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Nonlinear behavior analysis of spur gear pairs with a one-way clutch

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Abstract

Nonlinear behavior analysis of a paired spur gear system with a one-way clutch was used to verify whether a one-way clutch is effective for reducing torsional vibration. The dynamic responses were studied over a wide frequency range by speed sweeping to check the nonlinear behavior using numerical integration. The gear system with a one-way clutch showed typical nonlinear behavior, such as softening nonlinearity and jump phenomena. The oscillating component of the dynamic transmission error was reduced over the entire frequency range compared to a system without a one-way clutch, and double-side contact could be prevented, even with very small backlash. Installing a one-way clutch on both sides of the gear system was more effective at mitigating the negative effects of various parameter changes than installing one only on the input or output side.

1. Introduction

Gears are widely used in mechanical systems, and many studies have examined gear dynamics. To analyze the dynamic behavior of a gear system, the nonlinearity of mesh stiffness and backlash must be considered. Several works have modeled gear systems, including complicating effects, such as nonlinear mesh stiffness and backlash [1–3], nonlinear mesh damping [4], sliding friction force [5], and shaft flexibility [6]. Tooth modification [7], phasing in planetary gears [8], and optimizing boundary conditions [9,10] have been applied to reduce noise and vibration in gear systems.

One-way clutches are widely used in the serpentine belt systems of automobiles and heavy vehicles to mitigate the torsional vibration generated by periodic engine pulsations. A one-way clutch engages or disengages according to the relative angular speed between the driving and driven elements and acts as a vibration absorber [11]. It is reasonable to think that a one-way clutch might reduce the vibration in a gear system, such as a serpentine belt system, by suppressing the torsional vibration of the gear system. However, I am unaware of any other studies on the dynamics of a gear pair with a one-way clutch.

This study focused on the dynamic behavior of spur gear pairs with a one-way clutch. The dynamic responses were studied over a wide frequency range by speed sweeping to verify the nonlinear behavior, such

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as softening nonlinearity, around the natural frequency. Since the oscillating part of the dynamic transmission error (DTE) is the main source of noise and vibration in gear systems [12], the oscillating components of the DTE were compared as the main dynamic response.

2. Mathematical model

The system consisted of two gears mounted on input and output shafts through a one-way clutch (Fig. 1). The gears were perfect involute spur gears with no modifications. The torsional flexibility of both shafts was neglected, and the inertias of the input shaft and driver and those of the output shaft and load were lumped together, respectively.

Gears 1 and 2 had base circles of radii r_1 and r_2 and mass moments of inertia of I_1 and I_2, respectively; I_i and I_0 were the mass moments of inertia of the driver and load, respectively. The pair of gears was modeled using two disks coupled with a nonlinear mesh stiffness and mesh damping; $k(t)$ and c were the mesh stiffness and damping coefficient of the gear pair, respectively. The total backlash was $2b$; θ_i, θ_0, θ_1, and θ_2 represented the vibrations of the driver, load, and gears 1 and 2 about the nominal rigid body rotation, respectively. K_c and C_c were the stiffness and damping coefficient of the one-way clutch, respectively.

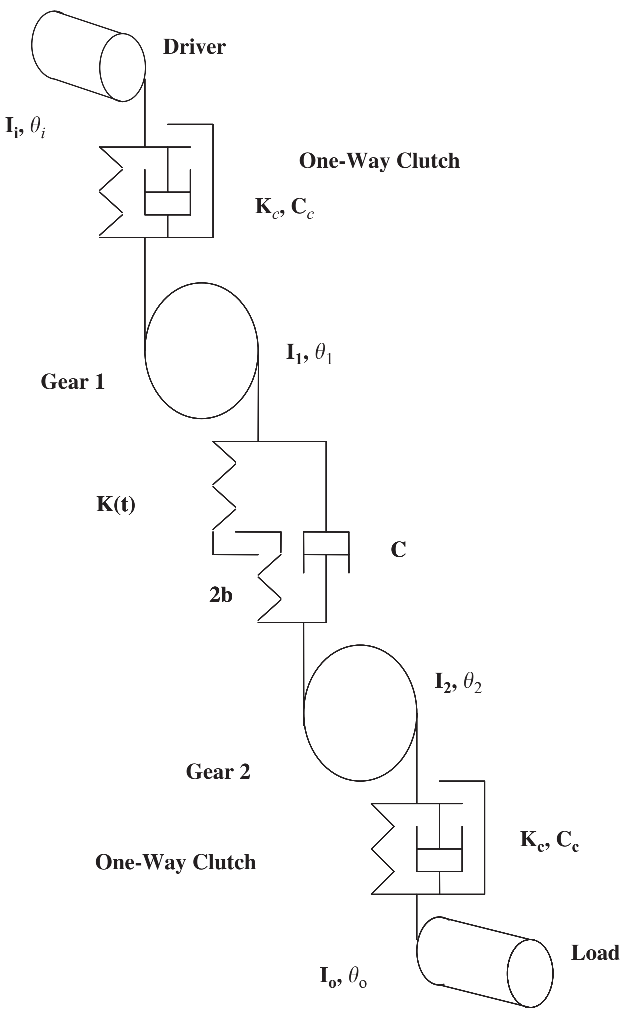

Fig. 1. Schematic diagram of a gear pair with a nonlinear one-way clutch at both sides of the driving and driven shafts.

The torque and speed of the input shaft were assumed to be constant and controllable, and the two gears and output shaft were assumed to show rotational displacement only. The one-way clutch was modeled as a nonlinear spring with discontinuous stiffness, i.e., zero stiffness for the disengaged clutch and finite linear stiffness for the engaged clutch. When the rotation of the driver exceeds that of gear 1, the clutch is engaged and clutch torque is transmitted from the driver to gear 1. When the rotation of the driver is less than the rotation of gear 1, then the clutch is disengaged and no torque is transmitted [11]. The same occurs between gear 2 and the load.

The equations of motion of the two gears and load are represented as

$$\begin{aligned} &I_1\ddot{\theta}_1 + c[r_1\dot{\theta}_1 - r_2\dot{\theta}_2] + k(t)\beta(t) = T_{c1}(t), \\ &I_2\ddot{\theta}_2 - c[r_1\dot{\theta}_1 - r_2\dot{\theta}_2] - k(t)\beta(t) = -T_{c2}(t), \\ &I_0\ddot{\theta}_0 = T_{c2}(t) - T_0. \end{aligned} \tag{1}$$

Forcing terms $T_{c1}(t)$ and $T_{c2}(t)$ are the torques transferred by clutch 1 to gear 1, and by clutch 2 to the load, respectively. They are piecewise linear functions and are represented as

$$\begin{aligned} T_{c1}(t) &= \left\{ \begin{array}{lll} K_c(\theta_i - \theta_1) + C_c(\dot{\theta}_i - \dot{\theta}_1) & \text{when} & \theta_i \succ \theta_1 \\ 0, & \text{when} & \theta_i \leqslant \theta_1 \end{array} \right\}, \\ T_{c2}(t) &= \left\{ \begin{array}{lll} K_c(\theta_2 - \theta_0) + C_c(\dot{\theta}_2 - \dot{\theta}_0) & \text{when} & \theta_2 \succ \theta_0 \\ 0, & \text{when} & \theta_2 \leqslant \theta_0 \end{array} \right\}, \end{aligned} \tag{2}$$

where T_0 is the static load torque. The vibration of the driver θ_i was assumed to be zero in this study.

