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Damage localization and severity estimate for three-dimensional frame structures

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Abstract

This paper newly develops a method for the damage localization and severity estimate for three-dimensional frame structures based on the employment of the cross modal strain energy (CMSE). While traditional modal strain energy methods must compare the modal information between the same mode for the baseline (analytical) and damaged (measured) structures, no such constraint is required for the CMSE method. Additionally, the CMSE method does not require the analytical and measured modes to be consistent in scale, or to be normalized. Numerical studies in this paper are conducted for three-dimensional five-story frame structures based on synthetic data generated from finite element models, and excellent results are obtained for both single-damage and multiple-damage scenarios.

1. Introduction

Any damage in the form of a loss of local stiffness in a structure would alter the dynamic properties of the structure, including the modal frequencies and mode shapes. In consequence, the change in the modal parameters, or quantities derived from them, can be used as indicators for damage diagnosis. Techniques based on these changes for diagnosing damage in a structure have attracted much attention in recent years, and many approaches have been developed.

The methods for damage diagnosis are commonly classified into four levels. While a higher level method always includes issues covered in a lower level method, specific focus of each level is generally accepted as follows: Level 1—determining whether damage occurs in the structure, Level 2—identifying the geometric location of the damage, Level 3—quantifying the severity of the damage, and Level 4—predicting the remaining service life of the structure. To date, vibration-based damage diagnosis methods that do not make use of some structural model primarily provide Level 1 and Level 2 damage diagnosis. When vibration-based methods are coupled with a structural model, Level 3 damage identification can be obtained in some cases. Level 4 prediction is generally associated with the fields of fracture mechanics, fatigue-life analysis, or structural design assessment [1].

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Nomenclature

A	cross-section area
$\mathbf{b}$	a column vector of size N_q
b_m	an element of $\mathbf{b}$
$\widehat{b}_m$	estimated b_m from $\widehat{\boldsymbol{\alpha}}$
$\mathbf{C}$	an N_q-by-N_d matrix, composed of $C_{n,m}$
C_{ij}	structural cross modal strain energy
$C_{n,ij}$	elemental cross modal strain energy associated with the stiffness matrix $\mathbf{K}_{\ell_n}$
$C_{n,m}$	same as $C_{n,ij}$ with the index m to replace ij
e_m	normalized residue for each equation
$\|\mathbf{e}\|$	norm of the vector $\mathbf{e}$
E	Young's modulus
I	moment of inertia
$\mathbf{K}$	system stiffness matrix
$\mathbf{K}_{\ell_n}$	the element stiffness matrix in global coordinate corresponding to element number ℓ_n
ℓ_n	the element number of the nth damaged element
$\mathbf{M}$	system mass matrix
N_d	the total number of the damaged members
N_i	the number of modes available for the baseline structure
N_j	the number of modes available for the damaged structure
N_q	total number of equations formed to estimate the damage extent
$\boldsymbol{\alpha}$	damage extent
$\widehat{\boldsymbol{\alpha}}$	estimate of $\boldsymbol{\alpha}$
α_n	the damage extent of the nth element of the structure
$\boldsymbol{\Phi}_i$	the ith mode shape of the undamaged system
λ_i	the ith eigenvalue of the undamage system
$*$	superscript used to indicate a damage version

Although the Level 3 damage diagnosis, including damage localization and severity estimate, has received much attention, there are only very few articles that address the damage diagnosis for three-dimensional frame structures. To detect the damage location and severity for a three-dimensional frame structure is a very challenging task. It has been demonstrated recently in a benchmark study organized by the Structural Health Monitoring Task Group, American Society of Civil Engineers [2]. An effective damage localization method specifically for three-dimensional frame structures is the *modal strain energy decomposition* (MSED) method developed by Li et al. [3]. The MSED method defines two damage indicators, axial damage indicator and transverse damage indicator, for each member. Analyzing the joint information of the two damage indicators greatly enhances the capability of localizing damage elements. However, the MSED method cannot achieve satisfactory estimate for the corresponding damage severity. To improve the accuracy of the damage severity estimation for three-dimensional frame structures, Hu et al. [4] developed the *cross modal strain energy* (CMSE) method which has been demonstrated to be able to accurately estimate the damage degree of multiple damaged members. However, the method was applicable only if the correct damage locations have been identified previously. As suggested in Ref. [4], one way to perform a Level 3 damage diagnosis is by applying one algorithm for the damage localization and implementing another algorithm for damage severity estimation after the damage location has been identified. A more attractive approach however is to perform the damage localization and damage severity estimate simultaneously. The primary objective of this article is to develop a method that can effectively identify the geometric locations of the damaged members, as well as accurately quantify their severity at same time for three-dimensional frame structures. The newly proposed method is an extension of the CMSE method that has been originally developed only for damage severity estimate [4]. The core of the CMSE method is to formulate simultaneous linear equations associated with modal strain energy-like terms that are product terms crossing over the baseline (analytical) model and the damaged (physical) structure, also crossing over various modes. Whereas traditional modal strain energy methods for diagnosing damage are either using an iterative solution procedure [5] or involving rough assumptions and significant approximations [6,7,3], the newly proposed CMSE method for damage localization and severity estimate is an exact, non-iterative solution method. Additionally, the derivation of the CMSE method dose not require the knowledge of the unchanged mass distributions of the baseline and damaged structures.

Numerical studies in this paper will be conducted for three-dimensional five-story frame structures based on synthetic data generated from finite element models. Two particular tasks, single-damage and multiple-damage scenarios, are to be performed. Task 1 considers three single-damage scenarios, with 5% stiffness loss at a column, a long-span beam, and a short-span beam, respectively. Task 2 investigates the capability of the CMSE method on diagnosing the damage for structures with multiple damaged members.

2. Cross modal strain energy (CMSE) method

The damage diagnosis method developed below formulates simultaneous equations involving quantities equivalent to *modal strain energy* (MSE) terms that cross baseline and damaged structures, thus the method is named as CMSE method [4], which is significantly different from any existing techniques that have applied the concept of ordinary modal strain energy [5,6,8,9].

