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PC-based pseudo-model following discrete integral variable structure control of positions in slider-crank mechanisms

Chin-Wen Chuang*

Department of Electrical Engineering, I-Shou University, 1, Section 1, Hsueh-Cheng Road, Ta-Hsu Hsiang, 840 Kaohsiung, Taiwan, ROC

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Abstract

This paper proposes a pseudo-model following (PMF) discrete integral variable structure control (DIVSC) scheme to control the position of a slider crank coupled with a pseudo-model (PM) synchronous motor. Applying the Grey modeling method, the reference model and plant are on-line converted to pseudo-reference model and accumulated model, respectively. The advantage is that the least data and memory are used in this modeling procedure. The DIVSC is introduced to guide the model following motion. The integral controller is used to reduce the steady-state error. The switching controller is used to finish the sliding motion. The two existing condition of the discrete sliding mode are introduced in this paper, too. Finally, the position control of the slider crank is used to show the performance of the proposed control scheme. The simulation and experimental results show the robustness and the practicability.

1. Introduction

The slider-crank mechanism is a basic structure in mechanical application. It can convert the rotational motion into translation motion. For examples, fretsaws, petrol and diesel engines are the typical application of its velocity control. Jasinski et al. [1], Zhu and Chen [2] and Badlani and Kleinhenz [3] have successfully solved the steady-state solutions of a slider crank few years ago. According to Ref. [4], the response of slider crank is dependent on length, mass, damping, external piston force and frequency. Based on the viewpoints of the ratios, length and speeds of the crank to the connecting rod, the transient responses have been investigated [5]. Taking the actuator into consideration, the dynamic model of the slider-crank mechanisms actuated by the field-oriented control pseudo-model (PM) synchronous [6,7,9] is published. With Hamilton principle and Lagrange multiplier method, model equations of the slider-crank mechanism coupling with PM synchronous are formulated [8–10]. Obviously, the mathematical model of slider-crank mechanism is nonlinear and complex. This characteristic also causes the difficulty of designing its controller. In order to simplify the controller design, the approximated linear model is proposed by many linearized procedures, such as regional partial differential (Bracket Lee's method) [11,12], etc. With the great advance in microprocessor, the calculation process is speeded up significantly. Hence, the related data acquisition and theory calculation can

*Fax: +886 7 6577205.
E-mail address: cwchuang@isu.edu.tw.

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be finished in a small period. The dynamic response can be approximated segment by segment to linear model. In this paper, a Grey modeling method is used to establish the approximated segment-by-segment model instantaneously. The most important advantage of Grey theory [13–15] is that it only needs at least four time-series data to establish the Grey model. Hence, the modeling calculation can be quickly finished.

After finishing the on-line modeling, the discrete integral variable structure control (DIVSC) [16,17] is applied to deal with tracking guidance. A previously defined model is used to be a reference model. By Grey theory operation, a pseudo-reference model is established. At the same time, by on-line Grey modeling process, an accumulated plant is established to approximate the linear plant. The controller's objective of DIVSC is to make the plant to track the reference model. The dynamic response of tracking work is guided by a previous defined sliding surface. The integral variable controller is added to eliminate the steady-state error. The switching controller is used to guide the tracking motion. At the same time, the control signal must satisfy the existing conditions [16,17] of the discrete sliding mode, that are

$$\Delta S(k) = S(k+1) - S(k) < 0, \tag{1a}$$

$$|\Delta S(k)| < \delta, \tag{1b}$$

where the δ is a small positive constant. The first condition is used to make sure that the sliding motion is moved toward the sliding surface. The second condition is used to guarantee the convergence of the sliding motion.

Finally, in order to examine the performance of the proposed controller, the experimental equipment is developed on a PC-based platform. The position of slider crank is controlled by the DIVSC based on Grey model. The simulation and experimental results will show the robust performance under external load existed.

2. Slider crank actuated by a PM synchronous

2.1. PM synchronous

Usually, the PM synchronous motor is coupled with a gear speed reducer with a gear ratio of $n = \theta/\theta_m$. If the driver of PM motor is well-designed, the dynamic equation of the PM motor can be simplified to be

$$n \cdot \tau = n \cdot K_t \cdot i_q = J_m\ddot{\theta} + B_m\dot{\theta}, \tag{2}$$

where i_q is the applied current command, K_t is the torque constant, J_m and B_m are the inertia and viscous constants, respectively. Hence, the model of a PM synchronous motor can be shown in the block diagram of Fig. 1.