In this study, both the external forces and the mesh stiffness and backlash were regarded as being nonlinear. The mesh stiffness variation $k(t)$ was the time-varying mesh stiffness obtained by assuming a rectangular wave [13].

$$\begin{aligned} &k(t) = k_0 + \sum_{r=1}^{R} k_r \cos(2\pi r f_m t - \phi_r), \\ &\frac{k_0}{k_{tp}} = \text{ICR}, \\ &\frac{k_r}{k_{tp}} = \frac{\sqrt{2 - 2\cos[2\pi r(\text{ICR} - 1)]}}{\pi r}, \\ &\phi_r = \frac{1 - \cos[2\pi r(\text{ICR} - 1)]}{\sin[2\pi r(\text{ICR} - 1)]}, \end{aligned} \tag{3}$$

Table 1
Dimensions of the gears

Teeth number	50
Module (m)	0.003
Pressure angle (deg.)	20
Face width (m)	0.02
Modulus of elasticity (N/m^2)	207×10^9
Density (kg/m^3)	7600
Base radius (m)	0.07047
Backlash (2b) (m)	400×10^{-6}
Mass (kg)	2.8
Mass moment of inertia ($I_1 = I_2$) (kg m^2)	7.875×10^{-3}

where f_m is the mesh frequency; ICR is the involute contact ratio, k_0 is the average mesh stiffness value and k_r and ϕ_r are the rth Fourier coefficient and phase angle of $k(t)$, respectively. Here, $R = 5$.

Gear backlash nonlinearity was modeled as a piecewise linear function.

$$\beta(t) = \left\{ \begin{array}{lll} r_1\theta_1 - r_2\theta_2 - b & \text{when} & r_1\theta_1 - r_2\theta_2 \succ b \\ r_1\theta_1 - r_2\theta_2 + b & \text{when} & r_1\theta_1 - r_2\theta_2 \prec -b \\ 0 & \text{when} & |r_1\theta_1 - r_2\theta_2| \leqslant b \end{array} \right\}. \tag{4}$$

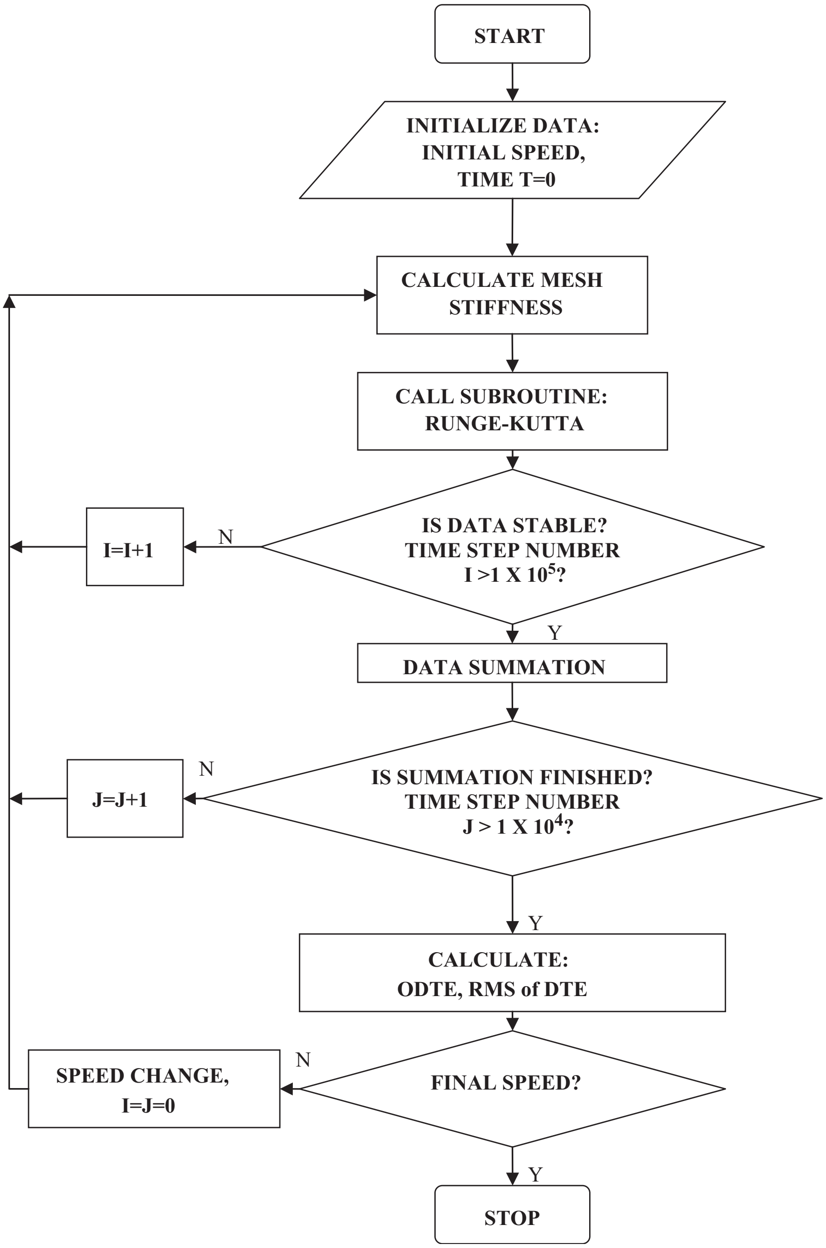

Fig. 2. Flowchart of the algorism for numerical integration.

The damping coefficients c of the tooth mesh and C_c of the clutch were calculated as

$$C = 2\zeta\sqrt{\frac{k_0}{\left[\left(r_1^2/I_1\right)+\left(r_2^2/I_2\right)\right]}},$$
$$C_c = 2\zeta_c\sqrt{K_c I_i}, \tag{5}$$

where ζ and ζ_c are the damping ratios of the tooth mesh and clutch, respectively [14].

For comparison, the behavior of a gear pair system mounted on the shaft directly without a one-way clutch was also analyzed. The equations of motion of gears 1 and 2 without a one-way clutch can be expressed as

$$(I_1 + I_i)\ddot{\theta}_1 + c[r_1\dot{\theta}_1 - r_2\dot{\theta}_2] + k(t)\beta(t) = T_i,$$
$$(I_2 + I_0)\ddot{\theta}_2 - c[r_1\dot{\theta}_1 - r_2\dot{\theta}_2] - k(t)\beta(t) = -T_0, \tag{6}$$

where T_i is the static input torque.