Denoting $\mathbf{M}$ and $\mathbf{K}$ as the mass and stiffness matrices for the baseline structure model, in the eigenanalysis for the structure, one writes

$$\mathbf{K}\mathbf{\Phi}_i = \lambda_i \mathbf{M}\mathbf{\Phi}_i, \tag{1}$$

where λ_i and $\mathbf{\Phi}_i$, denote the ith eigenvalue and eigenvector, respectively. Likewise, one writes the corresponding expression for the damaged structure as

$$\mathbf{K}^*\mathbf{\Phi}_j^* = \lambda_j^* \mathbf{M}^*\mathbf{\Phi}_j^*, \tag{2}$$

where $\mathbf{M}^*$ and $\mathbf{K}^*$ are the mass and stiffness matrices for the damaged structure, and λ_j^* and $\mathbf{\Phi}_j^*$ denote the associated jth eigenvalue and eigenvector. Throughout the paper, superscript "$*$" is used to indicate a damage version. In Eqs. (1) and (2), one can treat $\mathbf{\Phi}_i$ and λ_i as the analytical modal information for the baseline structure, and $\mathbf{\Phi}_j^*$ and λ_j^* as the measured modal information from the damaged structure.

The development of the CMSE method is under the assumption that the mass distributions of the baseline and damaged structures are not known, but without change, that is, $\mathbf{M}^* = \mathbf{M}$. In this method, quantities $\mathbf{\Phi}_i$, λ_i, $\mathbf{K}$, $\mathbf{\Phi}_j^*$, and λ_j^* are presumably given, the unknowns are $\mathbf{K}^*$ and $\mathbf{M}$. From Eqs. (1) and (2), the first step is to eliminate the mass matrices $\mathbf{M}$ and $\mathbf{M}^*$. Pre-multiplying Eq. (1) by $(\mathbf{\Phi}_j^*)^\mathrm{T}$ and Eq. (2) by $(\mathbf{\Phi}_i)^\mathrm{T}$ yields

$$(\mathbf{\Phi}_j^*)^\mathrm{T}\mathbf{K}\mathbf{\Phi}_i = \lambda_i(\mathbf{\Phi}_j^*)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_i, \tag{3}$$

$$(\mathbf{\Phi}_i)^\mathrm{T}\mathbf{K}^*\mathbf{\Phi}_j^* = \lambda_j^*(\mathbf{\Phi}_i)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_j^*. \tag{4}$$

Since $\mathbf{M}$ is a symmetric matrix, one shows that $[(\mathbf{\Phi}_j^*)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_i]^\mathrm{T} = (\mathbf{\Phi}_i)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_j^*$. Also noting the transpose of a scalar equals to itself, i.e., $[(\mathbf{\Phi}_j^*)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_i]^\mathrm{T} = (\mathbf{\Phi}_j^*)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_i$, one thus has

$$(\mathbf{\Phi}_j^*)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_i = (\mathbf{\Phi}_i)^\mathrm{T}\mathbf{M}\mathbf{\Phi}_j^*. \tag{5}$$

Theoretically, $\mathbf{\Phi}_i$ and $\mathbf{\Phi}_j^*$ are not orthogonal to the mass matrix even when $i \neq j$, unless there is no damage occurred in the structure and therefore $\mathbf{\Phi}_j^* = \mathbf{\Phi}_j$. Similarly, since $\mathbf{K}$ is a symmetric matrix, one shows that

$$(\mathbf{\Phi}_j^*)^\mathrm{T}\mathbf{K}\mathbf{\Phi}_i = (\mathbf{\Phi}_i)^\mathrm{T}\mathbf{K}\mathbf{\Phi}_j^*. \tag{6}$$

Dividing Eq. (4) by Eq. (3), and using the scalar identities of Eqs. (5) and (6), one obtains

$$\frac{(\mathbf{\Phi}_i)^\mathrm{T}\mathbf{K}^*\mathbf{\Phi}_j^*}{(\mathbf{\Phi}_i)^\mathrm{T}\mathbf{K}\mathbf{\Phi}_j^*} = \frac{\lambda_j^*}{\lambda_i}. \tag{7}$$

The above equation is defined only when $(\mathbf{\Phi}_i)^\mathrm{T}\mathbf{K}\mathbf{\Phi}_j^*$ is not zero. Otherwise, Eq. (7) should be written as $\lambda_i(\mathbf{\Phi}_i)^\mathrm{T}\mathbf{K}^*\mathbf{\Phi}_j^* = \lambda_j^*(\mathbf{\Phi}_i)^\mathrm{T}\mathbf{K}\mathbf{\Phi}_j^*$. Let the stiffness matrix of the damaged structure be written as

$$\mathbf{K}^* = \mathbf{K} + \sum_{n=1}^{N_d} \alpha_n \mathbf{K}_{\ell_n}, \tag{8}$$

where the index n is a counter for the *damaged elements*; N_d is the total number of the damaged elements; and α_n and ℓ_n are the damage extent and the element number of the nth damaged element, respectively.

From Eqs. (7) and (8), one shows

$$1+\frac{\sum_{n=1}^{N_d}\alpha_n(\mathbf{\Phi}_i)^{\mathrm{T}}\mathbf{K}_{\ell_n}\mathbf{\Phi}_j^*}{(\mathbf{\Phi}_i)^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_j^*}=\frac{\lambda_j^*}{\lambda_i} \tag{9}$$

or

$$\sum_{n=1}^{N_d}\alpha_n(\mathbf{\Phi}_i)^{\mathrm{T}}\mathbf{K}_{\ell_n}\mathbf{\Phi}_j^*=\left(\frac{\lambda_j^*}{\lambda_i}-1\right)(\mathbf{\Phi}_i)^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_j^*. \tag{10}$$

Define the *structural cross modal strain energy* between the ith mode of the baseline structure and the jth mode of the damaged structure as

$$C_{ij}=(\mathbf{\Phi}_i)^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_j^* \tag{11}$$

and the corresponding *elemental cross modal strain energy* for the stiffness matrix $\mathbf{K}_{\ell_n}$ as

$$C_{n,ij}=(\mathbf{\Phi}_i)^{\mathrm{T}}\mathbf{K}_{\ell_n}\mathbf{\Phi}_j^*. \tag{12}$$

Now, Eq. (10) is written as

$$\sum_{n=1}^{N_d}\alpha_n C_{n,ij}=\left(\frac{\lambda_j^*}{\lambda_i}-1\right)C_{ij}. \tag{13}$$