2.2. Displacement of slider-crank mechanism

The slider-crank mechanism system is shown in Fig. 2.

The slider-crank mechanism consists of three parts: crank, rod and slider. The θ is the angular from the reference zero degree position. In the closed triangular, the constraint equation [6,7,9] is shown to be

$$R\sin\alpha - L\sin\beta = 0, \tag{3}$$

where $\alpha = 180^\circ - \theta$. Hence, the angular between the rod and the vertical line can be found to be

$$\beta = \sin^{-1}\left(\frac{R}{L}\sin\alpha\right). \tag{4}$$

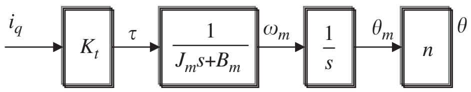

Fig. 1. Block diagram of a PM synchronous motor.

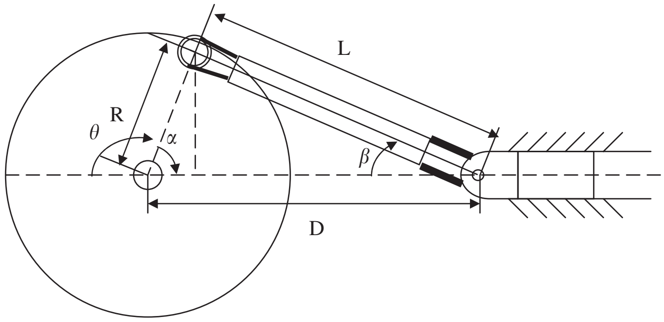

Fig. 2. Slider crank mechanism system.

The displacement of slider crank D will be described like

$$D = R\cos\alpha + L\cos\beta = R\cos\alpha + L\cos\left(\sin^{-1}\left(\frac{R}{L}\sin\alpha\right)\right). \tag{5}$$

Obviously, it is a nonlinear equation. The coupling effect will cause the difficulty to implement the position controller. The proposed control scheme can easily deal with the complex plant just like the linear systems real-time.

3. PMF-DIVSC

The structure of the PMF-DIVSC method is shown in Fig. 3. There are two steps while designing the PMF-DIVSC. The first step is the Grey modeling. The second one is the model following DIVSC rule.

The desired performance is chosen based on the reference model. In the step of Grey modeling, two second-ordered Grey models are established according to Grey theory. The Grey model of reference model is defined as a "Pseudo-Reference Model." The Grey model of the controlled plant is called "Accumulated Model".

The objective of controller is to make the accumulated model to follow the pseudo-reference model. The model following DIVSC rule, will play the role to achieve this goal.

3.1. The Grey modeling theory

The white systems are defined as known systems. The black systems are totally unknown. The Grey systems are the ones between these two systems. Only the parts of parameters, the order and the relationship of I/O are known in Grey systems. To solve the Grey systems, the Grey theory is proposed by Dr. Deng in 1982 [11]. It uses input and output data to set up the discrete Grey model to describe the I/O mathematical relationship. To apply the modeling theory, only four time-series I/O data are needed for calculation. Hence, this method can speed up the modeling procedure and reduce the usage of memory.

To avoid the singular point, firstly the data are biased by a proper constant, that is

$$X^{(0)}(i) = x(k+i-4) + \mathrm{DC}, \quad i = 1,2,3,4, \tag{6}$$

$$U^{(0)}(i) = u(k+i-4) + \mathrm{DC}, \quad i = 1,2,3,4, \tag{7}$$

where $x(k)$ and $u(k)$ are the output and input data, respectively. The k means this sampling time, and the $(k-1)$ is the previous sampling time. The accumulated generated operation (AGO) used to approximate the integral relationship is defined as