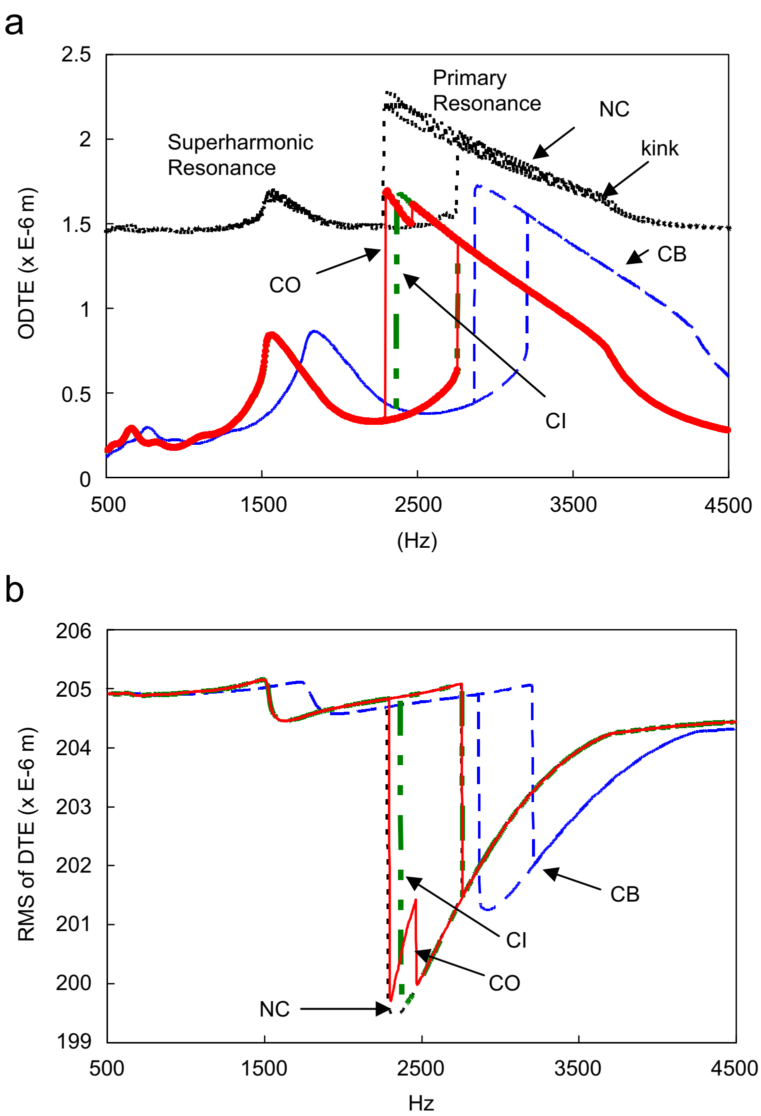

Fig. 3. DTE response of a gear pair system: (a) ODTE component and (b) DTE.

The DTE was defined as $(r_1\theta_1 - r_2\theta_2)$.

Because numerical integration gave results in good agreement with the analytical methods of nonlinear behavior analysis [1,3], the solutions were obtained using direct time domain numerical integration (fifth-order Runge–Kutta algorithm) in this study.

3. Parametric study

The primary purpose of this work was to examine the nonlinear dynamic response of a single gear pair with a one-way clutch. Table 1 shows the dimensions of an identical gear pair used in previous research [2,13] for comparison.

The average mesh stiffness k_0 was 462.1×10^6 N/m, and the contact ratio (ICR) was 1.75. The damping ratio of the tooth mesh and clutch were $\zeta = 0.07$ and $\zeta_c = 0.01$, respectively. The input torque T_i was 150 N m, and the load torque was $T_0 = r_2/r_1 \times T_i$. The stiffness of the one-way clutch was selected as 15×10^3 (N m/rad) to produce a torque equal to the input torque at a clutch displacement of 0.01 rad. The inertias of the driver and load were kept the same as those of the gears.

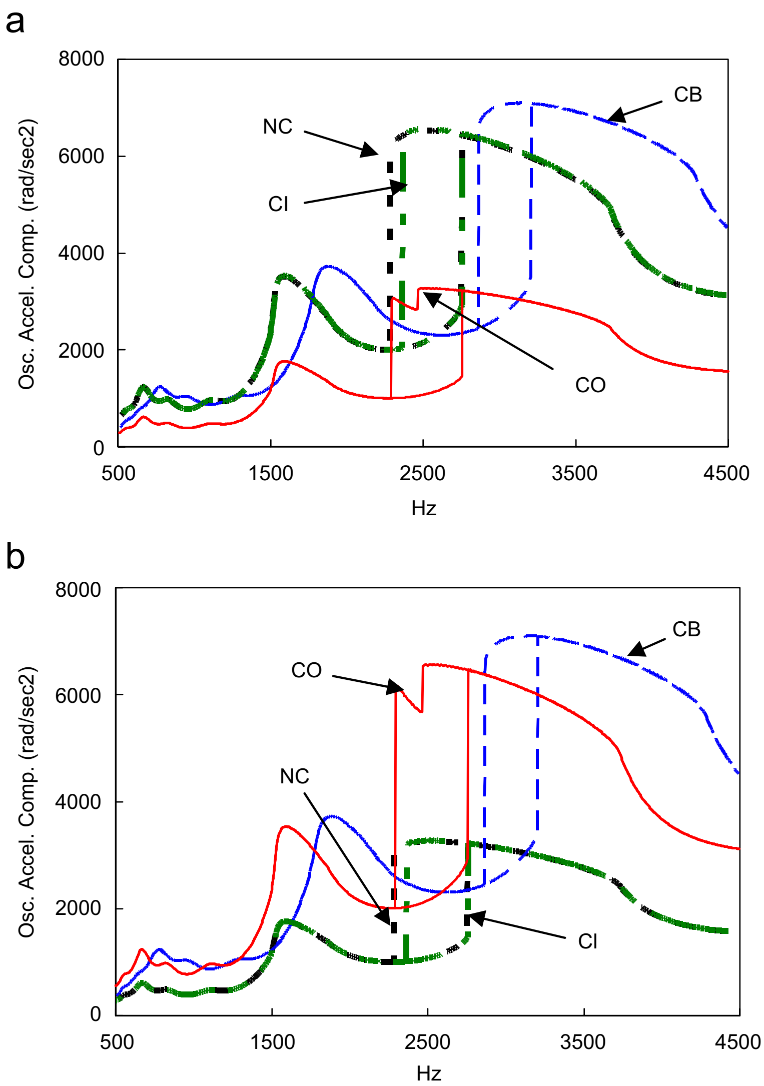

Fig. 4. Acceleration responses for various clutch conditions: (a) gear 1 and (b) gear 2.