Using a new index m to replace ij, Eq. (13) becomes

$$\sum_{n=1}^{N_d}\alpha_n C_{n,m}=b_m, \tag{14}$$

where

$$b_m=\left(\frac{\lambda_j^*}{\lambda_i}-1\right)C_{ij}. \tag{15}$$

When N_i and N_j modes are available for the baseline structure and damaged structure, respectively, totally $N_q=N_i\times N_j$ equations can be formed from Eq. (14). Written in a matrix form, one has

$$\mathbf{C}\boldsymbol{\alpha}=\mathbf{b}, \tag{16}$$

in which $\mathbf{C}$ is an N_q-by-N_d matrix, $\boldsymbol{\alpha}$ and $\mathbf{b}$ are column vectors of size N_d and N_q, respectively. When N_q is greater than N_d, a unweighted least-squares estimate for $\boldsymbol{\alpha}$, denoted $\widehat{\boldsymbol{\alpha}}$, is obtained as

$$\widehat{\boldsymbol{\alpha}}=(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\,\mathbf{C}^{\mathrm{T}}\mathbf{b}. \tag{17}$$

It is worthy to mention that those N_i and N_j modes of the baseline and damaged structures can be arbitrary modes in the sense that they are not required to start from the first mode. In practice, it is easy to obtain the analytical modes of the baseline structure, but difficult or expansive to extract the measured modes of the damaged structure, therefore one may choose a much larger N_i than N_j.

In the above derivation, using Eq. (8) implies that prior knowledge of the locations of the damage members must be given. When the damage locations are not known, it is suggested to perform a residual analysis for each suspicious scenario of the true damage locations. For each scenario, one follows the above procedure to estimate $\widehat{\boldsymbol{\alpha}}$ first. Afterwards, substituting $\widehat{\boldsymbol{\alpha}}$ into Eq. (16), one writes the corresponding $\mathbf{b}$ as

$$\widehat{\mathbf{b}}=\mathbf{C}\widehat{\boldsymbol{\alpha}}. \tag{18}$$

The normalized residue for each CMSE equation then is calculated as

$$e_m=\frac{\widehat{b_m}-b_m}{b_m},\quad m=1,\ldots,N_q. \tag{19}$$

Denoting $\mathbf{e}$ to be the vector for all e_m, one calculates the norm of $\mathbf{e}$ as

$$\|\mathbf{e}\| = \sqrt{\mathbf{e}^{\mathrm{T}}\mathbf{e}} = \sqrt{(e_1)^2 + \cdots + (e_{N_q})^2}. \tag{20}$$

The quantity $\|\mathbf{e}\|$ can be employed as an indicator to quantify the goodness of the "fitting" among all N_q equations. In principle, if the examined damage scenario is near to the true damage scenario, then the corresponding $\|\mathbf{e}\|$ would be small. Thus, a simple damage localization algorithm is based on finding the particular scenario that possesses the smallest residue norm among all suspicious scenarios. While the idea of using residue information has been commonly adopted by many other localization methods, a distinctive strength of the present CMSE method is that the accompanying severity estimate can be also used as an additional information to further judge the correctness of the damage localization because the damage severity estimates are expected to be very accurate if the true damage locations have been employed. Therefore, in the present CMSE damage diagnosis, one always expects a small $\|\mathbf{e}\|$ together with a reasonable $\widehat{\boldsymbol{\alpha}}$.

In comparison to other diagnosis method, a number of advantages of the CMSE method deserve to be mentioned: (1) The basic equations constructed to diagnose damage by the traditional modal strain energy methods must employ the modal information from the same mode of both the baseline and damaged structures. The basic equations formulated to diagnose damage by the CMSE method are based on cross modes, as shown in Eq. (10). There is no need for matching modes between the baseline and damaged structures. Since many equations can be formulated from a single measured mode (with many analytical modes), minimal modal information from measurements is needed. In addition, any particular measured mode could be applied. (2) The CMSE method uses "cross" terms between baseline and damaged structures, therefore, unlike many other methods, there is no need in this method for analytical and measured modes to be consistent in scale, or normalized in any particular way. There is no need to perform mass normalization. In fact, the mass matrix information is not required in the CMSE method. (3) The CMSE method is a direct (non-iterative), exact (without linearization or dropping higher-order terms) solution method. Almost all other existing MSE methods are either using an iterative solution procedure [5] or involving rough presumptions and significant approximations [6,7,3]. (4) It is clearly shown from Eq. (10) that the diagnosis for damage based on the CMSE method has ideally employed both the mode shapes and modal frequencies in each formulated equation.

3. Numerical studies

The structure adopted in the numerical studies is a three-dimensional five-story frame structure shown in Fig. 1. The structure is synthesized from a finite element model where each structural member is modelled as a three-dimensional uniform beam element, and is distinguished by assigning an element number. The essential geometrical and material properties of the frame structure are given below. The length of all horizontal members oriented in the x direction is 1 m, all horizontal members oriented in the y direction 3 m, and all vertical members 1 m. For all members, the Young's modulus $E = 2.1 \times 10^{11}$ Pa, the mass density per unit length $\overline{m} = 22.035$ kg/m, the cross-section area $A = 2.825 \times 10^{-3}\,\mathrm{m}^2$ and the moment of inertia $I = 2.89 \times 10^{-6}\,\mathrm{m}^4$. Performing the eigenanalysis, one obtains the first two mode shapes as exhibited in Fig. 2, where the first mode vibrates dominantly in the long-span y-direction with frequency 6.9105 Hz, and the second mode in the short-span x-direction with frequency 9.3615 Hz. It is realized that the first mode should vibrate in the 'weak' direction of the structure. In the current numerical example, the weak direction is in the long-span direction because when all cross sections and material properties remain identical for all beams, the stiffnesses of those beams are inversely proportional to their lengths.

In the following investigation, the synthesized damaged structures include single-damage and multi-damage scenarios. As far as the damage diagnosis is concerned, both damage localization and damage severity estimate will be performed simultaneously.