$$X^{(1)}(i) = \sum_{j=1}^{i} X^{(0)}(j), \quad i = 1,2,3,4, \tag{8}$$

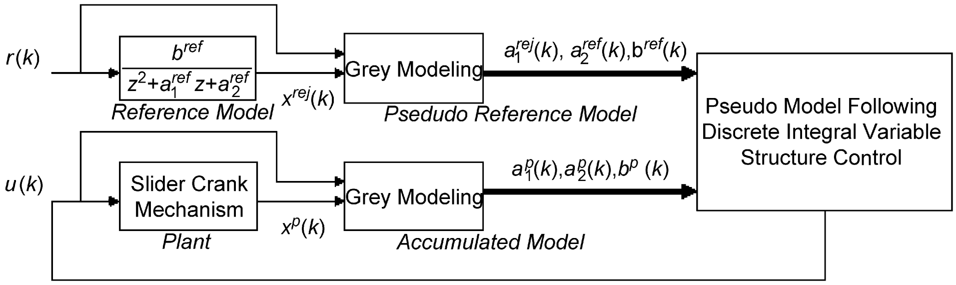

Fig. 3. Block diagram of PMF-DIVSC method.

$$U^{(1)}(i)=\sum_{j=1}^{i}U^{(0)}(j), \quad i=1,2,3,4. \tag{9}$$

By these data after AGO, the two variables and the second-ordered Grey model, called GM(2,2), is defined as

$$\frac{\mathrm{d}^2X^{(1)}(i)}{\mathrm{d}t^2}+a_2(i)\frac{\mathrm{d}X^{(1)}(i)}{\mathrm{d}t}+a_1(i)X^{(1)}(i)=b(i)U^{(1)}(i). \tag{10}$$

Because the Grey model is based on discrete sampling data, the differential Eq. (10) is transferred to difference equation, that is

$$\Delta^2X^{(1)}(i)-a_2(i)\Delta X^{(1)}(i)-a_1(i)Z^{(1)}(i)=b(i)U^{(1)}(i), \tag{11}$$

where $\Delta^2X^{(1)}(i)=X^{(0)}(i)-X^{(0)}(i-1)$ and $\Delta^1X^{(1)}(i)=X^{(0)}(i)$. The mean values of accumulated variable series are defined as

$$Z^{(1)}(i)=\tfrac{1}{2}[X^{(1)}(i)+X^{(1)}(i-1)], \quad i=2,3,4. \tag{12}$$

Finally, applying the data series to Eq. (11), the Grey modeling parameters can be obtained as

$$\begin{bmatrix} a_2(i) \\ a_1(i) \\ b(i) \end{bmatrix}=[(\mathbf{A}|\mathbf{B})^{\mathrm{T}}(\mathbf{A}|\mathbf{B})]^{-1}(\mathbf{A}|\mathbf{B})^{\mathrm{T}}\mathbf{Y}_n, \tag{13}$$

where

$$\mathbf{A}=\begin{bmatrix} -\Delta^1X^{(1)}(2) \\ -\Delta^1X^{(1)}(3) \\ -\Delta^1X^{(1)}(4) \end{bmatrix}, \quad \mathbf{B}=\begin{bmatrix} -Z^{(1)}(2) & U^{(1)}(2) \\ -Z^{(1)}(3) & U^{(1)}(3) \\ -Z^{(1)}(4) & U^{(1)}(4) \end{bmatrix} \quad \text{and} \quad \mathbf{Y}_n=\begin{bmatrix} \Delta^2X^{(1)}(2) \\ \Delta^2X^{(1)}(3) \\ \Delta^2X^{(1)}(4) \end{bmatrix}.$$

3.2. The PMF-DIVSC theory

After the first step of Grey modeling, the two real-time Grey models are established. Let the both models are described in discrete form as

Pseudo-reference model:

$$\Delta^2X_{\mathrm{ref}}^{(1)}(k)-a_2^{\mathrm{ref}}(k)\Delta^1X_{\mathrm{ref}}^{(1)}(k)-a_1^{\mathrm{ref}}(k)Z^{(1)}(i)=b^{\mathrm{ref}}(k)R^{(1)}(k). \tag{14a}$$

Accumulated plant model:

$$\Delta^2X_p^{(1)}(k)-a_2^p(k)\Delta^1X_p^{(1)}(k)-a_1^p(k)Z^{(1)}(i)=b^p(k)U^{(1)}(k), \tag{14b}$$

where $R^{(1)}(k)$ is the input command after the AGO.

