generated by moving harmonic loads, *American Society of Mechanical Engineers, Journal of Applied Mechanics* 72 (2005) 772–777.

[18] L. Sun, W.R. Hudson, Probabilistic approaches for pavement fatigue cracking prediction based on cumulative damage using Miner's law, *American Society of Civil Engineers, Journal of Engineering Mechanics* 131 (2005) 546–549.

[19] E.J. Huang, J.S. Arora, *Applied Optimal Design: Mechanical and Structural Systems*, Wiley, New York, 1979.

[20] J.K. Paeng, J.S. Arora, Dynamic response optimization of mechanical systems with multiplier methods, *American Society of Mechanical Engineers, Journal of Mechanisms Transmissions and Automation in Design* 111 (1989) 73–80.

[21] W.L. Winston, *Operations Research: Applications and Algorithms*, third ed., Duxbury Press, Belmont, CA, 1994.

[22] J.H. Holland, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, 1975.

[23] J.H. Holland, *Genetic Algorithms*, Scientific American, 1992.

[24] Z. Michalewic, *Genetic Algorithms + Data Structures = Evolution Programs*, third ed., Springer, New York, 1996.

[25] X. Cai, L. Lasdon, D.C. McKinney, Solving nonlinear water management models using a combined genetic algorithm and linear programming approach, *Advances in Water Resources* 24 (2001) 667–676.

[26] R.M. Chalasani, Ride performance potential of active suspension systems, part I: simplified analysis based on quarter-car models, *Proceedings of the American Society of Mechanical Engineers, Symposium on Simulation of Ground Vehicles and Transport Systems*, Anaheim CA, 1986, pp. 187-204.

[27] J. Palamas, O. Coussy, Y. Bamberger, Effects of surface irregularities upon the dynamic response of bridges under suspended moving loads, *Journal of Sound and Vibration* 99 (1985) 235–245.

[28] R.F. Harrison, J.K. Hammond, Evolutionary (frequency/time) spectral analysis the response of vehicles moving on rough ground by using 'covariance equivalent' modeling, *Journal of Sound and Vibration* 107 (1986) 29–38.
[29] D. Hrovat, Influence of unsprung weight on vehicle ride quality, *Journal of Sound and Vibration* 124 (1988) 497–516.
[30] T. Yoshimura, J. Hino, T. Kamata, N. Ananthanarayana, Random vibration of a non-linear beam subjected to a moving load: a finite element method analysis, *Journal of Sound and Vibration* 122 (1988) 317–329.
[31] D. Yadav, H.C. Upadhyay, Dynamics of vehicles in variable velocity runs over non-homogeneous flexible track and foundation with two point input models, *Journal of Sound and Vibration* 156 (1992) 247–268.
[32] J.C.O. Nielsen, A. Igeland, Vertical dynamic interaction between train and track—influence of wheel and track imperfections, *International Journal of Solid and Structures* 32 (1995) 825–839.
[33] J.K. Hammond, R.F. Harrison, Non-stationary response of vehicle on rough ground—a state space approach, *Journal of Dynamic System Measurement and Control* 103 (1981) 245–250.
[34] D. Cebon, Vehicle-generated road damage: a review, *Vehicle System Dynamics* 18 (1989) 107–150.
[35] K.B. Todd, B.T. Kulakowski, Simple computer models for predicting ride quality and pavement loading for heavy trucks, *Transportation Research Record* 1215 (1991) 137–150.
[36] M.S.A. Hardy, D. Cebon, Importance of speed and frequency in flexible pavement response, *American Society of Civil Engineers, Journal of Engineering Mechanics* 120 (1994) 463–482.
[37] D.E. Newland, *An Introduction to Random Vibration and Spectral Analysis*, second ed., Longman, New York, NY, 1984, pp. 104–129.
[38] W.Q. Zhu, *Random Vibration*, Scientific Press, Beijing, China, 1992.
[39] L. Sun, Developing spectrum-based models for international roughness index and present serviceability index, *American Society of Civil Engineers, Journal of Transportation Engineering* 127 (2001) 463–470.
[40] L. Sun, J. Su, Modeling random fields of road surface irregularities, *International Journal of Road Materials and Pavement Design* 2 (2001) 49–70.
[41] L. Sun, Z. Zhang, J. Ruth, Modeling indirect statistics of surface roughness, *American Society of Civil Engineers, Journal of Transportation Engineering* 127 (2001) 105–111.
[42] A.N. Heath, Application of the isotropic road roughness assumption, *Journal of Sound and Vibration* 115 (1987) 131–144.
[43] H. Honda, Y. Kajikawa, T. Kobori, Spectra of Surface Roughness on Bridge, *American Society of Civil Engineers, Journal of Structure Engineering* 108 (1982) 1956–1966.
[44] R.N. Iyengar, O.R. Jaiswal, Random field modeling of railway track irregularities, *American Society of Civil Engineers, Journal of Transportation Engineering* 121 (1995) 303–308.
[45] C.J. Dodds, J.D. Robson, The description of road surface roughness, *Journal of Sound and Vibration* 31 (1973) 175–183.
[46] T.D. Gillespie, *Heavy Truck Ride*: *SAE SP*-607, Society of Automotive Engineers, Warrendale, PA, 1985.
[47] International Organization for Standardization, Mechanical Vibration, Road Surface Profiles, Reporting of Measured Data, Draft Proposal ISO8606, 1995.
[48] D.E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading, MA, 1989.
[49] M. Mitchell, *An Introduction to Genetic Algorithms*, The MIT Press, Cambridge, MA, 1996.
[50] B.P. Buckles, F.E. Petry, *Genetic Algorithms*, Institute of Electrical and Electronics Engineers, Computer Society Press, California, 1992.