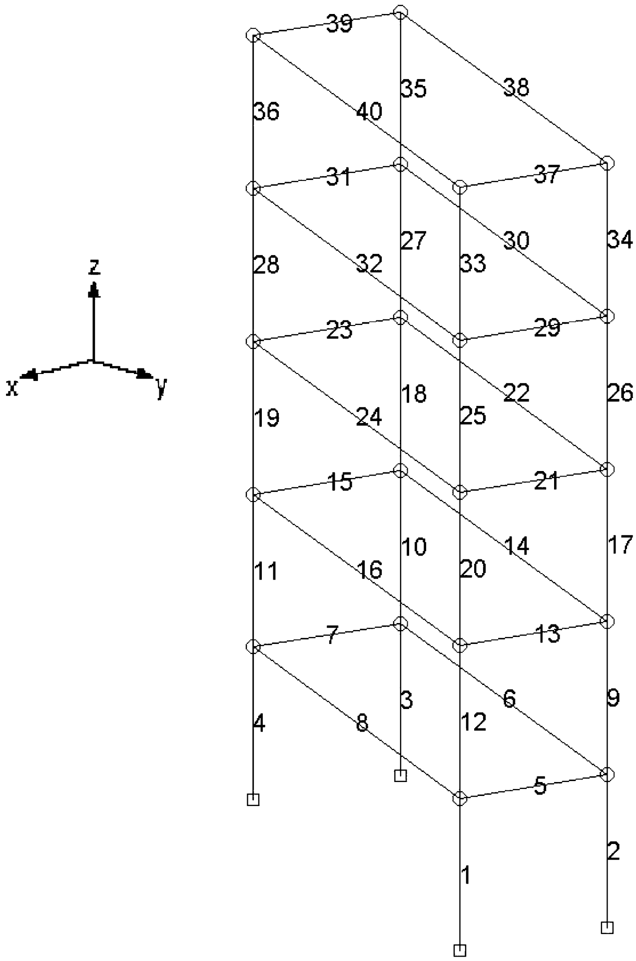

Fig. 1. The sketch of the three-dimensional five-story frame structure.

3.1. Single-damage scenarios

Three single-damage scenarios are considered, with the simulated damage element at: (A) member 18—a vertical column, (B) member 22—a long-span horizontal beam member that oriented longitudinally in the y-direction (for simplicity, named y-beam), and (C) member 23—a short-span horizontal beam member that oriented longitudinally in the x-direction (named x-beam), respectively. All simulated damages are with 5% stiffness loss. Table 1 summarizes the three single-damage scenarios. Listed also are the first two frequencies of the baseline and the three damaged structures. It is noticed that the first frequency of the damaged structure C (with a damaged short-span x-beam at 23), and the second frequency of the structure B (with a damaged long-span y-beam at 22), do not change noticeable from their counterparts of the baseline structure. This also suggests that a damaged (short-span) x-beam does not have discernible effect on the first mode shape, which predominantly vibrates in the (long-span) y-direction. Likewise, a damaged (long-span) y-beam would have negligible effect on the second mode shape that vibrates in the x-direction.

Theoretically, if no damage occurs, the CMSE under the situation $i=j$ is finite, but vanishes when $i\neq j$, due to the orthogonality property associated with the stiffness matrix. If a beam is damaged, the corresponding CMSE is no longer equal to zero when $i\neq j$, however, it tends to be a very small number, even more so when the two modes are vibrating in physically *orthogonal* directions. While analytically any two arbitrary modes can be used in the CMSE method, there are limitations in the numerical implementation.

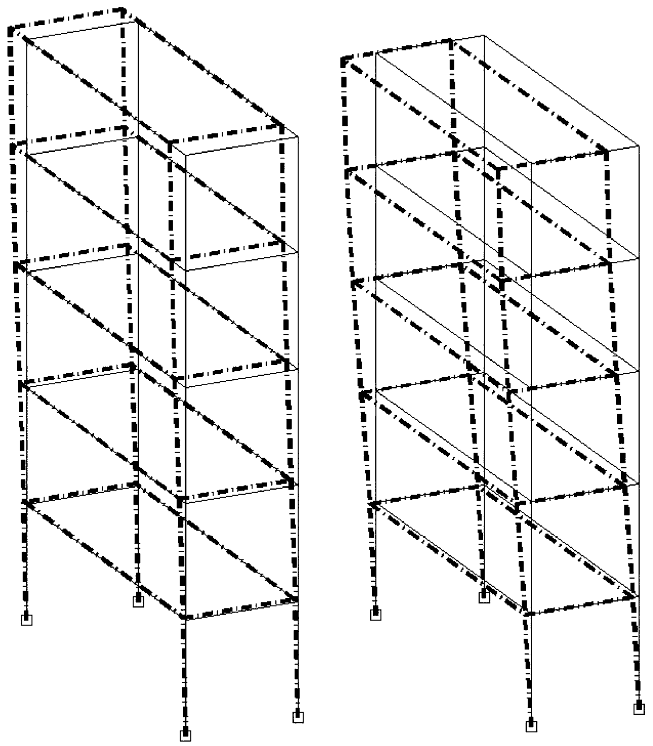

Fig. 2. The first mode is vibrating predominantly in the y-direction, and the second mode in the x-direction.

Table 1
A summary of the baseline and damaged structure

Structure	Damaged member	Stiffness loss (%)	1st frequency	2nd frequency
Baseline	None	0	6.9105	9.3615
A	Column 18	5	6.9084	9.3561
B	Long-span y-beam 22	5	6.8958	9.3615
C	Short-span x-beam 23	5	6.9105	9.3480

In particular, if only translational modes are utilized, then it is essential to choose the two modes vibrating in the same direction.

3.1.1. Damaged structure A—with a damaged column at element 18

In practice, only limited lower-order modes are relatively easy to be measured. Thus the applications of the CMSE method should employ minimal numbers of lower-order measured modes. Depicted in Fig. 3 is the third, fourth and fifth modes of the baseline structure, in which the third mode is a rotation model, and the fourth and the fifth modes are in the y- and x-direction, respectively. If only the first mode of damaged structure is measured, i.e., $j = \{1\}$, among the first five analytical (baseline) modes, one should not utilize the second and fifth modes of the baseline structure because of the mode orientation concerned. Taking the first, third and fourth mode for the baseline structure, i.e., $i = \{1, 3, 4\}$, together with $j = \{1\}$, one obtains the results shown in Fig. 4. The top panel of Fig. 4 is the damage severity estimate, α, for each element. One notices that it correctly estimates a 5% damage at element 18. However, if one presumes that the damage occurs at element 17, then a slightly more than 5% false damage is also estimated. To discern the true damage from a false