The tracking error of both pseudo-reference model and accumulated model is defined as

$$e(t) = X_p^{(1)}(t) - X_{\text{ref}}^{(1)}(t).$$

The error difference is obtained as

$$\Delta e(t) = \Delta X_p^{(1)}(t) - \Delta X_{\text{ref}}^{(1)}(t) = X_p^{(0)}(t) - X_{\text{ref}}^{(0)}(t).$$

The second-ordered error difference is obtained as

$$\Delta^2 e(k) = \Delta^2 X_p^{(1)}(k) - \Delta^2 X_{\text{ref}}^{(1)}(k) = -a_1^p(k)e(k) - a_2^p(k)e(k) + b^p(k)U^{(1)(k)} + (a_1^{\text{ref}}(k) - a_1^p(k))X_{\text{ref}}^{(1)}(k) + (a_2^{\text{ref}}(k) - a_2^p(k))X_{\text{ref}}^{(0)}(k) - b^{\text{ref}}(k)R^{(1)}(k). \quad (15)$$

An integral controller is introduced in order to decrease the steady-state error [14,15], that is

$$\Delta Y(k) = e(k). \quad (16)$$

Let the sliding surface according to the desired performance is defined as

$$S(k) = c_1(e(k) - K_I Y(k)) + \Delta e(k). \quad (17)$$

According to the Eqs. (14)–(16), the time difference of the sliding surface can be obtained as

$$\begin{aligned}\Delta S(k) &= S(k+1) - S(k) = -c_1 K_I \Delta Y(k) + c_1 \Delta e(k) + \Delta^2 e(k)\\ &= (-c_1 K_I - a_1^p(k))e(k) + (c_1 - a_2^p(k))\Delta e(k) + b^p U^{(1)}(k)\\ &\quad + [(a_1^{\text{ref}}(k) - a_1^p(k))X_{\text{ref}}^{(1)}(k) + (a_2^{\text{ref}}(k) - a_2^p(k))X_{\text{ref}}^{(0)}(k) - b^{\text{ref}} R^{(1)}(k)].\end{aligned} \quad (18)$$

The control signal is decided by satisfying the existing conditions of the discrete sliding mode, that are

$$\Delta S(k) = S(k+1) - S(k) < 0, \quad (19a)$$

$$|\Delta S(k)| < \delta, \quad (19b)$$

where the δ is a small positive constant. Let the switching controller is defined as

$$U^{(1)}(k) = K_1(k)e(k) + K_2(k)\Delta e(k) + K_3(k), \quad (20)$$

where

$$K_1(k) = \begin{cases} K_1^+ & \text{if} \quad S(k)e(k) \geqslant 0, \\ K_1^- & \text{if} \quad S(k)e(k) < 0, \end{cases}$$

$$K_2(k) = \begin{cases} K_2^+ & \text{if} \quad S(k)\Delta e(k) \geqslant 0, \\ K_2^- & \text{if} \quad S(k)\Delta e(k) < 0, \end{cases}$$

$$K_3(k) = \begin{cases} K_3^+ & \text{if} \quad S(k) \geqslant 0, \\ K_3^- & \text{if} \quad S(k) < 0. \end{cases}$$

Substituting Eq. (20) into Eq. (18), the ranges of the switching gains $K_i(k)$, $i = 1, 2, 3$ can be obtained by satisfying the first existence condition of discrete sliding mode [14,15], that is Eq. (19a).

$$\begin{aligned}S(k)\Delta S(k) &= (-c_1 K_I - a_1^p(k) + b^p(k)K_1(k))e(k) + (c_1 - a_2^p(k) + b^p(k)K_2(k))\Delta e(k)\\ &\quad + [(a_1^{\text{ref}}(k) - a_1^p(k))X_{\text{ref}}^{(1)}(k) + (a_2^{\text{ref}}(k) - a_2^p(k))X_{\text{ref}}^{(0)}(k)\\ &\quad - b^{\text{ref}} R^{(1)}(k) + b^p(k)K_3(k)] < 0.\end{aligned} \quad (21)$$

