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Vibration of visco-elastic rectangular plate with linearly thickness variations in both directions

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Abstract

The analysis presented here is to study the effect of linear thickness variations in both directions on vibration of visco-elastic rectangular plate having clamped boundary conditions on all the four edges. Using the separation of variables method, the governing differential equation has been solved for vibration of visco-elastic rectangular plate. An approximate but quite convenient frequency equation is derived by using Rayleigh–Ritz technique with a two-term deflection function. Logarithmic decrement, time period and deflection at different points for the first two modes of vibration are calculated for various values of taper constants and aspect ratio.

1. Introduction

Much work have been done on the vibrations of rectangular plate whose thickness varies in two directions [1–4], but none of them done on visco-elastic plate. Sobotka [5] has considered free vibrations of uniform visco-elastic orthotropic rectangular plates. Bhatnagar and Gupta [6,7] have studied the effect of thermal gradient on vibration of visco-elastic circular and elliptic plate of variable thickness.

The main objective of the present investigation is to study the effect of taper constant in both directions on the vibrations of visco-elastic rectangular plate having clamped support on all the four edges. The assumptions of small deflection and linear, isotropic visco-elastic properties are made. It is assumed that the visco-elastic properties of the plate are of the 'Kelvin Type'. Numerical calculations have been made using the material constants of alloy 'Duralium'. Deflection, time period and logarithmic decrement at different instants of time for the first two modes of vibration are calculated for various values of aspect ratio and taper constant and results are presented in tabular form.

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Nomenclature

a	length of the rectangular plate
b	width of the plate
x, y	coordinates in the plane of plate
h	plate thickness at the point (x, y)
M_x, M_y	bending moment
$M_{,xy}$	twisting moment
E	Young's modulus
G	shear modulus
v	Poisson's ratio
$\tilde{D}$	visco-elastic operator
D_1	$Eh^3/12(1-v^2)$, flexural rigidity
ρ	mass density per unit volume of the plate material
t	time
η	visco-elastic constant
$w(x, y, t)$	deflection of the plate i.e. amplitude
$W(x, y)$	deflection function
$T(t)$	time function
β_1, β_2	taper constants in x- and y-direction, respectively
$\wedge$	logarithmic decrement
K	time period

2. Analysis

The equation of motion of a visco-elastic isotropic plate of variable thickness is [6]

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} = \rho h w_{,tt}, \tag{1}$$

where

$$\begin{aligned} M_{,x} &= -\tilde{D}D_1[w_{,xx} + vw_{,yy}], \\ M_{,y} &= -\tilde{D}D_1[w_{,yy} + vw_{,xx}], \\ M_{,xy} &= -\tilde{D}D_1(1-v)w_{,xy}. \end{aligned} \tag{2}$$

A comma followed by a suffix denotes partial differentiation with respect to that variable. On putting the values of $M_{,x}$ $M_{,y}$ and $M_{,xy}$ from Eq. (2) in Eq. (1), one gets

$$\begin{aligned} &\tilde{D}[D_1(w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy}) + 2D_{1,x}(w_{,xxx} + w_{,xyy}) + 2D_{1,y}(w_{,yyy} + w_{,yxx}) \\ &\quad + D_{1,xx}(w_{,xx} + vw_{,yy}) + D_{1,yy}(w_{,yy} + vw_{,xx}) + 2(1-v)D_{1,xy}w_{,xy}] + \rho h w_{,tt} = 0. \end{aligned} \tag{3}$$

The solution of Eq. (3) can be taken in the form of products of two functions as

$$w(x, y, t) = W(x, y)T(t). \tag{4}$$

Substituting Eq. (4) into Eq. (3), we obtain

$$\begin{aligned} &\tilde{D}[D_1(W_{,xxxx} + 2W_{,xxyy} + W_{,yyyy}) + 2D_{1,x}(W_{,xxx} + W_{,xyy}) + 2D_{1,y}(W_{,yyy} + W_{,yxx}) \\ &\quad + D_{1,xx}(W_{,xx} + vW_{,yy}) + D_{1,yy}(W_{,yy} + vW_{,xx}) + 2(1-v)D_{1,xy}W_{,xy}]/\rho h W = -\ddot{T}/\tilde{D}T. \end{aligned} \tag{5}$$

Here, dot denotes differentiation with respect to t.