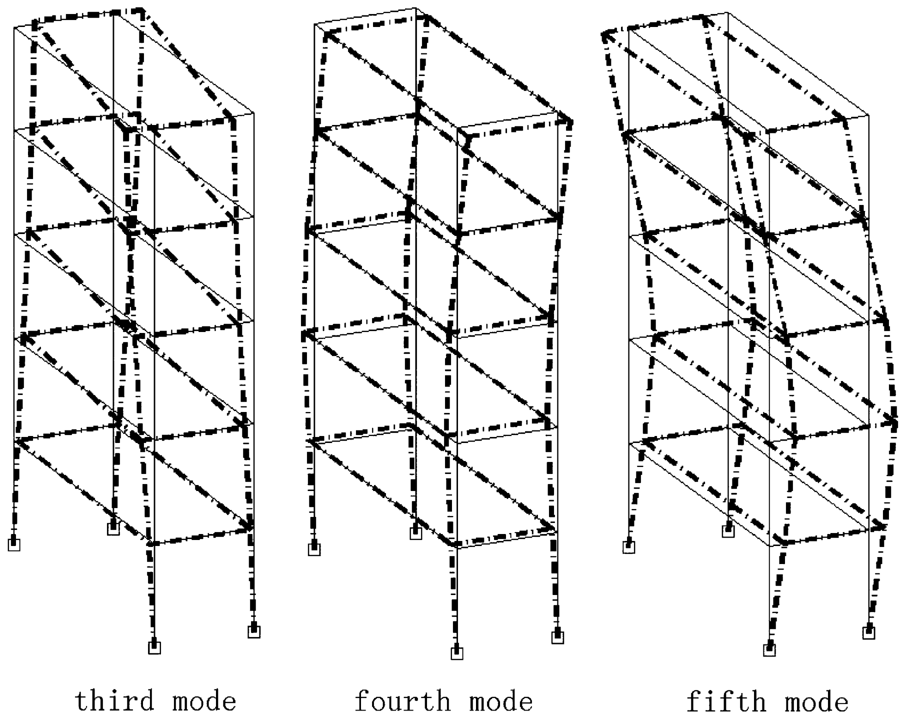

Fig. 3. The third mode is a rotation model, the fourth and the fifth modes are in the y- and x-direction, respectively.

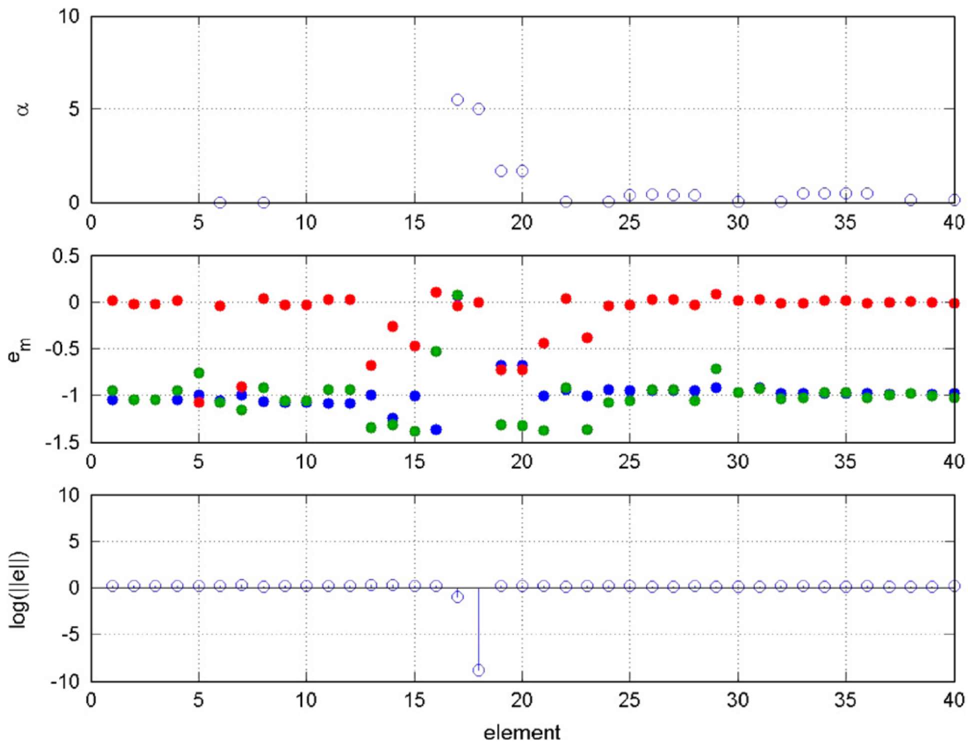

Fig. 4. Results of the CMSE method with a damaged column at element 18, using $i = 1, 3, 4$ and $j = 1$.

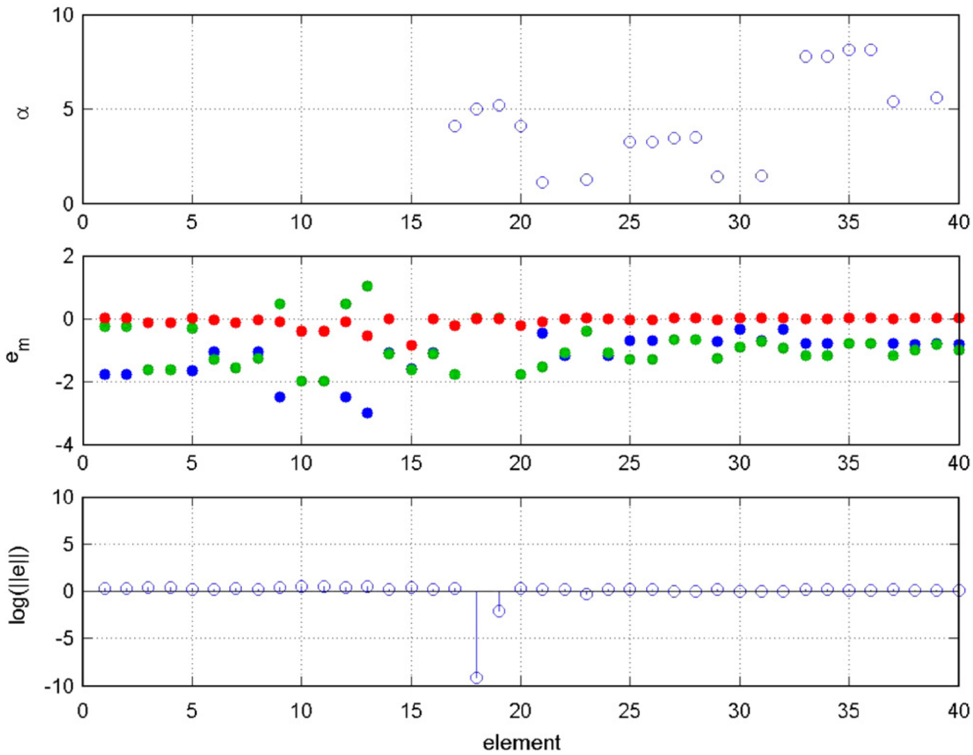

Fig. 5. Results of the CMSE method with a damaged column at element 18, using $i = 2, 3, 5$ and $j = 2$.