Hence,

$$K_1(k) = \begin{cases} K_1^+ < \min\left\{\dfrac{1}{b^p(k)}(c_1 K_I + a_1^p)\right\} & \text{if} \quad S(k)e(k) \geqslant 0, \\ K_1^- > \max\left\{\dfrac{1}{b^p(k)}(c_1 K_I + a_1^p)\right\} & \text{if} \quad S(k)e(k) < 0, \end{cases} \tag{22a}$$

$$K_2(k) = \begin{cases} K_2^+ < \min\left\{\dfrac{1}{b^p(k)}(-c_1 + a_2^p)\right\} & \text{if} \quad S(k)\Delta e(k) \geqslant 0, \\ K_2^- > \max\left\{\dfrac{1}{b^p(k)}(-c_1 + a_2^p)\right\} & \text{if} \quad S(k)\Delta e(k) < 0, \end{cases} \tag{22b}$$

$$K_3(k) = \begin{cases} K_3^+ < \min\left\{\dfrac{1}{b^p(k)}(-(a_1^{\text{ref}}(k) - a_1^p(k))X_{\text{ref}}^{(1)}(k) - (a_2^{\text{ref}}(k) - a_2^p(k))X_{\text{ref}}^{(0)}(k) + b^{\text{ref}} R^{(1)}(k))\right\} \\ \qquad \text{if } S(k) \geqslant 0, \\ K_3^+ > \max\left\{\dfrac{1}{b^p(k)}(-(a_1^{\text{ref}}(k) - a_1^p(k))X_{\text{ref}}^{(1)}(k) - (a_2^{\text{ref}}(k) - a_2^p(k))X_{\text{ref}}^{(0)}(k) + b^{\text{ref}} R^{(1)}(k))\right\} \\ \qquad \text{if } S(k) < 0. \end{cases} \tag{22c}$$

To satisfy the second existence condition of the sliding mode, that is Eq. (19b), the domains of the switching gains are found.

$$\begin{aligned} |\Delta S(k)| &\leqslant |(-c_1 K_I - a_1^p(k) + b^p(k) K_1(k)) e(k)| + |(c_1 - a_2^p(k) + b^p(k) K_2(k)) \Delta e(k)| \\ &\quad + |[(a_1^{\text{ref}}(k) - a_1^p(k)) X_{\text{ref}}^{(1)}(k) + (a_2^{\text{ref}}(k) - a_2^p(k)) X_{\text{ref}}^{(0)}(k) - b^{\text{ref}} R^{(1)}(k) + b^p(k) K_3(k)]| \\ &< \delta_1 + \delta_2 + \delta_3, \end{aligned}$$

where $\delta = \delta_1 = \delta_2 + \delta_3$ and δ_1, δ_2 and δ_3 are small positive constants.

Let

$$|(-c_1 K_I - a_1^p(k) + b^p(k) K_1(k)) e(k)| < \delta_1,$$

then

$$\max\left\{\frac{1}{b^p(k)}(c_1 K_I + a_1^p(k)) - \frac{\delta_1}{|e(k)|}\right\} < K_1(k) < \min\left\{\frac{1}{b^p(k)}(c_1 K_I + a_1^p(k)) + \frac{\delta_1}{|e(k)|}\right\}. \tag{23a}$$

The relation between the range and the domain of the switching gain $K_1(k)$ is shown in Fig. 4. The intersection area of the range and the domain is the allowed selected value of the switching gain $K_1(k)$.

By the same method, the domains of the other switching gains are found as

$$\max\left\{\frac{1}{b^p(k)}(-c_1 + a_2^p(k)) - \frac{\delta_2}{|\Delta e(k)|}\right\} < K_2(k) < \min\left\{\frac{1}{b^p(k)}(-c_1 + a_2^p(k)) + \frac{\delta_2}{|\Delta e(k)|}\right\}, \tag{23b}$$

$$\begin{aligned} &\max\left\{\frac{1}{b^p(k)}(-(a_1^{\text{ref}}(k) - a_1^p(k))X_{\text{ref}}^{(1)}(k) - (a_2^{\text{ref}}(k) - a_2^p(k))X_{\text{ref}}^{(0)}(k) + b^{\text{ref}} R^{(1)}(k)) - \delta_3\right\} < K_3(k) \\ &< \min\left\{\frac{1}{b^p(k)}(-(a_1^{\text{ref}}(k) - a_1^p(k))X_{\text{ref}}^{(1)}(k) - (a_2^{\text{ref}}(k) - a_2^p(k))X_{\text{ref}}^{(0)}(k) + b^{\text{ref}} R^{(1)}(k)) + \delta_3\right\}. \end{aligned} \tag{23c}$$