The preceding equation is satisfied if both of its sides are equal to a constant. Denoting this constant by p^2, we get two equations:

$$\begin{aligned} &[D_1(W_{,xxxx} + 2W_{,xxyy} + W_{,yyyy}) + 2D_{1,x}(W_{,xxx} + W_{,xyy}) + 2D_{1,y}(W_{,yyy} + W_{,yxx}) \\ &\quad + D_{1,xx}(W_{,xx} + vW_{,yy}) + D_{1,yy}(W_{,yy} + n_{W,xx}) + 2(1-v)D_{1,xy}W_{,xy}] - \rho h p^2 W = 0 \end{aligned} \tag{6}$$

and

$$\ddot{T} + p^2\tilde{D}T = 0. \tag{7}$$

Eqs. (6) and (7) are the differential equation of motion for isotropic plate and time function for visco-elastic plate of free vibration having variable thickness, respectively.

3. Equation of motion

The expressions for kinetic energy T and strain energy V are [8]

$$T = (1/2)\rho p^2 \int_0^a \int_0^b h W^2 \, \mathrm{d}x \, \mathrm{d}y \tag{8}$$

and

$$V = (1/2) \int_0^a \int_0^b D_1 \{(W_{,xx})^2 + (W_{,yy})^2 + 2v W_{,xx} W_{,yy} + 2(1-v)(W_{,xy})2\} \, \mathrm{d}x \, \mathrm{d}y. \tag{9}$$

We assumed that the thickness variation of the plate in both directions as

$$h = h_0(1 + \beta_1 x/a)(1 + \beta_2 y/b), \tag{10}$$

where β_1 & β_2 are taper's constants in x- & y-direction, respectively, and $h_0 = h$ at $x = y = 0$.

The flexural rigidity of the plate can now be written as (assuming Poisson's ratio v is constant)

$$D_1 = E h_0^3 (1 + \beta_1 x/a)^3 (1 + \beta_2 y/b)^3 / 12(1 - v^2). \tag{11}$$

4. Solution and frequency equation

In order to find solution, we use Rayleigh–Ritz technique. In this method, one requires maximum strain energy be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V - T) = 0 \tag{12}$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

For a rectangular plate clamped (c) along all the four edges, the boundary conditions are

$$W = W_{,x} = 0 \quad \text{at} \quad x = 0, a \quad \text{and} \quad W = W_{,y} = 0 \quad \text{at} \quad y = 0, b \tag{13}$$

and the corresponding two-term deflection function is taken as [3]

$$W = [(x/a)(y/b)(1 - x/a)(1 - y/b)]^2 [A_1 + A_2 (x/a)(y/b)(1 - x/a)(1 - y/b)], \tag{14}$$

which is satisfied Eq. (13).

Now assuming the non-dimensional variables as

$$X = x/a, \quad Y = y/a, \quad \bar{W} = W/a, \quad \bar{h} = h/a \tag{15}$$

and using Eqs. (11) and (15) in Eqs. (8) and (9), one obtains

$$T = (1/2)\rho p^2 \bar{h}_0 a^5 \int_0^1 \int_0^{b/a} [(1 + \beta_1 X)(1 + \beta_2 Y a/b) \bar{W}^2] \, \mathrm{d}X \, \mathrm{d}Y \tag{16}$$

and

$$V = Q \int_0^1 \int_0^{b/a} (1 + \beta_1 X)^3 (1 + \beta_2 Y a/b)^3 \{(\bar{W}_{,XX})^2 + (\bar{W}_{,YY})^2 + 2v \bar{W}_{,XX} \bar{W}_{,YY} + 2(1-v)(W_{,XY})^2\} \, \mathrm{d}X \, \mathrm{d}Y, \tag{17a}$$

where

$$Q = E \bar{h}_0^3 a^3 / 24(1 - v^2). \tag{17b}$$

Here limit of X is 0 to 1 and Y is 0 to b/a.