damage, one has to rely on the information of e_m and $\|\mathbf{e}\|$ for damage localization. The middle panel of Fig. 4 is the information of e_m which represents the residue of each equation formed by the CMSE method. Because the present application employs $i = \{1, 3, 4\}$ and $j = \{1\}$, three equations are formed. For each element number, there are three points shown in the figure, in which each point represents the residual from an equation. All three points of e_m at element 18 coincide at a value equal to zero that indicates a perfect match among all CMSE equations with the accurate 5% damage estimated. Displayed at the bottom panel of Fig. 4 is a single quantity of $\|\mathbf{e}\|$ for each element. The smallest quantity of $\|\mathbf{e}\|$ points to the damage location. From it, one can clearly see the method correctly obtain the damage location at element 18.

When the lowest measured mode in the x-direction, i.e. $j = \{2\}$ is taken, one should use the analytical modes of $i = \{2, 3, 5\}$. The results are shown in Fig. 5. Basically, the results demonstrate the same features shown in Fig. 4.

Due to space limitation, not shown are damage scenarios with a damaged column at other floors. The same features as shown in Figs. 4 and 5 were observed.

3.1.2. *Damaged structure B—with a damaged long-span beam at element 22*

Taking only the first measured mode, $j = \{1\}$, which vibrates in the long-span direction, one can use the analytical modes $i = \{1, 3, 4\}$ in the CMSE method. Excellent results for both damage severity and localization are achieved as shown in Fig. 6. However, while taking $i = \{2, 3, 5\}$ and $j = \{2\}$, one cannot properly localize damage, as shown in Fig. 7. Results presented above indicate that when the damaged member is oriented in the long-span direction, one should not employ modes that vibrate in the short-span direction. This is attributed to the fact that the damage of a long-span beam causes negligible change for the modes vibrating in the short-span direction as suggested in Table 1.

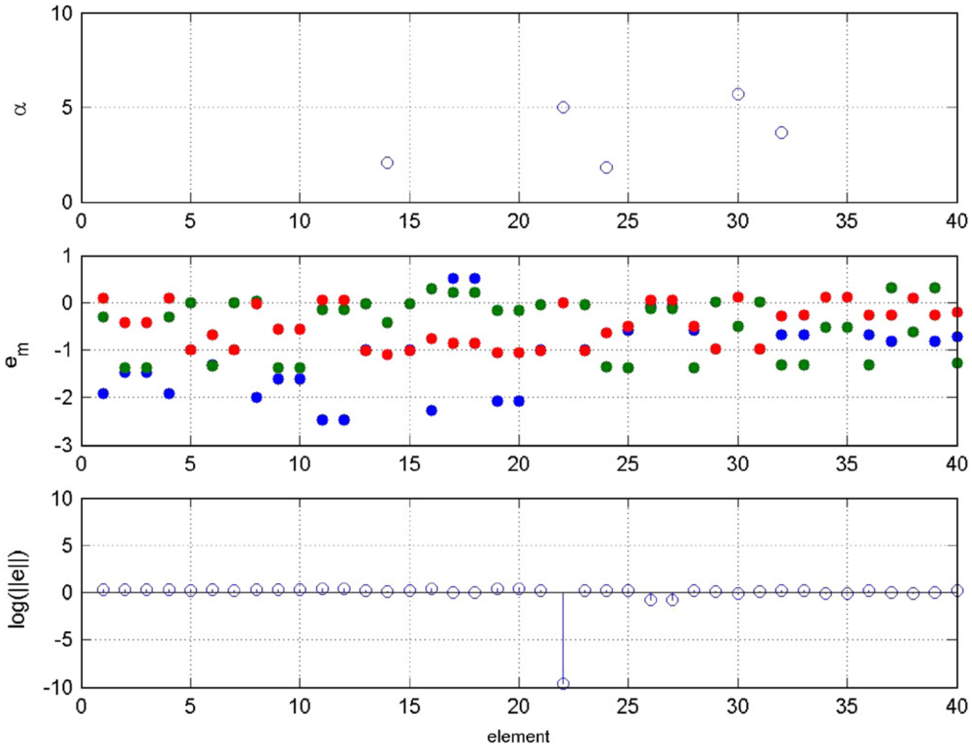

Fig. 6. Results of the CMSE method with a damaged long-span beam at element 22, using $i = 1, 3, 4$ and $j = 1$.

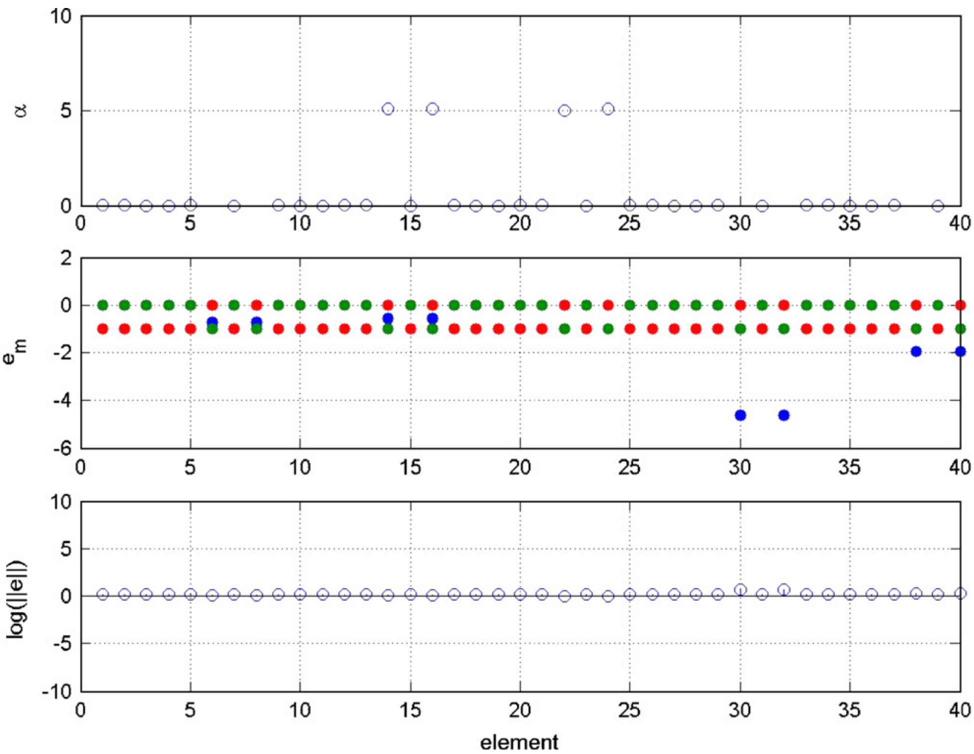

Fig. 7. Results of the CMSE method with a damaged long-span beam at element 22, using $i = 2, 3, 5$ and $j = 2$.