Once the allowed selected values of the switching gains are decided, the two switching values can be chosen to be the same values in absolute value. The switching is decided by the sign of the sliding surface. The control signal can be rewritten to be

$$U^{(1)}(k) = (K_1(k)|e(k)| + K_2(k)|\Delta e(k)| + K_3(k))\operatorname{sgn}(S(k)). \tag{24}$$

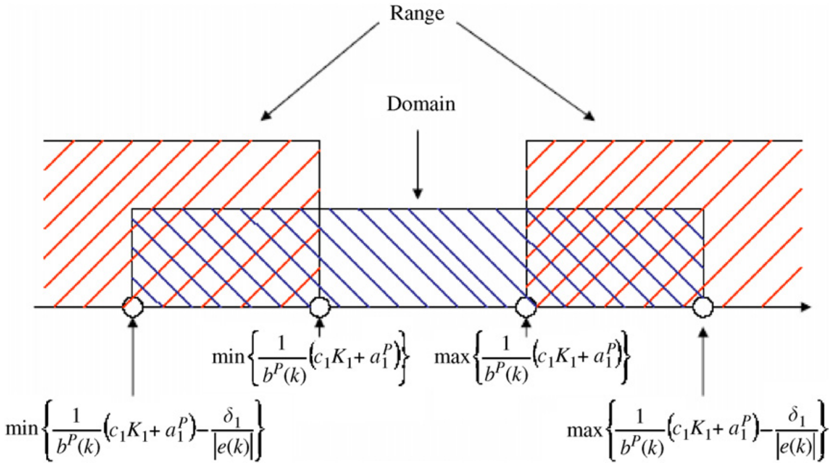

Fig. 4. The range and the domain of the switching gain.

However, the sign function will cause the chattering phenomena. Therefore, the sign function is replaced by the modified switching function, that is $M_\sigma(k) = S(k)/|S(k)| + \sigma$, where σ is small positive constant.

4. Simulation results

In this section, the numerical simulation results are used to show the potential of the proposed control rule. To demonstrate the performance of the proposed controller, this method is compared with a proportional-integral-derivative (PID) control.

In this paper, the actual slider-crank mechanism dimensions are $K_t = 0.6732\,\mathrm{kg\,m\,A^{-1}}$, $J_m = 0.00062\,\mathrm{kg\,m^2}$ and $B_m = 0.000153\,\mathrm{N\,m\,s^{-1}}$. The gear ration n is 1/30. The length of rod is 50 cm. The radius of the disk on motor is 10 cm. Let the initial angle be $\theta(0) = 1/4\pi$rad. The objective is to control the desired periodically translation position from 0.343 and 0.443 m. The desired specifications are settling time $t_s = 0.5$ s, maximum overshoot $M_P < 5\%$ and steady-state error $e_{ss} < 1\%$. The sampling time is 1 ms in this paper for both simulation and experiment.

Fig. 5 shows the response of PID controller, whose parameters $K_P = 3.5695$, $K_I = 2.9267$ and $K_D = 0.7727$ are chosen by Zigler–Nicholas rule. The blue line shows the response under no parameters variation and non-external load existed. The other lines show the responses of changed with parameter variation and load existed. Obviously, the overshoot and steady-state error have significant changes. It shows that the classical controller is not suitable for the nonlinear plant.

Fig. 6 shows the response of the proposed controller. The blue line is the response under nominal condition. The others are the responses under different conditions of the parameter varying and the load-existed. Due to no obviously varying in these response, the proposed controller can be concluded to be robust to the parameter varying and the load-existed.