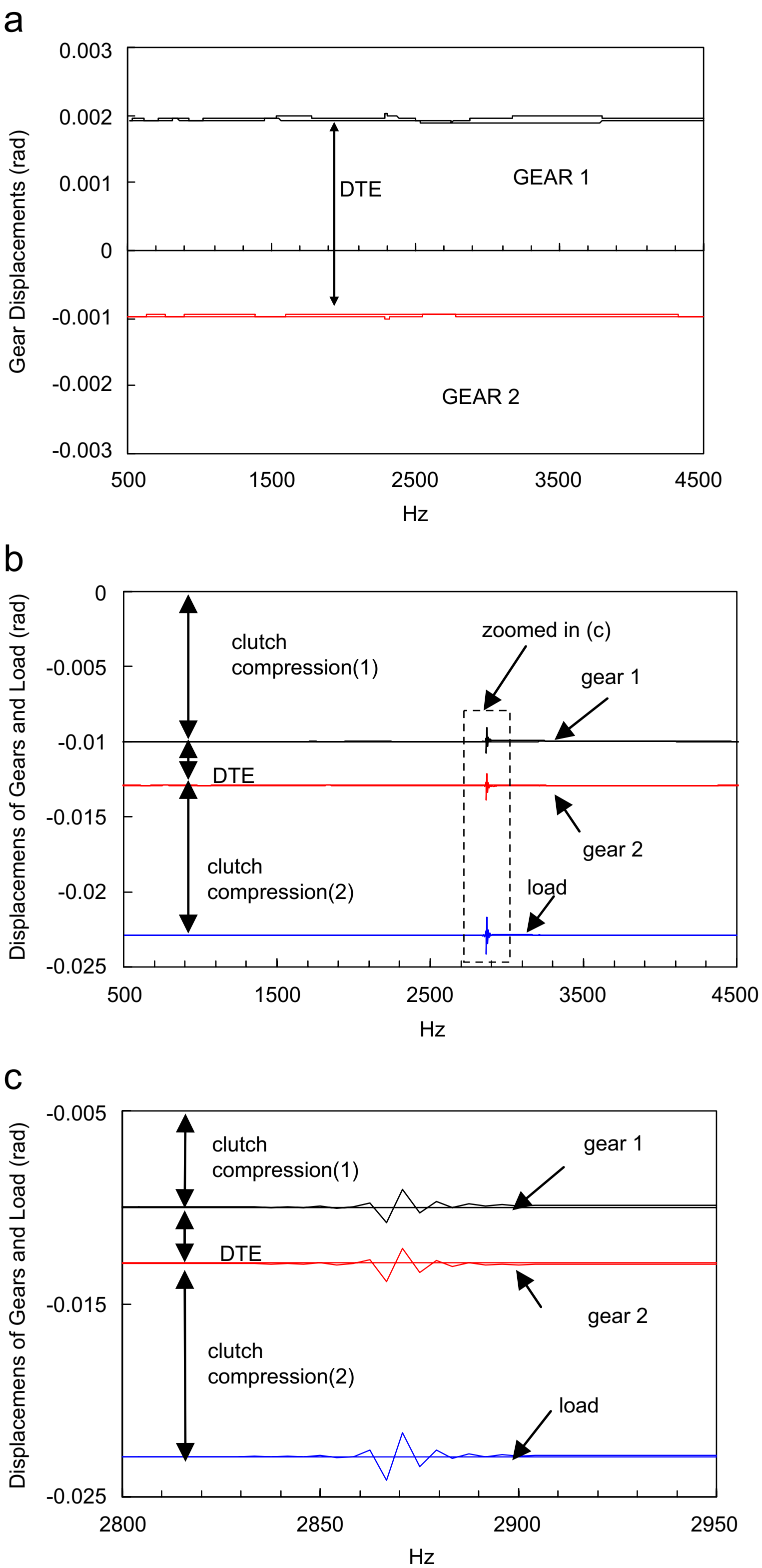

Fig. 5. Positions of the gears and load: Condition (a) NC, (b) CB, and (c) zoomed area of (b).

To detect jump phenomena, an increasing and decreasing frequency sweep was executed at a constant ratio for a wide range across the first natural frequency. To get stable data after a speed change, 1×10^5-time step data were discarded before averaging was performed. The time step was 1×10^{-5} s for all conditions. Fig. 2 shows the flowchart of the algorism for numerical integration used in this study.

The oscillating DTE (ODTE) component and RMS of DTE at a specific constant speed were defined as

$$\mathrm{ODTE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathrm{DTE}_{i+1} - \mathrm{DTE}_i)^2},$$

$$\text{RMS of DTE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathrm{DTE}_i)^2}, \tag{7}$$

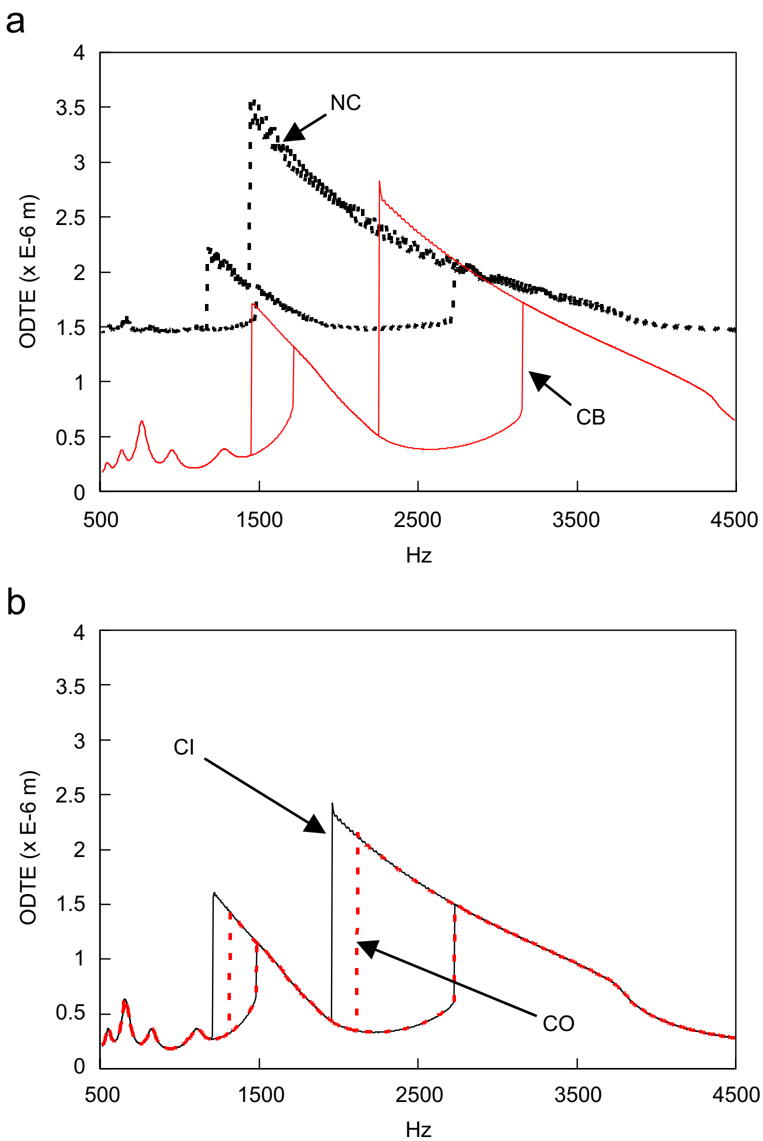

Fig. 6. ODTE responses of various clutch conditions for $\zeta = 0.03$: (a) NC and CB, (b) CI and CO.

where $\mathrm{DTE}_i = r_1\theta_1(t_i) - r_2\theta_2(t_i)$ and N is the total number of time steps used for averaging, which was 1×10^4 in this study. The calculated natural frequency was around 2700 Hz. Varying parameters such as gear damping ratio, contact ratio, backlash, and load inertia ratio, ODTE of various clutch conditions were compared with that of no clutch (NC) condition.

Fig. 3 shows the DTE response of the gear sets under four conditions: no clutch (NC), clutch at the driving shaft only (CI), clutch at the driven shaft only (CO), and clutch at both shafts (CB). The results predicted multiple resonances, softening nonlinearity, kink (onset of contact loss), and jump phenomena, which are typical in a nonlinear gear system, and generally agreed qualitatively with the results of Parker et al. [2]. A clear softening nonlinearity occurred as the peak bended to the left. The difference is thought due to the value of the shaft inertia and backlash not mentioned in their study.

Over the entire frequency range, ODTE components could be reduced if a one-way clutch were installed at either one shaft only or both shafts. Primary resonance was evident for a mesh frequency of $f_m \approx f_n \approx 2700$ Hz.

In condition CB, the multiple solution bands were narrower than in the other conditions, and the overlap range shifted to a higher frequency due to decreased inertia separated by the one-way clutch on both sides. In contrast, the overlap frequency range did not change very much when only one clutch was installed. The overlap frequency depends on the clutch conditions.

Condition CO showed unstable double-jump down when the speed was decreasing. Because this kind of double-jump down disappeared for other system parameters (Figs. 6, 8, 11 and 12), the double-jump down phenomena in condition CO appeared to depend on the system parameters.