On substituting the values of T & V from Eqs. (16) and (17) in Eq. (12), one obtains

$$(V_1 - \lambda^2 p^2 T_1) = 0, \tag{18}$$

where

$$V_1 = \int_0^1 \int_0^{b/a} [(1+\beta_1 X)^3(1+\beta_2 Ya/b)^3\{(\bar{W}_{,XX})^2 + (\bar{W}_{,YY})^2 + 2v\bar{W}_{,XX}\bar{W}_{,YY} + 2(1-v)(\bar{W}_{,XY})^2\}]\mathrm{d}X\,\mathrm{d}Y \tag{19}$$

and

$$T_1 = \int_0^1 \int_0^{b/a} [(1+\beta_1 X)(1+\beta_2 Ya/b)\bar{W}^2]\,\mathrm{d}X\,\mathrm{d}Y. \tag{20}$$

Here

$$\lambda^2 = 12\rho(1-v^2)a^2/E\bar{h}_0^2. \tag{21}$$

Eq. (18) involves the unknown A_1 & A_2 arising due to the substitution of W from Eq. (14). These two constants are to be determined from Eq. (18), as follows:

$$\partial(V_1 - \lambda^2 p^2 T_1)/\partial An = 0, \quad n = 1, 2. \tag{22}$$

On simplifying (22), one gets

$$b_{n1}A_1 + b_{n2}A_2 = 0, \quad n = 1, 2, \tag{23}$$

where b_{n1}, b_{n2} ($n = 1$, 2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of Eq. (23) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0, \tag{24}$$

where

$$\begin{aligned} b_{11} = {} & [(9\beta_1^3 + 32\beta_1^2 + 42\beta_1 + 28)(7\beta_2^3 + 36\beta_2^2 + 66\beta_2 + 44)/(485100) + (a/b)^4(7\beta_1^3 + 36\beta_1^2 + 66\beta_1 + 44) \\ & (9\beta_2^3 + 32\beta_2^2 + 42\beta_2 + 28)/(485100) + v(a/b)^2(\beta_1^3 + 6\beta_1^2 + 12\beta_1 + 8)(\beta_2^3 + 6\beta_2^2 + 12\beta_2 + 8)/(44100) \\ & + 4(1-v)(a/b)^2(\beta_1^3 + 4\beta_1^2 + 6\beta_1 + 4)(\beta_2^3 + 4\beta_2^2 + 6\beta_2 + 4)/(44100) - 2\lambda^2(2+\beta_1)(2+\beta_2)/(1260)^2], \end{aligned}$$

$$\begin{aligned} b_{12} = b_{21} = {} & [(\beta_1^3 + 4\beta_1^2 + 6\beta_1 + 4)(4\beta_2^3 + 21\beta_2^2 + 39\beta_2 + 26)/(840840) \\ & + (a/b)^4(4\beta_1^3 + 21\beta_1^2 + 39\beta_1 + 26)(\beta_2^3 + 4\beta_2^2 + 6\beta_2 + 4)/(840840) \\ & + v(a/b)^2(3\beta_1^3 + 17\beta_1^2 + 33\beta_1 + 22)(2\beta_2^3 + 15\beta_2^2 + 33\beta_2 + 22)/(10672200) \\ & + v(a/b)^2(2\beta_1^3 + 15\beta_1^2 + 33\beta_1 + 22)(3\beta_2^3 + 17\beta_2^2 + 33\beta_2 + 22)/(10672200) \\ & + (1-v)(a/b)2(5\beta_1^3 + 21\beta_1^2 + 33\beta_1 + 22)(5\beta_2^3 + 21\beta_2^2 + 33\beta_2 + 22)/(5336100) \\ & - 2\lambda^2(2+\beta_1)(2+\beta_2)/(5544)^2], \end{aligned}$$