3.1.3. Damaged structure C—with a damaged short-span beam at element 23

Results from using modes $i = \{2, 3, 5\}$ and $j = \{2\}$ are given in Fig. 8. Because the damaged member is oriented in the short-span direction, employing modes that vibrate in the short-span direction will yield satisfactory results, as expected.

3.1.4. Comparisons with other methods

For comparing the newly developed CMSE method with other MSE methods, the damage diagnosis for all three damaged structures above will be also conducted by using the traditional Stubbs damage index method [6] and the MSED method recently developed by Li et al. [3]. While both the CMSE and MSED methods can localize the damage members correctly for those three damaged structures, the traditional Stubbs damage index algorithm fails to localize the damage for structures with a damaged beam, long-span or short-span, as indicated in Table 2(a). The damage severity estimated by using either Stubbs damage index algorithm or MSED method varies with the number of modes being included in the calculation. For consistence, estimations of damage severity for all cases are based on using the first two modes of the damaged structure. Applying the MESD method yields two possible severity estimates, one from transverse modal strain energy, and the other from axial modal strain energy. Presented in Table 2(b) are the resulting damage severity estimates for damaged structures A, B and C, by using CMSE, MSED and Stubbs damage index methods. Because the total modal strain energy at a structural member is dominated by its transverse portion, the damage severity estimated by the Stubbs damage index method and that by the transverse modal strain energy of the MSED method are virtually identical. While applying MSED method to damaged structures B and C, one must use the axial modal strain energy on performing damage detection. Clearly, those estimates by MSED and Stubbs damage index methods all underestimate the true damage level significantly. Given the fact that rough approximations were made in the assumption and derivation for the MSED and Stubbs damage index methods, it is not particularly surprising to obtain poor severity estimates from those methods.

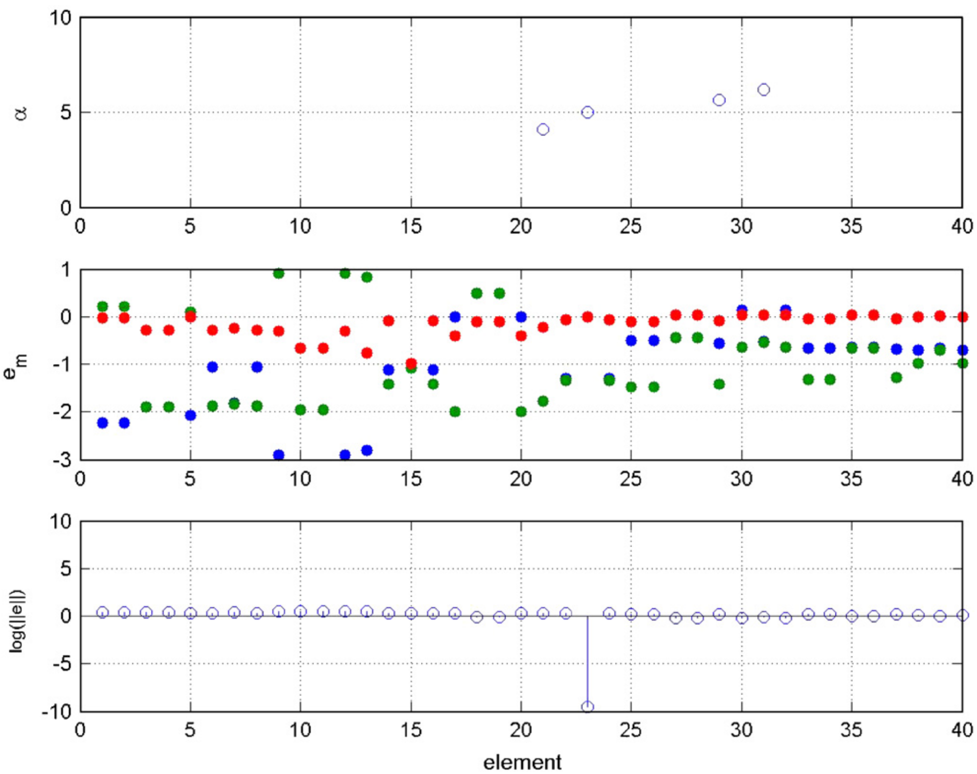

Fig. 8. Results of the CMSE method with a damaged short-span beam at element 23, using $i = 2, 3, 5$ and $j = 2$.

3.2. Multiple-damage scenarios

Assume that two damaged members presented in the previous single-damage scenarios take place simultaneously in the structure. The following study investigates the capability of the CMSE method on diagnosing the damage locations for multiple damaged members. Three multiple-damage scenarios, with the combination of damaged members at 18 and 22, 18 and 23, and 22 and 23, respectively, are investigated. Again, all damaged members are with 5% loss of stiffness. Table 3 summarizes the three two-damage structures and their first two frequencies.

First, perform the damage localization for the damaged structure D, with damaged column 18 and y-beam 22. Using the baseline modes $i = \{1, 3, 4\}$ together with the damaged (measured) mode $j = \{1\}$, one obtains the results of $\|\mathbf{e}\|$ shown in Fig. 9. In this figure, each $\|\mathbf{e}\|$ value corresponds to a trial of two presumed damage-elements. For the present 40-element structure, there are 780 possible two-damage combinations. The underlining damage localization algorithm is the search for the smallest $\|\mathbf{e}\|$ among all 780 suspicious scenarios. From Fig. 9, one can clearly observe that the CMSE method correctly point out the damage locations at elements 18 and 22. When modes associated with $i = \{2, 3, 5\}$ and $j = \{2\}$ are employed, a similar result as Fig. 9 has been observed (not shown here). Whereas the correct damage locations are identified, the corresponding damage severity are also estimated accurately. Repeating the same work for the damaged structures E (with damaged column 18 and x-beam 23) and F (with damaged y-beam 22 and x-beam 23) also yield excellent results.