Fig. 4 shows the acceleration response (oscillating component) of the driving and driven gears under various clutch conditions. The results predicted multiple resonances, stiffening nonlinearity, and jump phenomena. In contrast to the DTE responses, the peak value of the alternating components of acceleration in condition CB was higher than in condition NC due to the intermittent impulsive engagement of the elements by the clutch. The responses of both gears were the same in condition CB. In the other conditions, however, the responses of the gears differed. The values for gear 1 in conditions NC and CI were higher than in condition CO. The values for gear 2 in condition CO were higher than in conditions NC and CI. The dynamics of the one-way clutch had a dominant effect on the gear installed near the clutch.

Fig. 5 shows the displacements of the two gears in conditions NC and CB. With no clutch (a), the driving gear (gear 1) maintains forward displacement and the driven gear (gear 2) maintains backward displacement, and the total difference in the displacements equals DTE. With a clutch on both sides (b), all gears and the load maintain a backward displacement equal to the backlash plus clutch compression. The positions of the elements were almost invariant during frequency sweeping. Relatively large variations occurred at the jump down frequency (c). These trends were similar under conditions CI and CO, which were not presented here.

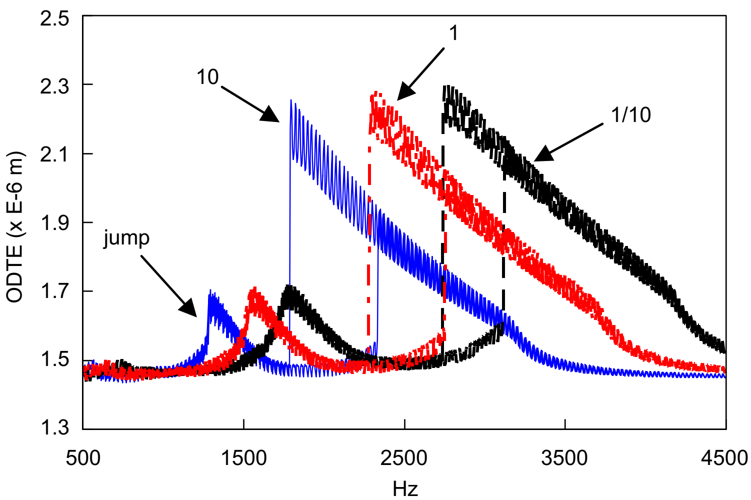

Fig. 7. ODTE responses of condition NC for various load inertia ratios.

Fig. 6 shows the DTE responses of various clutch conditions for a tooth mesh damping ratio of $\zeta = 0.03$, while keeping the other parameters nominal. With decreased damping, the width of the overlap band increased and the jump down frequency shifted toward lower frequencies. A new overlap band due to super-harmonic resonance occurred, the double-jump down in condition CO at the first resonance frequency disappeared, and decreased damping increased the nonlinearity of the system.

Fig. 7 shows the DTE responses of condition NC, demonstrating the effects of load inertia. Keeping the inertias of the driver and two gears constant at the same value, the inertia of the load varied from one-tenth to ten times that of the other elements. The DTE responses depended dominantly on the load inertia in condition NC. As the load inertia increased, the overlap band shifted to lower frequencies.

Fig. 8 shows the DTE responses of various clutch conditions, demonstrating the effects of load inertia. Keeping the inertias of the driver and two gears constant at the same value, the inertia of the load was varied from one-tenth to ten times that of the other elements. In conditions CB and CO, the load inertia had little effect on the DTE responses. In condition CI, however, the overlap range shifted to a much lower frequency as

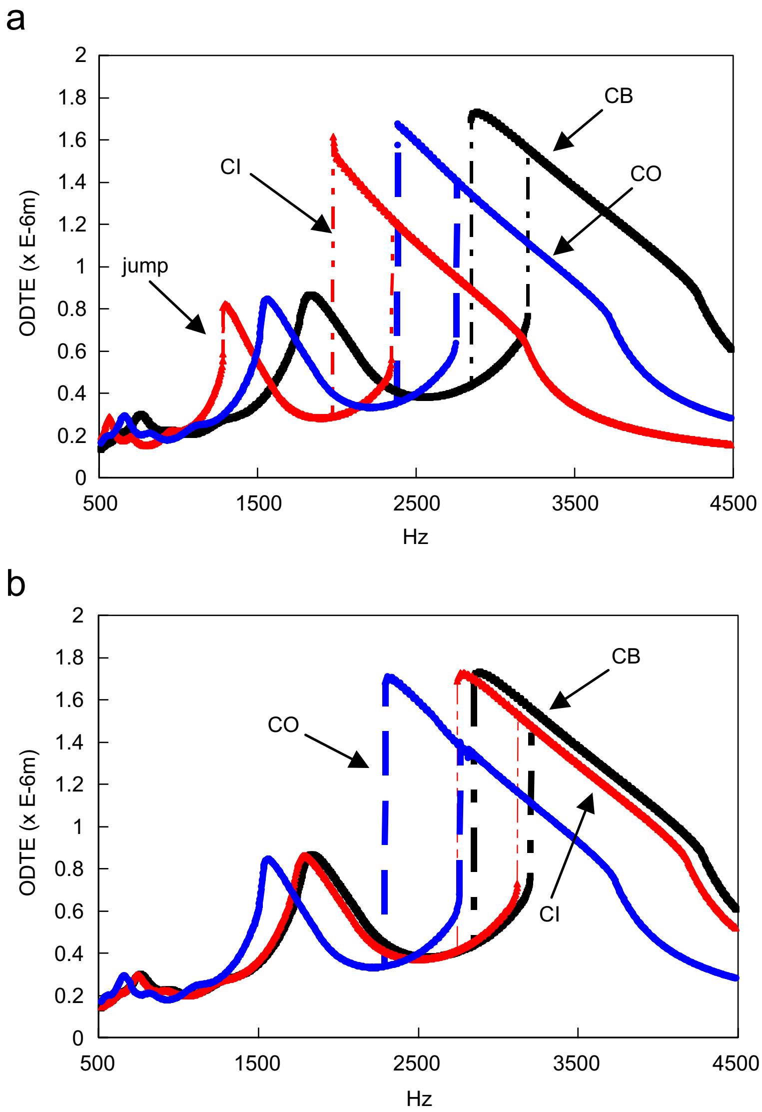

Fig. 8. ODTE responses of various clutch conditions for different load inertia ratios: (a) 10 and (b) 1/10.

the inertia increased, but the width did not change much. Installing clutches at both shafts effectively suppressed the negative effects of large load inertia. When the load inertia ratio was high, a new nonlinear jump occurred at the super-harmonic frequency in condition CI. A one-way clutch installed at the input shaft had little effect on the response of the gear system with load inertia variation. In condition CO, the double-jump down disappeared.