$$\begin{aligned} b_{22} = {} & [(5\beta_1^3 + 21\beta_1^2 + 33\beta_1 + 22)(3\beta_2^3 + 16\beta_2^2 + 30\beta_2 + 20)/(46246200) \\ & + (a/b)^4(3\beta_1^3 + 16\beta_1^2 + 30\beta_1 + 20)(5\beta_2^3 + 21\beta_2^2 + 33\beta_2 + 22)/(46246200) \\ & + v(a/b)^2(3\beta_1^3 + 19\beta_1^2 + 39\beta_1 + 26)(3\beta_2^3 + 19\beta_2^2 + 39\beta_2 + 26)/(100200100) \\ & + (1-v)(a/b)^2(11\beta_1^3 + 48\beta_1^2 + 78\beta_1 + 52)(11\beta_2^3 + 48\beta_2^2 + 78\beta_2 + 52)/(400800400) \\ & - 2\lambda_2(2+\beta_1)(2+\beta_2)/(24024)^2]. \end{aligned}$$

From Eq. (24), one can obtains a quadratic equation in p^2 from which the two values of p^2 can found. After determining A_1 & A_2 from Eq. (23), one can obtain deflection function W.

Choosing $A_1 = 1$, one obtains $A_2 = -b_{11}/b_{12}$ and then W comes out as

$$W = [XY(a/b)(1-X)(1-Ya/b)]^2[1+(-b_{11}/b_{12})XY(a/b)(1-X)(1-Ya/b)]. \tag{25}$$

5. Time functions of vibrations of visco-elastic plates

Time functions of free vibrations of visco-elastic plates are defined by the general ordinary differential Eq. (7). Their form depends on visco-elastic operator $\tilde{D}$.

For Kelvin's model, one can have

$$\tilde{D} \equiv \{1+(\eta/G)(\mathrm{d}/\mathrm{d}t)\}. \tag{26}$$

Using Eq. (26) in Eq. (7), one obtains

$$\ddot{T} + p^2(\eta/G)\dot{T} + p^2 T = 0. \tag{27}$$

Eq. (27) is a differential equation of order two for time function T.

Solution of Eq. (27) comes out as

$$T(t) = \mathrm{e}^{a_1 t}[C_1 \operatorname{Cos} b_1 t + C_2 \operatorname{Sin} b_1 t], \tag{28}$$

where

$$a_1 = -p^2\eta/2G \tag{29}$$

and

$$b_1 = p\sqrt{1-(p\eta/2G)^2} \tag{30}$$

and C_1, C_2 are constants which can be determined easily from initial conditions of the plate.

Let us take initial conditions as

$$T = 1 \quad \text{and} \quad \dot{T} = 0 \quad \text{at} \quad t = 0. \tag{31}$$

Using Eq. (31) in Eq. (28), one obtains

$$C_1 = 1 \quad \text{and} \quad C_2 = p^2(\eta/G)/2b_1 = -a_1/b_1. \tag{32}$$

Using Eq. (32) in Eq. (28), one have

$$T(t) = \mathrm{e}^{a_1 t}[\operatorname{Cos} b_1 t + (-a_1/b_1)\operatorname{Sin} b_1 t]. \tag{33}$$

Thus, deflection w may be expressed, by using Eq. (33) and (25) in Eq. (4), to give

$$\begin{aligned} w = {} & [XY(a/b)(1-X)(1-Ya/b)]^2[1+(-b_{11}/b_{12})XY(a/b)(1-X)(1-Ya/b)] \\ & \times [\mathrm{e}^{a_1 t}\{\operatorname{Cos} b_1 t + (-a_1/b_1)\operatorname{Sin} b_1 t\}]. \end{aligned} \tag{34}$$

Time period of the vibration of the plate is given by

$$K = 2\pi/p, \tag{35}$$

where p is frequency given by Eq. (24).

Logarithmic decrement of the vibrations given by the standard formula

$$\wedge = \log_\mathrm{e}(w_2/w_1), \tag{36}$$

where w_1 is the deflection at any point on the plate at time period $K = K_1$ and w_2 is the deflection at same point at the time period succeeding K_1.