5. Experimental results

In order to demonstrate the proposed control rule, the PC-based experimental equipment is setup in this paper. The experimental instrument of slider crank is divided into three parts: actuator, slider crank and controller. The photographic is shown in Fig. 7. The first part consists of a PM Synchronous motor, driver. The driver is worked on 3-phase, 220 V and 60 Hz. The slider crank is coupled with the PM motor. The translation position is measured by a photometer whose resolution is 1 p/mm. To test the robustness, the different external loads are added by external mass (7 and 3 kg) on slider. The Pentium PC is used to handle the hardware I/O process and controller software calculation. The interface card of NI PCI-7344 is installed in the PC to feedback the photometer signal. The LabJack U12 with the USB interface is used to output the

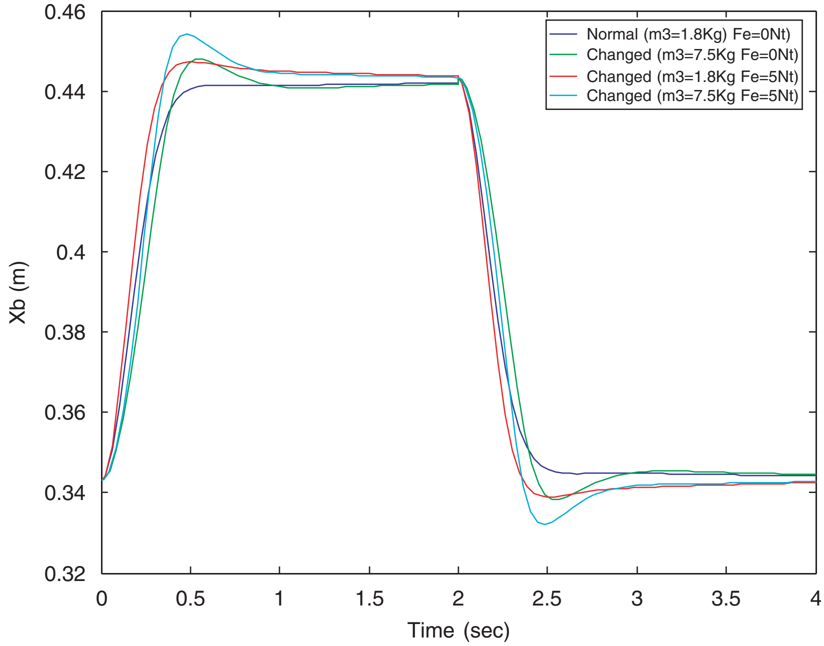

Fig. 5. Response of fixed PID control.

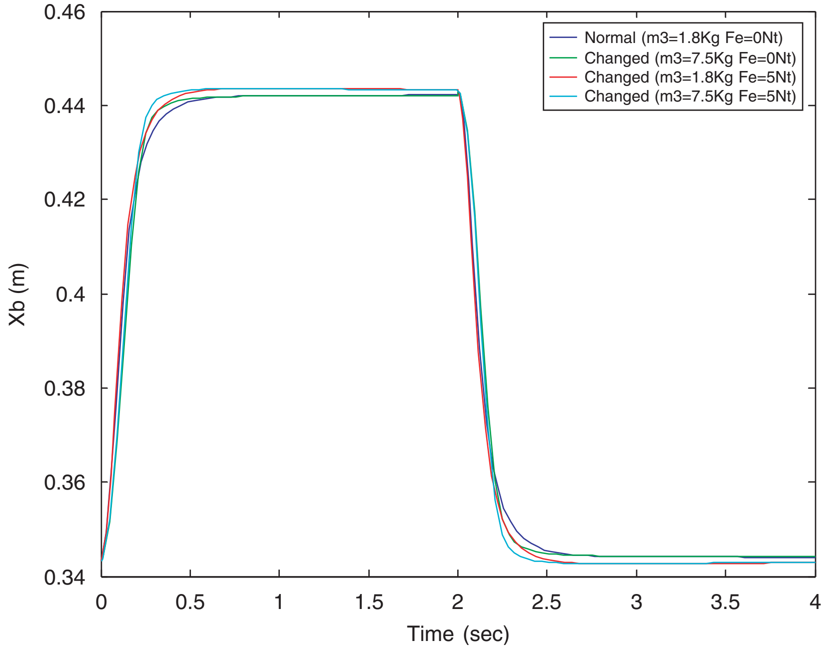

Fig. 6. Response of fuzzy PID control.

analog control signal. The proposed control scheme is implemented by the LabVIEW software. At the same time, the feedback signal is connected to a digital oscilloscope.