Fig. 9 shows the DTE response in the no-clutch condition for various backlashes, while keeping the other parameter values nominal. The oscillating DTE components decreased with the backlash. When the backlash was very small (5 μm), hardening nonlinearity due to double-side contact appeared, demonstrating that torsional vibration greater than the backlash causes double-side contact. However, such double-side contact

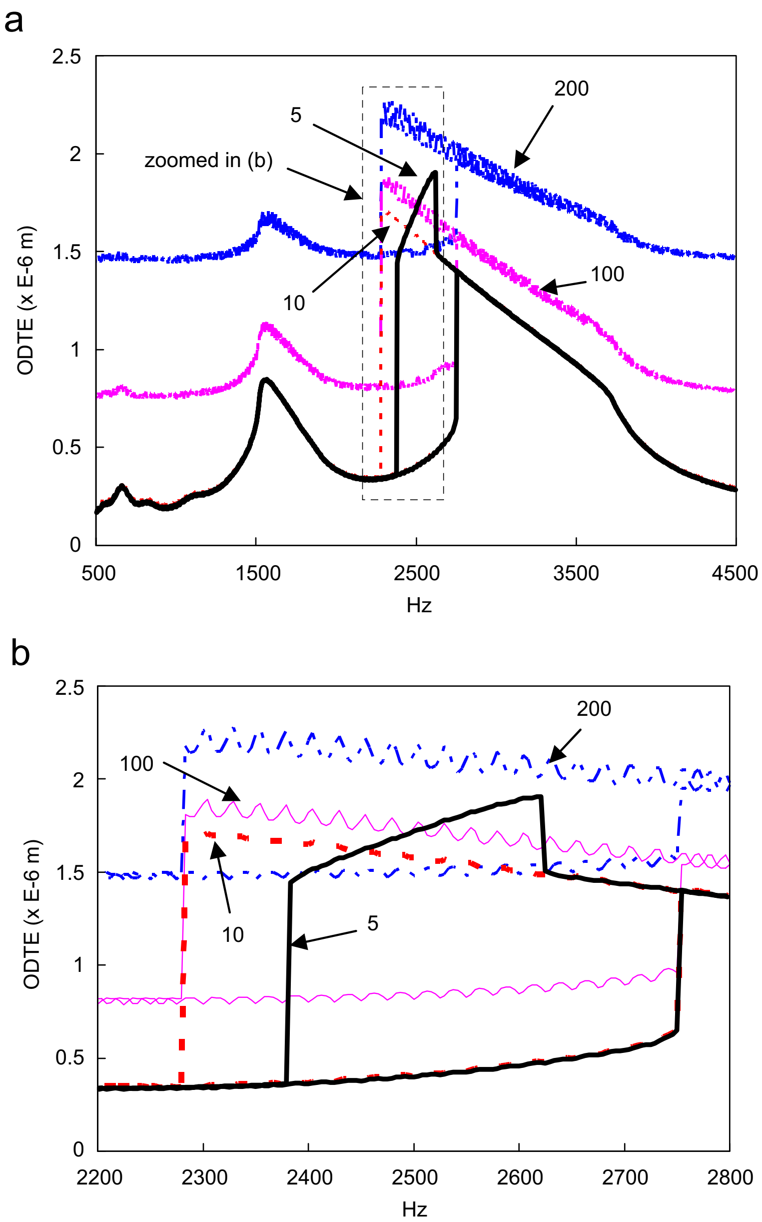

Fig. 9. ODTE responses in the no-clutch condition for various backlashes (b: μm): (a) ODTE and (b) zoomed area of (a).

would rarely occur in a real system, because the backlash recommended by design handbooks (such as the AGMA [American Gear Manufacturers Assoc.]) for the gear dimension used in this study was much greater than 5 μm. The overlap range and jump frequency were barely affected by backlash.

Fig. 10 shows the DTE responses of various conditions for a backlash of 5 μm. The responses under all clutch conditions, except for condition CB, showed double-side contact. In contrast, double-side contact nonlinearity did not occur in condition CB. The clutches installed at both shafts seemed to be effective in preventing potential double-side contact.

Fig. 11 shows the DTE response under four clutch conditions for various contact ratios, while keeping the other parameters nominal. The effects of the contact ratio on the DTE were the same in the clutch and no-clutch conditions. There should be an optimum contact ratio in order to minimize ODTE.

Fig. 12 shows the DTE responses in condition CO for various load inertia ratios. It shows that the unstable double-jump down phenomena occurs in a specific range of the load inertia ratios.

Though every clutch conditions are effective for reducing ODTE, CB condition is more effective for mitigating the negative effects of various parameter changes (Table 2). Hence, CB condition only was simulated hereinafter.

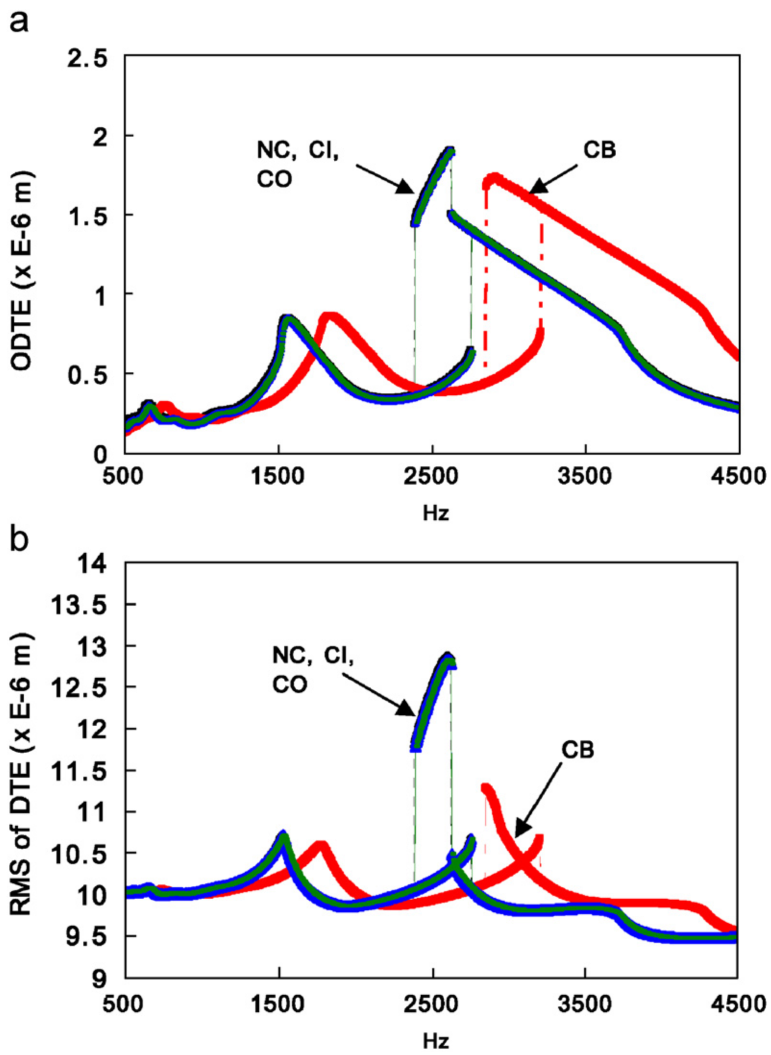

Fig. 10. DTE responses of various clutch conditions for a backlash of 5 μm: (a) ODTE and (b) RMS of DTE.