6. Numerical evaluations

Computations have been made for calculating the values of logarithmic decrement ($\wedge$), time period (K) and deflection (w) for a isotropic visco-elastic rectangular plate for different values of taper constants β_1 & β_2 and aspect ratio a/b at different points for first two modes of vibrations.

In calculations, the following material parameters are used:

$$E = 7.08 \times 10^{10}\,\mathrm{N/M^2},$$
$$G = 2.632 \times 10^{10}\,\mathrm{N/M^2},$$
$$\eta = 14.612 \times 10^{5}\,\mathrm{N\,s/M^2},$$
$$\rho = 2.80 \times 10^{3}\,\mathrm{kg/M^3},$$
$$v = 0.345.$$

These values have been reported [9] for 'Duralium'. The thickness of the plate at the center is taken as $h_0 = 0.01\,\mathrm{m}$.

7. Results and discussion

Tables 1 and 2 contains numerical results for logarithmic decrement $\wedge$ and time period K, respectively, for aspect ratio $a/b = 1.5$ for first two modes of vibration for different values of taper constants β_1 & β_2. It can be seen from tables that as taper constant increases, logarithmic decrement and time period decrease. Also, the effect of taper constant is more in y-direction in comparison to x-direction.

Table 3 depicts values of time period K for first two modes of vibration for different values of aspect ratio a/b for the following two cases: (i) $\beta_1 = \beta_2 = 0.0$ and (ii) $\beta_1 = \beta_2 = 0.6$.

It is interesting to note that as aspect ratio increases, time period decreases in the above two cases for both modes of vibration.

Tables 4–7, respectively, contains numerical values of deflection w (i.e. amplitude of vibration mode) for aspect ratio $a/b = 1.5$ for first two modes of vibration for different values of X and Y for the following:

Table 4: $\beta_1 = \beta_2 = 0.0$ and time is 0 K,
Table 5: $\beta_1 = \beta_2 = 0.0$ and time is 5 K,
Table 6: $\beta_1 = \beta_2 = 0.6$ and time is 0 K,
Table 7: $\beta_1 = \beta_2 = 0.6$ and time is 5 K.

One can get results for higher modes of vibration by adding more terms to expression (14).

Table 1
Logarithmic decrement ($\bigwedge$) if a clamped visco-elastic rectangular plate for different values of taper constants (β_1 & β_2) and a constant aspect ratio ($a/b = 1.5$)

	β_2											
	0.0		0.2		0.4		0.6		0.8		1.0	
β_1	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.0	−0.164188	−0.652611	−0.181522	−0.722762	−0.200325	−0.799689	−0.220184	−0.881917	−0.240808	−0.968431	−0.261993	−1.058551
0.2	−0.181000	−0.720298	−0.200110	−0.798014	−0.220836	−0.883348	−0.242727	−0.974714	−0.265460	−1.071037	−0.288813	−1.171622
0.4	−0.198447	−0.790672	−0.219396	−0.876352	−0.242109	−0.970567	−0.266093	−1.071635	−0.291001	−1.178444	−0.316588	−1.290307
0.6	−0.216371	−0.863253	−0.239209	−0.957267	−0.263957	−1.060820	−0.290085	−1.172157	−0.317217	−1.290154	−0.345090	−1.414168
0.8	−0.234664	−0.937716	−0.259428	−1.040427	−0.286250	−1.153784	−0.314561	−1.275984	−0.343957	−1.405930	−0.374158	−1.543073
1.0	−0.253244	−1.013853	−0.279965	−1.125632	−0.308892	−1.249282	−0.339418	−1.382989	−0.371112	−1.525734	−0.403675	−1.677132

Table 2
Time period ($K \times 10^{-5}$) of a clamped visco-elastic rectangular plate for different values of taper constants (β_1 & β_2) and a constant aspect ratio ($a/b = 1.5$)