The experimental position response of the proposed controller without the external load is shown in Fig. 8. The 2 voltages are mapped to 10 cm of the translation position. The rising time and steady-state error are similar to the simulation results. By the same controller, the response with 7 kg load added is shown in Fig. 9. Obviously, the rising time are kept within a small varying range. The steady-state error is the same as the

Fig. 7. Experimental instrument of slider crank.

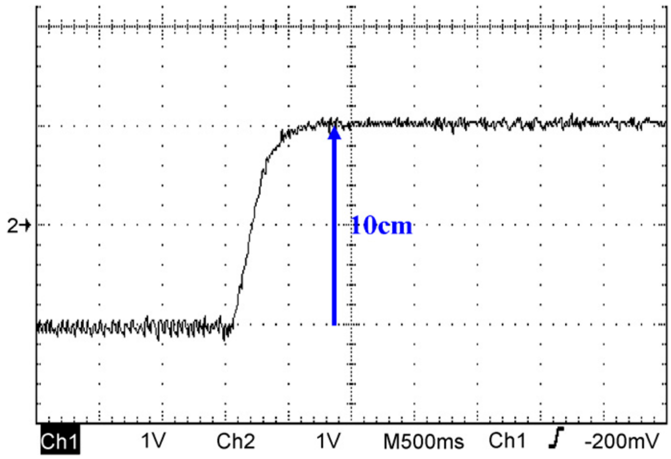

Fig. 8. Experimental position response without external load.

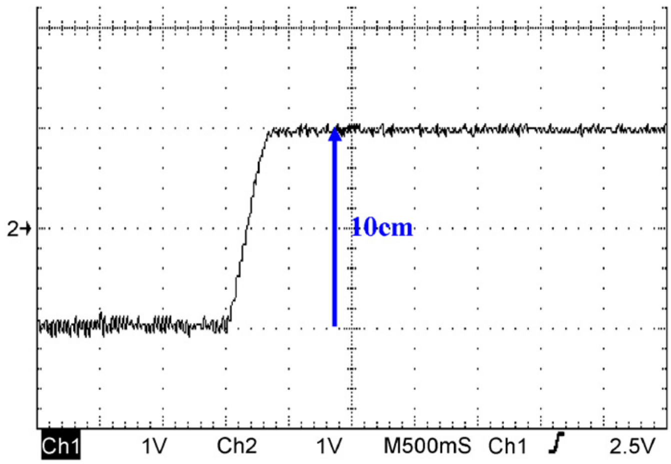

Fig. 9. Experimental position response without external load of 7 kg.

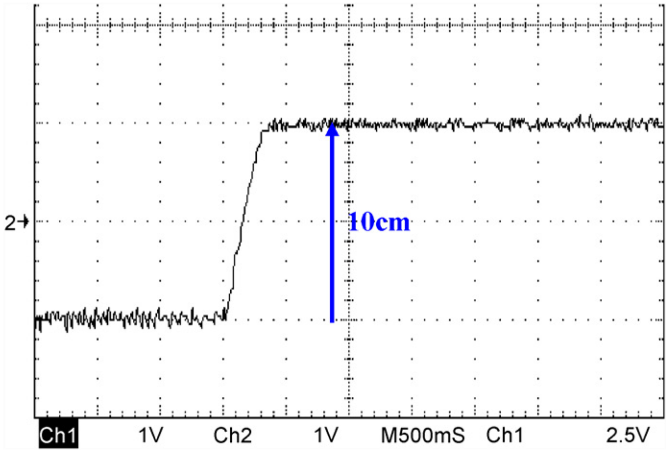

Fig. 10. Experimental position response without external load of 3 kg.

response without load. The response of the different load (3 kg) is shown in Fig. 10. The curve is almost the same as the response of 7 kg load existed. Obviously, the robustness of the proposed controller can be proved by these experimental results.

6. Conclusion

This paper has presented the Grey theory to establish the pseudo-reference model and the accumulated model. Applying the DIVSC rule, the model following work between these two models is achieved. The relevant design procedures are detailed described, too. The position control of slider-crank mechanisms using the proposed rule is implemented to compare with the classical PID rule. Simulation results show the PMF-DIVSC approach is more robust than the PID approach. Experimental results show the proposed rule can achieve accurate positioning control. Both results show the proposed rule is robust to plant parameter variation and external load disturbances.

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