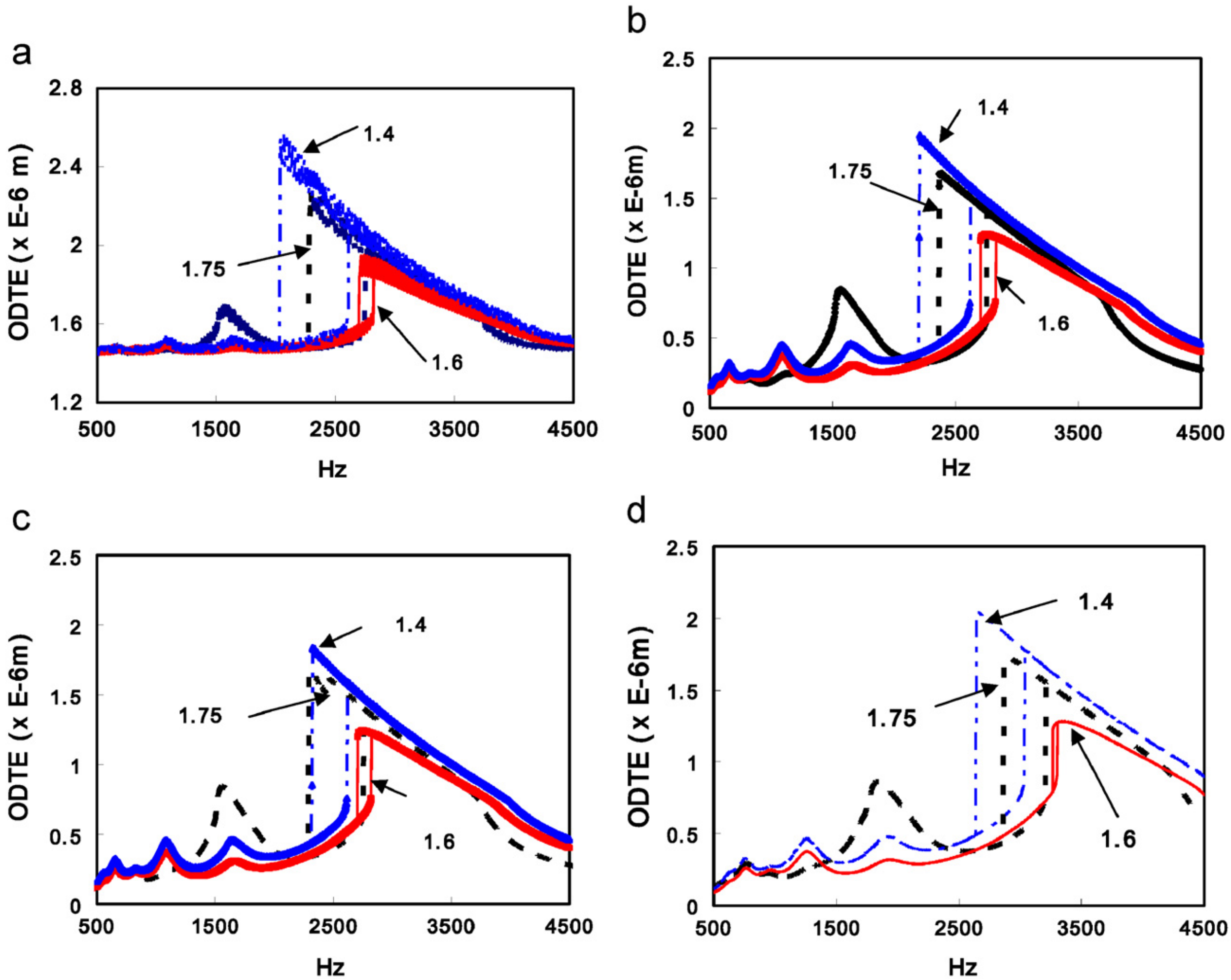

Fig. 11. ODTE responses of four clutch conditions for various contact ratios: (a) NC, (b) CI, (c) CO, and (d) CB.

Fig. 13 shows the DTE responses in condition CB for various backlashes. The DTE responses barely depended on the backlash variation in condition CB.

Fig. 14 shows the DTE responses of condition CB for various torques, while keeping the other parameters nominal. The overlap range and frequency of jump and kink did not depend on the torque. The peak value was proportional to the torque. These trends were similar under other clutch conditions, which were not presented here.

Fig. 15 shows the DTE and displacement responses in condition CB for various clutch stiffnesses, while keeping the other parameters at nominal values. The stiffness was normalized relative to the nominal value 1.5×10^4 N m/rad. When the clutch stiffness exceeded a specific range, the response lost its typical nonlinear phenomena and became very unstable. The gear displacement increased almost linearly as a function of time during frequency sweeping. Increased displacement resulted from the impulsive torque induced by large clutch stiffness. Hence, the clutch stiffness should be kept below a specific value. These trends were similar under conditions CI and CO, which were not presented here.

Fig. 16 shows the DTE responses of condition CB for various clutch-damping ratios, while keeping the other parameter values nominal. Except for zero damping, there was no difference among the responses of various clutch-damping ratios. The DTE responses in condition CB depended little on clutch damping. These trends were similar under conditions CI and CO, which were not presented here.

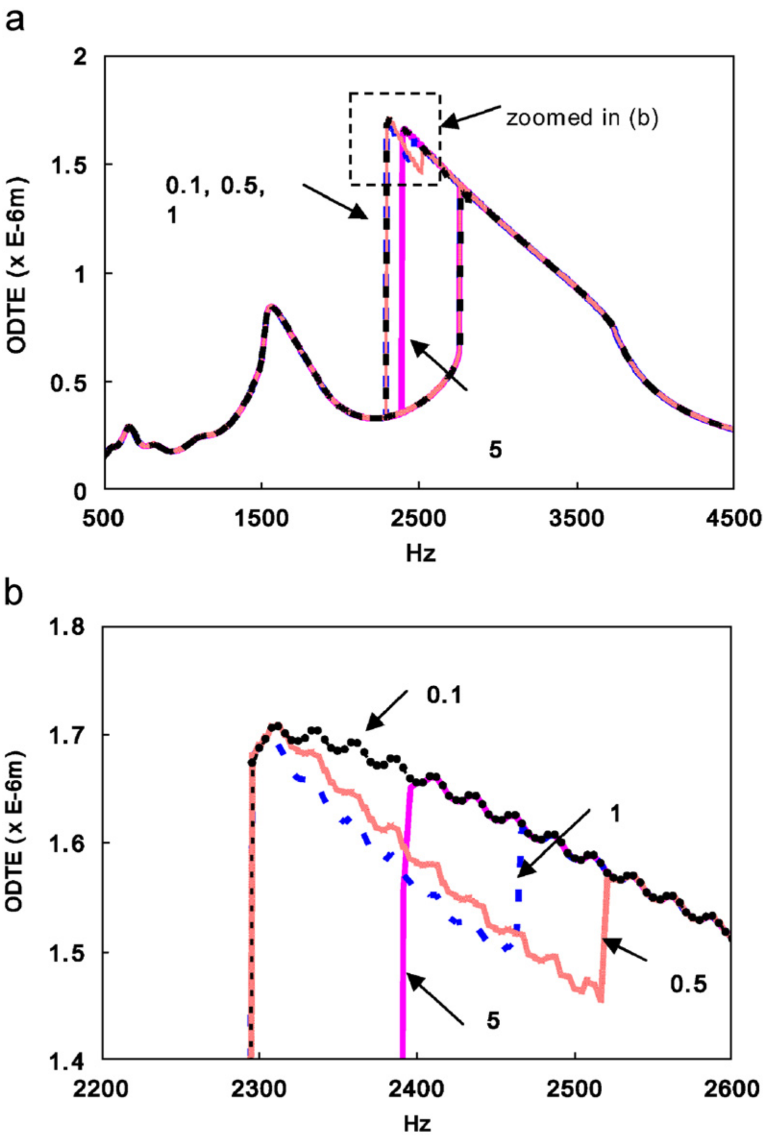

Fig. 12. ODTE responses in condition CO for various load inertia ratios: (a) ODTE and (b) zoomed area of (a).