One advantage of using the graphic way as shown in Fig. 9 is that it provides a visual result. If a damaged structure has 3 or more damage elements, due to a higher-order dimensional plot being required, one cannot use a similar way to show the damage locations. Alternatively, one can search for the numerical minimum of $\|\mathbf{e}\|$ among all suspicious scenarios, and marked the damaged elements directly at the three-dimensional sketch of the structure, as demonstrated in Fig. 10.

For a 40-element structure with multiple-damage locations, if all possible 3-damage combinations are considered, there are 9880 scenarios that must be examined. The above direct search algorithm can become

Table 2
The capability of damage localization, and the estimated damage severity by Stubbs damage index, MSED (modal strain energy decomposition) and CMSE (cross modal strain energy) methods

(a) *The capability of damage localization*

Structure	Stubbs	MSED	CMSE
A	Yes	Yes	Yes
B	No	Yes	Yes
C	No	Yes	Yes

(b) *The estimated damage severity by Stubbs damage index*

Structure	Stubbs	MSED (transverse)	MSED (axial)	CMSE (%)
A	0.0073%	0.0073%	0	5
B	0	0	0.43%	5
C	0	0	0.48%	5

Table 3
A summary of the baseline and damaged structures

Structure	Damaged member	Stiffness loss (%)	1st frequency	2nd frequency
Baseline	None	0	6.9105	9.3615
D	Column 18 and y-beam 22	5	6.8938	9.3561
E	Column 18 and x-beam 23	5	6.9084	9.3426
F	y-beam 22 and x-beam 23	5	6.8958	9.3480

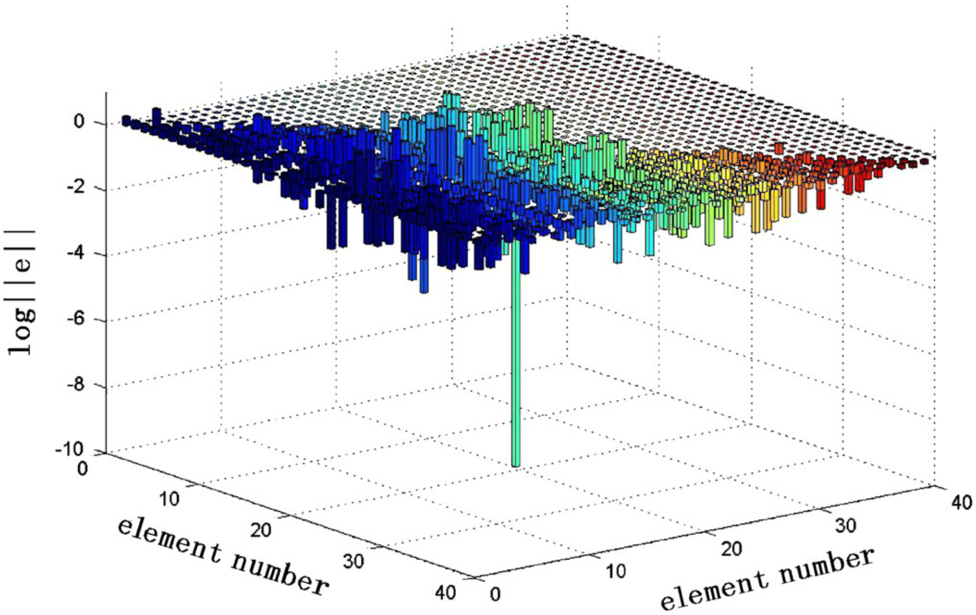

Fig. 9. Results of $\|\mathbf{e}\|$ with damaged elements 18 and 22, using $i = 1, 3, 4$ and $j = 1$.

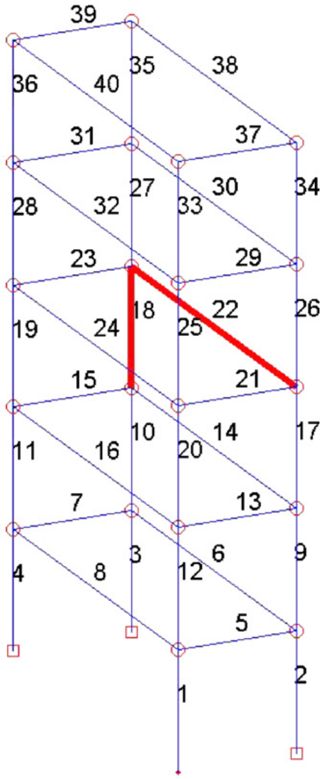

Fig. 10. Display of the damaged elements directly at the three-dimensional sketch of the structure.

time consuming for a structure with too many suspicious scenarios. In such a situation, a better way to search for multiple-damage locations perhaps should be employed. For instance, one can first use single-damage scenarios for a multi-damage structure to narrow the potential damaged elements. Sequentially, by eliminating many undamaged elements, one can significantly reduce the number of suspicious multiple-damage scenarios.

4. Concluding remarks

Assuming that the damage locations are known a priori, Hu et al. [4] developed a CMSE method for the damage severity estimate. The present paper extends the usage of CMSE to the localization of damage members for three-dimensional frame structures. One of the major advantages associated with the proposed CMSE method is that minimal measured modal information is required—often a single measured mode is sufficient. While traditional modal strain energy methods must pair the modal information from the same mode for the baseline (analytical) and damaged (measured) structures, no such constraint is required for the CMSE method. Additionally, the CMSE method does not require the analytical and measured modes to be consistent in scale, or to be normalized in any specific way.

Numerical studies are conducted for damage localization on several simulated damage scenarios associated with a three-dimensional five-story frame structure. Those damaged scenarios include single-damage and multiple-damage structures. Results indicate that the developed CMSE localization method is effective and robust to locate single and multiple damages in a structure. While analytically any two arbitrary modes can be used in the CMSE method, there are limitations in the numerical implementation. In particular, when translational modes are utilized, it is suggested that the two modes—one analytical mode and one measured mode—are not vibrating in physically orthogonal directions.

The numerical studies in the present article localize and assess the damaged elements under the assumption that the measured modes and frequencies are free of errors. In reality, it is unavoidable that measurement errors must exist. Thus, future research effort should emphasize on a thorough investigation about the robustness of the CMSE method under noisy measurements.

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