	β_2											
	0.0		0.2		0.4		0.6		0.8		1.0	
β_1	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.0	667.9	169.0	604.2	152.8	547.5	138.4	498.2	125.8	455.6	114.8	418.8	105.4
0.2	605.9	153.4	548.1	138.7	496.7	125.6	452.0	114.1	413.4	104.2	380.0	95.6
0.4	552.7	139.9	500.0	126.5	453.2	114.6	412.4	104.1	377.1	95.1	346.7	87.3
0.6	507.0	128.4	458.6	116.1	415.7	105.2	378.3	95.6	346.1	87.3	318.2	80.1
0.8	467.5	118.5	422.9	107.1	383.4	97.0	349.0	88.2	319.2	80.5	293.6	73.9
1.0	433.3	109.9	392.0	99.3	355.4	90.0	323.5	81.8	296.0	74.7	272.2	68.5

Table 3
Time period ($K \times 10^{-5}$) of a clamped visco-elastic rectangular plate for different aspect ratio (a/b)

a/b	$\beta_1 = \beta_2 = 0.0$		$\beta_1 = \beta_2 = 0.6$	
	First mode	Second mode	First mode	Second mode
0.5	1650.1	412.6	934.9	233.2
1.0	1129.0	288.5	639.4	163.3
1.5	667.9	169.0	378.3	95.6
2.0	412.5	103.2	233.7	58.3
2.5	274.4	68.1	155.5	38.5

Table 4
Deflection ($w \times 10^{-5}$) of a clamped visco-elastic rectangular plate for different values of X and Y, a constant aspect ratio ($a/b = 1.5$) and $\beta_1 = \beta_2 = 0.0$ and time = 0 K

	Y							
	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
X	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.2	66.3	33.1	150.0	37.9	150.0	37.9	66.3	33.1
0.4	150.0	37.9	340.5	−37.9	340.5	−37.9	150.0	37.9
0.6	150.0	37.9	340.5	−37.9	340.5	−37.9	150.0	37.9
0.8	66.3	33.1	150.0	37.9	150.0	37.9	66.3	33.1

Table 5
Deflection ($w \times 10^{-5}$) of a clamped visco-elastic rectangular plate for different values of X and Y, a constant aspect ratio ($a/b = 1.5$) and $\beta_1 = \beta_2 = 0.0$ and time = 5 K

	Y							
	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
X	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.2	29.2	1.3	66.0	1.4	66.0	1.4	29.2	1.3
0.4	66.0	1.4	149.8	−1.4	149.8	−1.4	66.0	1.4
0.6	66.0	1.4	149.8	−1.4	149.8	−1.4	66.0	1.4
0.8	29.2	1.3	66.0	1.4	66.0	1.4	29.2	1.3

Table 6
Deflection ($w \times 10^{-5}$) of a clamped visco-elastic rectangular plate for different values of X and Y, a constant aspect ratio ($a/b = 1.5$) and $\beta_1 = \beta_2 = 0.6$ and time = 0 K

	Y							
	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
X	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.2	68.5	33.1	157.6	38.1	157.6	38.1	68.5	33.1
0.4	157.6	38.1	365.8	−37.2	365.8	−37.2	157.6	38.1
0.6	157.6	38.1	365.8	−37.2	365.8	−37.2	157.6	38.1
0.8	68.5	33.1	157.6	38.1	157.6	38.1	68.5	33.1

Table 7
Deflection ($w \times 10^{-5}$) of a clamped visco-elastic rectangular plate for different values of X and Y, a constant aspect ratio ($a/b = 1.5$) and $\beta_1 = \beta_2 = 0.6$ and time = 5 K

	Y							
	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
X	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.2	16.1	0.083	36.9	0.096	36.9	0.096	16.1	0.083
0.4	36.9	0.096	85.8	−0.093	85.8	−0.093	36.9	0.096
0.6	36.9	0.096	85.8	−0.093	85.8	−0.093	36.9	0.096
0.8	16.1	0.083	36.9	0.096	36.9	0.096	16.1	0.083

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