Table 2
Response of three clutch conditions compared to condition NC for various parameters

Parameters	Clutch conditions		
	CI	CO	CB
ODTE	Improved	Improved	Improved
Influence by mesh damping	Same as NC	Same as NC	Same as NC
Influence by load inertia ratio	Same as NC	Improved	Improved
Influence by backlash	Same as NC	Same as NC	Improved
Influence by contact ratio	Same as NC	Same as NC	Same as NC

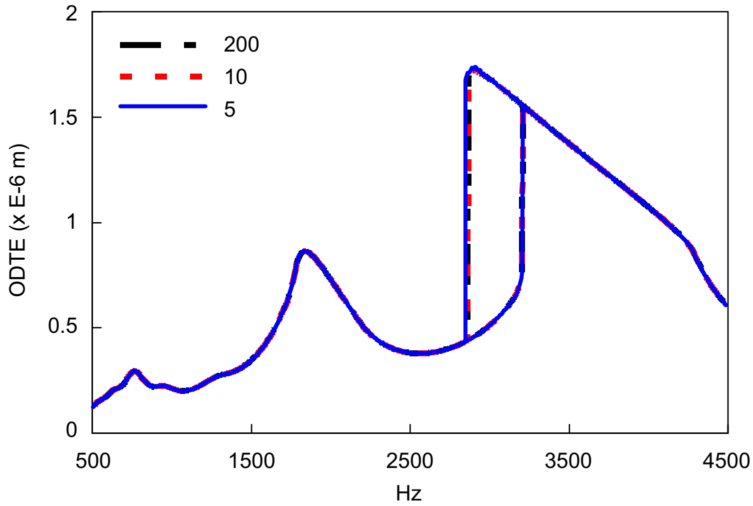

Fig. 13. ODTE responses of condition CB for various backlashes (b: μm).

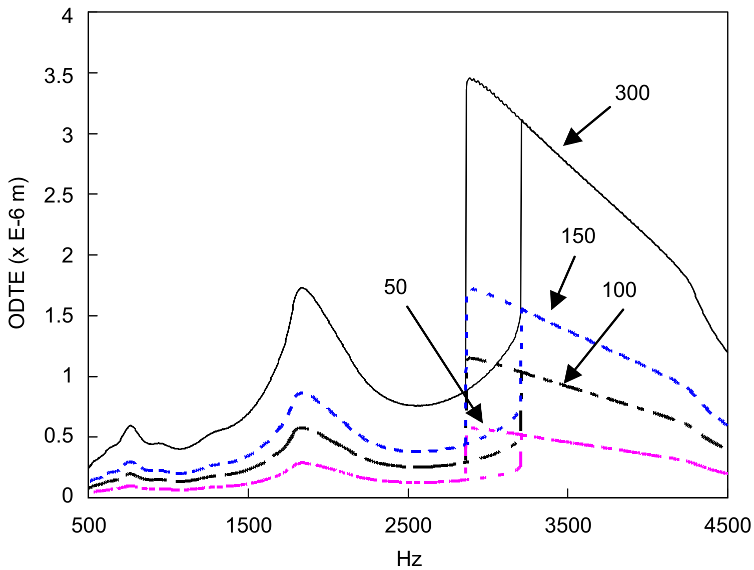

Fig. 14. ODTE responses of condition CB for various input torques (N m).

4. Conclusions

This study examined whether a one-way clutch is effective for reducing the torsional vibration of a gear system using a paired gear model considering only rotational motion. A numerical integration method was used to solve the equations of motion.

The gear system with a one-way clutch showed typical nonlinear behavior, such as softening nonlinearity and jump phenomena, as in the system without a one-way clutch. However, the oscillating DTE component of the system with a one-way clutch was reduced over the entire frequency range. Installing a one-way clutch on both sides of the gear system shifted the overlap frequency range to higher frequencies, and was more effective at mitigating the negative effects of variation in several parameters than installing a clutch at the input or

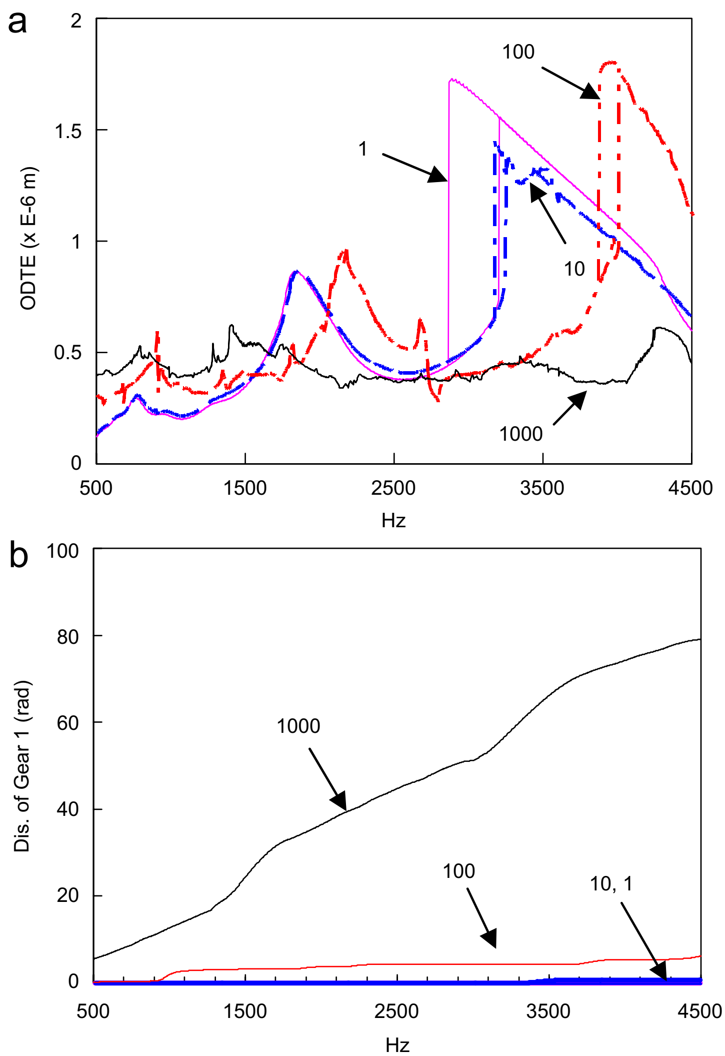

Fig. 15. ODTE and relative gear displacement in condition CB for various clutch stiffnesses: (a) ODTE and (b) gear displacement.

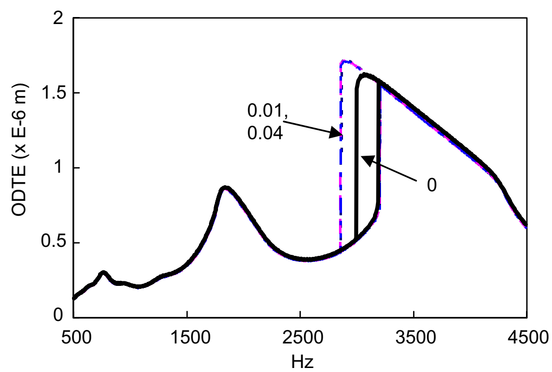

Fig. 16. ODTE responses of condition CB for various clutch-damping ratios.

output side only. The stiffness of the one-way clutch had a great effect on the behavior of the gear system, and it must be kept under a specific value for a stable response. Nevertheless, clutch damping had little effect on the behavior of the gear system. The effects of mesh damping, the contact ratio, and torque on a gear system with a one-way clutch were the same as in a gear system with no clutch. All analytical results presented in this study should be verified through further experimentation.

Acknowledgments

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